MANUAL
OF
PHOTOGRAMMETRY

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(Preliminary Edition)

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PITMAN PUBLISHING CORPORATION
NEW YORK CHICAGO

Preliminary Edition

PUBLISHED
BY
PITMAN PUBLISHING CORPORATION

Printed in the United States of America

PREFACE

THE publishing of a book of any kind is a most difficult task. The majority of us shy away from creative work and are inclined to procrastinate when confronted with the problems of putting our thoughts on the printed page. It is easy to "talk" a grand job of writing "this paper," or of presenting "these notes," on subjects with which we are conversant. However, few of us do anything constructive about it

The men who have contributed material for this manual are the exception. They are of that select few who translate their dreams into actualities Many of them are extremely busy on vital war work. In spite of this, they have been sufficiently self-sacrificing to devote some of their time to the preparation of material for this manual. They are to be congratulated for their drive and initiative.

At this time I wish to express, through my office, the gratitude of the American Society of Photogrammetry for their fine contribution.

COL. MINTON W. KAYE, A.A.F.
President, American Society of Photogrammetry

INTRODUCTION

CREATIVE genius has, throughout the centuries, developed new tools intended solely for the benefit of mankind Some of these tools, however, have had the unfortunate quality of proving destructive in their applications as well as constructive. Aerial photogrammetry is no exception to the rule, as is evidenced by its use in modern war-fare, for it is the most important weapon available for defensive and offensive operations

Rather than delve into its destructive value, let us project ourselves into the future and discuss the tremendous potentialities of aerial photogrammetry as a means of making our planet a place worthy of our trust, where the riches of the earth can be used for the common good

Accurate topographic maps and hydrographic charts are essential for the maximum cultural and material development of any nation The first, form the basis on which the internal highways of progress are mapped; the second, pave the ways on which mankind must depend for its external ties with its neighbors across the seas

When we consider mapping in its fullest meaning, we are faced with the fact that vast areas at home and throughout the world remain dormant because of lack of knowledge, or because the existing maps are totally inadequate This has been due to lack of vision on the part of many legislators who have never recognized the need for good maps, nor have they realized that maps form the basis for all engineering projects They have forgotten too, that maps not only portray the outward physical characteristics of the earth, but to the trained geologist, reveal the hidden riches underground.

This war has brought home the fact that, as far as mapping was concerned, we were unprepared for global operations. Had it not been for mapping agencies, aerial cameras, new techniques used, and thousands of photogrammetrists which we are fortunate to have in America, our armed forces would have been at a serious disadvantage

Now that we are so map conscious, it is time to make plans for the future It is our duty to urge that the Americas be mapped in their entirety and that every nation in this hemisphere bend every effort to see that necessary surveying work is undertaken without delay. Photogrammetry can make this come true in a comparatively short time because it is the most efficient, accurate and economical method developed as yet for mapping vast areas.

The American Society of Photogrammetry, who for the past ten years has been devoting every effort to the development and application of this comparatively new science, decided to publish this preliminary edition of its "Manual of Photogrammetry," not only as its contribution to the war effort, but also as a contribution to ultimate peace and friendship among all nations. Every effort has been made to describe methods, equipment and procedures in as clear a manner as practicable, with the idea in mind that it will serve as a text for all those interested in Aerial Photogrammetry—that comparatively new field of engineering, which has already revolutionized the science of Surveying and Mapping

It is well to keep in mind while reading the manual that it is not intended as a finished or perfect coverage of the subject. At present, pressure of war makes

it impracticable to cover the field in its entirety, but it is planned that a first edition with a revision and addition of all pertinent information will be forthcoming as soon as time permits. It is for this purpose that the Editor includes at the back of the manual several pages for the reader's suggestions and criticisms When the material is revised these suggestions will be called for and should be forwarded to the revision committee. In this way it is hoped that when the first edition of the "Manual of Photogrammetry" is prepared it can be a complete and up to date presentation of the entire field Until that time the best means available of keeping abreast with new developments is through the articles regularly published in "Photogrammetric Engineering."

The Society is deeply indebted to those of its members who have given so much of their time to make this manual possible and principally to the Editor in Chief of the Manual and to the other outstanding American Photogrammetrists who have prepared the chapters presented in this edition.

GUILLERMO MEDINA, *Head Engineer,*
U. S. Hydrographic Office

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CHAPTER I

PRINCIPLES OF SURVEYING

MANUAL OF PHOTOGRAMMETRY

MAPS, SURVEYING AND AIR PHOTOGRAPHS

O. S. Reading

MAN'S most distinct vision is limited to a few inches and his viewpoint generally is such that the most distant object he can see is only a few miles away. To transcend these limitations he has invented maps—maps which place any part of the earth within the scope of his glance, that he may plan his actions in accordance with the relative positions of its features and their importance to his purposes. These purposes differ widely and a great variety of maps is made to meet those which are sufficiently important to warrant separate maps. Table I lists the principal types of maps.

TABLE I. PRINCIPAL TYPES OF MAPS

Scale	*Type*	*Purpose*
1:600 to 1:2,500	Plans	Construction; fortification; land registration or cadaster; tax inventory and assessment, insurance risk, real estate development, city planning and administration.
1:2,500 to 1:31,680	Large-scale topographic maps	Military defense of positions to be strongly held, battle maps; engineering purposes preceding final location and construction surveys; administrative purposes of every sort, as parks, reservations, minor civil divisions, irrigation or drainage areas, etc., detailed geographic, geologic and ecologic studies, base for special maps, as zoning maps, population density maps, land use maps, erosion maps, etc.
1:40,000 to 1:125,000	Medium-scale maps	Military tactical maps, strategical and administrative maps in thickly settled country, general area planning, routes for highways, etc.; county maps, maps of larger parks, forests, reservations, quarantine, and other administrative districts and purposes, guidance for hiking, hunting, and fishing.
1:200,000 to 1:500,000	Intermediate-scale maps	Same as above for very sparsely settled areas of little economic importance and requring little topographic details; military planning maps for movement and supply of troops, strategic maps; state maps, road maps, regional planning, geographic, and geologic studies.
1:1,000,000 to 1:7,000,000	Small-scale maps	Military strategic maps; regional and national planning and geographic studies; atlas maps; wall maps.
		CHARTS
		Maps made especially for navigation and kept up to date as to aids and dangers
1:5,000 to 1:40,000	Harbor and Airport charts	Close navigation in crowded areas, depiction of special facilities, aids, and dangers, designation of permissible channels, anchorages or parking spaces.

1 80,000 to 1:250,000	Coast charts—approach charts	Coastal navigation by piloting, instrument approach aeronautical charts; contact flying in congested areas.
1 500,000	Sailing charts Aeronautical contact flying charts	Coastal navigation where shore is not approached closely, contact flying of settled areas.
1·1,000,000 to 1·5,000,000	Regional charts Planning charts	Sailing charts for offshore navigation, regional air navigation, radio and flight planning charts

Map Accuracy

It will be noted that the maps were classified according to scale. For convenience in use and for economy of space and cost, maps are always made on the smallest scale which will show clearly the information necessary for their purpose The probable error of first class drafting or measurement on a map is about one tenth millimeter (0.004 inch), or the thickness of a fine line or dot. This distance multiplied by the scale of the map represents the maximum precision with which objects can be plotted, and also the smallest difference in shape that can be shown to scale, working on a drafting table under favorable light conditions and with low power magnification. The values of this distance for common maps scales and the area covered by maps and charts of customary sizes at these scales are shown in Table II.

Table II Accuracy—Scale—Area Conditions

Scale of Map	*Graphic tolerance* *Closest practicable* 0 004″ or 0 1 mm	*Ordinary use* 0 016″ or 0 4 mm		*Dimensions of area on map* *G S Quadrangle* 14″×18″		*War Dept Quadrangle* 22″×29″		*C & G. S. chart* 36″×48″	
1″=50 feet	0 2 foot	8 foot	Mile	13	17	21	27	34	45
1 600	0 1 meter	2 meter	Km	21	.27	34	44	55	73
1″=200 feet	8 foot	3 2 feet	Mile	53	68	83	1 10	1 36	1 82
1 2,400	2 meter	1 0 meter	Km	85	1 1	1 34	1 77	2 19	2 23
1″=833 feet	3 3 feet	13 1 feet	Miles	2 2	2 8	3 5	4 6	5 7	7.6
1 10,000	1 0 meter	4 0 meters	Km	3 6	4 6	5 6	7 4	9 1	12.2
1″=2,000 feet	7 9 feet	31 5 feet	Miles	5 3	6 8	8 3	11 0	13 6	18 2
1 24,000	2 4 meters	9 6 meters	Km	8 5	11 0	13 4	17 7	21 9	29 3
1″=1 mile	20 8 feet	83 2 feet	Miles	14 0	18 0	22 0	29 0	36 0	48 0
1·63,360	6 3 meters	25 3 meters	Km	22 5	29 0	35 4	46 7	57 9	77 2
1″=15 78 miles	328 0 feet	1312 0 feet	Miles	221 0	284 0	347 0	458 0	568 0	757 0
1 1,000,000	100 0 meters	400 0 meters	Km	356 0	457 0	559 0	736 0	914 0	1219 0

This table illustrates several points of interest. First, no higher precision of measurement or location in the field than the tolerance shown is usable on the maps or obtainable from them. (Actually under less favorable conditions of plotting, errors of four times the tolerance are not uncommon) Second, it is evident that objects the size of ordinary houses and roads cannot be shown clearly on scales smaller than 1:24,000 without exaggeration in size, and even on this scale some exaggeration is usually necessary for clarity and emphasis. Third, a very large number of maps or photographs is necessary to cover any

considerable area unless the generalization and the elimination of detail necessary on smaller scales can be tolerated It will be appreciated from Table I that the same area must be mapped on many different scales if the various purposes listed are to be adequately served Ordinarily this can be done at moderate expense by office compilation from accurate large-scale survey sheets For some areas the smaller scales are adequate until development necessitates resurveys on larger scales

Coordination of Maps

It is evident that some means of identifying and coordinating the many maps is essential For the world and large areas, or detached islands, latitude and longitude have long been used and are satisfactory For areas small enough so that the curvature of the earth does not materially affect the plotting, plane rectangular coordinates simplify the surveying and plotting considerably The U S. Coast and Geodetic Survey has computed the values of latitude and longitude and plane coordinates of all control stations in the United States The plane coordinates on the state systems may be used with an error not exceeding 1 part in 10,000 For some purposes extended rectangular coordinates are considered so useful that errors as high as one part in five hundred are accepted, as in the edges

Table III Projections Most Used for Mapping

Name and characteristics	*Advantages*	*Disadvantages*	*Principal uses*
Polyconic, cone for each parallel	High accuracy of scales and direction for small areas One table can be used anywhere without additional computations	Distortion of scale and direction at edges where area is large Series of maps have rolling fit only	Survey sheets, topographic atlas of U. S., wall maps International map of world (modified polyconic)
Lambert conformal, cone with two standard parallels	Scale and direction errors small for areas long in east-west direction Maps of series on same standard parallels fit together	Cannot be extended far north and south without large errors	Aeronautical charts of world and U S, East-west areas in state systems of plane coordinates
Transverse mercator, cylindrical axis east-west, conformal	Scale and direction errors small for areas long in north-south direction Maps of series fit together.	Cannot be extended far east and west without large errors	Maps of British Isles, north-south areas in state systems of plane coordinates
Mercator, cylindrical, axis north-south, meridian-parallel ratio constant	Rhumb line (constant compass course) appears as straight line	Scale variations large, directions are rhumb lines not great circles except on meridians and equator	Nautical charts and for purposes where allowance for convergence of meridians is inconvenient
Stereographic, plane tangent to sphere.	Distortions small near center, valuable for polar regions	Cannot be extended far without large errors	Maps and charts of polar regions
Orthographic, vertical to a plane.	Satisfactory for plans.	Large distortions of any considerable area of spheroid.	Engineering and architectural drawings, construction plans on plane grid.

of the seven zones of the military grid for the United States The use of rectangular coordinates is more completely discussed in the fourth paper of this chapter

Everyone is aware, in these days of world strategy and illustrated magazines, that large portions of the curved surface of the earth cannot be shown on a flat sheet of paper without considerable distortion Various "projections" or systems of plotting the lines of latitude and longitude on a flat piece of paper are in use, each devised to hold certain desired properties while accepting distortion of others The important characteristics of the projections most used for mapping are shown in Table III A detailed discussion of projections and their construction is given in the third paper of this chapter and the publications referred to therein.

Map Quality Depends on Surveys

The world has been mapped for thousands of years, but the users of the early maps doubtless had some difficulty in finding the Garden of Eden or the mermaids and sea serpents often shown in what would be otherwise vacant areas, to say nothing of the difficulties encountered due to the distortions of terrain The maps improved as the mariners' compass and astronomical instruments came into general use and again improved markedly during the past two centuries with the advent of geodetic control. Modern maps show the area they portray with all the accuracy and completeness practicable on the scale adopted and successive sheets covering a large area join each other without discrepancies or appreciable distortions This high accuracy, of course, is practicable only when the area shown has been accurately surveyed by modern methods

Surveys of Large Areas with Uniform Precision

In all measuring or surveying a succession of measurements accumulates the constant or systematic error of the measuring device, plus or minus the accidental errors of the measuring operations The amount of error is likely to increase with the distance measured and usually is expressed as a part of it, as 1 part in 25,000 Ordinarily, the more precise methods of measurement require more precautions and more expensive instruments, and, therefore, tend to cost more than the less precise For these reasons the most efficient surveying practice, when a certain tolerance of error is specified, requires that the longer distances be measured by the method yielding sufficient accuracy at the lowest cost and that during these measurements a number of positions be determined at shorter distances apart Methods of lower precision and less cost may then be used to locate intermediate positions Thus, if a tolerance of 1 meter in position be specified for an area 25,000 meters square, methods given a precision of 1 part in 25,000 would be used to determine the relative positions of points say about 10,000 meters apart over the area Then methods having a precision of 1 part in 10,000 would be used to locate positions between the first-order positions at about 5,000 meters apart Methods yielding a precision of 1 part in 5,000 would then be used to locate third-order stations between the second-order points at about 1,000 meters apart. The remaining detail would be located between the third-order stations with precision of 1 part in 1,000. In this way all points in the area would be located within 1 meter of their correct positions at much less cost than if the methods yielding a precision of 1 part in 25,000 were used throughout Obviously, if the methods yielding a precision of one part per 1,000 were used throughout, discrepancies and distortions up to 25 meters would arise which could only be eliminated by more precise methods of survey It is a cardinal principle of surveying that the more precise measurements be made first and

the less precise follow to be checked and adjusted to them, working from the whole to the parts. The more precise positions thus control the less precise surveys and are called control stations.

Geodetic Control Surveys

In order to avoid troublesome and expensive discrepancies in their surveys and maps, nearly all civilized governments have established national geodetic surveys capable of locating control stations with the highest practicable precision. These surveys are so accurate that they measure and take into account the curvature of the earth, the deflection of the plumb line or level surface by variations in the force of gravity, and the irregularities in the earth's crust. A close international cooperation exists between the scientists in charge of these surveys (with the exception of a few nations in time of war), and close study is made of all scientific discoveries and techniques which may lead to higher precision in measurement. For example, if researches in electronics and interferometry should make it readily practicable to determine the distance traveled by a beam of light with higher precision than a carefully prepared base can be taped over the ground, the method would be immediately adopted by the geodesists.

At present the most precise and efficient method of determining the horizontal positions is by triangulation. The distance between two stations favorably situated for measurements of high precision is measured in sections with three carefully standardized invar tapes, so that each tape is compared with the other two The probable error of the measurements very seldom exceeds 1 part in 1,000,000 and usually is between 1 in 3,000,000 and 1 in 4,000,000 The angles between the stations at the ends of the base and the other stations forming triangles at either side of it are then measured with such precision that, after allowing for spherical excess, the average error of the sum of the three angles composing a triangle seldom exceeds one second of arc.

The sides of the triangles are then computed and become in turn the lengths from which the sides of other triangles, whose angles have been measured, can

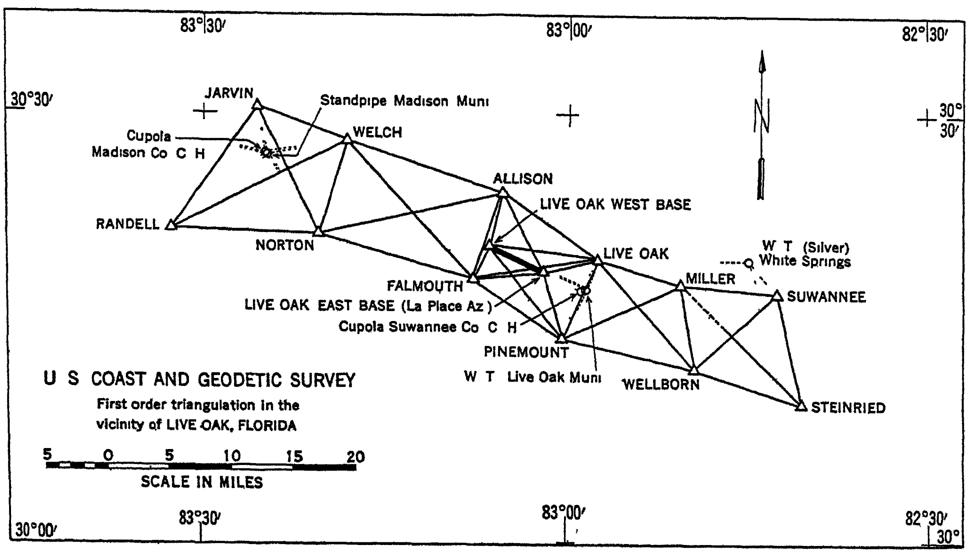


FIG. 1. Typical section of arc of first-order triangulation.

be computed. The process is continued until a network of triangles has been extended over the area to be surveyed. Figure 1 shows a typical base expansion net. When the areas to be covered are very large, the triangles are arranged in chains or "arcs," usually with the triangles overlapped to form quadrilaterals, with the spaces between the arcs to be filled in later, as conditions warrant. The quadrilaterals with diagonals observed are the simplest figures which give two independent determinations of every length. A new base length is measured about every 100 to 200 miles, and azimuths with a probable error not greater than 0.3 second are measured on circumpolar stars at intervals sufficiently short to eliminate appreciable swing Values for latitude and longitude are also obtained from the astronomic observations. The longitudes being used to correct the azimuths for deflection of the vertical and the latitudes and longitudes are used in studies of the shape of the earth. The whole network of triangles, base lengths, and azimuths is so arranged that there are at least two ways of determining every quantity. The geometrical conditions are such that quantities having large errors can be immediately isolated and remeasured, and those with small errors can be adjusted to their most probable values. In this way, no matter how rough or irregular the terrain may be, it is practicable to determine relative positions with an accuracy such that the distance between any two of them will be correct within 1 part in 100,000. If the object marking such a position be destroyed or lost, the position can be remarked with that accuracy. If, as occasionally happens when the urgency for accomplishing a large amount of work stretches the tolerances, the bases do not check by as much as one part in 25,000, additional bases can be measured and the more questionable angles or azimuths of the scheme can be re-observed to bring any weak part of the network up to the desired precision.

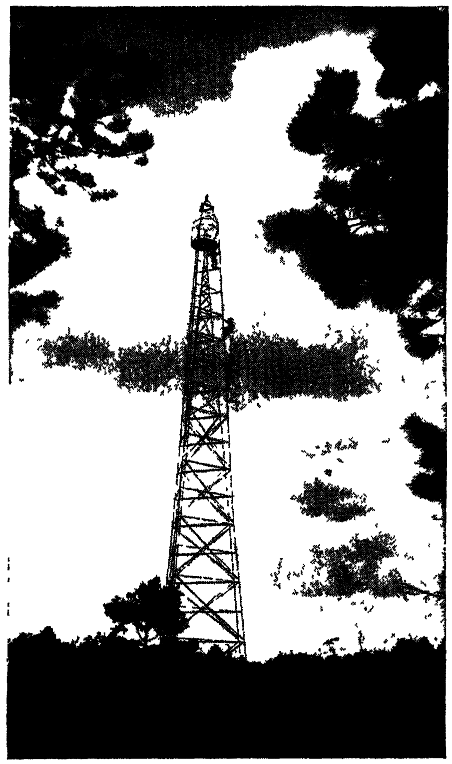


FIG. 2. Portable (knockdown) steel towers used in triangulation.

One of the main objectives of a national geodetic survey is to so distribute the first-order triangulation stations over the country that all boundaries can be determined from them with the accuracy warranted by their importance. Thus in cities where an inch or so is of importance, the stations should not be more than a mile and a half to three miles apart. In country where a foot more or less will not bother, they can be ten to twenty miles apart.

For mapping and other purposes where such accuracy of position is not essential, second- and third-order triangulation, or traverse measurements, are used to determine the positions of additional control stations and prominent objects. Since triangulation requires that the stations be intervisible (or at least intervisible when instruments and light or target are elevated by towers of

moderate height, see figure 2), the stations must be on the higher and often less accessible points, perhaps some distance from objects which can be unmistakably identified in air photographs In traverses the distances are measured in straight lines with tapes along the ground and the angles between the successive straight lines or tangents are observed with a transit Traverses between the main control stations are ordinarily run along roads because of the cleared lines and even ground they afford Objects such as bridges, crossings, and buildings, unmistakable on photographs, may be readily located by the traverse party For this reason traverses are preferred for the final breakdown of control for mapping from photographs where roads are available in level or moderately hilly terrain In some areas where traversing becomes difficult, triangulation on small captive balloons, with observations coordinated by radio, has been found more efficient for the final breakdown of the horizontal control

Vertical control is also necessary for maps showing the elevation of the terrain and for many engineering purposes Vertical control is obtained by spirit levels (excepting that vertical angles are ordinarily used for the final breakdown in rough or mountainous country) Vertical control also varies in precision for reasons of economy. In the United States there is a network of first-order levels of the highest practicable precision (plus or minus 4 millimeters times the square root of the distance in kilometers), subdivided by second- and third-order levels The first and second-order level net of the United States is shown in figure 3

Both the horizontal and vertical control stations established by the national surveying agencies are marked with monuments of an enduring nature, usually a cement post or block with a bronze tablet showing the exact point Descriptions and coordinates for first-order control and many of the lower order stations may be obtained by request addressed to the Director, U. S. Coast and Geodetic Survey. Figure 4 shows the first- and second-order Horizontal Control Net of the United States The Intelligence Division, Office of the Chief of Engineers, is making an index of all control surveys in the United States The U S. Geological Survey and other agencies also furnish data on supplemental control established by them in connection with their surveying activities

One of the first steps in any extensive mapping project is, of course, to obtain full information as to the control available, in order that standard accuracy and coordination with other surveys may be maintained with minimum expense The bibliography at the end of this paper mentions several manuals and texts which describe fully the methods of executing control surveys.

Where geodetic control is not available, modern radio-time equipment and the primatic astrolabe are capable of obtaining positions with a probable error of 1 second of latitude and longitude (in the torrid and temperate zones) plus or minus the deflection of the vertical at the place of observation. This is most useful for small-scale mapping, such as aeronautical charts, or for exploratory or reconnaissance surveys, where time and effort must be at a minimum. Near mountains or ocean deeps the deflection of the vertical may amount to as much as 40 seconds or more (56 seconds between the north and south coast of Puerto Rico, for example) For the type of surveys mentioned, a mile error is not of great importance, particularly as such errors gradually diminish as the distance from the cause of the deflection increases The equi-altitude observations grow rapidly weaker as the poles are approached. However, sufficiently accurate results are obtained up to 75 degrees from the equator The observation technique is quite simple and can be readily learned, but great care is necessary in the time measurements, as an error of 1 second of time means an error of 15 seconds in

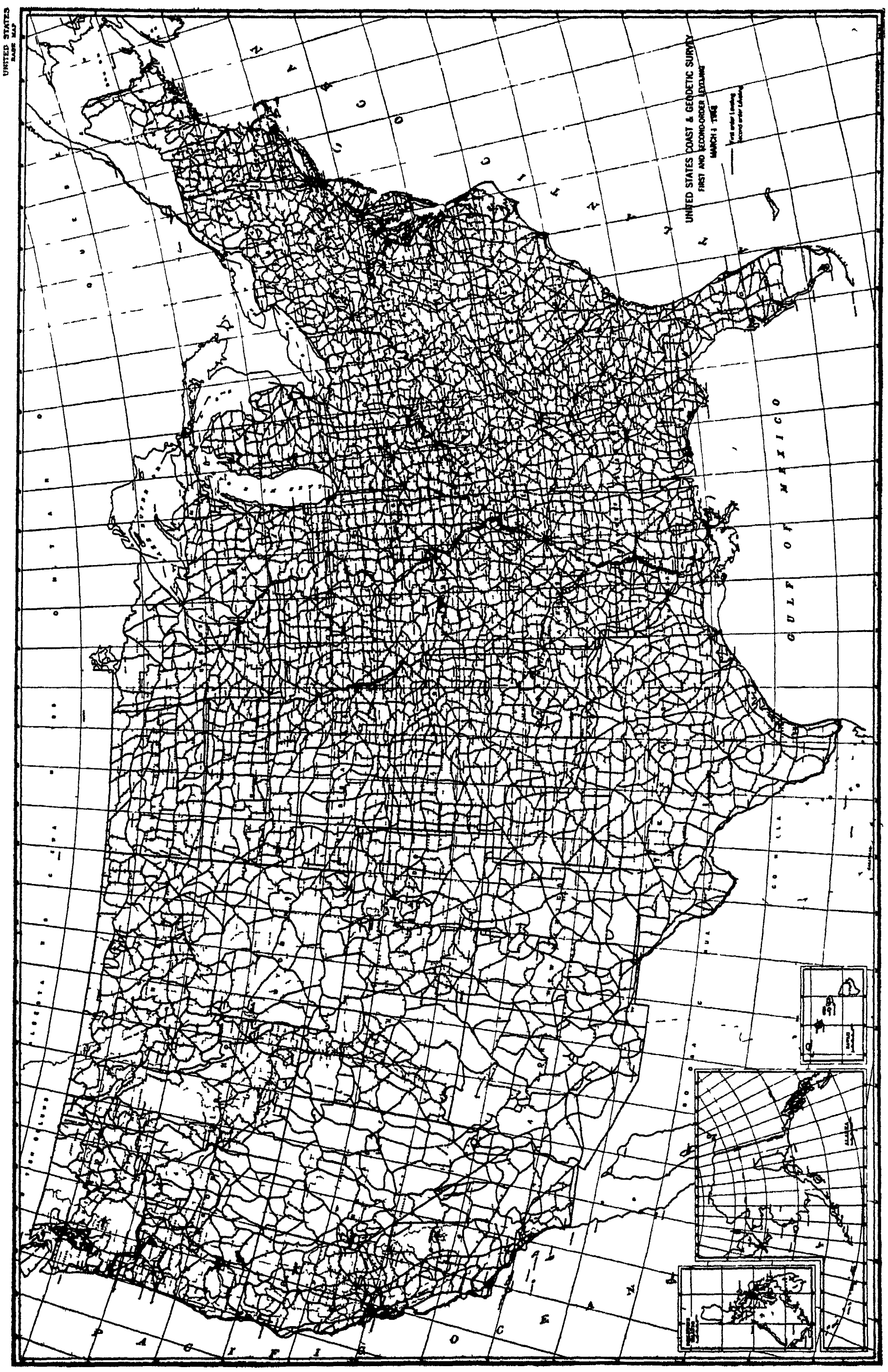


FIG 3. First- and second-order level net of the United States 1943. SOURCES OF CONTROL DATA

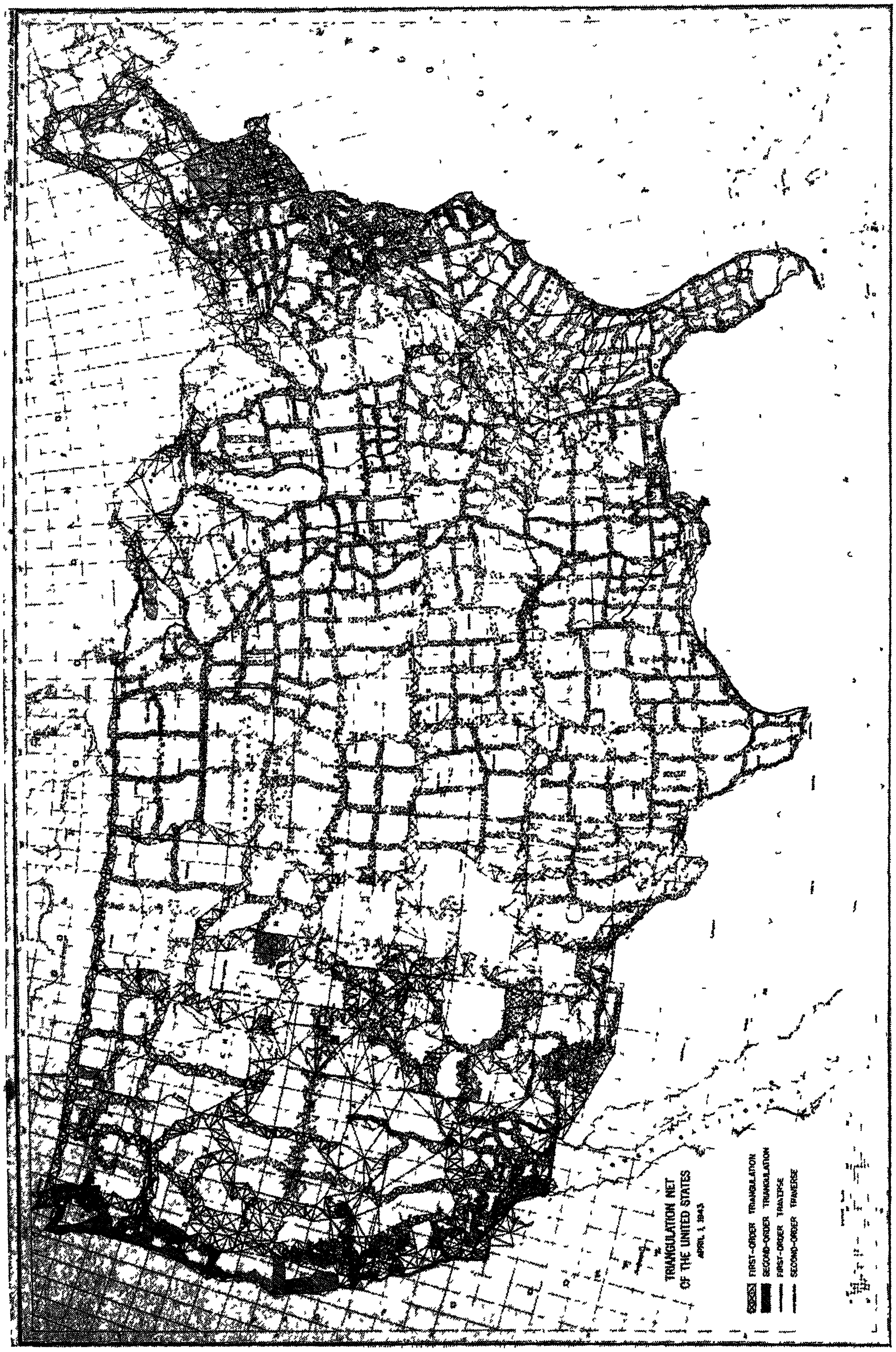


FIG. 4. First- and second-order triangulation net of the United States 1943.

CONTROL FOR URGENT SMALL SCALE SURVEYS

longitude. This method, which has become of much importance in connection with a program for aeronautical charts of the world, is described in the second paper of this chapter.

Building the Map on the Control Framework

Until the first world war accelerated the development of aircraft and aerial photography, the planetable was considered the ideal instrument for building the map on a framework of control. In using the planetable, the map was made in the field with the terrain before it as a model, and the surveyor could measure and map as much detail as needed for the purposes of the survey. However, the limited view from the side of objects (which must be mapped as if viewed from above) required many set-ups and measurements, with attendant high cost, if there was much detail to be accurately shown. With continued improvement in air-craft, lenses, cameras, and photographic materials, and the ensuing increased availability of air photographs, the planetable has been gradually replaced. It is now used only for contouring on photographs of terrain of low relief and for small projects for which air photographs may not be available, or for small areas of very large scale.

Place of Air Photographs in Surveying

For surveying purposes air photographs may be considered extraordinarily complete and detailed *records of directions* from undetermined air stations. The stations are undetermined in the surveying sense because the airplane moves so rapidly and is subject to such variable accelerations as to prevent accurate determinations of the space coordinates of the exposure stations as the exposures are made. There are, however, numerous methods of determining the exposure stations from the directions to objects of known position which appear on the photographs.

The directions are registered on the sensitized surface of a glass plate or film held in a plane perpendicular to the optical axis. In mapping cameras, the plane is fixed at the distance from the lens which gives the best average definition over the whole photograph for distant objects.

The intersection of the optical axis with the plane perpendicular to it is called the *principal point* of the photograph. The position of the principal point is indicated directly by a mark registered by the mapping camera on each photograph during the exposure, or by the intersection of lines from marks registered on the margins of the photograph. The relative directions of all objects from the exposure station may be determined by means of the calibrated focal length of the lens and position of the principal point. An aerial photograph may be considered as a record of directions projected through a point (the lens) at a known distance (calibrated focal length) perpendicularly from the marked principal point of the photograph. See Figure 5. The distance and angle relations are fixed in each modern air camera and are determined by calibration. The calibrated focal length is computed to give as small and as uniform distortion as practicable with the lens used and is not necessarily the same as the distance from the internal perspective center of the lens as set in the camera. For some uses this length is further corrected to compensate, as far as practicable, for changes in dimensions of the photographic materials, and is then called the principal distance of the photograph.

The accuracy with which directions may be determined from aerial photographs varies with the aberrations of the lenses, the grain of the high-speeed

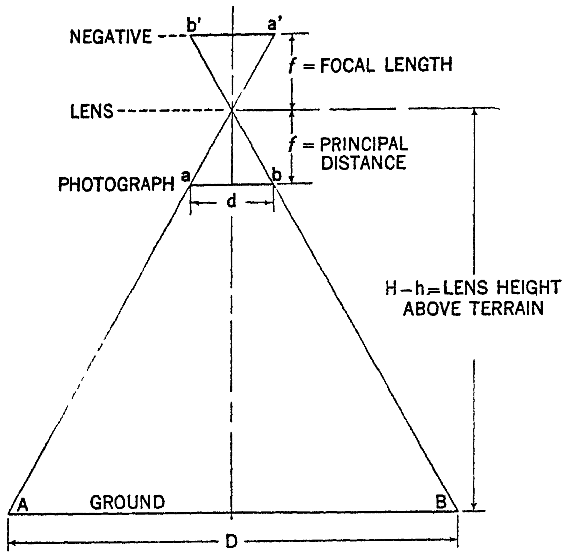


FIG. 5. Diagram of photograph

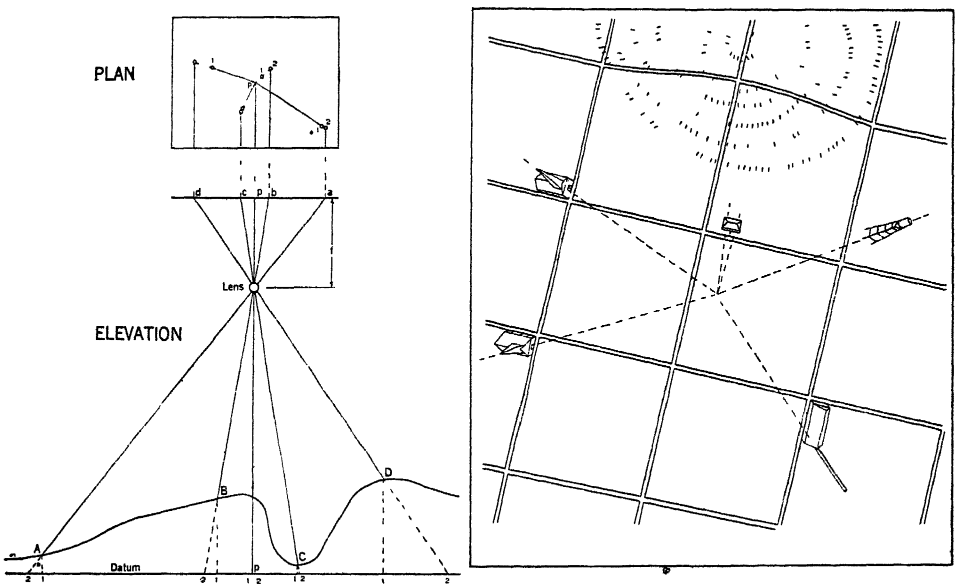
emulsions, non-uniform shrinkage or distortion of the sensitized material (sometimes as much as 1 part in 150 for paper prints), and other sources of error In general, directions may be determined with an error of about 12 seconds when all precautions and the best equipment are used Errors as great as 6 or 7 minutes may occur in ordinary paper prints. The amount of displacement caused by small angular errors at typical radii are as follows

TABLE 4 PROBABLE ERROR OF ONE POINTING (DIRECTION MEASUREMENT)

Radii	*First-order theodolite*	*Third-order theodolite*	*12" air camera glass plate*	*6" air camera low-shrink film*	*Tilt distortion 3 degrees*	*Tilt and paper distortion*
	ONE SECOND	FIVE SECONDS	TWELVE SECONDS	ONE MINUTE	THREE MIN	SIX MIN
3" 76 2 mm	000015" 000381 mm	000075" 00191 mm	00018" 00457 mm	0009" 0229 mm	0027" 0686 mm	0054" 137 mm
6" 152 4 mm	00003" 00076 mm	00015" 0038 mm	00036" 0091 mm	0018" 046 mm	0054" 137 mm	0108" 274 mm
12" 304 8 mm	.00006" 0015 mm	0003" 0076 mm	.00072" 0183 mm	0036" 0914 mm	0108" 274 mm	0216" 549 mm
1 mile 1 609 km.	0256 ft 780 cm	128 ft 3 90 cm.	.307 ft 9 36 cm	1 536 ft 46 82 cm	4 608 ft 14 05 cm	9 216 ft 280 9 cm
5 miles 8 047 km	128 ft 0390 m	644 ft 196 m	1 536 ft 468 m	7 68 ft 2 34 m	23 04 ft 7 02 m	46 08 ft 14 05 m
40 miles 64 374 km	1 03 ft .314 m	5 14 ft 1 57 m	12 32 ft 3 76 m	61 43 ft 18 72 m	184 29 ft 56 18 m	368 58 ft 112 34 m

Note that 1 minute of arc means an error of only about .002 inch (a difference barely visible to the unaided eye) on a photograph made with a 6-inch focal length lens at a radius of 6", yet a first-order theodolite is pointed with sixty

times this accuracy on the targets or signal lights used in triangulation It will be appreciated from the relative size of these errors that aerial photographs at present are not surveying instruments of geodetic precision, but rather suited to fourth-order control and graphic mapping. For graphic mapping, however, photographs give advantages of speed, economy, freedom from large errors or blunders, and an extraordinarily complete, detailed, and permanent record of topographic features Their advantageous view from the air shows practically everything not covered by trees or deep shadows, while modern lenses and photographic emulsions record detail that the eye cannot see without the aid of a magnifying stereoscope.



1 Shows map position
2 Shows photo position

FIG 6. Effects of relief on location of objects in photographs.

How Photographs Differ from Maps

If the axis of the air camera could be held truly vertical, and the terrain were all practically at sea level, the photographs would give map positions directly. There are three unavoidable departures (generally small in amount) from these special conditions, the effects of which must be eliminated in the compilation of maps from photographs: (1) differences of elevation or relief; (2) tilt or inclination of the optical axis of the camera from the vertical; and (3) variations in scale due to fluctuations in the height of the airplane at the exposure stations of the successive photographs

In general, maps show the positions of objects as projected vertically upon the datum of mean sea level A photograph shows the positions as projected through a single point (the lens), that is, in perspective. The difference is illustrated in Figure 6. Displacements, due to elevation, radiate from the point vertically beneath the exposure station, called the ***plumb or nadir point*** (which in truly vertical photographs coincides with the principal point) Note in the

plan diagram that the displacement of the tops of the chimney, tank, and spire all radiate from the plumb point. The hilltop is displaced as much, but the displacement is often not so apparent in photographs. The straight road running across the slope of the hill from side to side appears curved, while the straight road running radially across the slope appears straight.

Since photographs record directions, at least two overlapping photographs recording intersecting directions are necessary to determine the position of a point See Figure 7. The trace of a plane passing through the object, the nadir

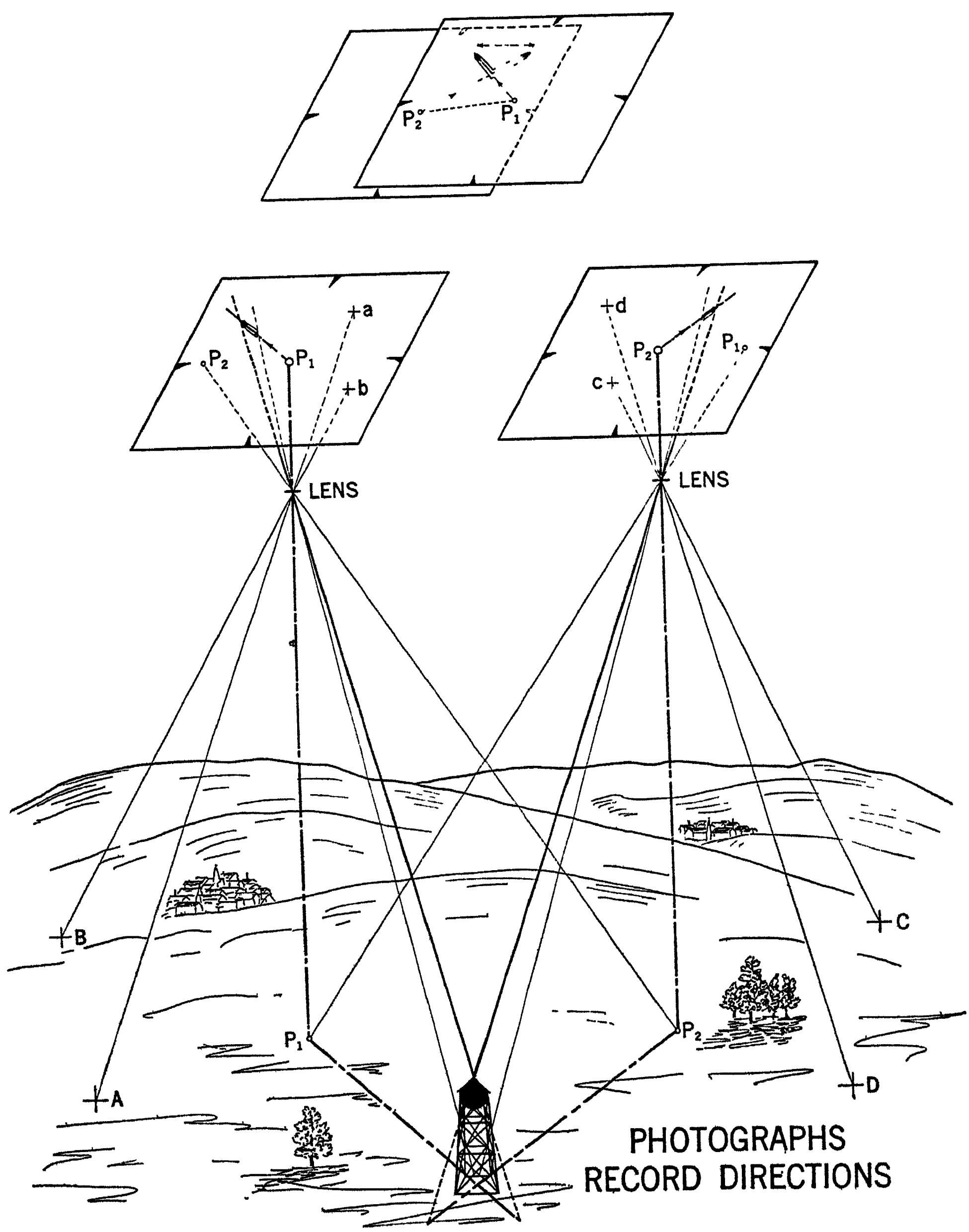


FIG. 7. Radial intersections from photographs.

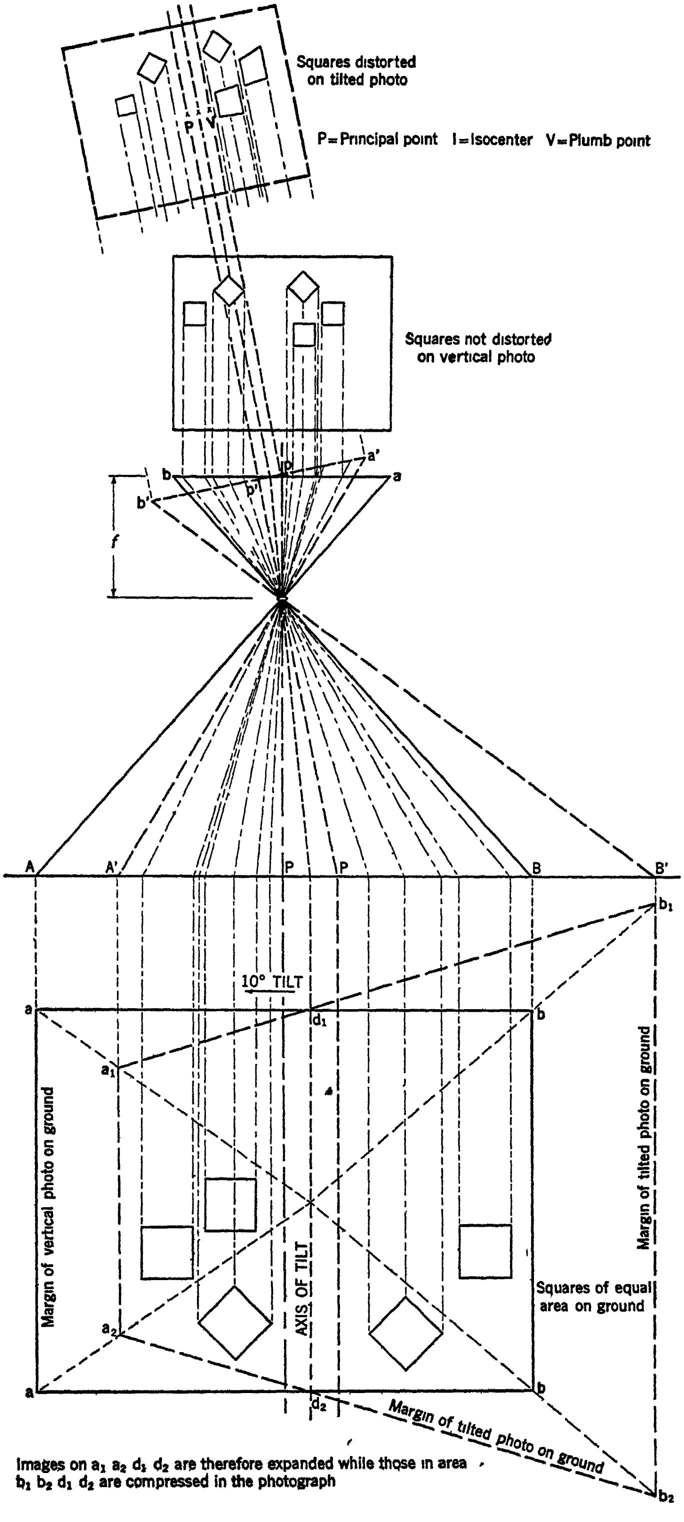


FIG. 8 Effects of tilt on photographs.

point, and the lens, is a radial from the nadir point as projected on the photograph. This property of the photographs is used for a system of compiling maps from photographs, called the radial method (described fully in chapter IX) The displacements, due to relief (note the difference between the positions of the bottom and top of tank in the plane sketch of the overlapping photographs in Figure 7, are annoying when compiling planimetry However, they are measured and used by a number of stereoscopic devices and plotting machines to map relief as explained in chapter XI.

Figure 8 illustrates the effect of tilt of the lens axis from the vertical on photographs. The scale of the tilted photographs agrees with that of an untilted one from the same exposure station only along a line perpendicular to the maximum tilt and passing through the *isocenter* (a point at the foot of a perpendicular from the lens to the line of intersection between the planes of an untilted and the tilted photograph). The area above the isocenter included on the positive print is

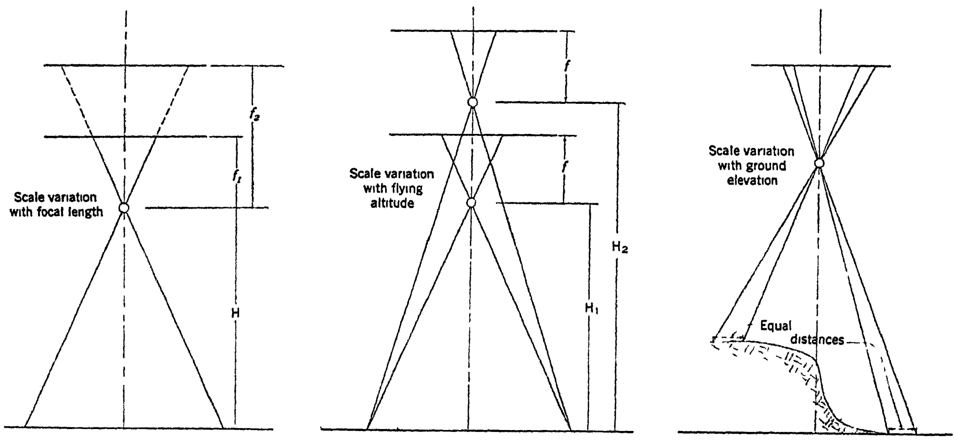


FIG. 9 Variation in scale with focal length, altitude and height

much larger and its images are, therefore, compressed, while the area included below the isocenter is smaller and its images are, therefore, expanded Characteristic distortions in different areas of a photograph tilted 10 degrees for squares of equal size on the ground are diagrammed in Figure 8. If there be no appreciable differences in elevation of the terrain, radials drawn from the isocenter of a tilted photograph will be true in direction, but displacements, due to relief, radiate from the nadir point. If marked tilt and relief are both present there is no one origin for radials which will be graphically correct

Scale variations in nearly vertical photographs, with altitude and focal length, are illustrated in Figure 9. The scale varies with the flying altitude, using the same focal length, with the focal length of the lens at the same altitude, and with the elevation of the terrain if focal length and flying altitude remain constant The variation is proportional in each case (similar triangles). Radials are not affected by changes in scale due to the above sources

The geometric characteristics of photographs, including the effects of relief, tilt, and scale variations, mentioned in the foregoing paragraphs are more fully discussed with formulas for computations in chapter VI Complete analytical computations for problems of mapping from photographs are given in chapter VIII and the references therein.

Oblique Photographs

If the axis of the camera be tilted more than a few degrees from the vertical, the photographs are called obliques; high obliques if the photographs include the horizon, and low obliques if they do not Obliques include more area than verticals from the same altitude When they show the horizon near the top of the photographs, they include very much more area, although the images grow smaller, practically to the vanishing point, as the distance from the camera increases As the camera is pointed farther from the vertical, the areas screened by high ground or trees also increase

Because of their larger coverage, obliques are often used when flying time is limited. Under certain conditions they are valuable for directions to hills, mountain peaks, or prominent features for the computation of relative elevations, or for the control of the aximuth of a strip of photographs The use of obliques in mapping is discussed in chapter XIII

Multi-lens cameras, combining vertical and oblique views, are used to minimize the flying time and ground control that would be required for single-lens photography These include trimetrogron, which photograph from horizon to horizon, and various three-, four-, five-, and nine-lens cameras giving coverages over fields 100° to 150°

Oblique photographs show the terrain more nearly as we are accustomed to viewing it than verticals do with their view from directly above Obliques are, therefore, more pictorial and more readily interpreted. For these reasons they are often preferred for illustrating the approaches to airports or targets, and for publicity purposes.

Light and Shade—Standard Symbols

Figure 10 illustrates another set of differences between maps and photo graphs The photographs must show their information by variations of light and shade, which require local knowedge or much training and study to interpret correctly; whereas the maps show information by standard symbols, obtainable with the map or printed on it The photographs emphasize detail according to its light-reflecting power—the maps according to its importance to man. Maps are easier to read in poor light and cheaper to reproduce in large quantities than the photographs On the photographs, trees often practically obscure details, such as roads, streams, and buildings; while on maps such important details stand out plainly When maps are to be compiled from the photographs, it is ordinarily necessary to visit the field to locate, with at least as high a precision as they are to be plotted, the control stations on the photographs. At the same time correct local names, information for the interpretation of doubtful detail, and supplemental measurements of detail which is obscured or partially obscured are obtained. Under certain circumstances it is preferable to obtain much of this information on a field-completion survey after the compilation has been made.

Quality of the Photographs

The quality of the photographs has a marked effect on the efficiency with which maps may be made from them Any improvement in definition directly affects the precision with which images of the same points can be identified on successive photographs, and, therefore, the accuracy of the direction measurements. Distortion of lenses, if considerable and not compensated for in the plotting instruments, is difficult to eliminate in the compilation processes Unequal or irregular shrinkages in photographic materials are one of the main limitations

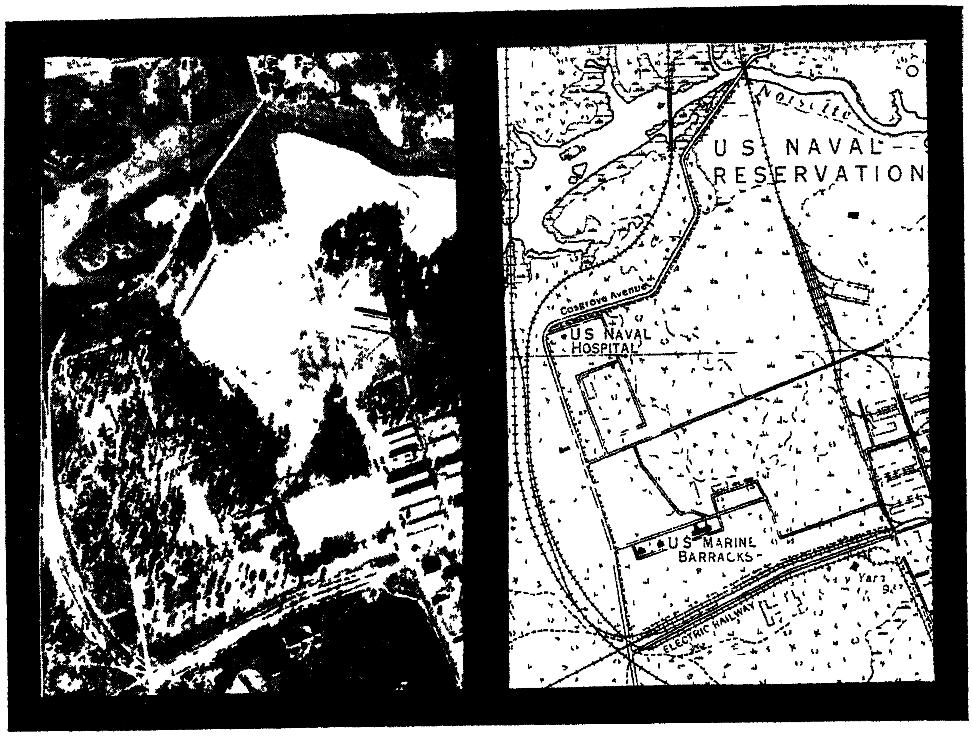


FIG. 10. Differences between map and photograph.

to accuracy. Many of the plotting methods (particularly those not using the more expensive instruments) are greatly complicated if there is excessive tilt or variation in altitude of the exposures. Marked reduction in tilt and altitude variations are to be expected from the development of gyroscopic and photo-electric aircraft and camera stabilizing controls during the present war. This will tend to enhance the usefulness of the simple methods of compilation. In chapter IV are given the specifications resulting from the experience gained during the execution, before the present war, of over five million dollars worth of aerial photography. Every effort during this time was made to obtain the highest quality in the photographs that could be obtained without excessive increase in cost.

DIRECT USES OF PHOTOGRAPHS

Aside from furnishing the most efficient means of making maps on a skeleton of ground control, air photographs are highly useful for many other purposes. Two properly made air photographs, correctly adjusted under a suitable stereoscope (see chapter VII), furnish a special or three-dimensional model of the earth they portray, which is unapproached by any other method for observing and recording complete detailed information. Modern aerial cameras and aircraft take these models in a few minutes and later they can be made available at any number of offices for many special studies as well as for mapping. The product of one photographic flight may be studied by agriculturists for crops, soil types, and for erosion control; by foresters for forestation; by geologists for structures indicating possible mineral wealth; by assessors for adjustment of taxes; by engi-

neers for development of cities, water power, navigability of streams, and routes for highways, railroads, or transmission lines; by soldiers for military cover, obstacles, and all the many aspects of the terrain which affect modern warfare. Each specialist can examine the photographs for the particular data he needs, data all too likely to be omitted or generalized into insignificance when a county is shown on a general map by hand-drawn lines and conventional symbols

This ungeneralized, almost unlimited detail of the photographic record, is one of its principal advantages In fact, even though engineers make a survey on the ground or compile maps with a special use in mind, in trying to save time or expense, they are likely to omit information which would be of great assistance in solving problems not foreseen, but which arise later in the course of the work. If the model of the terrain furnished by good aerial photographs under a stereoscope be available, the information may often be obtained from them without another trip to the locality.

Mosaics

The area covered by a pair of photographs, even those from modern wide-angle cameras, is rather small The photographs may be joined together (after being rectified as may be necessary to a common datum plane) to form a composite picture or mosaic of larger areas (see chapter X). The mosaic retains the photographic detail, but an extra set of prints is necessary if it is desired to use a stereoscope to study the third dimension Marked differences in elevation of the terrain cause displacements of position which can be minimized only by a complicated restitution, or a large increase in the number of exposures However, much of the earth is sufficiently level to permit making satisfactory mosaics with simple manipulation of paper prints at considerably less cost than a line map. If laid so as to fit over an accurate map or radial-plot for control, scale differences can be held to those caused by relief displacements only. Although special machines have been built which are fairly successful in removing scale differences due to relief from the photographs, the machines are somewhat complicated and have not attained marked commercial success as yet. The photographic detail supplied by mosaics and the ease with which they are understood by those familiar with the terrain but unaccustomed to map conventions, make them particularly valuable for visualizing certain problems, especially those of city planning and tax adjustment. Mosaics form one of the most important commercial uses of air photographs for purposes for which exact scale is not important, and where no up-to-date large-scale maps exist.

Military Uses of Air Photographs

It is not uncommon in the present war for aircraft on photographic missions to carry several cameras in order to obtain large-scale intelligence photographs as well as wide-angle photographs for mapping on a single flight. The enemy's works are photographed from the air every day or as often as weather and other circumstances permit The changes in the successive sets of photographs as revealed by careful study of stereoscopic pairs give very detailed information as to what the enemy is doing and expects to do. Bombing is much increased in efficiency, not only by maps showing precise detail obtained from photographs to aid in identifying targets, but also by photographs taken after the bombs have fallen to analyze the pattern obtained, correct the aiming technique, and assess the damage resulting from the various attacks.

Modern cameras and photographic emulsions obtain such excellent photographs, even from altitudes of 30,000 feet, that it is possible to pick out mine

fields, gun emplacements, and even to distinguish between different types of guns and tanks Other cameras with moving film or high-speed shutters photograph, at very large scales, the details of the terrain passed over by the aircraft from altitudes of only a hundred feet or so, and at speeds of 350 miles per hour and upwards Whenever weather or other conditions permit of obtaining good photographs, the data they furnish are so complete, and can be so accurately interpreted by specialists, as to enable operations to be carried out with great economy of life and ammunition—economies so great that in many theatres of the present war only emergency operations are conducted until the information from air photographs becomes available Millions of square miles of other countries are now being photographed by the Army Air Forces with Trimetrogon cameras in order that aeronautical charts adequate for fighting a world war may be constructed. It is difficult to appreciate, without personal experience, how readily any part of the world may be traversed by air, or the ease with which whole continents can be crossed or the world encircled by modern aircraft. Henceforth, the unsurpassed, detailed record of air photographs will be made available for any part of the earth, wherever sufficient needs arises Man has indeed placed the earth beneath the scope of his much sharpened glance.

BIBLIOGRAPHY*

Special Publications of U S Coast and Geodetic Survey on Control Surveys and Coordinates.

No 227 Horizontal Control Data
No. 14 Astronomical Observations
No 120 Manual of First Order Triangulations
No 145 Manual of Second and Third Order Triangulation
No 138 Manual of Triangulation Computation and Adjustment
No. 140 Manual of First Order Leveling
No. 193 Manual of Plane Coordinate Computations
No 194 Manual of Traverse Computations on Lambert Grid
No 195 Manual of Traverse Computations on Transverse Mercator
Serial No 584 Azimuths from Plane Coordinates
No. 59 Progressive Military Grid System for the United States
No 68 Elements of Map Projections

Cartography:

Cartography, C. H. Deetz, SP-205 U S Coast and Geodetic Survey, Superintendent of Documents, $ 60
Maps and Survey, A. R Hinks, Cambridge Univ Press
A Key to Maps, Winterbotham, Blackie & Son London.
Topographic Instructions, U S. Geological Survey, Bull. 788, Superintendent of Documents, $1 00

* For a more complete listing see "Ready Reference List" at back of manual

USE OF THE PRISMATIC ASTROLABE FOR ASTRONOMIC POSITIONS

L. M Samuels

THE Prismatic Astrolabe with a 60-degree prism was developed by French engineers in 1903. Later on British engineers brought out a similar instrument with a 45-degree prism, for which certain advantages are claimed The principles of both types are analogous in determining astronomic latitude and longitude by equal altitudes of heavenly bodies when the time of a prime meridian, such as that of Greenwich, is known at the place of observation. This method of equal altitudes originated with Gauss, the noted astronomer, who in 1808 demonstrated that if the right ascensions and declinations of three or more stars be known, and equal altitudes of these stars be observed at times recorded, then from these times there may be computed· the observed altitude, the error of the chronometer on local time, and the latitude of the observer. Nowadays, with radio time signals and an approximate value of the latitude, both latitude and longitude may be deduced precisely by this process of equal altitudes.

Customarily in the Navy, astronomic position origins for surveys formerly were determined with the transit-zenith telescope From actual observing with each I am convinced of many worth-while advantages of the prismatic astrolabe over the transit—zenith telescope. As far as accuracy is concerned, the astrolabe is definitely a precision instrument as will be discussed further on. There is nothing at all complicated about the observations, computations, or adjustments of the astrolabe.

The French "Geodetic Model" of Claude and Driencourt with a 60-degree prism, which we use, weighs 30 pounds, for the instrument, with case and accessories about 45 pounds, and can be carried by one man Whenever possible we build a pier for observing and for a permanent marker, but a tripod support is part of the equipment furnished by the manufacturer It serves its purpose well and weighs only 11 pounds The instrument is set up and adjusted without much difficulty or exertion, and the observer can be seated No more care is required for leveling the astrolabe than for an ordinary surveyor's theodolite. Orientation in the meridian closely enough for a start is done with an attached magnetic compass, applying the local magnetic declination Collimation, which the makers term "auto-collimation," is effected with two adjusting screws on the exterior of the alidade and requires but a moment's work. The instrument having been leveled, oriented, and collimated, there remains but to pour a small amount of mercury in the pan for an artificial horizon, and the observations are in order.

In appreciation of our success with the 60-degree astrolabe we feel greatly indebted to Colonel Bagley, U. S. Army, to Mr. Shea of the Massachusetts Institute of Technology, and to Major Weld Arnold, U. S Army, formerly of the Institute of Geographical Exploration, for the help they have all given us in astrolabe work, and also for Major Arnold's compilation of a very comprchensive list of stars for each degree of latitude from 60 degrees south to 60 degrees north, without which our astrolabe observations could hardly have been attempted. On occasions when no program of stars had been prepared in advance, we have read them off directly from Major Arnold's catalogue, taking them in sequence as tabulated for local sidereal times and corresponding azimuths. A pre-selected list of stars to observe is, however, desirable, so as to get the proper distribution of azimuths in each quadrant, but no special preparation of ob-

serving programs, such as required for the transit-zenith telescope, is necessary in astrolabe procedure. No reference to old catalogues, such as the Boss, need be made for the astrolabe, as current star publications suffice without having to work up star positions to any date.

As to the dependability of the Astrolabe, we had an excellent opportunity a year or so ago to verify this at the Naval Observatory by accepting for standard of comparison the astronomic position of the pier we used as determined by Naval Observatory astronomers. In latitude, two of our student observers obtained results with the astrolabe that differed respectively six-tenths of a second of arc and zero seconds of arc from the established value; in longitude each came within one and one-tenth of a second of arc. An experienced observer in the same test checked to two-tenths of a second of arc in latitude and five-tenths, also arc, in longitude. On one occasion two of our engineers received instruction in the astrolabe during the winter under adverse weather conditions permitting but three clear nights. Only seven days in all could be spared for the period of instruction, but nevertheless their subsequent determinations have been very gratifying, with probable errors of observations not more than two-tenths of a second of arc for both latitude and longitude No test for personal equations of the observers has been possible as yet, but from an inspection it appears that the results of our observers at the Naval Observatory, as mentioned, would have been even closer with a small correction to the longitude on the assumption of a slight lag in the general or usual personal reaction. The makers of the astrolabe have developed a machine for testing personal equation which is reputed to be very efficient, but owing to war conditions we have been unable to get any of them.

Procedure in observing with the 60-degree astrolabe is simple and not difficult to learn. Our junior hydrographic engineers, without previous assignment to astronomical work, have received a fair understanding of the observing and computations in a few days when pressed for time. Our experience with the astrolabe has proven the field operations to be economical and expedient. An astronomical expedition consisting of two officer observers, completed 24 stations for latitude and longitude in less than six weeks. This time included ship transportation between stations, averaging about 60 miles apart, and time for landing gear through surf, making and breaking camp, and for small surveys to connect the piers to lighthouses and nearby portions of the shore line and topography.

Optics

The optical arrangement of the 60-degree astrolabe is illustrated on Figure 1.

R and R' are rays from a star. R passes normally through the upper face of the prism and is reflected from the lower face through the object glass to the focal plane of the objective at f. R' is first reflected from the mercury surface to the upper face, thence to f. The diagram is drawn to show the convergence of R and R' at the instant when the two star images coincide at the altitude governed by the angle of the prism. The three figures at the bottom indicate the images on entering the field, at passage, and on leaving the field. Of the two eyepieces shown, the upper one is the finder eyepiece, of 30 diameters, having a field of 1 degree 36 minutes, for training the alidade on the stars. A small prism actuated at will by a lever transmits the star images from f to this finder eyepiece, and after centering the images therein the observer throws back the lever and the images are then set to appear for observing in the lower eyepiece of 80 diameters and 36-minute field.

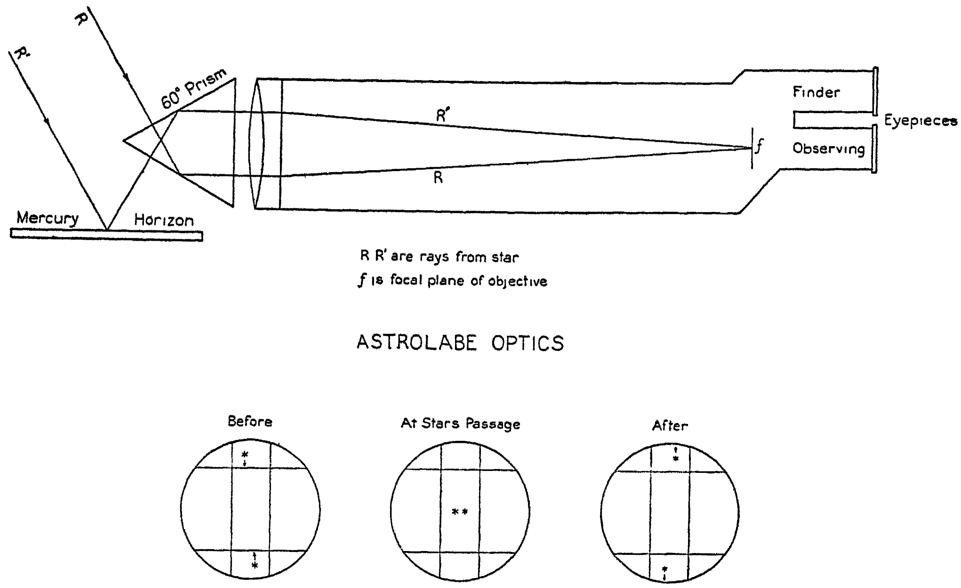

R
R'
60° Prism
Finder
Eyepieces
Observing
f
R'
R
Mercury
Horizon
R R' are rays from star
f is focal plane of objective
ASTROLABE OPTICS
Before
At Stars Passage
After

FIG. 1

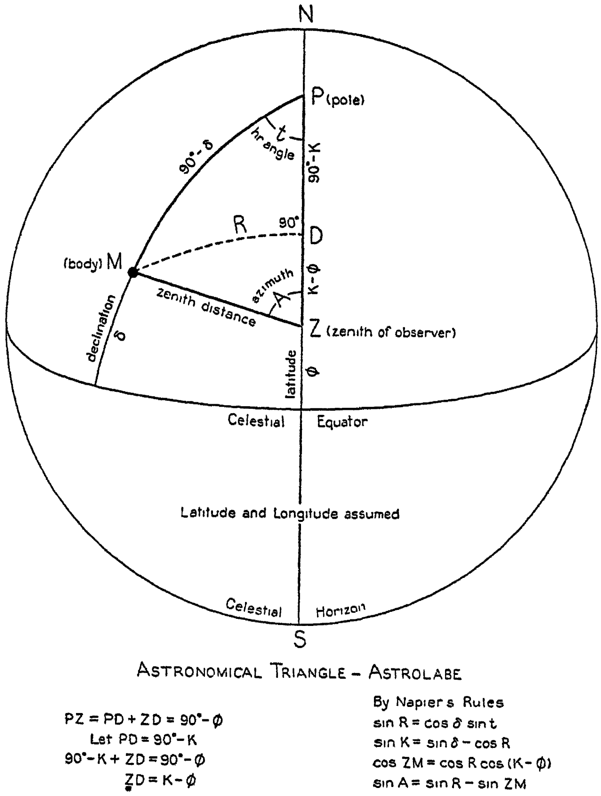

N
P (pole)
90°-δ
t
hr angle
90°-K
R
90°
D
(body) M
azimuth
A
K-φ
zenith distance
Z (zenith of observer)
declination
δ
latitude
φ
Celestial Equator
Latitude and Longitude assumed
Celestial Horizon
S
ASTRONOMICAL TRIANGLE – ASTROLABE
PZ = PD + ZD = 90°-φ
Let PD = 90°-K
90°-K + ZD = 90°-φ
ZD = K-φ
By Napier's Rules
sin R = cos δ sin t
sin K = sin δ ÷ cos R
cos ZM = cos R cos (K-φ)
sin A = sin R ÷ sin ZM

FIG. 2

Astronomical Triangle

The astronomical triangle concerned in the computation, shown on Figure 2, is projected on the plane of the observer's horizon extended to the celestial *NS* is the observer's meridian through his zenith point *Z* and *P* the elevated pole. *M* is the body observed and delta (δ) is declination *R* by construction is a perpendicular from *M* to the meridian at *D*

The latitude and longitude are assumed The Greenwich sidereal time and the body's right ascension are known, from which the body's hour angle (t) is deduced. The declination of the body is known The two spherical right triangles

ZENITH DISTANCES AND AZIMUTHS

Greenwich Sidereal Time of Observation Known

Star	*214 β Aql	*218 41 Cyg	*221 ε Aqr	*234 ι Cet	*236 ε And	*246 η Psc
Chro. time	1 53 36 74	2 06 11 49	2 14 10 01	2 46 52 75	2 54 44 21	3 27 28 20
Chro. Corr.	+ 11 57 65	+ 11 57 65	+ 11 57 65	+ 11 57 65	+ 11 57 65	+ 11 57 65
G. S. T.	2 05 34 39	2 18 09 14	2 26 07 66	2 58 50 40	3 06 41 86	3 39 25 85
λ	4 13 00 00	4 13 00 00	4 13 00 00	4 13 00 00	4 13 00 00	4 13 00 00
L. S. T.	21 52 34 39	22 05 09 14	22 13 07 66	22 45 50 40	22 53 41 86	23 26 25 85
R. A. Star	19 52 27 84	20 27 02 07	20 44 31 99	0 16 27 34	0 35 28 41	1 28 21 30
(Time) t	2 00 06 55	1 38 07 07	1 28 35 67	1 30 36 94	1 41 46 55	2 01 55 45
(Arc) t	30 01 38 25	24 31 46 05	22 08 55 05	22 39 14 10	25 26 38 25	30 28 51 75
Star's δ	+ 6 15 46 12	+30 10 29 78	- 9 42 27 67	- 9 08 45 97	+ 28 59 33 52	+15 02 37 21
Log sin t	9 699 3281	9 618 2166	9 576 3534	9 585 6456	9 633 0926	9 705 2248
(add) Log cos δ	9 997 4004	9 936 7624	9 993 7363	9 994 4431	9 941 8502	9 984 8550
Log sin R	9 696 7285	9 554 9790	9 570 0897	9 580 0887	9 574 9428	9 690 0798
R	note Convert log sin R to log cos R and enter second line down.					
Log sin δ	9 037 7815	9 701 2587	9 226 9132	9 201 2674	9 685 4707	9 414 2297
(subtract) Log cos R	9 938 2727	9 970 0558	9 967 7296	9 966 0839	9 966 9416	9 940 4149
Log sin K	9 099 5098	9 731 2029	9 259 1836	9 235 1835	9 718 5291	9 473 8148
K	+ 7 13 26 70	+32 34 58 94	-10 27 52 63	- 9 53 46 25	+31 32 09 31	+17 19 15 57
Ø	+10 40 15 00	+10 40 15 00	+10 40 15 00	+10 40 15 00	+10 40 15 00	+10 40 15 00
K - Ø	3 26 48 30	21 54 43 94	21 08 07 63	20 34 01 25	20 51 54 31	6 39 00 57
Log cos (K - Ø)	9 999 2137	9 967 4341	9 969 7562	9 971 3974	9 970 5430	9 997 0681
(add) Log cos R	9 938 2727	9 970 0558	9 967 7296	9 966 0839	9 966 9416	9 940 4149
Log cos Z	9 937 4864	9 937 4899	9 937 4858	9 937 4813	9 937 4846	9 937 4830
Computed Z	30° 00′ 36 38	33″50	36″85	40″58	37″83	39″17
Assumed Z	30 00 10 00	10 00	10 00	10 00	10 00	10 00
Correction = ΔZ	26″38	23′50	26″85	30″58	27 83	29 17
Log sin R	9 696 7285	9 554 9790	9 570 0897	9 580 0887	9 574 9428	9 690 0798
(subtract) Log sin Z	9 699 1027	9 699 0922	9 699 1043	9 699 1179	9 699 1079	9 699 1128
Log sin A	9 997 6258	9 855 8868	9 870 9854	9 880 9708	9 875 8349	9 990 9670
Computed A	264° 01	314° 09	227° 59′	130° 31′	48° 42	78° 21
Star List A	263¼°	313¼°	227¼°	131¾°	49¾°	79°

Ø = Assumed Latitude
t = Hour Angle
A = Azimuth
λ = Assumed Longitude

δ = Declination
Z = Zenith Distance
R & K are auxiliary angles

$\text{Sin R} = \text{Sin t Cos } \delta$

$\text{Sin K} = \frac{\text{Sin } \delta}{\text{Cos R}}$

$\text{Cos Z} = \text{Cos (K - Ø) Cos R}$

$\text{Sin A} = \frac{\text{Sin R}}{\text{Sin Z}}$

North Latitude and Declination are regarded (+), South as (-). If Latitude and Declination are both (+) or both (-) then K-Ø = the numerical difference of K and Ø. If Latitude and Declination are of different names, K-Ø = the numerical sum of K and Ø. The plotting requires a constant "Assumed Z" to be less than the smallest "Computed Z".

FIG. 3

PMD and *DMZ*, are successively solved, from which the values of *ZM*, the zenith distance, and *A*, the azimuth of the body, are computed in relation to the assumed position.

Computations

The form of Figure 3, filled out to illustrate its use, is arranged for computation of stars observed with the Astrolabe. Practically all of the stars in Major Arnold's "60° Star Lists," previously mentioned, are tabulated in the British catalogue, *Apparent Places of Fundamental Stars*, which now is being published each year. Referring to Figure 3, the chronometer times of the star passages at 60° altitude are carefully scaled from the chronograph sheets. The right ascensions and declinations are interpolated from the British catalogue.

The identity of each star used may be checked by its right ascension as approximated in the "60° Star Lists"

As already mentioned in the solution of the astronomical triangle, the zenith distances and azimuths of the stars are computed from assumed co-ordinates. The position sought is then plotted graphically, using the various zenith distance corrections and azimuths in somewhat similar manner as in plotting lines of position in navigation, except that for the astrolabe the differences between observed and computed zenith distances are plotted instead of the altitude differences.

One "wrinkle" in the computations is concerned with the "Assumed *Z*," which appears at the lower part of Figure 3. In this connection it should be noted that the absolute value of the prism angle does not enter into the computations As a rule the variation from 60° is only a few seconds. The observations being based upon a constant observed zenith distance, any value for a constant assumed zenith distance ("Assumed Z") may be arbitrarily chosen for convenience in plotting. For the computations of Figure 3, the constant of 30°00′10″ was selected, so that the plotting of zenith distance corrections (ΔZ) would permit a scale of two divisions to one second of arc in the plotting of Figure 6, such a scale being sufficiently large for accurate graphic determination of this particular computed position. Another consideration in selecting the value of "Assumed Z", is to choose a value less than the smallest "Computed Z", so that all star position lines may be plotted from the assumed position in

NHO-750

ASTROLABE COMPUTATIONS

LATITUDE AND LONGITUDE

GREENWICH SIDEREAL TIME OF OBSERVATION KNOWN

SET No 6

PLACE *Carupano, Venezuela* ASSUMED LAT *10° 40′ 15″00 N* ASSUMED LONG *63° 15 00 00 W* ASSUMED Z D *30°00′10″00*

SHIP OR EXPEDITION *U S S Sumner* DATE, 1941 MEAN EPOCH OF OBSERVATIONS *July 21 3*

SIGNALS FROM *Arlington* FREQUENCY *113 KC* OBSERVER *W S Davis* COMPUTER *W. S. Davis*

LATITUDE AND LONGITUDE OBSERVATIONS
Carupano, Venezuela
July 1941

The latitude and longitude of a concrete pier near E. Guayacan Point at Carupano, Venezuela, was determined with a 60° astrolabe using a sidereal chronometer. The rate of the chronometer was checked with radio time signals from Arlington. Both clock ticks and radio signals were recorded by means of a chronograph. Six sets of from 15 to 20 stars each were observed and plotted. The scaled values of these 6 sets are as follows

	Longitude		Latitude	
Set No. 1	63-14	56.6	10-40	13.1
2	63-14	57.1	10-40	12.8
3	63-14	58.0	10-40	12.0
4	63-14	56 8	10-40	13.0
5	63-14	58.2	10-40	12.6
6	63-14	57.9	10-40	12.8

See Fig. 3 for computations of six stars of set No.6

The means of these 6 sets were taken as the observed position of this pier

Longitude	Latitude
63-14 57.4 W.	10-40 12.7 N.

The probable error was found to be :
± 0.2″ in longitude ± 0.1″ in latitude

The probable error was computed from the formula:

$$P\ E. = \pm\ 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}}$$

CLOCK CORRECTIONS

Greenwich Date, *July 21, 1941*	h	m	s	h	m	s	h	m	s
G C T of Signal	6	00	00 00	7	00	00 00	8	00	00 00
G S T of Greenwich 0^h	19	53	28 25	19	53	28 25	19	53	28 25
Corr for Int since 0^h	+		59 14	+	1	9 00	+	1	18 85
G S T of Signal	1	54	27 39	2	54	37 25	3	54	47 10
Clock Time of Signal	1	42	29 70	2	42	39 63	3	42	49 50
Clock Corr on G S T at Signal (F−, S+)	+	11	57 69	+	11	57 62	+	11	57 60

Average Hourly Rate between Signals	Interval between Signal and Mean Epoch	Change in Error in Interval
Gaining / Losing 0.045 s ×	1 0 Hours =	0.045 s Gaining / Losing

Chronometer Correction for Set = + $0^h\ 11^m\ 57^s65$

LATITUDE DIFFERENCE	Divisions	° ′ ″	
Assumed Latitude, ϕ_a		10 40 15 00	
Y-coordinate (N, S)	2.2 =	- 2 20	
Observed Latitude		10 40 12 80	

LONGITUDE DIFFERENCE	Divisions	° ′ ″	Log
X-coordinate (W, E)	2.1		0 322 2193
Log Cos Lat (subtract) or Nat Sec Lat (multiply)			9 992 4250
			0 329 7943
Corr to assumed Long		2.10	
Assumed Longitude		63 15 00 00	
Observed Longitude		63 14 57 90	

COMPUTER'S NOTES

FIG. 4

direction *toward* the stars. Increasing or decreasing the assumed zenith distance affects plotting of the computed circle of position only to the extent of lengthening or shortening the radius without displacing the center.

Figure 4 shows a form and illustrative computation such as is used for recording data in general concerning the observations of one set of stars, and includes the results of the entire observations at one station.

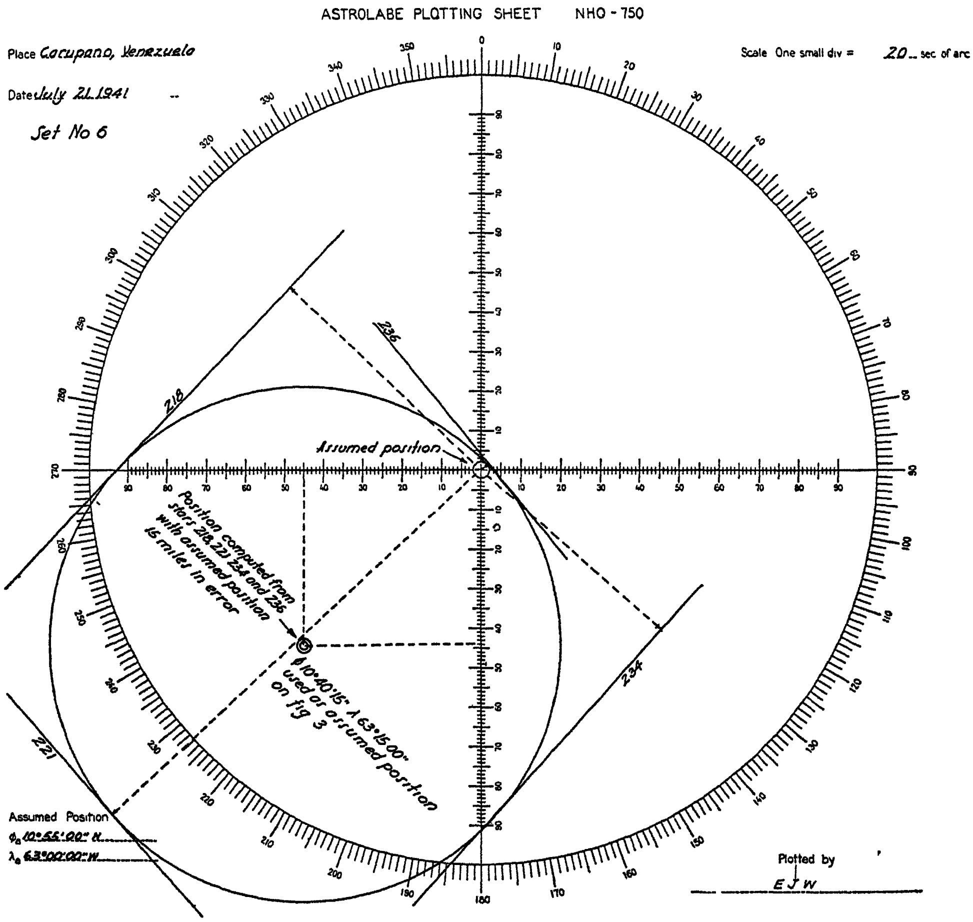


FIG. 5

PRELIMINARY POSITION

A preliminary computation is first made from four stars of an observed set, in order to get an approximate position within 5 or 10 seconds of arc of the final position, for use in the final computations and plotting. The four stars are selected in azimuth as nearly 90 degrees apart as possible, preferably near the intercardinal points, and with the latitude and longitude to the nearest minute from available maps or charts, the zenith distance corrections and azimuths are computed in the same manner and form as the final position. Illustrating how the approximate position is derived, Figure 5 was prepared assuming a latitude and longitude purposely grossly in error about 15 miles. This position is represented at the center of the diagram. The four dotted lines are the plotted

azimuths of the four selected stars. The normals to these dotted lines are drawn at their respective zenith distance corrections from the diagram center, and by construction are tangents to a circle the center of which is the observed point. The scale of this figure is 3 divisions of Figure 5 to one minute of latitude, and the derived position proved to be within 5 seconds in arc of the final position. A close value of the longitude may be found directly if any star of the set is observed on both sides of the meridian. In such a case, the longitude is the differ-

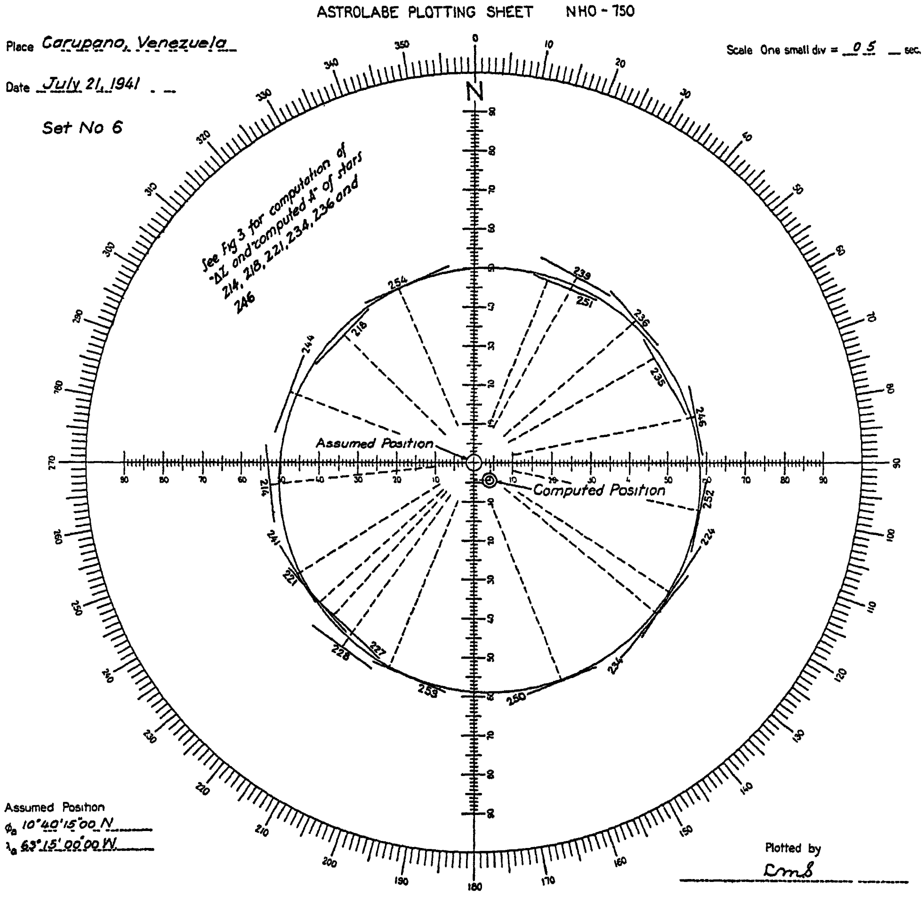


FIG. 6

ence between the mean of the Greenwich sidereal times of the star's passages, and the star's right ascension. The computation procedure of the four stars, for an approximate assumed position, is identical with the computation for the final position as on Figure 3.

Final Observed Position

Figure 6 is an example of the final plotting. The approximate position, found from the four selected stars of the set, and shown at the diagram center, becomes the assumed position for the final computations and plotting. The azimuth lines

of the various stars of the set, including the four selected stars recomputed for the new assumed position, are shown with their corresponding normals, these normals being tangents to a circle the center of which is the final observed position and indicated by the two small concentric circles The value of this final observed position is referred to the assumed position as derived from the preliminary computation, the latitude correction in arc being scaled direct, the correction for longitude arc being the scaled difference, times the secant of the latitude.

Opinions differ as to the number of stars which should be observed for each set. Our system is, whenever possible, to control the set with three radio time signals, an hour apart, and within this two-hour period to observe as many stars as practicable, consistent with careful work Generally a greater number of stars are observed than are utilized in the computations, thus permitting a selection

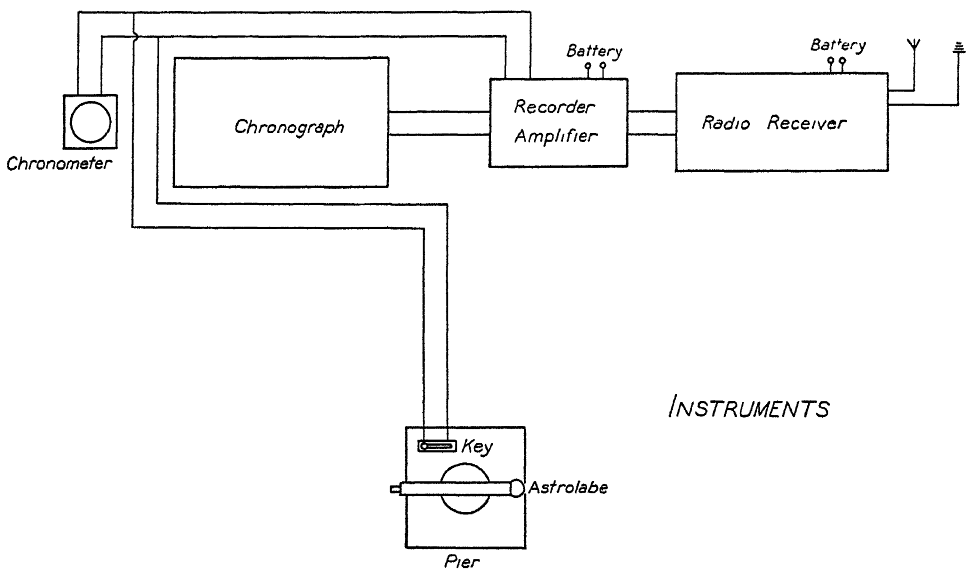


FIG. 7

for distributing the position-circle tangents to advantage in the quadrants, and also providing additional stars for checking if needed. We prefer not less than five stars to each quadrant. As a rule, stars closer than 10 degrees to the meridian are avoided as they change too slowly in altitude for accurate timing, and time is the deciding element for precision in astrolabe operations.

A few minutes before commencing the observations, it is advisable to take out several stars from the catalogue and point on them to check the compass orientation of the azimuth circle, and also to adjust the focus of the observing eyepiece. This focus must not be altered during any set, as so doing will materially vary the constant measure of the altitude.

A unique feature in observing with the Prismatic Astrolabe is that the two images of the star, as seen in the telescope, approach from opposite directions, and therefore at twice the apparent motion, so if there be any displacement in the observer's estimate of the coincidence of the images his error is halved.

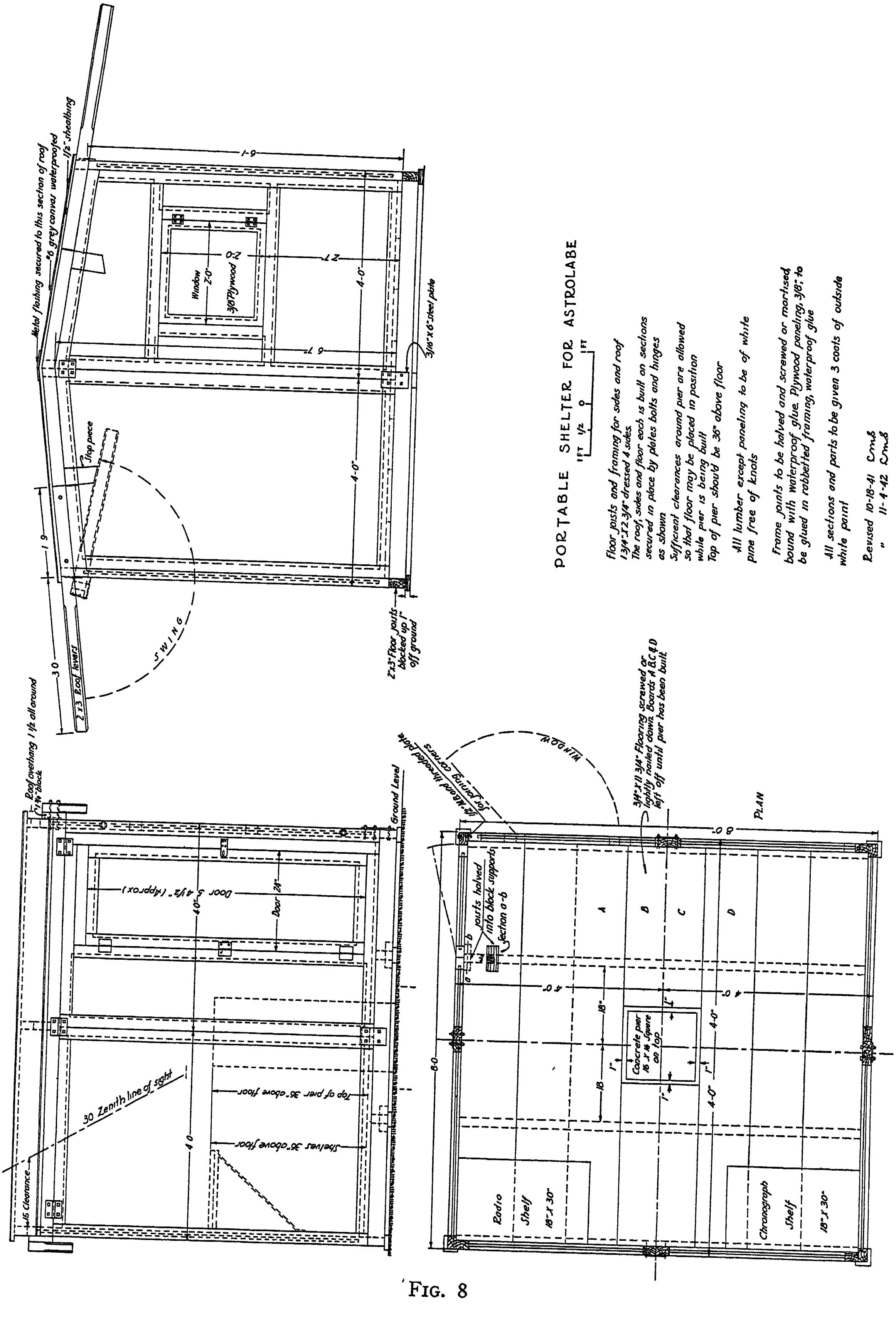

PORTABLE SHELTER FOR ASTROLABE
1 FT 1/2 0 1 FT
Floor joists and framing for sides and roof 1 3/4" X 2 3/4" dressed 4 sides.
The roof, sides and floor each is built on sections secured in place by plates bolts and hinges as shown
Sufficient clearances around pier are allowed so that floor may be placed in position while pier is being built
Top of pier should be 36" above floor
All lumber except paneling to be of white pine free of knots
Frame joints to be halved and screwed or mortised, bound with waterproof glue. Plywood paneling, 3/8" to be glued in rabbetted framing, waterproof glue
All sections and parts to be given 3 coats of outside white paint
Revised 10-18-41 CmS
" 11-4-42 CmS
Metal flashing secured to this section of roof
#6 grey canvas waterproofed
1/2" sheathing
Window
2'-0"
3/8" Plywood
3/16" X 6" steel plate
Stop piece
SWING
2 X 3 Roof levers
2" X 3" Floor joists blocked up 1" off ground
Roof overhang 1 1/2 all around
Ground Level
Door 5' 4 1/2" (Approx.)
Door 24"
Top of pier 36" above floor
Shelves 36" above floor
30 Zenith line of sight
15 Clearance
PLAN
WINDOW
1/2" M bolt threaded plate for joining corners
3/4" X 11 3/4" Flooring screwed or lightly nailed down. Boards A, B, C & D left off until pier has been built.
Joists halved into block supports
Section a-b
Concrete pier 16" X 16" Square on top
Radio Shelf 18" X 30"
Chronograph Shelf 18" X 30"

FIG. 8

INSTRUMENTS

Figure 7 shows our usual arrangement of instruments for astrolabe observations. At the bottom of the figure is the pier with astrolabe and observer's break-circuit key. In the upper part of the diagram at the left is the break-circuit sidereal chronometer, and thence towards the right are the chronograph, the radio recorder-amplifier and the radio receiver. Current for electrical needs is supplied from flashlight cells and a 6-volt storage battery.

ASTROLABE SHELTER

Figure 8 shows the portable type of shelter we use for the astrolabe. It is 8 feet square, 6½ feet high, and designed so that it may be set up at the same time the pier is being built. The sides and roof are each in two sections, 4 feet

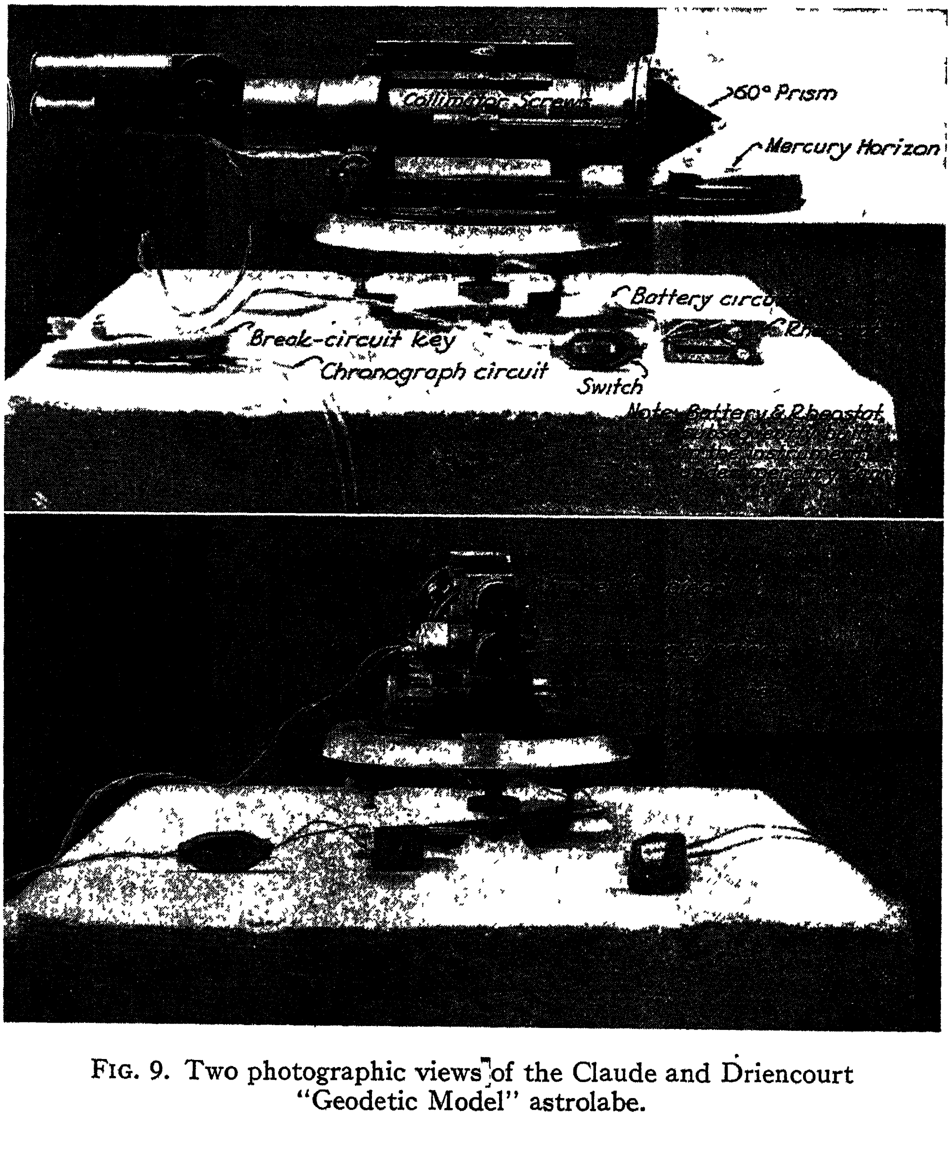


FIG. 9. Two photographic views of the Claude and Driencourt "Geodetic Model" astrolabe.

wide and 6 to 8 feet long, and in the erection are bolted together in place. The paneling is $\frac{3}{8}$-inch plywood rabbetted into 2-inch by 3-inch white pine framing. The roof is in two sections as mentioned. Each section is hinged at the side and can be thrown back with two levers to clear the ceiling for observing. All sections of the structure are thoroughly waterproofed and painted, and the whole may be towed as a raft in the water for landing if desired. There is sufficient room inside the shelter for a couple of cots and also for the observer's living and computing quarters.

RECTANGULAR COORDINATES AND STANDARD HORIZONTAL DATUM

R. M. Wilson

PLANE rectangular coordinate systems to represent standard datum, which may be extended over large areas without excessive loss of accuracy, have been used in the United States since 1935. They have been legally adopted by some of the States, beginning with New Jersey in 1935[1] In order that individual systems may be State-wide without introducing objectionable distortion they must be planned carefully and must have their foundations laid in exact mathematics. The United States Coast and Geodetic Survey, in certain of its publications,[2,3,4] sets forth the theory of these plane coordinate systems and by numerical examples illustrates the different computations involved. It is not intended here to outline the theory again, nor to provide a detailed guide to the use of the systems.

These systems are needed for the urgent purpose of welding together upon a common horizontal datum the various individual local surveys conducted by engineers in all branches of the profession.[5,6] With that use in mind, it is intended in the following paragraphs to discuss certain features of the subject, perhaps from a new point of view, without the distraction of mathematical derivations.

The standard plane coordinate systems may be regarded as parts of the fundamental world-wide geodetic system that have been translated into a form convenient for general everyday use In the geodetic system distances measured on the ground are reduced to sea level so that a survey may be considered as lying on the surface of the geodetic reference spheroid. This assumption, taken for granted in the following discussion, implies that the plane coordinates are also expressed in terms of sea-level distances.

A true rectangular coordinate system must be contained in a plane; it is not possible to arrange such a system so that it will be form-fitted to the curvature of the earth or of the reference spheroid. The problem, therefore, as in mapping, is to take the details of a survey from the curved surface of the spheroid and show them as projected into a plane, flat surface. A great many different kinds of map projections are in use, each having been devised or chosen to meet some particular requirement.[7] Fortunately, the mathematics developed in the study of projections for maps, already available, applies equally well to projections for rectangular coordinate systems. In any system designed to be useful to the surveyor, one of the fundamental requirements is that angles shown by the

[1] Kissam, Philip, New Jersey Adopts Plane-Coordinate System *Civil Engineering*, November 1935

[2] Adams, Oscar S , and Claire, Charles N , Manual of Plane-Coordinate Computation U. S Department of Commerce, Coast and Geodetic Survey, *Special Publication No 193*, 1935.

[3] Adams, Oscar S , and Claire, Charles N , Manual of Traverse Computation on the Lambert Grid. U S Department of Commerce, Coast and Geodetic Survey, *Special Publication No. 194*, 1935

[4] Adams, Oscar S., and Claire, Charles N , Manual of Traverse Computation on the Transverse Mercator Grid: U S. Department of Commerce, Coast and Geodetic Survey, *Special Publication No. 195*, 1935

[5] Adams, Oscar S., Plane Coordinate Systems· U S. Department of Commerce, Coast and Geodetic Survey, *Serial No. 562*, 1933.

[6] Adams, O. S , Development of State Grid Systems: *Civil Engineering*, January 1937.

[7] Deetz, Charles H , and Adams, Oscar S , Elements of Map Projection with Applications to Map and Chart Construction· U. S Department of Commerce, Coast and Geodetic Survey, *Special Publication No. 68*, fourth edition, Revised April 2, 1934

projection in the plane coordinate system shall be as nearly equal as possible to the corresponding angles measured on the ground. Conformal projections are designed to meet this requirement: their derivation provides that horizontal angles between lines at any geodetic point shall not be changed in the projection and that scale factors at any projected point must be equal in all directions about that point Thus infinitesimally small areas are projected without change in shape, although they may be changed in size Another requirement is that the projection chosen shall remain as true to scale as possible over large areas The Lambert conformal and the transverse Mercator projections appear most nearly to meet these requirements

In defining the relation between rectangular and geodetic coordinates there are, therefore, two distinct surfaces to consider—the surface of the spheroid upon which are situated the points used in geodetic computations, and the plane surface of the projection, on which lie the corresponding points that are referred to the rectangular coordinate system. Therefore the change from geodetic to plane rectangular coordinates, or the reverse, is not merely a transformation of coordinates from one kind to the other for an identical point to which both kinds of coordinates refer. Instead, the change is really a computation of the rectangular coordinates for a separate point, lying in the plane, which is to represent the point given on the spheroid for which the geodetic coordinates are known. Of course this procedure is reversed if it is the rectangular coordinates that are known. Even though the plane is imagined as being placed into the closest possible juxtaposition with the surface of the spheroid, the two corresponding points may still be separated very widely in space because of the divergence between the different surfaces upon which they lie Their positions are related to one another only through the mathematics and the constants of the projection that is being used, generally the accurate relationship cannot be represented geometrically. Some confusion may be avoided by remembering these two separate surfaces, and by visualizing two sets of points for any survey —the points on the spheroid and the points representing them in the plane of the projection.

Now consider what happens when a simple geodetic triangle on the spheroid is projected to a plane. The two types of projection used for the standard systems under discussion are both conformal. Therefore, in using either one, the three angles of the geodetic triangle should be projected to the plane without any change in size. But the sides of the geodetic triangle generally will not be projected to the plane as straight lines. So before using the projected triangle in plane computations its sides must be straightened. When this is done it is evident that the true angles as projected must be changed slightly, so the quality of exact conformality cannot be maintained in considering as a whole a triangle or other geodetic figure covering an appreciable area. Moreover, the length of the projected sides will not be the same nor even exactly in the same proportion as the lengths of the sides of the geodetic triangle The projected and straightened triangle is suitable for use in any kind of plane computation in the rectangular coordinate system. The same general considerations apply also to the elements of a projected traverse line.

The mathematics of the adopted projections provides formulas for obtaining the plane coordinates of projected points from given geodetic coordinates of points on the spheroid, or vice versa. Formulas and scale factors are provided to show the difference between lengths on the spheroid and the corresponding lengths as projected in the plane. Still other formulas are provided to determine the slight changes in direction at the ends of projected geodetic lines when these are straightened in the plane.

The small changes just described provide the means to prepare the observations of ordinary surveys for computation directly in the plane of rectangular coordinates, thus avoiding the complications of passing through the geodetic phase By the application of these changes to the observed angles and distances (assumed as reduced to sea level first), corresponding angles and distances between projected points in the plane are obtained just as if the measurements had been made there originally. The observed values contain the accidental errors that always occur in any survey, the same accidental errors are still contained in the angles and distances as prepared for plane computation. To distribute such errors, adjustments by least squares or by other methods should be made as in any plane survey. Simple plane computations suffice to continue the work.

It may seem that these coordinate systems, subjecting the observations to many alterations, distortions, and projections between surfaces, will so seriously juggle the results of a survey as to damage its usefulness. So perhaps the magnitude of the changes introduced into the observed quantities should be indicated in order to show that the plane coordinate positions are still reasonable for expressing the results of a survey.

Any geodetic triangle contains spherical excess at the rate of one second for each 75 square miles of its area, approximately. It may be flattened to a plane by reducing the sum of its angles by the amount of its spherical excess. The manner in which that reduction is shared among the three angles is determined by the rules of the projection employed. Any other figure or survey needs to absorb changes among its observed angles only at the same small rate to eliminate its spherical excess and so flatten it to a plane.

Corrections to lengths due to the varying scale of the adopted projections rarely exceed one part in ten thousand within the areas for which they are used. The State systems (with one exception) have been chosen and arranged to remain within this limit of scale difference. Scale corrections usually apply in unequal ratios to different lines of the same survey, thereby producing slight distortion. But this distortion can be no greater than is implied by the change in ratio of length corrections. At the most this ratio is only 1:10,000, and its changes are gradual over large areas. It is not the magnitude of the scale-correction ratio itself but rather the gradual change of that ratio in any area that produces the distortion.

In simple plane surveying a certain degree of tolerance has always been invoked to allow the plane computing of measurements taken on the curved surface of the earth; that is, spherical excess and slight local distortions customarily have been ignored in such work. If an engineer is content to continue with the same degree of tolerance, he may still ignore these same effects in using the standard State systems so long as he operates in a compact area of only a few square miles. But corrections due to spherical excess and distortion certainly should be taken into consideration for surveys over extended areas, or where long triangulation lines are involved.

The scale changes are of greater magnitude and importance. In the established practice of ordinary plane surveying, measured horizontal distances are not usually reduced to sea level. But this should be done in preparing to use the standard systems, and the further factor due to the scale of the projection should also be applied. Where a local survey is conducted in relatively flat areas it will be found practicable to combine these two factors into a single constant ratio. It is easy to apply this proportional correction to all measured distances. Then, except for the probably insignificant changes of these factors themselves, within the limited area, all dependent or unmeasured lengths computed within

the local survey will also be correct upon the projection scale. Thus the local survey may be adjusted in overall size so that it will fit accurately and consistently into its allotted place on the plane of the projection.

The feature that may bother the engineer most is the necessity of including in his survey at least one station (but preferably two or more) of the fundamental control net of the United States. One such station, having rectangular coordinates correctly referred to the standard system, provides starting values of X and Y to introduce into the local survey. If other basic stations can be included they will provide the means to check and adjust the local survey more safely and rigidly into place. When tying to only one basic station, it may be difficult to obtain an initial direction which will orient the local survey harmoniously with the standard coordinate system, and, also, no check is then available. However, many of the more recently established triangulation stations of the first-order net have been provided with auxiliary azimuth marks, and at such stations a reference direction in terms of grid azimuth may be obtained.[8] Lacking such an azimuth mark, an approximate initial direction may be obtained by reducing to grid azimuth the astronomic azimuth obtained from an observation on Polaris. The reduction is not difficult, as it is simply the application of the "mapping angle" in the Lambert projection, or the ordinary convergence from the central meridian in the transverse Mercator projection.

If two or more basic stations are included in the survey, the grid azimuths between them provide the best initial directions, and these are easily computed by the ordinary formulas of analytic geometry from the rectangular coordinates of the stations.

The preparation confronting an engineer before he can use a State system as the reference datum for his survey now has been outlined in its four parts. These relate to shape, scale, place, and orientation, all four being adjustments of his survey to fit others upon the plane of the projection After attending to these preliminary arrangements, *any ordinary survey may be conducted in both field and office by the usual methods of simple plane surveying, and it will be based at the same time upon standard datum*

Transformations between geodetic coordinates and plane coordinates on the State systems are made much more easily with a 10-bank calculating machine than with logarithms. Because large numbers are used, the most precise work requires logarithms of numbers, sines, and cosines to not less than eight decimal places, or natural functions to eight or ten places. Seven-place tables are sufficient for many practical transformations in ordinary surveying, however, if there is no need to obtain mathematically or legally perfect agreement between hundredths of feet in the rectangular coordinates and ten-thousandths of seconds in the corresponding geodetic coordinates.

The basic manual already mentioned,[2] issued by the United States Coast and Geodetic Survey, describes the rigorous computation using logarithms. A shorter and easier method for use with a calculating machine, which is rigorous with the Lambert projection but permits the use of certain very close approximations in the transverse Mercator formulas, is given in a pamphlet published by the United States Geological Survey,[9] which is a practical and convenient guide to the computer.

[8] Adams, Oscar S , Azimuths from Plane Coordinates· U S. Department of Commerce, Coast and Geodetic Survey, *Serial No. 584*, 1936.

[9] Speert, J. L., Formulas and Tables for the Transformation of Geodetic to Plane Coordinates on the Lambert and Transverse Mercator Projections. U. S. Department of the Interior, Geological Survey, second edition, 1943.

Elements of the individual projections are required in computing transformations or scale changes in distances. They are tabulated and issued in lithographed pamphlets, separately for the several States, and may be obtained from the Coast and Geodetic Survey. The projections adopted for the different States are indicated in the following table.

LIST OF STATES WITH THEIR RESPECTIVE GRIDS

Lambert System

1. Arkansas
2. California
3. Colorado
4. Connecticut
5. Iowa
6. Kansas
7. Kentucky
8. Louisiana
9. Maryland
10. Massachusetts
11. Minnesota
12. Montana
13. Nebraska
14. North Carolina
15. North Dakota
16. Ohio
17. Oklahoma
18. Oregon
19. Pennsylvania
20. South Carolina
21. South Dakota
22. Tennessee
23. Texas
24. Utah
25. Virginia
26. Washington
27. West Virginia
28. Wisconsin
29. Long Island
30. Nantucket and Marthas Vineyard

Transverse Mercator System

1. Alabama
2. Arizona
3. Delaware
4. Georgia
5. Idaho
6. Illinois
7. Indiana
8. Maine
9. Michigan
10. Mississippi
11. Missouri
12. Nevada
13. New Hampshire
14. New Jersey
15. New Mexico
16. New York
17. Rhode Island
18. Vermont
19. Wyoming

Both Systems

1. Florida

A very desirable coordination of information is achieved when the results of all surveys—private, State, and Federal—are expressed in common terms. Small surveys then expand into harmonious contact with others adjacent to them. The results of one survey made for some specific purpose are supplemented perhaps by information already available from preceding surveys made for other purposes. Successive surveys of the same area do not need to repeat all of the previous work, or possibly a new survey may be wholly unnecessary because of the accumulation of information already available for that area, referred in an orderly manner to a datum common to all surveys.

CHAPTER II

PHOTOGRAMMETRIC OPTICS

ELEMENTS OF PHOTOGRAMMETRIC OPTICS

Reynold E Ask

ABSTRACT

In practically every step of map compilation by photogrammetric methods, instruments are used in which optical elements play a very necessary part Among these are aerial cameras, ratio printers, stereoscopes, projectors, transits, plotting apparatus, etc Although many books on geometrical optics have been published, it is still difficult for the photogrammetrist to find a single book which presents those phases of the subject which are of chief interest to him. It is the purpose of this chapter to partially fill this need Necessarily in a single chapter it is impossible to go into each of the many subjects in detail Numerous references are given, however, to aid those who may be further interested in the subject Throughout the chapter mention will be made of the practical applications of the various principles presented.

Contents

I. Introduction to Lenses

THE study of optics is usually subdivided into two main fields, physical and geometrical. Physical optics concerns such subjects as wave motion, diffraction, interference, and polarization, and need not be considered in this discussion, although certain aspects of it are important. Practically all of this chapter concerns geometrical optics Following are the postulates upon which the subject of geometrical optics is based. (1) Light travels in a straight line in a homogeneous medium; (2) when a ray is reflected from a surface, the angle of reflection equals the angle of incidence (the angles are measured from a normal to the surface); (3) when a ray is refracted (on passing from one medium to another medium of different density), the sine of the angle of incidence (i) divided by the sine of the angle of refraction (r) equals a constant called the "index of refraction" (N)

$$\frac{\sin i}{\sin r} = N \qquad \text{eq. (1)}$$

It is this property of refraction which makes lens action possible

A lens may be defined as an optical medium bounded by spherical surfaces (a plane surface being considered a spherical surface of infinite radius). On entering a lens, light is refracted in such a manner as to form either a real or a virtual image depending on whether a converging (positive) or a diverging (negative) lens is used respectively (see Fig. 1).

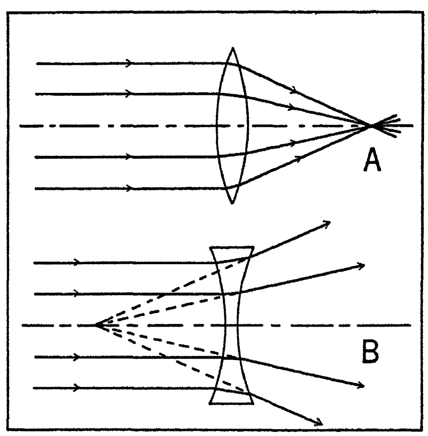


FIG 1. Lenses. (A) Converging or positive (B) Diverging or negative

1. Thin Lenses

The term "thin lens" is used in approximate computations to indicate that the thickness of a lens is ignored, all distances being measured from the lens center The relationship between the object distance (O), the image distance (I), and the focal length (F) is given by the basic lens equation.

$$\frac{1}{I} = \frac{1}{O} + \frac{1}{F} \qquad \text{eq. (2)}$$

In the above equation the following sign convention is used: All distances (except focal length) measured from the lens in the direction of the incident light are considered positive; all distances measured against the direction of the incident light are considered negative; the focal length of a converging lens is considered positive, the focal of a diverging lens is considered negative The distances (O) and (I) are also called conjugate distances. The magnification (M) is defined as a ratio of image to object distance and a minus sign indicates that the image is inverted.

$$M = \frac{I}{O} \qquad \text{eq. (3)}$$

The following examples illustrate the manner in which equations (2) and (3) are used

Example (1): Given an object distance of 25 inches and a focal length of +10″. Compute the image distance and magnification (see Fig 2)

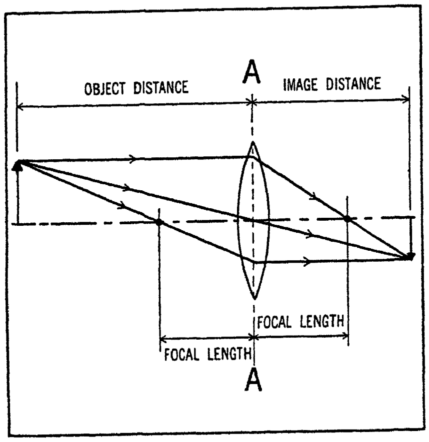


FIG. 2. Image formation positive lens.

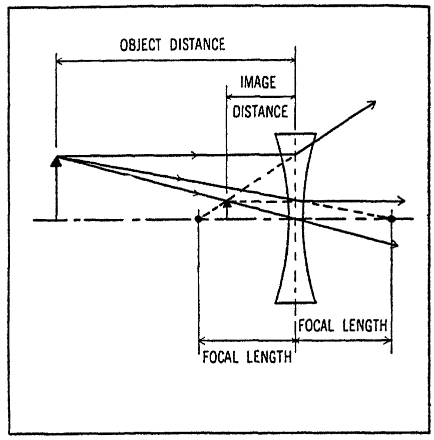


FIG 3. Image formation negative lens

$$\frac{1}{I} = -\frac{1}{25} + \frac{1}{10}$$

$$= -0\ 040 + 0\ 100 = +0\ 060$$

$$I = +16\ 67''$$

$$M = \frac{I}{O} = \frac{+16\ 67}{-25} = -0\ 667$$

Thus it has been found that a *real inverted* image was formed which is *smaller* than the object Note the simple graphical construction used in the figure to obtain a good approximation of the image position.

Example (2)· Same conditions as in example (1) except F is $-10''$ (see Fig. 3).

$$\frac{1}{I} = -\frac{1}{25} - \frac{1}{10}$$

$$= -0\ 040 - 0.100 = -0.140$$

$$I = -7.14''$$

$$M = \frac{I}{O} = \frac{-7\ 14}{-25} = +0.286$$

Thus in this case we have a *virtual erect* image *smaller* than the object.

If the object distance is infinitely great, it can readily be seen from equation (2) that the image distance equals the focal length. This is essentially the relationship in an aerial camera (see Fig. 1A) where the object distance (flying height) is very large compared to the focal length.

When two thin lenses having focal lengths (f_1 and f_2) are placed in contact, the approximate focal length (F) of the combination is given by the equation

$$F = \frac{f_1 f_2}{f_1 + f_2} \qquad \text{eq. (4)}$$

When working with photographic lenses (or other positive lenses forming a real image) the following equations, in which the magnification (M) is considered, are usually found to be more convenient than equation (2) because no reciprocals are used:

$$\text{Object distance } (O) = F + \frac{F}{M} \qquad \text{eq. (5)}$$

$$\text{Image distance } (I) = F + FM \qquad \text{eq. (6)}$$

Example (3): The focal length of the lens in a ratio printer is 240 mm. Compute the object and image distances for a magnification of 4.

$$O = 240 + \frac{240}{4} = 300 \text{ mm}$$

$$I = 240 + 240 \times 4 = 1200 \text{ mm}$$

$$\text{Check:} \qquad \frac{1200}{300} = 4$$

In some ratio printers the object the image distances are linked to a cam mechanism which causes these distances to vary as in equations (5) and (6). It is therefore possible to obtain sharp focus at various magnifications or scale ratios merely by turning one handwheel The term "auto-focus" is given to such an arrangement.

In spectacle optics the dioptry system is used to express the power of a lens. The power in diopters (Δ) being equal to the reciprocal of the focal length (F) in meters:

$$\Delta = \frac{1}{F \text{ (meters)}} \qquad \text{eq. (7)}$$

Thus a 5 diopter lens has a focal length of $\frac{1}{5}$ meter or 20 cm. The advantage of this system is that the power of a series of lenses in contact equals the algebraic sum of the individual powers, thus: $+5\Delta - 1\Delta = +4\Delta$, giving a focal length for the combination of $\frac{1}{4}$ meter or 25 cm. If focal lengths values had been given instead of diopters it would be necessary to use equation (4) which requires a little more computation Due to their cheapness, spectacle lenses are often used in photogrammetry where great accuracy is not essential, such as in simple lens stereoscopes, magnifiers, and for experimental work. All opticians carry a complete assortment of spectacle lenses. They also are equipped to edge these lenses to any specified diameter.

2. Thick Lenses

The term "thick lens" is used in accurate computations to indicate that the thickness of a lens is considered, the object and image distances being measured from the nodal points N and N' instead of lens center (see Fig. 4). These distances are computed from the same equations as for thin lenses. The nodal points of most photographic lenses (except the telephoto type—discussed later) lie rather close to the lens center and their location is determined from optical bench measurements. It should also be noted that the nodal points

occasionally overlap, that is, they are reversed in position as shown by the dotted lines in the upper part of the figure. The letter (S) will be used in this discussion to indicate the nodal point separation. Referring again to the figure, it is obvious that the total distance from object to image is the sum of· object distance (O), nodal point separation (S), and image distance (I). The graphical ray construction used for a thin lens (Fig 2) can be modified so that it also will apply to a thick lens To do this we split the figure along line A–A and then separate the two parts a distance equal to the nodal point separation.

In a thick lens it is necessary to distinguish between the following types of focal length (see Fig 4): The equivalent focal length being the distance from the rear node to image plane; the back focal length being the distance from rear vertex of the lens to image plane, and the front focal length being the distance from the front vertex to image plane, all distances being measured parallel to optical axis and the object position in all cases being at infinity. For any lens or lens system it is possible to determine by computation the location of the nodal points and values for the various focal lengths. References are given at the end of the chapter to aid those who may be interested in such computations.[1]*

In addition to the above focal lengths, the following two related terms are also used in photogrammetry: (1) *Calibrated focal length* is an adjusted value of the equivalent focal length so selected as to minimize the effect of lens distortion over the entire field. The calibrated focal length is used when determining the

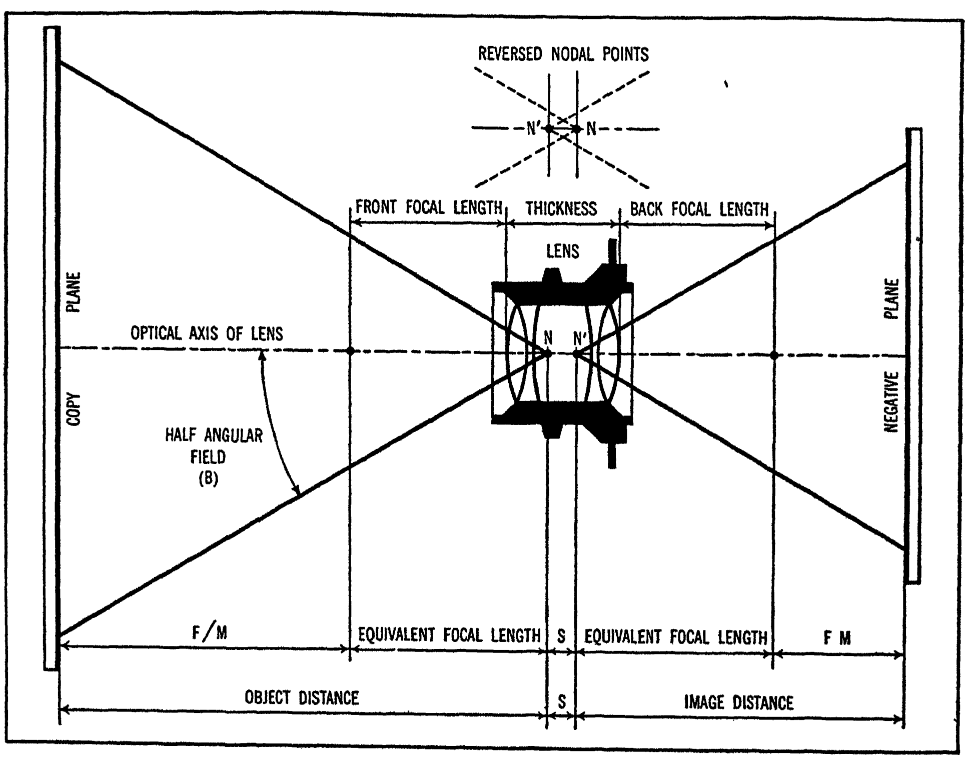


FIG. 4. Important optical dimensions of a photographic objective. The diagram is arranged as if the lens were used in a typical copying camera.

* See References at end of chapter, page 74.

setting of diapositives in certain plotting instruments and in photogrammetric computations based on measurements on the negative (such as those made with a precision comparator); (2) *Principal distance* is a geometrical property of a negative or print and equals the calibrated focal length corrected for both the enlargement or reduction ratio and the film or paper shrinkage. It re-establishes the same angular cone of rays as existed at the taking camera at the moment of exposure.

3 Lens Aberrations

An aberration is a defect in an optical image caused by the fact that essentially no lens system can form a perfect image A brief description of the aberrations considered in lens design follows.

Spherical aberration (see Fig. 5): Rays from various zones of a lens focus at different places along the axis, this results in an object point being imaged as a blur circle It is caused by the spherical shape of the lens surfaces. Its effect decreases as the lens aperture is reduced.

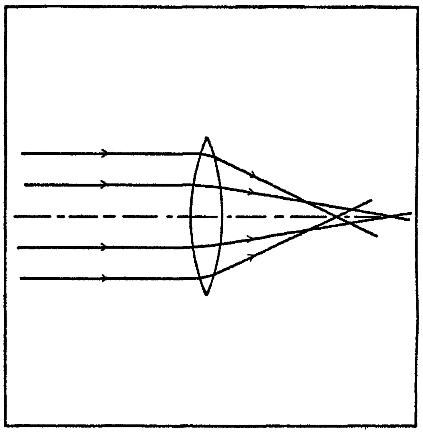


Fig. 5. Spherical aberration.

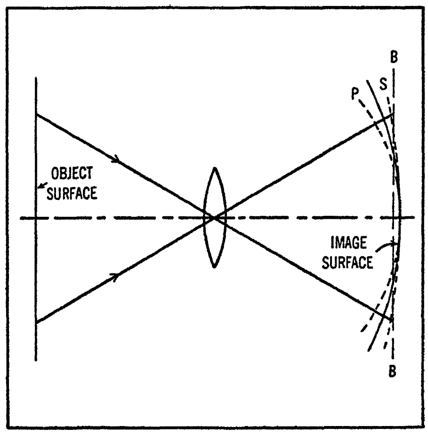


Fig. 6. Astigmatism (dotted curves) and curvature of field (solid curve).

Coma: A comet-shaped blur of light formed around image points off the axis. It is partly due to spherical aberration of oblique rays.

Astigmatism and curvature of field: Astigmatism is an aberration which causes a point object off the axis to be imaged as two mutually perpendicular short lines located at different distances from the lens One of these lines is radial and the other tangential with respect to the center of the field In Figure 6 the tangential image surface (also called primary) is marked (P) and the radial image surface (also called secondary) marked (S). The surface of best definition is located midway between these two (solid curve) and its departure from flatness is termed "curvature of field." A positive value indicates that the field is concave towards the lens. If a lens exhibiting curvature of field is used in a camera, it is obvious that the film cannot be dished to assume the surface of best definition However, the film plane can be placed in such a position (dashed line B–B in Fig. 6) as to obtain the best average definition over the entire field. In an aerial camera the distance from the rear node of lens to plane of film corresponds to the focal length obtained by the above method.

Longitudinal chromatic aberration (see Fig. 7). Rays of various wave lengths or colors focus at different distances from the lens (the distances being measured parallel to optical axis). This defect can be practically eliminated in a lens consisting of two or more elements by using glasses of different dispersive powers. The term "achromatic" is applied to such a lens.

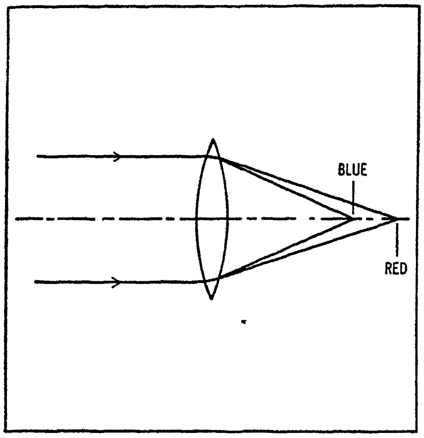


FIG. 7. Longitudinal chromatic aberration.

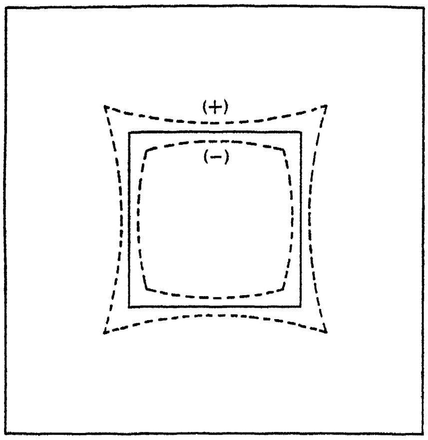


FIG. 8. Effect of distortion on the image of a square object.

Lateral chromatic aberration: A difference in image magnification for various colors caused by chromatic aberration of oblique rays Due to this defect, images near the outer portion of the field will be fringed with color. In a visual instrument (such as a telescope) the colors are readily apparent to the eye.

Distortion (see Fig. 8). A displacement of an image point radially to or from the center of the field. A positive distortion is considered away from the center It also may be expressed as a variation in magnification in different parts of the field.

A lens designer is interested in each of the above defects individually. The lens user, however, is mainly interested in distortion and "definition " The latter term is used to express the ability of a lens to record fine detail and really represents the combined effects of all the above mentioned errors except distortion. Definition may be expressed mathematically as "resolving power" which refers to the maximum number of lines per millimeter that can just be resolved (that is, seen as separate lines) in the image. Theoretically the resolving power increases with lens aperture, but practically this is only true when a very small angular field is used (such as in a telescope). As far as photographic lenses are concerned the resolving power is limited by residual aberrations which in most cases become smaller as lens aperture is reduced. However, a reduction in aperture smaller than $f/22$ may reduce the resolving power due to diffraction effects.

II. Photographic Lenses

1. Design

In Figs. 9 to 12 are examples of several aerial lenses. No attempt will be made to discuss the detailed design[2] of such lenses, although a few remarks are necessary. Photographic lenses are designed for specific types of work. A lens designed for copying maps, for instance, would not be satisfactory for use in a

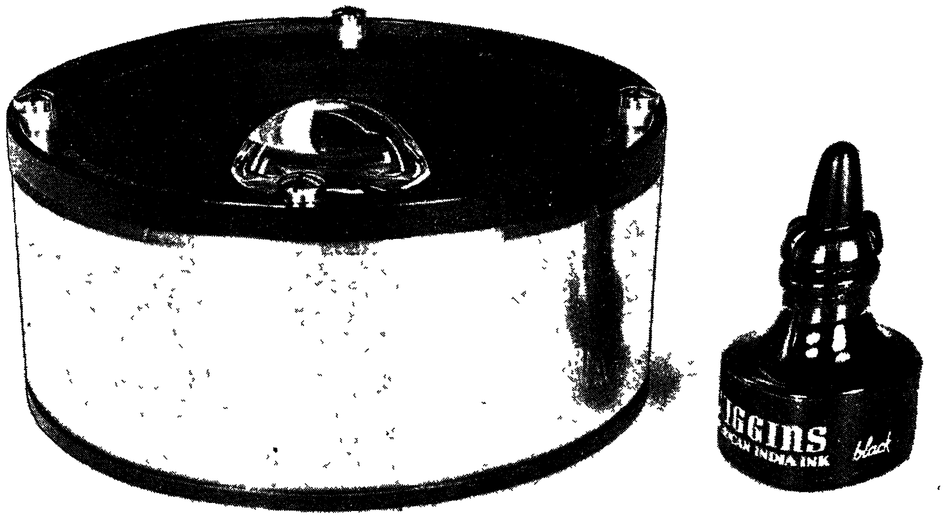


FIG. 9. Bausch & Lomb wide-angle Metrogon lens mounted in special test barrel (focal length 6 in.)

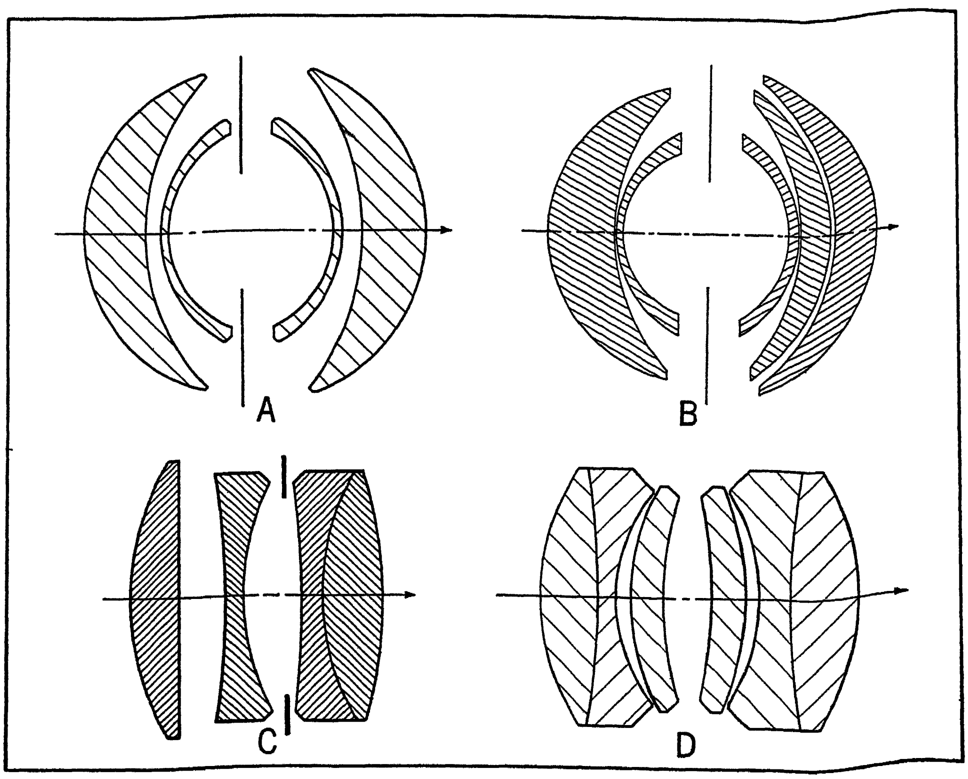


FIG. 10. Sections through typical aerial camera lenses. (A) B & L wide-angle Metrogon—$f/6.3$ aperture and 93° field. (B) Ross wide-angle—$f/5.5$ aperture and 95° field. (C) B & L Aero Tessar—$f/6.0$ aperture and 45° field. (D) B & L Altimar—$f/4.0$ aperture and 65° field. (All diagrams except "B" courtesy Bausch & Lomb Optical Co.)

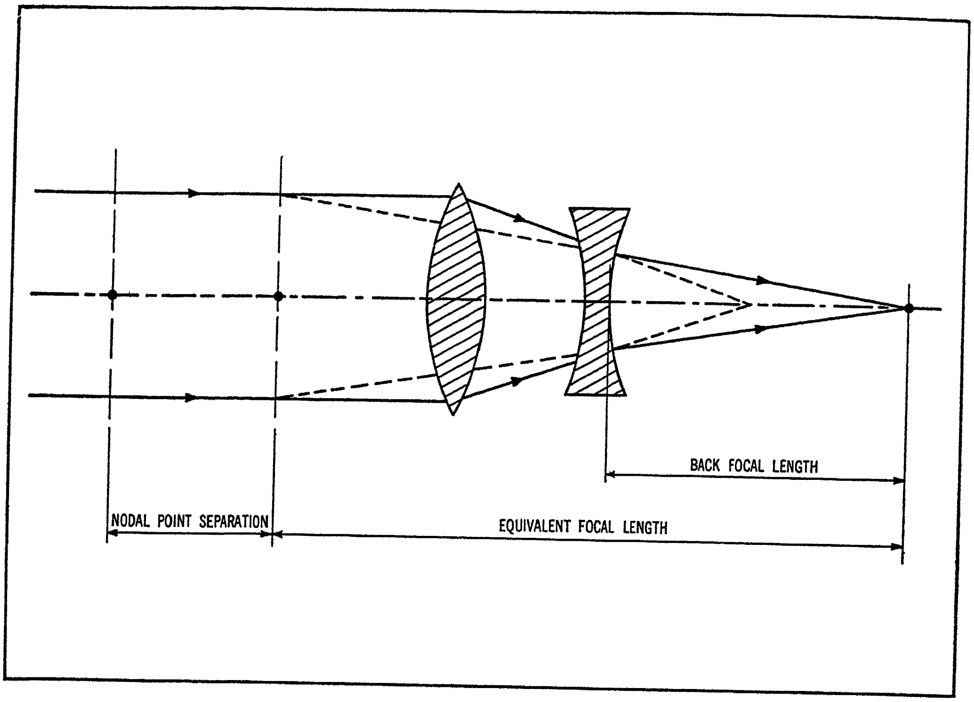


FIG. 11. Schematic diagram of a telephoto lens.

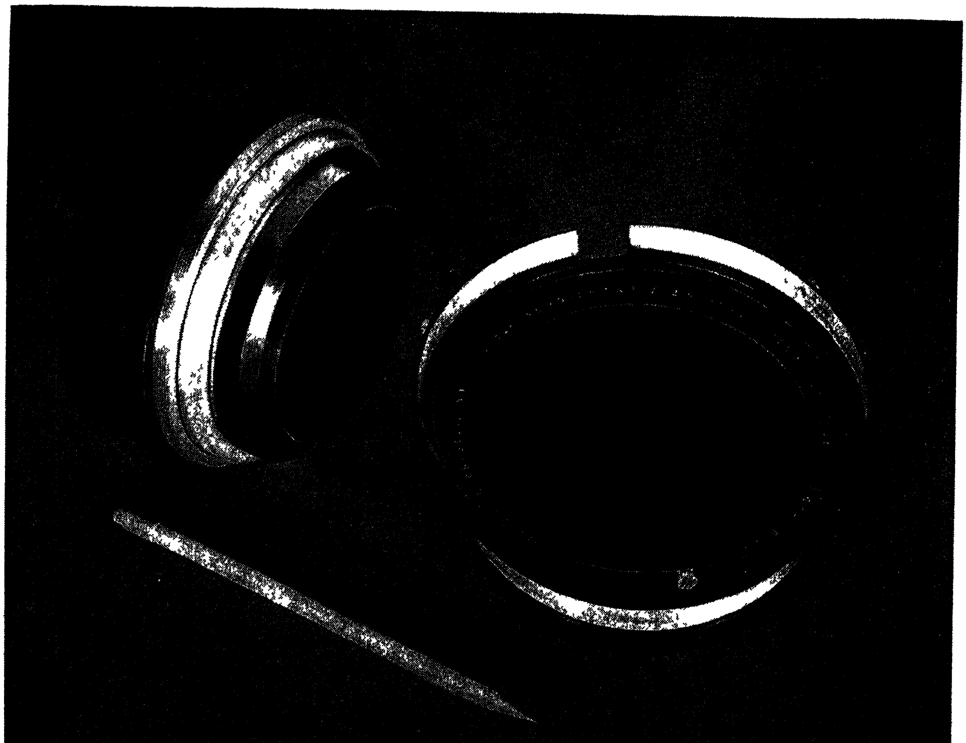


Fig. 12. Front and rear cells of Kodak Aero-Ektar lens (12″—$f/2.5$). Used for night flash bomb photography. Courtesy Eastman Kodak Co.

precise aerial mapping camera In the former case the lens is designed to give good definition and low distortion values for finite object distances (say for scale ratios of 0.25 to 2.00) while in the latter case the lens is designed for an infinite object distance only.

In attempting to obtain a "near perfect" lens, designers have tried hundreds of different combinations. The variables at the disposal of the designer being: the index of refraction of the glass (1 50 to 1 72), the dispersive power of the glass, the curvature of the surfaces, and the number, thickness, and spacing of the elements. The labor involved in designing a new lens may mean months of trigonometric ray tracing Since it is impossible to design a perfect lens, the best that can be done is to reduce those errors which would impair the work which the lens is intended to perform; in other words a balance is sought between the most desired properties Examples of this balancing are found in the wide-angle aerial camera lenses illustrated, where low distortion, good definition, and a large angular field are secured at the expense of maximum aperture

Future developments will probably bring an increase in the angular field of aerial lenses An interesting wide-angle lens is the Goerz Hypergon*. Although this lens covers a field of 135°, its slow speed (*f*/22) and poor marginal definition make it unsuitable for aerial photography Another defect of all wide-angle lenses is the falling-off in marginal illumination In the Hypergon lens a partial illumination compensation is obtained by placing a rotating star-disk in front of lens during exposure, another method makes use of a graduated density filter With the development of "super-speed" films, it is possible that the extreme wide-angle lens of small aperture may some day be used in aerial photogrammetry. Other promising wide-angle aerial lenses are those described in U S. Patents 2,037,017 and 2,126,126.

Although the wide-angle lens is the preferred type to use for mapping work, certain types of aerial photography require lenses of different design. For instance to obtain maximum haze penetration in oblique photography, infra-red film and filters may be used To secure optimum definition from such a combination it is necessary that the lens be corrected for infra-red light. For military reconnaissance on the other hand, lenses of long focal lengths (up to 50 inches) are necessary to obtain a desirable image size from a great altitude. Such lenses require the use of long cumbersome camera bodies. One method of avoiding this inconvenience is to use a telephoto lens (see Fig. 11). This lens consists of a positive front element and a negative rear element, the separation of the two being rather large. The result of such a design is to place the rear nodal point of the lens system far out in front of the positive element, thereby obtaining a large equivalent focal length (say 40 inches) with a much smaller overall camera length (say 24 inches) Another special lens for military reconnaissance is the large aperture type used in night photography with flash bombs (see Fig. 12).

2. MANUFACTURE[4]

The glass used in photographic objectives, as in other optical elements, is specially made for that purpose It differs from ordinary plate glass both in composition and in the manner in which it is made. The composition of the various types of optical glass differ greatly, but the basic ingredient is usually silica in the form of a fine white sand. Recently a new type of optical glass has been developed in which compounds of the "rare elements" (lanthanum, tantalum, and tungsten) have replaced the silica. This new glass has a higher refractive index for a given dispersive power than was previously obtainable. All

* The Hypergon lens was first developed by Goerz in 1900, it is now manufactured by Zeiss.

the materials that go into a batch of optical glass must be chemically pure, and must be weighed and mixed under rigid scientific control The mixture is fed into a large clay pot (3 to 5 feet in diameter) which was preheated in a gas furnace. The batch is stirred while the mixture is introduced and also for a considerable period afterward so as to insure uniformity of composition. The pot is then removed from the furnace, covered with an insulating material, and allowed to cool slowly This is termed "annealing" and usually requires one week. During this cooling period the glass cracks up into irregular chunks which are later inspected for such defects as bubbles, undesirable color, stones, striae and strain. Fig. 13 shows various stages in the making of a lens.

The next step is to place these irregular chunks in a gas furnace, heating until plastic, and then pressing into a mold of the desired size and shape. The resulting piece of optical glass is now termed a lens blank. These blanks are now fine annealed for about 30 days to remove internal strains.

The lens blank is then ready to be ground. In this operation the blank is cemented to a grinding block and ground with a cast iron tool of the proper curvature, water and emery being used as the abrasive. Finer and finer emery is introduced as the surface approaches the desired shape. When the grinding has been completed the emery is thoroughly washed off, and the grinding tool replaced with a pitch-lined polishing tool. The polishing is accomplished by rouge and water During this operation the surface is checked periodically by placing a test glass of the opposite curvature on the lens and noting the resulting pattern of interference fringes This fringe pattern is really a contour map of the air film between the two glasses, the contour interval being one-half a wave length of light (0.00001″). When the polishing of one surface is satisfactory, the lens is turned over and the other side completed.

The next operation is centering, which consists of grinding the edge of the lens element concentric with the optical axis. Small errors in centering of successive elements will cause unsymmetrical distortion in the completed photographic lens. Avoidance of such errors is important in precision photogrammetry.

Most photographic lenses consist of 4 to 8 lens elements, some of which are cemented together. The cementing operation consists in gradually heating the elements on a hot plate, placing a drop of Canada balsam at the center of the desired surface, and pressing the elements firmly together until all air bubbles and excess balsam are squeezed out. The lens elements are now ready to be mounted in their respective metal cells. These cells have been machined very accurately so as to keep the lenses at the designed separation, and also hold them firmly but without strain. The front and rear cells are now mounted in the lens barrel which usually contains the iris diaphragm and a between-the-lens shutter.

3. Testing

Before a given photographic lens is mounted in a camera, it is subjected to a series of suitable tests. These fall into two categories: optical laboratory and camera tests.

In the laboratory an optical bench is used to obtain quantitative measurements of back, front and equivalent focal lengths, nodal point separation, distortion, and definition or resolving power. The optical bench used in these tests may be of the visual or photographic type. In the visual type,[5] the accuracy of the test depends upon the visual ability of the observer to interpret images of suitable test targets This type is chiefly used for measuring the focal lengths on the axis, nodal point separation, and distortion for various scale ratios. It is

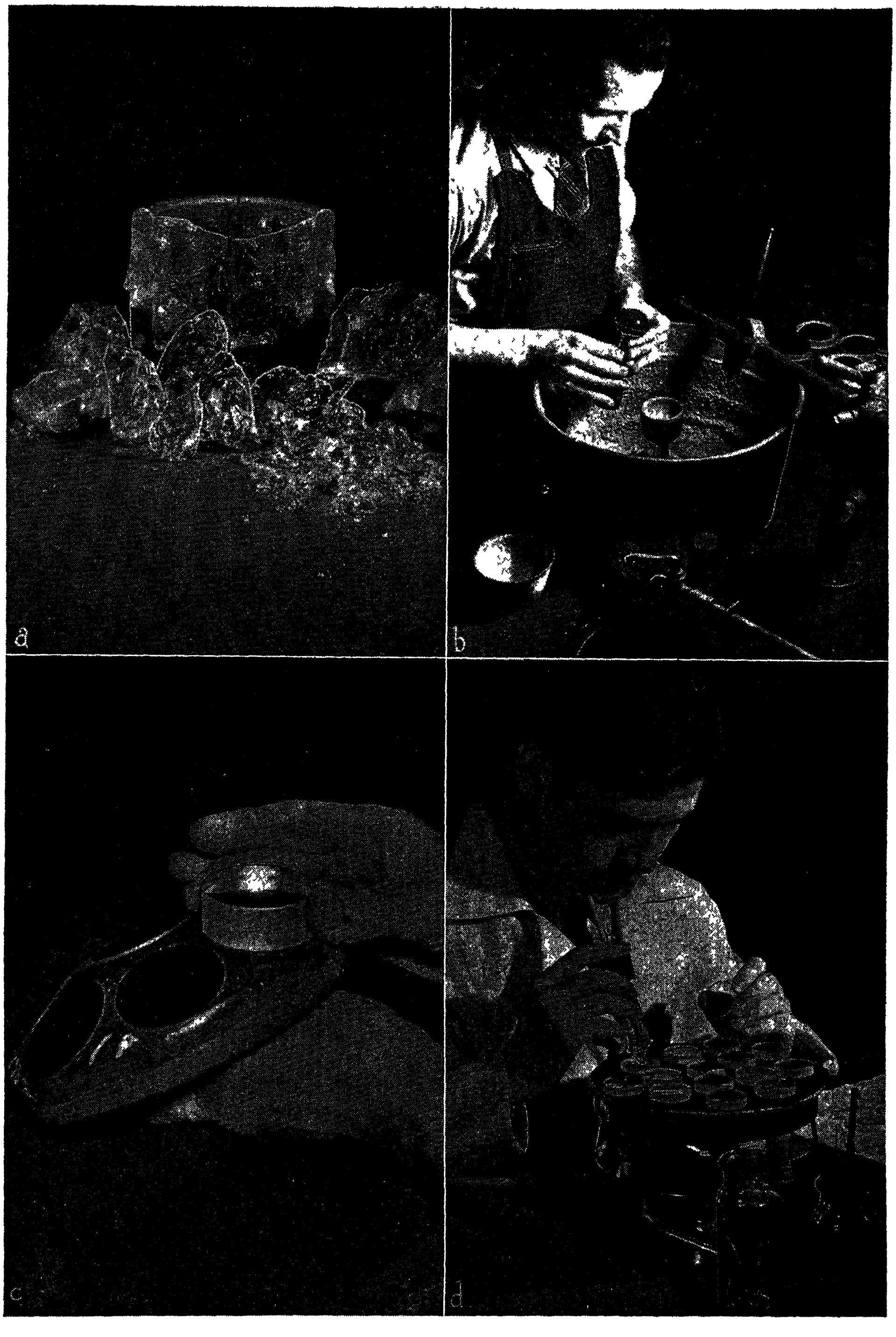


FIG. 13. The manufacture of photographic lenses. (a) Broken pot of optical glass. (b) Fine grinding and polishing operation. (c) Using an optical test glass to check the surface quality. Limits of accuracy of 0.0000058″ are common. (d) Cementing of the lens elements. (Above photographs courtesy Bausch & Lomb Optical Co.)

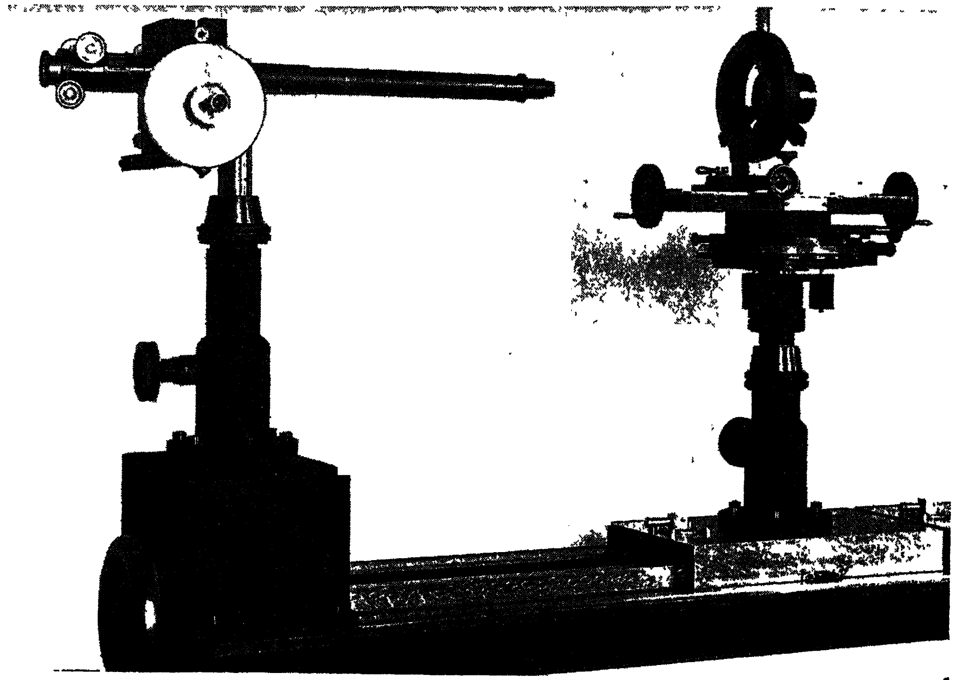


FIG. 14. Lens testing bench—A target is placed in focal plane of a collimator (not shown but located beyond right edge). The lens, mounted in nodal slide, forms an image of this target which is examined by microscope.

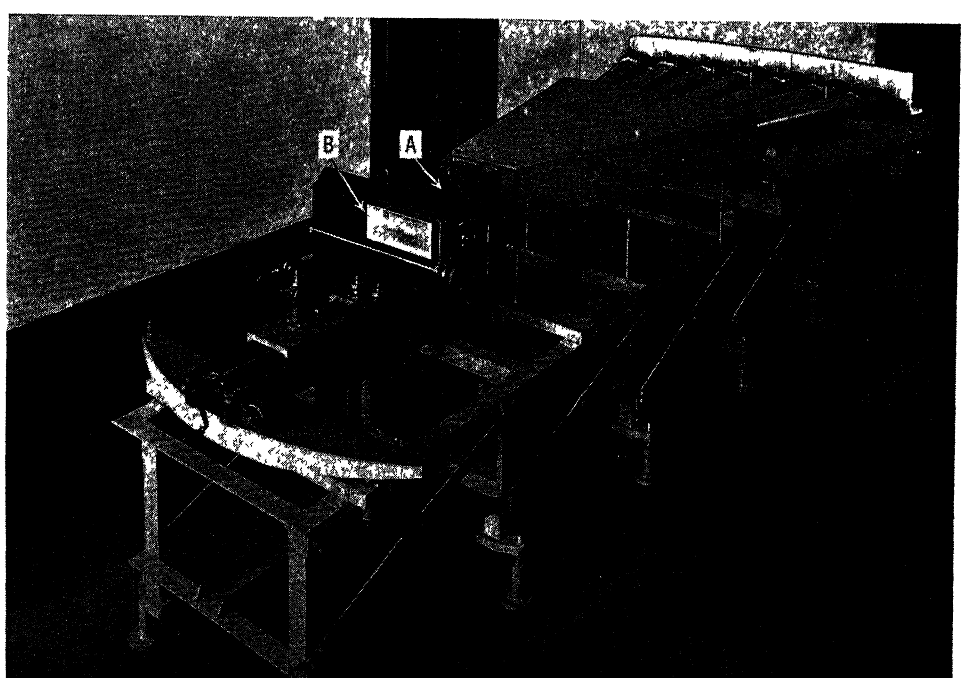


FIG. 15. Precision lens testing camera—The collimators containing the test charts are at the right, the lens to be tested at (A), and the photographic plate at (B) (above photographs courtesy National Bureau of Standards).

very difficult in the visual type of bench (see Fig. 14) to determine the value of the focal length which will give the best average definition over the entire field.

Aerial camera lenses are usually tested in the photographic type of testing equipment (see Fig 15) [6] A photographic plate records the images of a series of test charts arranged to cover the lens field in five degree steps. A chart similar to those used is shown in Figure 16, the numbers opposite each group of lines indicating the number of lines per millimeter that will be imaged on the test plate by a lens of given focal length Each chart is placed in the focal plane of a collimator (see page 65) for the purpose of obtaining an infinite object distance. A series of exposures is then made with slightly different lens-plate distances. The resulting photographic plate is then studied with a microscope to determine what value of the focal length will give the best average definition (or resolving power) over the entire field. In Figure 17 the horizontal line through "0" represents the row of images exposed for best axial definition. However, an examination of test plate may indicate that the best average definition over the entire field is obtained by shortening the axial focal length (say by 0 2 mm.) The resolving power obtained by the above test may be higher than that obtained under service conditions. The reason being that the emulsion of the test plate is finer grained than that of the panchromatic film used in the aerial camera. Also measurements can be made on the plate to determine the distortion for an infinite object distance

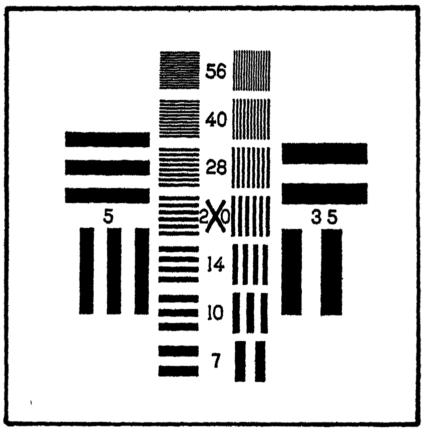


FIG. 16. Chart for testing lens resolving power

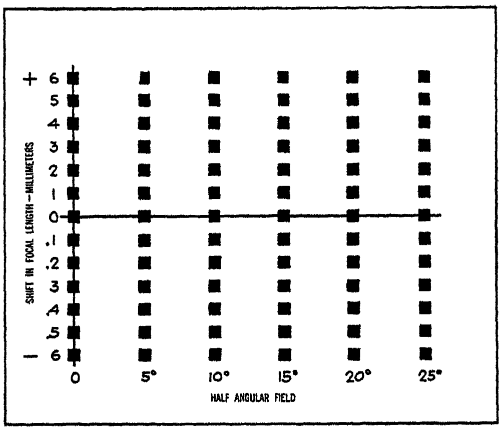


FIG 17. Schematic diagram of plate obtained with lens testing camera.*

It might be desirable to mention that only a few organizations have available precise optical bench equipment for testing photographic lenses. Among these are large optical manufacturers, the National Bureau of Standards, and a few universities. Lens manufacturers usually conduct tests in their own laboratories and the prospective purchaser can often obtain the results of such tests. In submitting a lens to a testing laboratory it is necessary to specify at what scale ratios and apertures the tests are to be conducted. For instance a lens used in a copying camera or ratio printer should be tested for the scale ratios at which it is to be used, while an aerial camera lens is tested for an infinite object.

A typical test report issued by the National Bureau of Standards on a wide angle aerial camera lens is given below:

* Each small black square on this plate contains the complete chart shown in Fig. 16.

Lens	FOCAL LENGTHS	
	Back focal length	Equivalent focal length
	mm.	mm
3244465	119 91	152 18

The values of the focal lengths have been selected to give best average definition across the entire negative and do not necessarily correspond to those values of focal lengths which give best definition on the axis The probable errors of these determinations of focal length are approximately ±0 10 mm

Lens	DISTORTION								
	5°	10°	15°	20°	25°	30°	35°	40°	45°
3244465	0 00	0 00	+0 02	+0 04	+0 08	+0 11	+0 12	+0 07	−0 15

The values of the distortion are measured in millimeters and indicate the displacement of the image from its distortion-free position A positive value indicates a displacement from the center of the plate. The probable error is approximately ±0 02 mm

Lens	RESOLVING POWER									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
3244465										
Tangential	55	55	55	39	28	28	28	28	28	14
Radial	55	55	55	55	39	39	39	39	39	28

The values of the resolving power are given at 5° intervals from the center of the field and are obtained by photographing suitable test charts comprised of patterns of parallel lines The series of patterns of the test chart are imaged on the negative with the lines spaced 5, 7, 10, 14, 19, 28, 39, 55, and 77 lines to the millimeter. The row marked "tangential" gives the number of lines per millimeter in the image on the negative of the finest pattern of the test chart that is distinctly resolved into separate lines when the lines lie perpendicular to the radius drawn from the center of the field. The row marked "radial" gives similar values for the pattern of test lines lying parallel to the radius.

All measurements were made with parallel light incident on the lens The effective wave length was approximately 575 millimicrons.

This is a Bausch and Lomb Metrogon lens, nominal focal length 152 4 mm , maximum aperture $f/6$ 3 It was tested at maximum aperture During the test, the lens was mounted in a special test barrel submitted with the lens

Washington, D C , August 26, 1942

The above results show that the distortion for this particular lens is very small. Also the resolving power is considered good for a lens covering such a wide field

Sometimes a lens is tested after it is mounted in an aerial camera. Such a test may be performed on the photographic testing equipment previously described. The results of such a test would be a determination of distortion and definition for the focal length actually set in the camera at the time the test was made. The values of the distortion and focal length thus obtained being used in the computation of the "calibrated focal length" previously mentioned on page 42. Also the position of the four fiducial marks may be checked to see if lines connecting opposite marks intersect on the optical axis of the lens, thus

locating the principal point of the photograph [7] Another method of obtaining an indication of definition would be by a flight check, in which exposures are made over terrain containing intricate detail

4 Image Brightness

In a camera the amount of light that the sensitized surface receives from an object of given light intensity depends upon several factors, such as · the relative aperture of the lens, the magnification or scale ratio, the angular field, and the light loss in the lens.

The relative aperture of a lens (also called speed or f-number) is defined as the ratio of equivalent focal length to aperture, thus a lens of 8 inch focal length and 1 inch aperture would have a relative aperture of $f/8$. The more common f-numbers engraved on lenses are as follows. 1—1.4—2—2 8—4—5 6—8—11—16—22—32—45—64 This series is usually followed in American made lenses except in the case of maximum aperture which may fall somewhere between the given values (such as $f/3.5$). Assuming a constant shutter speed, the exposure varies inversely as the square of the f-number and the above series is so selected that the exposure is approximately doubled as you go from one number to the next larger, thus the exposure at $f/11$ would be twice that of $f/8$ ($121/64=2$ approx)

In ratio printing or copying work the "effective" aperture is the ratio of image distance to aperture. It can be found by simply multiplying the f/number engraved on lens by the quantity $(M+1)$ where (M) is the magnification or scale ratio. If you assume the lens stop to be constant, the exposure will vary as $(M+1)^2$. Example: When $M=2$ the exposure is 10 seconds; what will it be when $M=4$? $(4+1)^2=25$, $(2+1)^2=9$, $25/9\times 10=28$ seconds. If an exposure meter is used in copying work it is necessary to multiply the exposure as indicated on the meter by the quantity $(M+1)^2$ to obtain the correct exposure. This must be done because the meter is calibrated for an infinite object distance.

A subject related to lens aperture is depth of definition Space does not permit a complete discussion of this subject, but references are given to aid those who may be interested in the mathematical relationships [8] We have seen from equation (2) that for a given object distance we obtain a corresponding image distance at which points are imaged with optimum sharpness At a greater or lesser distance, points will be imaged as blurred circles, becoming larger the further we depart from the optimum position In a particular instrument, the maximum circle diameter which will yield satisfactory definition is determined by experimentation. With this diameter fixed, it is possible to compute the usable depth of definition for any given lens aperture. It also can be mathematically proved that this depth increases as the lens aperture is reduced, other conditions being constant A practical use of the above relationships is found in the multiplex projector. In one model of this equipment the object distance (lens to diapositive) is 30 mm. With a lens of 27.7 mm. focal length, this gives an optimum image distance of 360 mm. By selecting a proper lens aperture (about $f/22$) it is possible to obtain an image depth sufficient to take care of all the relief usually encountered in the stereoscopic model (see page 67—Fig. 31). Another practical use of depth of definition is found in that type of ratio printer equipped with a tilting easel. Such equipment is used for approximate rectification of prints for mosaic work (see page 55—Fig. 19).

The illumination on the negative in a camera is strongest at the center of the field. Except for the vignetting effect of the lens mount, the illumination varies as $\cos^4 A$ where (A) is half the angular field.[9] This falling off in illumina-

tion is rather serious in wide angle aerial cameras If the half angular field of the camera is 45° the illumination at the edge of the negative will only be 25 per cent of what it is at the center ($\cos^4 45° = 0\ 71^4 = 0\ 25$) Subtracting the light loss due to vignetting would probably reduce this to only 15 per cent (the vignetting loss becomes less as lens aperture is reduced). This uneven illumination makes it impossible to obtain a proper exposure over the entire negative in spite of the fact that modern film emulsions have considerable latitude In actual practice the center is purposely overexposed in order to obtain a printable image on the edges. The resulting negative, however, is rather difficult to print, requiring the center to be exposed much longer than the edges, also the image definition is seriously impaired. There has recently been developed a variable density camera filter (dense in center) which reduces this uneven illumination.

When light passes through glass, part of it is lost by absorption and part by reflection. The amount lost by absorption is very small, about one-half per cent per centimeter of thickness The amount lost by reflection is considerable, about 5 per cent for each air-glass surface. This reflection loss not only means a longer exposure but also may cause flare spots on the negative due to multiple internal reflections. If a photographic lens contained eight air-glass surfaces then the light transmitted would only be 66 per cent ($0.95^8 = 0\ 66$). If it were possible to eliminate this reflection loss the effective speed of the lens would be increased.

Within the past few years methods have been developed commercially[10] which reduce the reflection loss by coating the lens surface with a thin transparent film one-quarter wave length thick To thoroughly understand the theory of this film one would have to study that branch of physical optics which concerns light interference Briefly, however, it may be said that light on striking the surface of a coated lens, is partially reflected from both surfaces of the film. Since this film is one-quarter wave length thick, the two reflected rays will be out of phase by one-half wave length and therefore interfere with each other and no reflection can take place A characteristic of the present types of coatings is that the more durable ones eliminate only a part of the reflection loss while the less durable ones are almost 100 per cent effective A partial solution to this problem is obtained by coating the inner surfaces of the lens with the less durable type, and leaving the outer surface uncoated

III. Optical Rectification

Optical rectification is the process of projecting the image of a tilted photograph into a horizontal plane. In the case of a multiple lens camera, the projection of the images of the various chambers into a common plane is usually called transformation. The optical and mathematical relationships are similar for both cases, the chief difference being that in rectification we are concerned with small accidental tilts (say under 5°), whereas in transformation we are dealing with large tilts (maybe 40°) whose values are definitely known from the camera calibration data.

Rectifying apparatus is often designed so that a change in scale and tilt removal can be made simultaneously. The resulting prints are used in laying down controlled mosaics, in the measurement of land areas, and in certain types of stereoscopic plotting equipment in which parallax measurements are made directly on the photographs.

1. Approximate Rectification

An approximate rectification, which is satisfactory for mosaic work, may be obtained by tilting the easel of a ratio printer (see Fig. 18 and 19). Contrast-

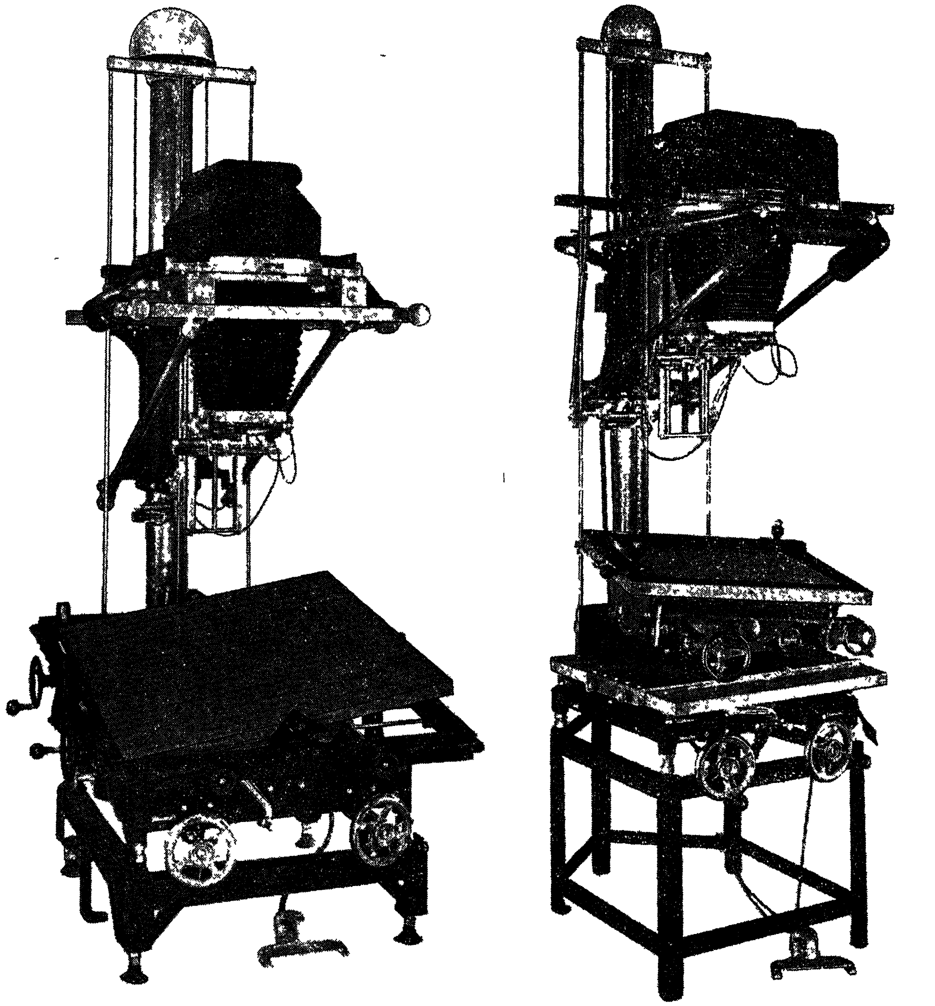
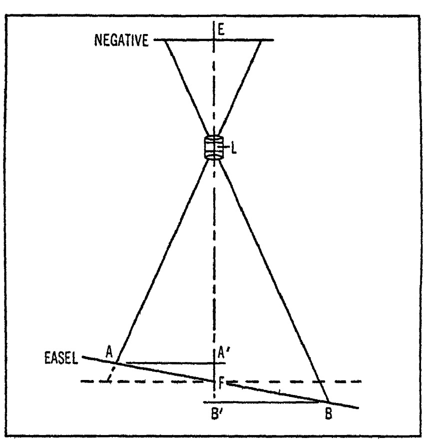


FIG. 18 Saltzman printers with tilting easels (Courtesy J G Saltzman, Inc.)

FIG. 19 (Right) Approximate rectification obtained by tilting the easel of a ratio printer.



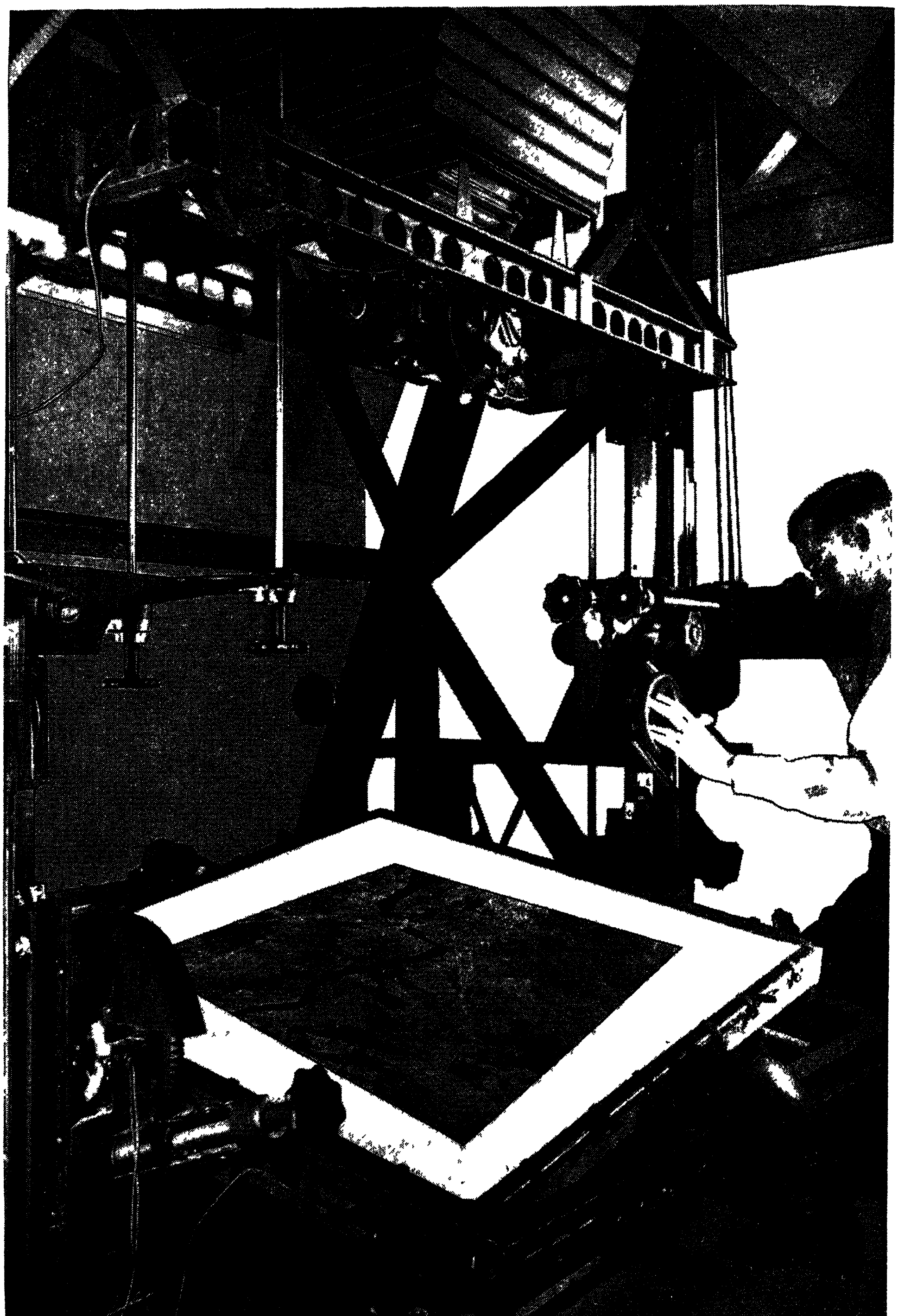


FIG. 20. Rectifying camera of the U. S. Coast & Geodetic Survey.

ing this method with exact rectification the following items should be noted: (1) Due to mathematical relationships, to be discussed later, the removal of tilt distortion is not theoretically correct; (2) in order to secure sufficient image definition it is necessary to stop the lens down considerably. If in Figure 19, EL and FL are conjugate distances, it can readily be seen that EL and $A'L$ (or EL and $B'L$) are not conjugate distances and therefore points A and B will not be in sharp focus. However, if the easel tilt is not excessive, a sufficiently sharp image is obtained by reducing the lens aperture to about $f/32$ (see depth of definition, page 53). On some ratio printers it is possible to restore the focus by tilting the lens board.

The operation procedure for using a ratio printer in the above manner will now be explained. It is first necessary to determine the tilt of the photograph. The necessary data for this determination may be obtained from a previously run radial line plot by measuring the distance from the principal point to several properly distributed radial control points, both on the photograph and on the plotting sheet An approximate value of the tilt is obtained by analyzing the variations in these distances. For a ratio printer of a given focal length, tables are usually prepared to speed this operation. If the easel of the ratio printer cannot be rotated in its own plane, it will be necessary to resolve the tilt into two components one of which is parallel and the other perpendicular to the length of the film These two tilt components (corrected for scale ratio changes) are then set on the tilt arcs of the easel and print exposed.

The rectification of photographs for the accurate measurement of land areas is accomplished in a manner similar to that described for mosaic work with the following exceptions: (1) A more accurate determination of tilt is necessary. The Anderson method, which requires the measurement of three ground distances, is often used. (2) The ratio printer must be of precise mechanical construction and the easel settings for any degree of air tilt must be based upon extensive calibration data. Tests on printers of this nature have indicated that for air tilts under 4°, results compare very favorably with those obtained by "exact rectification" which is discussed below. This statement is based on an extensive series of tests conducted in the photographic laboratory of the Agricultural Adjustment Agency, U. S. Department of Agriculture

2. Exact Rectification

In order to obtain precise results using plotting instruments of the stereocomparator type, it is necessary that the photographs used in such instruments be tilt-free to a high degree of accuracy and of equal scale. The removal of tilt and change in scale is accomplished in a specially designed rectifying camera such as illustrated in Fig. 20. This camera was designed for rectifying large multiple-lens photographs to be used for stereoscopic plotting. An approximate change of scale is accomplished by an auto-focus linkage between the lens and positive planes, with the final setting being made on a pair of metric scales. The negative plane (located beyond top of the figure), the lens plane, and the positive plane can each be tilted about two axes at right angles to each other. The tilt angles are set on graduated circles reading to one minute of arc. This combination of linear motions and tilts satisfies the conditions for exact rectification which will now be discussed.

In Fig. 21 is shown a schematic diagram of a rectifying camera. In order to simplify this figure, the principal line (CN) is assumed to be parallel to edge of tilted negative, a condition which seldom exists in practice. The procedure in operating a rectifying camera is as follows. (1) Determine the tilt of the photo-

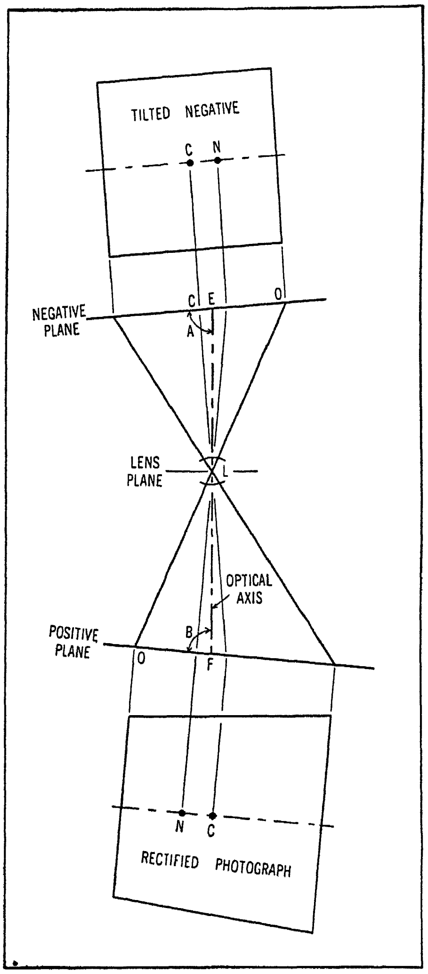


FIG. 21. Schematic diagram of a rectifying camera.

graph by any method which gives results within the desired degree of accuracy. The nadir point (N) should be marked on the negative in such a manner that its image will be visible on the rectified print; (2) Compute angles (A) and (B) and lengths EL, FL, and EC according to equations given below; (3) Set these values on the camera scales. The mathematical relationships between these settings are such that any pair of corresponding points (as O and O') when projected to the optical axis, form a pair of conjugate distances. Therefore, it is not necessary to use a small lens aperture to secure good definition, as was the case when only the positive (easel) was tilted. Another point of interest in true rectification is that the negative, lens and positive planes, when extended, intersect in a common line (4) Place principal point (C) of negative on point (C) in camera. (5) Rotate negative in its own plane until its axis of tilt is parallel to axis of tilt of negative holder. In some types of equipment this rotation cannot be accomplished but the negative holder and easel can be tilted in two directions at right angles to each other. In using such equipment the angles (A) and (B) will have to be resolved into similar components. (6) Expose the print. The only image displacement that now remains in the print is due to relief, which radiates from the nadir point. The image of this point appears on the print because it previously was marked on the negative. A pair of these prints, which have been corrected for both tilt and scale, may now be used in the stereocomparator type of plotting equipment for the accurate determination of elevations.

Equations for computing the items mentioned above are as follows[11].

$$\cos A = \frac{G \sin T}{MF} \qquad \text{eq. (8)}$$

$$\cos B = \frac{G \sin T}{F} \qquad \text{eq. (9)}$$

$$EL = \frac{G \sin (A + B)}{\sin A \cos B} \qquad \text{eq. (10)}$$

$$FL = \frac{G \sin (A + B)}{\cos A \sin B} \qquad \text{eq (11)}$$

$$EC = \frac{F}{\sin T}\left(\frac{\sin B}{\sin A} - \cos T\right) \qquad \text{eq. (12)}$$

G = focal length of rectifying camera* (in regards to focal length selection the following should be noted. (1) the shorter the focal length the larger the angular field, (2) the longer the focal length the more acute will be the intersection between the rays and the negative and positive planes)
F = focal length of air camera.
T = tilt of air camera
M = scale ratio (less than 1 for reduction, more than 1 for enlargement).

In using the above equations, it may be necessary to shift the light source when rectifying various photographs having different degrees of tilt, due to a variation in distance EC In order to avoid this difficulty, it is possible to base the design of the rectifying camera on another set of equations[11] which refer all measurements to the line of centers (that is, a line connecting principal point of negative to node of rectifying lens). Although the use of these equations allow the light source to be fixed, they complicate the camera design because the lens also must be tilted

In the actual design of a rectifying camera many refinements must be considered which are not discussed here Among these are the nodal point separation of the lens, and the errors caused by the glass pressure plate used to hold the film flat. In order to facilitate the operation of a rectifying camera, various mechanical linkages have been designed[12] which reduce the number of necessary settings to a minimum One of these is an auto-focus mechanism in which a single control wheel moves both negative and positive planes so as to maintain sharp focus for all scale ratios and tilts. Another linkage is designed to manipulate all tilting motions from a single control wheel. Instruments equipped with these two items are given the classification of "automatic."

Sometimes the setting of the various motions of a rectifying camera are accomplished by trial. The negative, containing at least three ground control points, is placed in its holder and a grid or map, with the same control plotted thereon, is placed on the easel. The various tilts and distances are so manipulated that the images of the control on the negative fall on their plotted positions However, if the ground control points are at various elevations it will be necessary to make repeated corrections for relief displacements until the coincidence of the points are within the desired degree of accuracy. This method is particularly used in machines of the automatic type because of their simple control mechanism.

IV. Errors Caused by Glass Plates

Plane parallel glass plates are often used in the optical path of various photogrammetric instruments. These plates may be in the form of pressure plates for holding film or paper flat during exposure, filters, or reflecting prisms. The latter may be considered as the optical equivalent of a thick plane parallel plate plus one or more plane mirrors depending upon what type of prism is used. To obtain the best possible performance from a photographic lens, it is necessary that filters or prisms used in the system be of a high optical quality. For pressure plates the optical quality need not be so great because the plate is so close to the focal plane that any small irregularities in its surface would affect the image definition very slightly. Carefully selected commercial plate glass is usually satisfactory for pressure plates. The use of the above mentioned plates cause certain errors which may be classed as: displacement of the focal plane; image distortion; and aberrations which impair the lens definition.

* Note special case· If $G=F$ and $M=1$, then $A=B=90-T$. And $EL=FL=2G$ Also the isocenter of negative will fall on optical axis at E

1. Displacement of the Focal Plane

The insertion of a glass plate between lens and focal plane displaces the focal plane away from the lens, its amount along the axis being about one-third the thickness of the plate (In Fig. 22 this displacement is the distance 00′.) This axial displacement is

$$\text{displacement} = T - \frac{T}{N} \qquad \text{eq (13)}$$

in which (T) is the plate thickness and (N) its index of refraction. For most commercial plate glass, filters and prisms this index is about 1.52. The above displacement formula is not absolutely correct for "off-axis" rays in that the plate introduces curvature of field which causes images of points near the edge of the field to have a different displacement than the image of an axial point, thus giving a "dished" image field.

2. Image Distortion

The distortion introduced by the insertion of a glass plate into an optical system results in a radial displacement of an image point toward or away from the center of the field depending on whether the plate is inserted between lens and image or between lens and object respectively (see Fig. 22) The magnitude of this distortion can be found from Table I, which gives the distortion error, expressed as a decimal part of the plate thickness, for rays at various angular distances (in degrees) from the center of the field The distortion varies with the

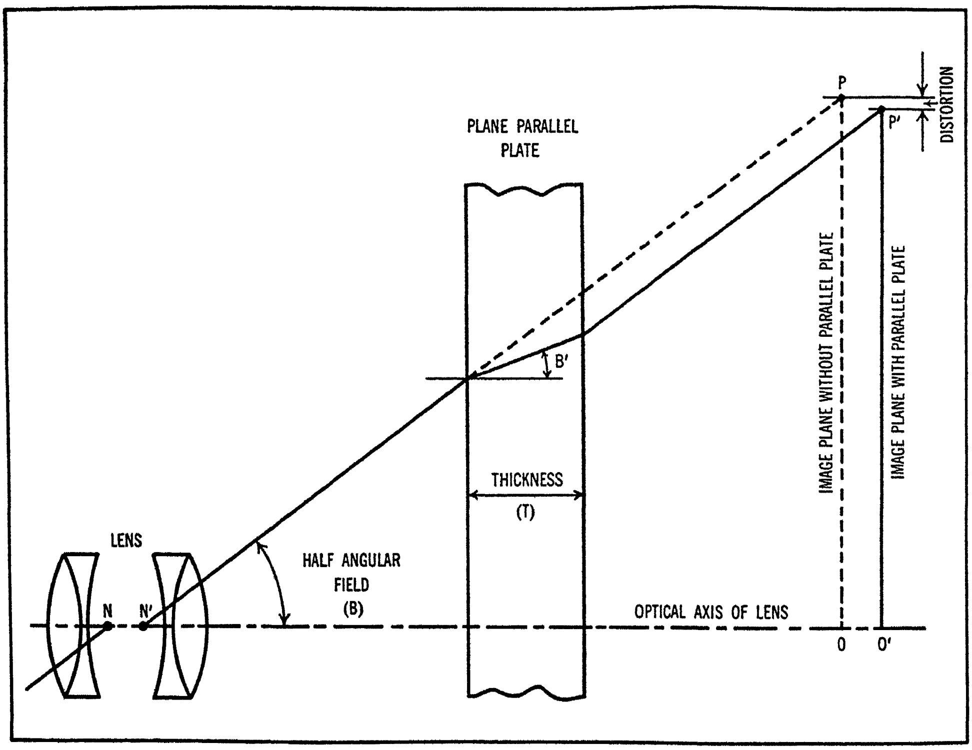


Fig. 22. Image distortion and displacement caused by the insertion of a plane parallel plate.

index of refraction (N) of the glass. The index used in computing the table was 1.52. Since most glass plates, filters, etc., have an index near this value, the results obtained by using Table I will be accurate enough for practical purposes. Computations were made to determine the change in the distortion with change in index. The results indicated that for values of the index between 1.48 and 1.56, the table values are correct to 3 in the 4th place for angles up to 25 degrees, and 6 in the 4th place for angles between 25 and 50 degrees.

TABLE I DISTORTION INTRODUCED BY A PLANE PARALLEL PLATE EXPRESSED AS A DECIMAL PART OF THE PLATE THICKNESS

(B) Half angular field in deg	Distortion $N=1$ 52	(B) Half angular field in deg	Distortion $N=1$ 52
1	0 0000	26	0 0197
2	0 0000	27	0 0222
3	0 0000	28	0 0251
4	0 0001	29	0 0281
5	0 0001	30	0 0315
6	0 0002	31	0 0352
7	0 0003	32	0 0391
8	0 0005	33	0 0434
9	0 0007	34	0 0481
10	0 0010	35	0 0532
11	0 0014	36	0 0587
12	0 0018	37	0 0646
13	0 0022	38	0 0710
14	0 0028	39	0 0779
15	0 0035	40	0 0854
16	0 0043	41	0 0934
17	0 0051	42	0 1021
18	0 0061	43	0 1114
19	0 0073	44	0 1215
20	0 0085	45	0 1324
21	0 0099	46	0 1441
22	0 0115	47	0.1566
23	0 0133	48	0.1702
24	0 0152	49	0 1848
25	0 0173	50	0 2006

In computing the distortion error the following two items should be noted: (1) A plate inserted anywhere between lens and object causes a radial displacement of an image point away (sign assumed +) from the center of the field, its magnitude being equal to the value in Table I multiplied by both the plate thickness and the scale ratio In the case of an aerial camera the scale ratio, that is, image size to object size, is very small (say 1:20,000). For the purpose of distortion computation it may be considered to be zero, hence the image distortion caused by a filter placed over the *front* surface of the camera lens will be zero. Another way of stating this would be to say that if the object is essentially at infinity, the light rays striking the filter are parallel and therefore there will be no image distortion. (2) A plate inserted anywhere between lens and image causes a radial displacement of an image point toward (sign assumed −) the center of the field, its magnitude being equal to the value in Table I multiplied by plate thickness only.

In certain types of photogrammetric equipment it is necessary to compensate for the distortion introduced by a filter or pressure plate. This can be done by inserting a compensation plate, of the proper thickness and index, in the system in such a position as to neutralize the distortion that is present. Sufficient information has been given in the above discussion to enable one to determine the thickness and location of such a plate.

3. Aberrations Affecting Lens Definition

A lens designer, after tedious computations, finally arrives at a design in which he has balanced the aberrations in the best possible manner. Care should be taken lest the insertion of a glass plate into the optical path of this lens upset his elaborate calculations.[13] The introduction of curvature of field has already been mentioned Fortunately the effect of this error, as well as others, is reduced by stopping the lens down (due to increase in depth of definition). In some cases, however, it may not be possible to do this because of the resulting decrease in illumination. It might be added that the distortion error previously discussed, is not reduced by stopping the lens down.

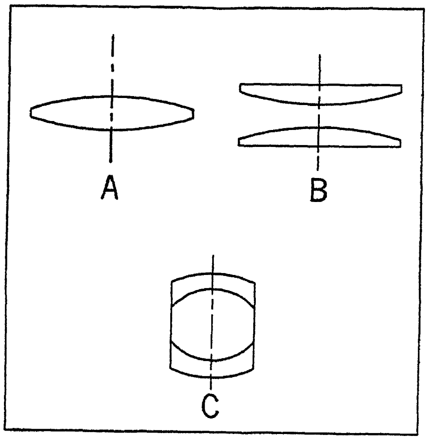


Fig 23 Types of magnifiers. (A) Simple reading glass (mag. 2 or 3). Definition is good only in center of field (B) Doublet as used in photo-engraving glasses (mag. up to 5). Gives a large field relatively free from distortion. (C) Cemented triplet (mag. up to 20). Gives a large field free from both distortion and color.

V. Simple Optical Systems

In photogrammetry use is made of a great variety of optical systems. Only the more elementary ones will be described here. A complete study of this subject would bring one into the field of optical instrument design. Those further interested in the subject should consult the references at the end of the chapter.[2]

1 Magnifiers and Eyepieces

A magnifier forms a virtual image of a real object, while an eyepiece forms a virtual image of a real image formed by some other part of the optical system, the image in both cases being erect In general eyepieces have a shorter focal length and higher optical quality than magnifiers The image quality obtained by a magnifier depends upon the number and shape of the lens elements. The chief characteristics of several types are shown in Figure 23. The magnifying power (M) of a magnifier or eyepiece does not have a definite value since it depends upon the viewing distance of the observer. An approximate value is given by the equation

$$M = \frac{10}{\text{focal length in inches}} \left(\text{or } \frac{250}{\text{focal length in mm}} \right) \qquad \text{eq. (14)}$$

This relationship is based on the following assumptions: (1) that the normal viewing distance for most observers is 10 inches and that the purpose of the lens is merely to enable one to accommodate the eye to a shorter viewing dis-

tance thereby causing the object to subtend a larger angle at the eye; (2) that the lens is placed at its focal length away from the object, thus forming a virtual image at infinity. Most persons, however, prefer to use an object distance less than the focal length, thereby causing the virtual image to be formed at some finite distance (say about 10 inches). For low power magnifiers the percentage change in magnification with change in object distance is considerable (see problem 11 at end of chapter), but for the shorter focal lengths used in most eyepieces the change is negligible.

Eyepieces (or oculars) are used on such photogrammetric instruments as certain types of lens stereoscopes, transits, levels, and many stereoplotting instruments The chief function of an eyepiece is to magnify the image. They are also used to erect the image formed by another part of the optical system, to enable cross hairs to be introduced, and to correct for residual aberrations of the system. The two more common types of eyepieces are the Huygenian and the Ramsden, the characteristics of which are discussed below

The Huygenian type consists of two plano-convex lenses arranged as shown in Figure 24. Both lenses are made from the same type of glass with the focal length of the field lens being about twice that of the eye lens The separation of the two lenses is made equal to one-half the sum of their focal lengths. A study of the ray path through this eyepiece may be of interest If the eyepiece was removed, the remainder of the optical system would form a real image at (*D*). This real image is the virtual object for the eyepiece, which in turn forms a virtual image at infinity If a reticle or cross hair is used it must be placed at (*C*). The use of a reticle in this type of eyepiece is not very satisfactory, since the field is very small and therefore the reticle would have to be very short. Also the eye lens is fixed in reference to the reticle so that it cannot be focused for different observers. In the Huygenian eyepiece the longitudinal chromatic aberration and curvature of field are large, but the distortion and lateral chromatic are small

The Ramsden eyepiece consists of two plano-convex lenses of equal focal length arranged as shown in Figure 25 The separation of the lenses is usually made equal to two-thirds the focal length of either lens. The rest of the optical system forms a real image at (*H*) and the eyepiece uses this image as an object and forms a virtual image at infinity. When cross hairs are used they are placed at (*H*) in which position it is possible to adjust the eyepiece to accommodate

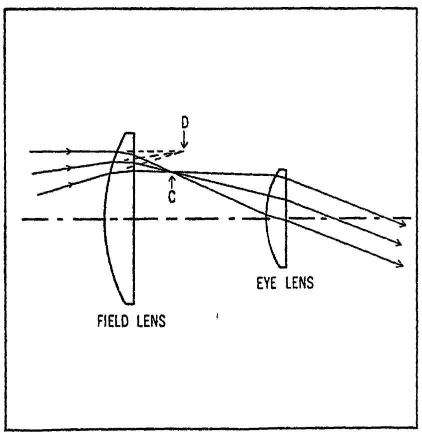


FIG. 24. Huygenian eyepiece.

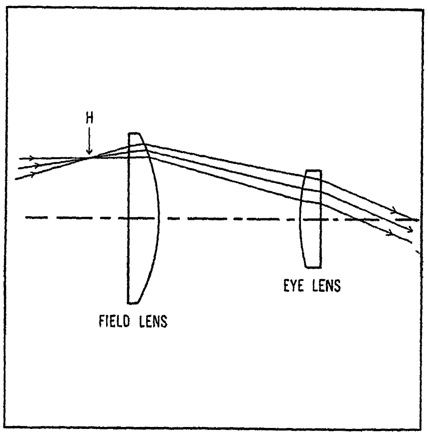


FIG. 25. Ramsden eyepiece.

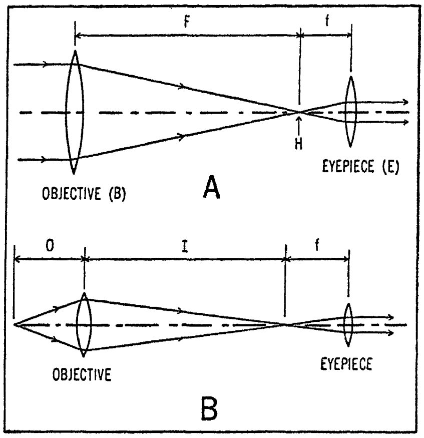


FIG 26. Optical systems. (A) Telescopic. (B) Microscopic.

different observers. In the Ramsden eyepiece the distortion, curvature of field and longitudinal chromatic aberration are small, but the lateral chromatic is a little large. In order to reduce this latter defect the single eye lens is sometimes replaced by an achromatic doublet, in which case it is known as a Kellner eyepiece.

All of the eyepieces described above form an erect image of the object that they magnify. If the rest of the optical system forms an inverted image, it may be desirable to erect this image. This may be done with an erecting eyepiece using a lens system (see Fig. 27B) or by means of a porro prism system as used in binoculars (see page 70—Fig 39).

In high power microscopes a "compensating" eyepiece is sometimes used. Such an eyepiece is designed to have aberrations which are opposite in sense to those of the objective, thereby securing a high correction for the entire optical system.

2. TELESCOPIC AND MICROSCOPIC SYSTEMS

There is no definite boundary between these two systems, although the term microscope is usually reserved for cases where the object distance is less than ten inches. The most obvious use of telescopes in photogrammetry is for surveying instruments. Equally important, however, is their use in the viewing systems of stereoscopic apparatus. A simple refracting telescope (Fig. 26A) consists of an objective lens (B) and an eyepiece (E). When the object is at infinity, a real image is formed at H. The eyepiece uses this image as an object and forms an inverted virtual image at infinity. The magnification (M) of such a system is

$$M = \frac{F}{f} \qquad \text{eq. (15)}$$

in which (F) is the focal length of the objective and (f) the focal length of the eyepiece. If the object is relatively close to the objective (as in microscope, Fig. 26B) the magnification is obtained as follows·

$$\text{magnification due to objective} \quad M_B = \frac{I}{O}$$

$$\text{magnification due to eyepiece} \quad M_E = \frac{10}{f\,(\text{inches})}$$

$$\text{magnification due to system} \quad M = M_B \times M_E \qquad \text{eq. (16)}$$

The purpose of Figure 26A was merely to show the metrical relationships between the various elements. In actual practice the optics of a transit telescope[14] may be as shown in Figure 27A, in which an achromatic objective is used with a Ramsden eyepiece. Since this system gives an inverted image it may be

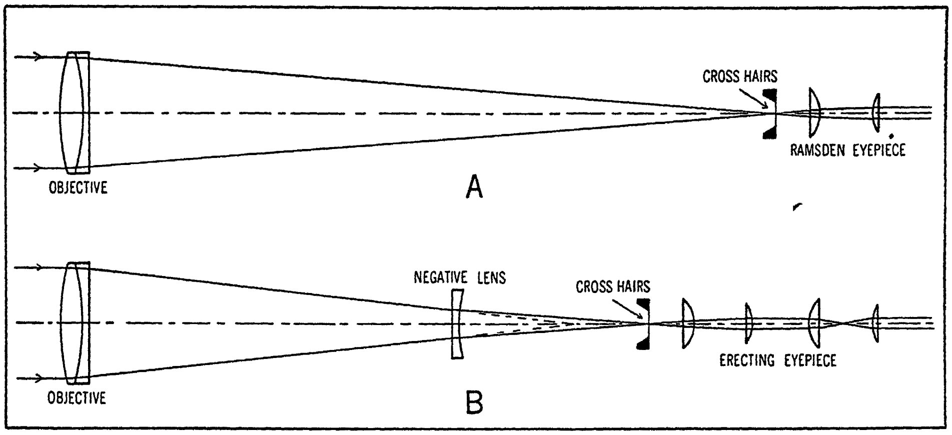


FIG. 27. Optics of a transit telescope. (A) External focusing type with Ramsden eyepiece giving an inverted image (B) Internal focusing type with erecting eyepiece.

desirable to replace the Ramsden eyepiece with one of the erecting type shown in Figure 27B. A majority of the modern transits are equipped with the "internal focusing" telescope which is also shown in Figure 27B. In this type, focusing is accomplished by moving an internal slide carrying a negative lens. This eliminates the wear caused by dust collecting on an external slide and also eliminates a constant from stadia computations.

The proper way to focus a telescope or microscope is as follows: (1) Focus the eyepiece on the cross hairs; (2) focus the objective on the object until the image appears sharp, in which case the plane of the image and cross hairs should coincide; (3) check the focus by slightly moving the eye from one side to the other. If the cross hairs appear to move over the object the focus is imperfect and parallax is said to exist; (4) refocus the objective to remove this parallax.

3. COLLIMATORS

A collimator is an optical laboratory device which establishes in space a fixed line of sight. It consists of a telescope objective with an illuminated slit or cross hair at its focal point (see Fig. 28). After passing through the lens, rays from each point of the object are made parallel and the object is virtually at infinity. Since the emergent rays are parallel, the established line of sight is in the same direction regardless of what portion of the objective is used.

Any telescope can be converted into a collimator merely by removing the eyepiece, inserting a small lamp to illuminate the cross hairs, and then focusing the objective for infinity. The requirements of the telescope thus converted being that

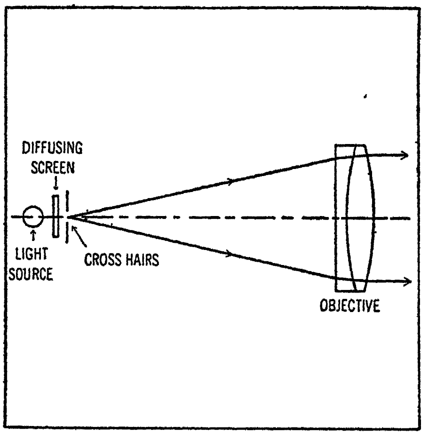


FIG. 28. Optical elements of a typical collimator.

it have a higher optical quality and a larger aperture than the instrument to be tested. The focusing operation may be accomplished by the following methods: (1) Replace the eyepiece and then focus on a distant object (such as a star). The objection to this method is that the adjustment may be disturbed when the eyepiece is removed and the illuminating system attached; (2) Focus a theodolite on a distant object, then point the instrument toward the objective of the collimator and change the latter's focus until the image of the cross hairs, as seen in the theodolite, appear sharp, (3) Use an autocollimating or Gauss eyepiece.[15] Other methods of focusing which require the use of two or three collimators are also available.[16]

Collimators are widely used in the laboratory for testing various optical systems such as aerial camera lenses (see Fig 15), surveying instruments, etc

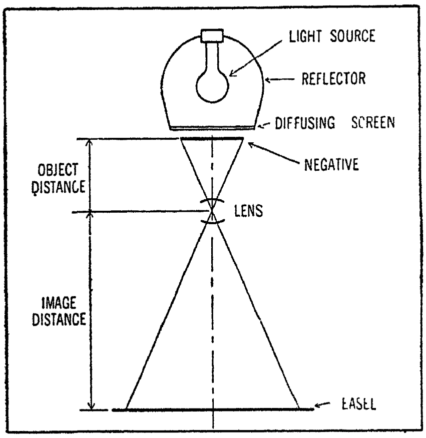


FIG. 29 Ratio printer with diffuse illumination

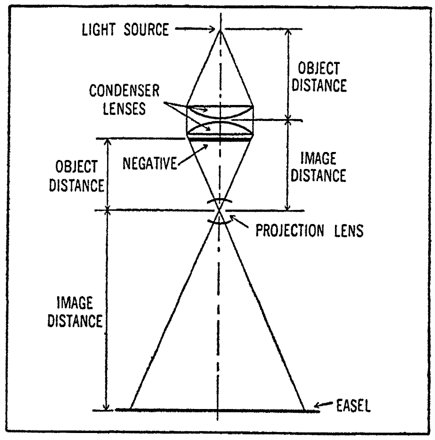


FIG. 30 Ratio printer with condenser illumination.

4. PROJECTION SYSTEMS

Projection systems are used in such equipment as ratio printers, multiplex projectors, reflecting map projectors, etc The purpose of these projection systems is to form a real image on a receiving screen of sufficient brightness to satisfy the particular work to be done.

Several methods of illumination are used in projectors. The most common type being the diffusion system of a ratio printer shown in Figure 29. The chief parts of such a system are a light source, reflector, and diffusing screen. The light source may be either an incandescent, mercury vapor, or argon lamp. The last two types are preferred because they emit less heat. The light source, mounted in a reflector of suitable design, serves to illuminate an opal or ground glass. Because of its diffusing properties, this glass becomes a secondary source of light which illuminates the entire negative area. Plastic diffusing materials have recently been developed which appear to be superior to glass since they are practically unbreakable and also have a higher light transmission for a given degree of diffusion. The contrast in a print made in a ratio printer equipped with a diffusion system is practically the same as that obtained in a contact print.

Another method of illumination requires the use of a condensing lens. The purpose of such a lens is to concentrate the illumination from a light source upon a limited area, rather than to form an optically perfect image. For this reason high optical quality is usually not necessary. The condensing system

often consists of two plano-convex lenses arranged as in Figure 30 The focal length of this system is so selected that a crude image of the light source is formed at the diaphragm of the projection lens. As the scale ratio of the printer is changed it will be necessary to refocus the condenser The size of the condenser should be slightly larger than the negative diagonal, thus for 9″×9″ aerial negatives, a 14″ diameter would be required. The chief advantages of a condensing system over a diffusing system are that a higher screen illumination is obtained from a given light source, and also a higher image contrast, although negative scratches will be emphasized. These advantages, however, do not offset such items as increased cost, bulk, and weight, and therefore condensers are seldom used in ratio printers (except in the small amateur sizes).

The illumination problem in the multiplex projector is much different from that in a ratio printer. In the latter case it is usually possible to compensate for the light lost by the diffuser by merely using a larger lens aperture. As previously mentioned, the aperture of the multiplex projector lens necessarily must be small (about $f/22$) in order to secure sufficient depth of definition to take care of the relief in the spatial model Also considerable light is absorbed by the red or green anaglyphic filter. For these reasons it is absolutely necessary to use a condensing lens system in order to obtain sufficient screen illumination for stereoscopic observation. Since the light source must be concentrated on an aperture only 1 mm. in diameter, a specially designed system of high optical quality is required (see Fig. 31). The surfaces of these condenser lenses have been coated with a non-reflecting film in order to increase the light transmission.

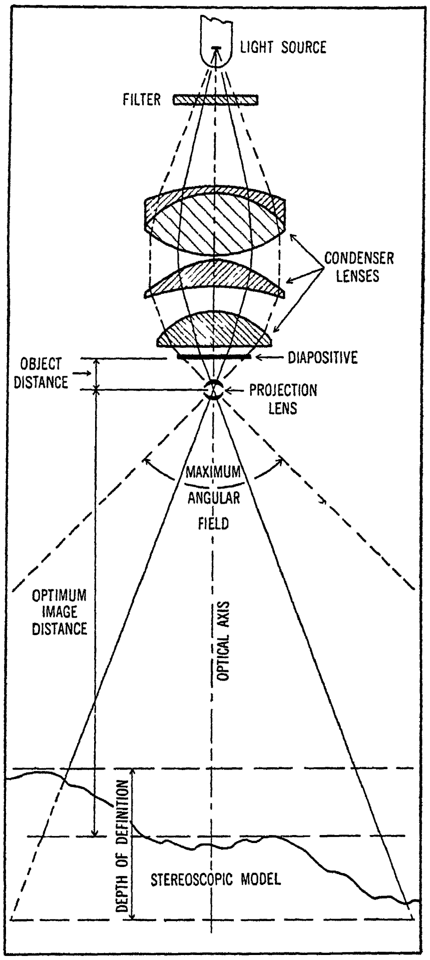


FIG. 31. Optical system of a wide-angle multiplex projector.

In the usual map projector the problem is to illuminate an opaque object (a photograph or map). A light source is usually mounted on either side of the object (see Fig. 32). Each of these sourses consists of a projection lamp (500 watts) mounted in a spherical glass reflector. Of the three methods of illumination discussed, this is the least effective, as only a small portion of the light striking the matte surface of the photograph or map is reflected toward the projection lens.

VI. COLOR FILTERS

A filter may be considered as a colored plate which transmits certain portions of the spectrum and absorbs the remainder. In general it may be said that a filter transmits light of its own color. For photographic purposes it is convenient

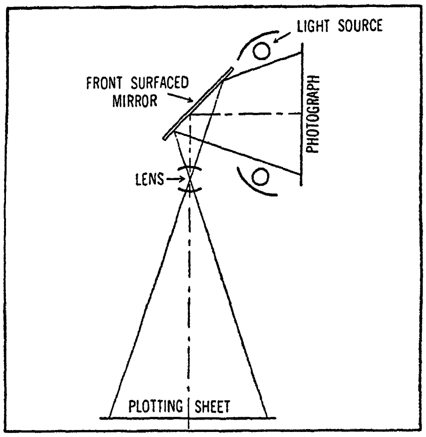


FIG 32. Reflecting map projector.

to subdivide the spectrum into the following five regions: ultra violet, blue, green, red, and infra red. Figure 33 gives the approximate wave length limits of each region and also indicates the range of sensitivity of various photographic emulsions. By the use of a filter it is possible to select those portions of the spectrum which are best suited for each type of work.

1. CONSTRUCTION

A filter may consist of an unmounted colored gelatin film, a colored gelatin film cemented between two plates of optical glass, or a colored plate of optical glass Due to its fragility, the first type mentioned is seldom used for other than experimental purposes. The gelatin film mounted between glass has the one disadvantage that the cement deteriorates due to age or heat causing the image definition to be impaired. Filters made of colored glass are considered the best type to use for accurate work. It is very important that a glass filter introduced into the path of a photographic lens have an optical quality equal to that of the lens itself, if critical image definition is to be maintained. By this we mean that the surfaces should be plane and parallel and the glass itself free from defects. Even if a filter is of good optical quality, the items discussed under "Errors Caused by Glass Plates" still apply. Note: It was previously mentioned that in the case of an aerial camera there will be no distortion or displacement of the image if the filter is mounted in *front* of the lens.

2. USES IN AERIAL PHOTOGRAPHY

In aerial photography the purpose of filters is either to eliminate the effects of atmospheric haze[17] (not fog or smoke) or to increase the contrast of certain desired features of the terrain. Atmospheric haze consists of small particles of dust and water vapor which have the property of scattering blue light. That portion which is scattered in the direction of the camera produces a slight fog on the panchromatic film. By placing a blue absorbing filter (yellow in color) over the lens this undesired light is eliminated, the exposure being made by green and red rays reflected from ground objects

For maximum haze penetration (especially in oblique photography) infrared film and filters are sometimes used. Until recently this type of film was too slow for aerial photography. However, the type II Infra-red Aero Film intro-

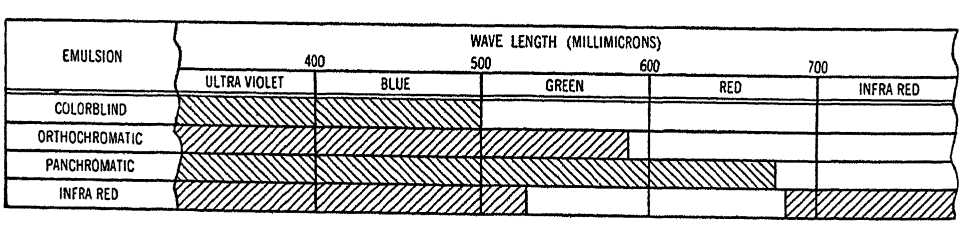


FIG. 33. Approximate sensitivity range of various photographic emulsions.

duced by Eastman Kodak Company has a speed such that satisfactory exposures can be made at 1/150 sec. at $f/11$ (using Wratten filter No. 25 or 89A at an altitude of 5000 feet). This film and filter combination is also useful in military intelligence because it increases the contrast of terrain features, thus aiding in detecting camouflage.

A polarizing screen (which is not a color filter) is occasionally used in aerial photography to eliminate undesirable reflections from bodies of water, thereby aiding in the detection of submerged objects, reefs and shoals. According to the wave theory, ordinary light (unpolarized) is said to vibrate perpendicular to the direction of propagation in all planes, while plane polarized light vibrates in a single plane. Ordinary light on passing through a polarizing screen becomes plane polarized. Such a screen may be compared to a series of parallel slits and only light which vibrates in the plane of these slits is allowed to pass through, thus two such screens placed with their planes of vibration at right angles to each other would allow no light to pass. Sunlight, upon reflection from water, becomes partially plane polarized. If a polarizing screen is placed on the camera (in front of lens) with its plane of vibration at right angles to the plane of vibration of the reflected light, then the latter would be completely cut out.

The type of filter chosen for any particular job depends on several items, such as: flying height, density of the atmospheric haze, type of film used, and character of the terrain. In Table II are listed several filters used in aerial photography. Also given are the filter factors, which indicate how many times the exposure must be increased in order to compensate for the light absorbed by the filter when used with a particular type of film. The filter numbers listed in table below refer to Wratten light filters manufactured by Eastman Kodak Company.[18]

TABLE II. FILTERS USED IN AERIAL PHOTOGRAPHY

Type	Wratten number	Color	Filter factors*	Conditions when used
Aero 1	3	Light yellow	1.5	Very light haze
Aero 2	5	Yellow	2	Light haze
Minus Blue	12	Deep yellow	2	Medium haze
A	25	Red	4	Heavy haze at high altitudes

* For Eastman Super-XX panchromatic aero film.

3. PROJECTION FILTERS

Another use of filters in photogrammetry is for projection purposes, such as the anaglyphic principle used in the multiplex equipment. This is a method of stereoscopic viewing in which one image of a stereoscopic pair is projected and viewed in one color and the other in a complementary color. This is accomplished by placing a red filter in front of light source on the right projector and also over the right eye of the observer, likewise a blue green filter on the left projector and left eye. Since the filters are complementary each eye sees only one image, this of course being a necessary condition for stereoscopic vision. Often used for this purpose are Wratten gelatin filters Nos. 26 (stereo red) and 55 (stereo green). Other suitable filters of dyed glass are also available.

The polarizing screens previously mentioned in this section may also be employed for stereoscopic projection and viewing.[19] These screens have the advantage over the anaglyphic method in that natural color photographs may be used.

4. Safelights

Filters are also used for safelights in photographic laboratories. The surface quality of such filters can be very poor since they are only used for general illumination. The necessary requirement being that they absorb only that portion of the spectrum which would affect the sensitive material being used. For various types of sensitive material the following Wratten safelights are used.

Contact paper	—series 00 (Yellow)
Projection paper	—series 0 (Orange)
Non-color-sensitized film	—series 1 (Reddish Orange)
Orthochromatic film	—series 2 (Deep Red)
Panchromatic film	—series 3 (Green)
Infra-red film	—series 7 (Green)

Safelights should be used with caution. A check on their "safeness" can readily be determined by exposing a test strip of the sensitive material to be used, and inspecting it for fog

VII Prisms

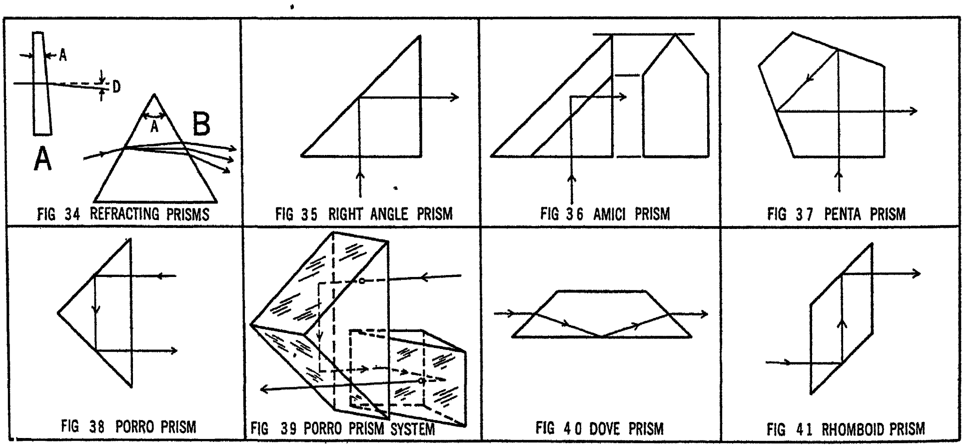
Prisms are used in photogrammetric instruments for several purposes. They may be used to deviate a beam of light through a certain angle, to laterally displace a beam without introducing any deviation, and to invert, revert, or rotate an image in its own plane.

1. Refracting Prisms

Refracting prisms, when used to produce deviations of a few minutes, are called "wedges" (see Fig. 34A). This type cannot be used for large deviations because it disperses white light into its constituent colors (see Fig 34B). For small deviations, the dispersion of a refracting wedge can be practically eliminated by constructing the wedge from two pieces of glass having different index and dispersion. Such a combination is termed an achromatic wedge. The deviation (D) produced by a wedge is given by the equation

$$D = A(N - 1) \qquad \text{eq. (17)}$$

in which (A) is the refracting angle and (N) the index Another method of expressing this deviation is by use of prism diopters. A deviation of 1 centimeter



Figs. 34 to 41. Typical prisms used in photogrammetric instruments.

in a distance of one meter is one prism diopter. A refracting wedge is sometimes used in a simple lens stereoscope (lens and wedge ground as one piece of glass) in order to use photographs having an air base larger than the observer's eyebase.

2. Reflecting Prisms

Reflecting prisms do not cause the troublesome dispersion mentioned above and therefore can be used for large deviations. Front surfaced plane mirrors could be used for the same purpose but the surface coating deteriorates with age and has to be replaced. Mirrors are generally used in the larger sizes, however, because prisms of this nature are very expensive and difficult to obtain It must be remembered that the items discussed under "Errors Caused by Glass Plates" applies also to prisms and for this reason mirrors may be preferred. A brief discussion of a few of the more common types of reflecting prisms follows [20]

(a) The *right angle prism* shown in Figure 35 produces a deviation of 90° and reverts the image. By reversion we mean that the top and bottom of the image will be interchanged without altering the relative position of the right and left sides This type sometimes replaces the small eye mirrors in a mirror stereoscope.

(b) The *Amici "roof angle" prism* shown in Figure 36 produces a deviation of 90° and inverts the image. It resembles a right angle prism except the hypotenuse face has been replaced by two surfaces inclined at 90° to each other to form a "roof." Entering rays striking the right side of the roof are reflected over to the left side and then out while those striking the left side follow the reverse course. Thus, the final image is formed with rays that have crossed over from one side to the other. If the two reflecting surfaces do not intersect within a few seconds of 90° the beams reflected from them will not match, and a double image will be formed. For this reason the prism is very difficult to manufacture and rather expensive.

(c) The *penta prism* shown in Figure 37 produces a deviation of 90° but does not change the image. The two reflecting surfaces of this prism must be silvered. A particular property of this type is that a 90° deviation is produced even if the beam does not strike the entering face exactly normal.

(d) The *Porro prism* shown in Figure 38 produces a deviation of 180° and inverts the image. It has the same shape as a right angle prism but the ray path is different Two such prisms arranged as in Fig. 39 form an erecting system often used in optical instruments.

(e) The *Dove prism* shown in Figure 40 does not produce any deviation or displacement but reverts the image. It is also known as a rotating prism, the image rotating 180° when the prism rotates 90°. It is used in the optical system of the aerocartograph to keep the images as seen in the eyepiece, in proper fusion while the floating mark travels over the spacial model.

(f) The *rhomboid prism* shown in Figure 41 produces no deviation or image change but merely displaces the axis parallel to itself. Sometimes used in eyepieces of plotting instruments to obtain a varying interocular distance for different observers.

VIII. Mirrors

Mirrors are used in such photogrammetric instruments as stereoscopes, reflecting map projectors, and in certain types of plotting instruments. The surface quality of the mirror will vary with the type of work to be performed. When used for visual inspection, as in the ordinary mirror stereoscope or reflecting map projector, the surface quality of selected commercial plate glass is sufficient.

Such glass (say for a 12 inch square) will probably be flat to within one or two thousandths of an inch On the other hand if the mirror is placed into the optical system of a plotting machine, in such a location as to influence its accuracy of measurement, it may be desirable to have its surface flat to within one-fifth of a wavelength of light (0.000004 inch) In order that the surface retain this flatness it is necessary to make its thickness at least one-tenth of its diameter A few remarks concerning materials used for making mirrors follows.

1. Glass

The most common mirror of this type is one which has its rear surface silvered and then backed with an opaque enamel to prevent it from tarnishing and to make it durable. The chief disadvantage of such a mirror is that a "ghost" image appears due to a partial reflection from the front surface. This secondary image is noticeable in a stereoscope especially when the prints contain contrasty detail, such as a white concrete road winding through a pine forest. In order to eliminate this double reflection, attempts were made to deposit the silver on the front reflecting surface, and to protect it from tarnishing and abrasion by coating it with a thin layer of transparent lacquer. This coating of lacquer impaired the definition of the reflected image and also reduced the reflectivity from 95 per cent to about 70 per cent. (Note: The reflectivities for the above and following materials are given for light with a wavelength of about 550 millimicrons.)

Within the past few years an excellent process of making front surface mirrors has been developed commercially It consists of depositing a thin non-tarnishing film (about one-quarter wavelength thick) of high reflectivity upon a glass surface. The coating is accomplished by thermal evaporation in a high vacuum Apparently the most successsful coating is made by first depositing a thin layer of chromium, which exhibits a strong adhesion to glass, followed by a thin layer of aluminum. The resulting composite film has high reflectivity (90 per cent), tenacity and durability.

2. Speculum Metal

This is an alloy of 68 per cent copper and 32 per cent tin. It has a reflectivity of about 60 per cent. Its chief disadvantage is that it tarnishes rather readily in the presence of certain atmospheric gases and cases are known where the reflectivity has been reduced to two-thirds of its original value within a few months. It is seldom used in photogrammetric instruments.

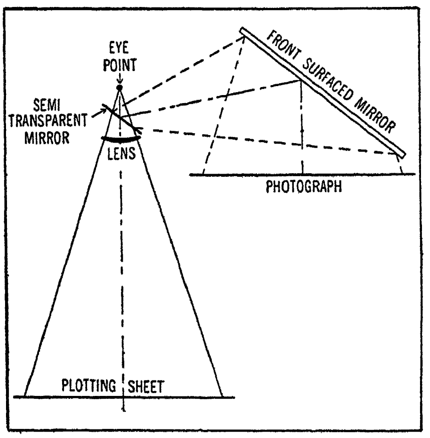


Fig. 42. Schematic diagram of a vertical sketchmaster.

3. Stellite

This is a trade name for an alloy of chromium, cobalt, and tungsten. It takes a high polish, does not tarnish, is very hard, and has a reflectivity of 60 per cent. However, it is very difficult to machine and must be shaped by grinding.

4. Stainless Steel

This is an alloy of varying composition; one type contains 18 per cent chromium and 8 per cent nickel. This material takes a high polish, does not tarnish when pol-

ished, and has a reflectivity of about 61 per cent. Some stainless steels may be machined readily.

Metal mirrors should be used instead of glass if there is danger of frequent breakage It is interesting to know that metal mirrors can have their reflectivities increased to about 90 per cent by the deposition upon their surfaces of an evaporated metallic film as explained above.

5. SEMI-TRANSPARENT

In contrast to the above-described mirrors, in which the greatest possible reflectivity is desired, the semi-transparent type is one which both reflects and transmits the light that falls upon it. The ratio of reflection to transmission can be varied by controlling the density of the evaporated metallic film which is deposited on the surface of the glass. It is often used in instruments of the camera lucida type. An example of such an instrument is the "Vertical Sketchmaster" (see Fig. 42) for transferring detail from vertical aerial photographs to the plotting sheet The mechanical parts of this instrument are arranged to take care of changes in scale and approximate tilt removal

IX. PROBLEMS

1 A ray of light strikes a plane polished glass plate at an angle of incidence of 42°12′ Using eq (1), compute the angle the refracted ray makes with the normal (Ans 26°14′)

2 Using equations (2) and (3), compute the image distance and magnification for the following conditions Also state whether the image is real or virtual, erect or inverted
   (a) Positive lens of 8″ focal length and object distance of 10″
   (Ans· $I=+40''$, $M=-4$, real inverted image)
   (b) Positive lens of 8″ focal length and object distance of 6″
   (Ans· $I=-24''$, $M=+4$, virtual erect image)
   (c) Negative lens of 8″ focal length and object distance of 12″
   (Ans. $I=-4\ 8''$, $M=+0\ 4$, virtual erect image)
   (d) Negative lens of 8″ focal length and object distance of 5″.
   (Ans· $I=-3\ 08''$, $M=+0\ 62$, virtual erect image)

3. Two thin lenses, one having a focal length of +10″ and the other a focal length of +5″, are placed in contact Using eq. (4) compute the approximate focal length of the combination. (Ans +3 3″)

4 Two thin lenses, one having a power of +5 diopters and the other a power of −2 diopters, are placed in contact What is the approximate focal length of the combination in cm ? Refer to discussion following eq (7) (Ans +33 3 cm)

5. The focal length of an aerial camera is 6 24″. A negative made with this camera was analysed for tilt and found to have a value of 3°42′. Compute the necessary values for setting a rectifying camera having a focal length of 8.24″ Also assume that the negative is to be enlarged to a scale ratio of 1 080. Use eq (8) to (12). (Ans $A=85°28'29''$, $B=85°06'43''$, $EL=15.865''$, $FL=17\ 143''$, $EC=0.151''$.

6 Given the following data for a multiplex reduction printer; Focal length of lens 50 24 mm., nodal point separation 2 60 mm , thickness of glass plate in front of negative 7.90 mm , index of refraction of glass plate 1 520, scale ratio setting on printer 0 250 Compute the total distance from negative to diapositive using eq. (5), (6), and (13) (Ans. 319.30 mm)

7. Given a single lens aerial camera with a focal length of 6 24″ and a negative size of 9″×9″.
   (a) Compute the angular field at the extreme corner of the negative. (Ans 91°04′)
   (b) If the light intensity at the center of the negative is 100% what is it at the extreme corner? Note except for the vignetting effect of the lens mount, the light intensity varies as $\cos^4 A$, where (A) is half the angular field. (Ans. 24%)

8. Referring to problem (7), assume that a 0.25″ thick glass plate is pressed against the film to hold it flat Using table I, compute the amount and direction of the image distortion caused by this plate at the corner of the negative. (Ans: −0.035″)

9. The focal length of a ratio printer lens is 10.25″. Using eq (5) and (6), compute the following:
   (a) Conjugate distances for a ratio of 2.45. (Ans.: O=14.43″, I=35 36″)
   (b) Conjugate distances for a ratio of 3.68 (Ans.: O=13 04″, I=47.97″)
   (c) If the correct exposure is 12 seconds for part (a), what will it be for part (b)? Note· The exposure varies as $(M+1)^2$ where (M) is the magnification or scale ratio. (Ans.: 22 Sec)

10. Referring to problem (9), assume that a 0.375″ thick glass plate is pressed against the film

(9″×9″) to hold it flat Using table I, compute the amount and direction of the image distortion caused by this plate at the corner of a print made at a scale ratio of 3 68 (Ans +0 027″)

11. A magnifying glass used for examining aerial photographs has a focal length of 2 5″ Compute the following.
    (a) The magnifying power, using eq (14), which assumes that the lens is placed its focal length away from the object thereby forming a virtual image at infinity (Ans 4)
    (b) The magnifying power, using eq (2) and (3), assuming a virtual image distance of 10″ (Ans 5) The purpose of this problem is to show that the magnifying power is not a fixed value but depends upon the object distance used by the individual observer.
12. A transit telescope has an objective lens of 10″ focal length and an eyepiece of 0 50″ focal length Using eq (15), compute the magnification (Ans : 20)
13. A microscope has an objective lens of 1 00″ focal length and an eyepiece of 0 80″ focal length Compute the magnification of the system (see eq 16), assuming an object distance of 1 25″ (Ans 50)

X References

1—Jacobs, D H, "Fundamentals of Optical Engineering," page 17.
2—Gardner, I C., "Application of the Algebraic Aberration Equations to Optical Design," National Bureau of Standards scientific paper no 550 (1927)
3—Kingslake, R, "Lenses for Aerial Photography," Journal of the Optical Society of America, March 1942, page 129
4—Melson, E W, "Making Photographic Objectives," Photo Technique, July 1941, page 44
5—Hardy and Perrin, "The Principles of Optics," first edition (1932) page 378
—Kingslake, R, "A New Bench for Testing Photographic Lenses," Journal of the Optical Society of America, April 1932, page 207.
6—Gardner, I C, and Case, F A, "Precision Camera for Testing Lenses," Journal of Research of the National Bureau of Standards, April 1937, page 449
7—Washer, F E., "Locating the Principal Point of Precision Airplane Mapping Cameras," Journal of Research of the National Bureau of Standards, October 1941, page 405
8—Mack, J E, and Martin, M J, "The Photographic Process," (1939), page 75
9—Washer, F. E., "Characteristics of Wide-Angle Airplane Camera Lenses," Journal of Research of the National Bureau of Standards, September 1942, page 233.
10—Cartwright, C H, "Treatment of Camera Lenses with Low Reflecting Films," Journal of the Optical Society of America, March 1940, page 110
—Turner, A. F, "Coating Lenses," Photo Technique, October 1940, page 48
11—Report of the Air Survey Committee (The War Office, London), no 1 (1923), page 41
12—Von Gruber, O., "Photogrammetry" (1932 English translation), page 297
13—See reference no 2, page 160.
14—Taylor, E. W., "The Surveyor's Telescope," Empire Survey Review (London), January 1941, page 2.
15—Lawes, A. V., "The Application of the Gauss Method of Collimation to the adjustment of Survey Instruments," Empire Survey Review (London), July 1940, page 429.
16—Medlen, E. M., "The Use of Collimators for Testing and Adjusting Instruments," Photogrammetric Engineering, vol VI (1940), page 13.
17—Clark, Walter, "Atmospheric Haze," Photo Technique, July 1941, page 50.
18—Eastman Kodak Co., "Wratten Light Filters."
19—Rule, J. T., "Polaroid and its Application to Aerial Photography and Photogrammetry," Photogrammetric Engineering, vol. VII (1941), page 31
20—See reference no 2, page 175. Also reference no. 1, page 151.

XI. Bibliography

General Geometrical Optics

Gardner, I C., "Elementary Optics and Application to Fire Control Instruments," Government Printing Office, 1924. (Out of print, but copies available in some libraries).
Gleichen, A., "Theory of Modern Optical Instruments." English translation by H. H. Emsley and W. Swaine, London, 1921.
Hardy and Perrin, "The Principles of Optics" (1932).
Jacobs, D. H., "Fundamentals of Optical Engineering" (1943).
Martin, L. C., "Applied Optics." 2 vol. (1932)

Books with chapters on Photographic Optics

Henney, K., and Dudley, B., "Handbook of Photography" (1939).
Eastman Kodak Co., "Kodak Reference Handbook" (1941).
Mack, J. E., and Martin, M. J., "The Photographic Process" (1939).
Neblette, C. B., "Photography" (1942).

CHAPTER III

CAMERAS

DESIGN AND CONSTRUCTION OF AERIAL CAMERAS

Revere G Sanders

INTRODUCTION

A STATEMENT to the effect that aerial photogrammetry would never have come into being if aerial cameras had not been developed is so elementary as to be an almost childish statement. However, photogrammetrists see and handle scores and scores of aerial photographs daily until they take on the routine appearance of any other of the expendable supplies of a drafting room or mapping room Although the aerial photograph may lose something of its importance through daily contact and familiarity, it must not be forgotten that the characteristics of the aerial photograph which enables it to be used in a routine, perhaps production line manner, are derived from the carefully designed features, construction, and method of operation of the surveying instrument of the air; the aerial camera.

It is probably true that the majority of people who think of aerial photogrammetry think of it as a subject which involves surveying and mapping only. It must be remembered, however, that photogrammetry means by definition, measurements made from photographs. Consequently, anyone who looks at a photograph and estimates the size of an object or the distance between objects, or the height of an object, is practicing photogrammetry regardless of the purpose for which he is seeking the information. The so called routine reconnaissance military or naval aerial photography and the subsequent evaluation or photo interpretation of the photographs is just as much in the realm of photogrammetry as is a photographic mission undertaken under closely controlled flying conditions and using precision cameras with a subsequent evaluation of the photographs through use of such elaborate instruments as the stereoplanigraph or the Multiplex. The only difference lies in the objective which is being sought. This broad consideration of the scope of aerial photogrammetry precludes limiting this chapter to consideration only of precise aerial mapping cameras, but extends consideration to general military and naval types of equipment as well As a matter of fact, more varied types of equipment are used in military and naval aerial photography than in precise mapping photography, which accounts for the fact that a large percentage of the equipment referred to in this chapter is the so-called military or naval type.

An interesting fact with regard to the development and manufacture of aerial cameras is that there are so few manufacturing concerns in the business Only three countries have produced any considerable quantities of aerial cameras and these are: Germany, Great Britain and the United States. Furthermore, in each of these countries, only one manufacturing concern seems to be responsible for almost all the development and camera production. In Germany it is the Carl Zeiss Company, in Great Britain it is the Williamson Manufacturing Company, Ltd., and in the United States it is the Fairchild Camera and Instrument Corporation The lack of a multiplicity of aerial camera manufacturers in the various countries is probably due to the following reasons:

a. Small normal (peace time) demand
b. Extensive development necessary
c. Unusually precise workmanship required
d. Need for expert and experienced designers and craftsmen
e. High cost of marketing such a specialized device

In the United States the Fairchild Camera and Instrument Corporation, has for

the past 25 years been responsible for 90% of the aerial cameras developed and produced However, the development which this company has carried on has been in the nature of what might be called detailed development. The United States Army Air Forces and the United States Navy Bureau of Aeronautics have, from the very start been responsible for the broad principles of design by virtue of the requirements which they have demanded be met by aerial camera manufacturers It is to these armed forces of the United States that the credit must go for the outstanding performance of American aerial cameras in the present conflict.

The aerial cameras produced by each of the above mentioned countries have their own very definite characteristics. The German aerial cameras are rather complex, particularly with respect to operating procedure. The aerial photographer must, of necessity, be quite expert and even so it is frequently easy to make mistakes. The British aerial cameras are quite different from the German in that they are more simple to operate. However, the most striking feature of British aerial cameras is the unit design which makes it possible to produce any major component of a camera in a multiplicity of factories without incurring trouble in the final assembly of the completed unit. This feature is of far more importance in time of war than in more normal times. The American aerial cameras are characterized by a greater ruggedness, longer life, and far more simple operating procedure than either British or German aerial cameras. However, this simplicity and strength is achieved at the expense of size and weight.

Prior to World War II, there were relatively few types of American aerial cameras The demands of the war, however, brought forth a large number of special types of cameras for as many special purposes. A few years ago, any article on the subject of aerial cameras would explain that there existed two general classifications of cameras; namely, single lens cameras and multiple lens cameras. At the present writing, the multiple lens camera type is militarily obsolete. However, in view of the great value of multiple-lens cameras in the recent past some data is included later in this chapter.

In general it can be said that all types of single lens cameras consist of the following main components:

a. Lens
b. Lens Cone
c. Shutter
d. Camera Body
e. Camera Drive Mechanism
f. Film Magazine

The *Lens*, obviously, needs no further description.

The *Lens Cone* in its truest form is a truncated cone which contains the lens and shutter in one end, and a means of attachment to the camera body on the other end.

The *Shutter* serves to admit or exclude light from the camera. The 'between-the-lens' shutter is most common and is so called because the leaves of the shutter operate between the elements of the lens. Also contained in the shutter and operating between the lens elements is the diaphragm which serves to close down the aperture when the sunlight is too strong. The haze cutting filters are attached to the front of the shutter by means of a positive snap lock. This whole assembly (lens, shutter, and filter) mounts in the end of the lens cone. In most cases lenses of the same focal length can be interchanged in the shutters in event

of breakage. Shutters are likewise interchangeable in cones of the same type, providing the focusing-posts are kept with the shutter

The *Camera Body* houses the camera drive mechanism, driving motor, operating handles and levers electrical connections and switches, and other details which may be necessitated by special requirements The lens cone bolts to the bottom of the camera body while the top of the camera body provides a seating surface for the magazine.

The *Camera Drive Mechanism* is the power unit and power distributor for the entire camera. The electric motor causes the various cams, gears, and shafts to turn. By means of rods and couplings the power is routed to the shutter and to the magazine. When the cycle is complete, the camera drive upon receipt of an electrical or mechanical impulse, operates the shutter thus taking a photograph This component can be considered as the brains of the camera

The *Film Magazine* is primarily a container for the film. Besides this function, however, it has other important duties. The film magazine contains a driving mechanism which receives power from the camera drive mechanism and thereby winds the film after each picture has been taken The mechanism in the magazine is also designed to wind exactly the right amount of film without resulting in double exposures or in film waste This function is known as 'metering' Furthermore, the magazine contains a means for holding the film exactly in the focal plane while the photograph is being taken. The magazine fastens to the top of the camera body with a form of latch, thus completing the camera

There are several exceptions to the above which may be confusing if not explained In the type F-56 camera with its semi-interchangeable components, the foregoing description of the lens, shutter, filter, camera drive mechanism, and film magazine remain the same. However, the lens cone and the camera body are not separated, but form one integral casting

For the type K-20 the foregoing description holds only for the lens, filter, and shutter The lens cone, camera body, and magazine are all merged into one continuous casting The camera drive mechanism, the drive mechanism of the magazine, the metering means, and the film flattening means are all intermingled and not separated one from the other.

Aerial Camera Lenses

A thorough discussion of the characteristics of photographic lenses is given in Chapter II, and therefore will not be repeated here Listed in Table I (in order of increasing focal length) are a group of available aerial camera lenses. The focal length lens to use for any particular purpose is generally governed by the desired scale of the photograph Methods for computing the scale are discussed in Chapter VI.

Shutters

In general there are three types of aerial camera shutters. The three types are known as:

a. Between-the-lens
b. Focal Plane
c. Louvre

The function of all three is the same; namely, to prevent light from reaching the film emulsion except for the brief instant during which the photograph is being made. The important characteristics of the above shutters are:

a. Speed
b. Speed accuracy

TABLE I. AVAILABLE AERIAL CAMERA LENSES

Focal Length	Aperature	Manufacturer	Maximum Negative Size	Camera Used In	Camera Angle (film diagonal)	Type
5¼″	f/6 3	Bausch and Lomb	7″×7″	F-56	86°38′	Wide Angle Metrogon
6″	f/6 3	Bausch and Lomb	9″×9″	K-17 K-22 T-5	93°23′	Wide Angle Metrogon
6″	f/6 8	Goertz	5½″×6″	T-3A	68°18′	Conventional
6⅜″	f/4 5	Bausch and Lomb Eastman Kodak Ilex	4″×5″	K-20 K-25	53°19′	Conventional
7″	f/2 5	Eastman Kodak	5″×7″	K-21 K-24	63°07′	Night Photography
8¼″	f/4 0	Bausch and Lomb	7″×7″	F-56	61°56′	Conventional
8¼″	f/6 8	Goertz	7″×9″	K-3B	69°18′	Conventional
10″	f/4 5	Eastman Kodak	5″×7″	F-8	46°32′	Conventional
12″	f/2 5	Eastman Kodak	9″×9″	K-19B	55°53′	Night Photography
12″	f/5 0	Eastman Kodak	9″×9″	K-17 K-22	55°53′	Conventional
13½″	f/3 5	Eastman Kodak	9″×9″	K-12 K-19	50°29′	Night Photography
15″	f/5 6	Wollensak	5″×7″	F-8	32°00′	Telephoto
20″	f/5 6	Bausch and Lomb	7″×7″	K-16 F-56	27°48′	Telephoto
24″	f/6 0	Bausch and Lomb Eastman Kodak	9″×18″	K-17 K-18	45°30′	Conventional
40″	f/5 0	Baker	9″×9″	K-22	18°5′	Conventional
40″	f/8 0	Bausch and Lomb	7″×7″	K-15A F-56	14°7′	Telephoto

c. Efficiency
d. Life

The 'speed' of an aerial camera shutter refers to the duration of the exposure. It is the length of time expressed as a fraction of a second from the instant when the shutter just begins to admit light to the film until the instant when the shutter finally cuts off the light. In other words it is the elapsed time from the start of the opening of the shutter until the completion of the closing of the shutter. In aerial camera between-the-lens shutters this time interval (omitting a few special purpose cameras) ranges from about 1/150 of a second to about 1/500 of a second Most aerial camera shutters of this type can be adjusted to give at least three different speeds such as 1/125 of a second, 1/250 of a second,

and 1/500 of a second This adjustability is by means of a knob directly on the shutter or remotely by some mechanical or electrical linkage to the shutter.

The importance of shutter speed in an aircraft camera is due to the fact that the camera is moving while a picture is being taken of a stationary object. The importance of shutter speed grows greater each year as newer and faster flying aircraft are used in aerial photography It is not uncommon for aerial cameras to be carried in aircraft having a cruising speed in the neighborhood of 300 miles per hour This corresponds to a speed of approximately 450 feet per second If the aerial camera shutter is set for a 'speed' of 1/100 of a second it can be seen that the aircraft would travel 4.5 feet over the ground during the exposure. If the camera in the aircraft is a K-25 camera equipped with a $6\frac{3}{8}''$ lens mounted vertically and if the aircraft is flying at an altitude of 500 feet (not unusual for combat photography) the image on the focal plane of an object on the ground will move approximately 05″ during the exposure. Thus, all object points on the ground become lines about 05″ long on the photograph This condition can be perceived by the naked eye and is called 'image movement ' This 'movement' can easily be identified because the stretching out of images is always in the direction of flight In spite of this very considerable movement mentioned, the photographs are still quite usable insofar as large features on the ground are concerned. However, for purposes of photo interpretation for detailed intelligence reports or for highly accurate mapping, a far higher quality photograph is required

For a given aircraft flying speed, improvement in photograph quality is obtained by reducing the image movement by speeding up the camera shutter The improvement is in proportion to the increase in shutter speed. Also for a given shutter speed and aircraft speed, the 'movement' can be reduced by flying at a higher altitude, although percentage distortion due to movement is still the same. Obviously the photograph quality can be improved by slower flying speeds.

The ultimate goal in the reduction of image movement is to reach a point where it is equal to the resolution of the lens at its most used diaphragm opening At present, practically all aerial camera lenses resolve point objects on the ground to minute circular images on the film not exceeding .002″ in diameter The physical and mechanical limitations of the between-the-lens shutters are such that this goal is not yet achieved in cases of high aircraft speeds and long focal length cameras.

Shutter speed also serves to minimize the effect of aircraft vibration on photograph quality. Vibration is always present in aircraft to varying degrees. By properly designed shock absorbing mounts, the aerial cameras can be insulated from these vibrations. However, if the mounts are inefficient the vibration may impair the quality of the photograph The presence of vibration effect in a photograph is recognized by a general lack of sharpness and sometimes blurred appearance of ground objects This is distinguished from 'image movement' by the fact that the stretching out of the images is not consistently in the direction of flight.

High shutter speeds will reduce the deleterious effect of vibration on photograph quality. However, the real solution with respect to vibration lies in properly designed aerial camera mounts.

Shutter Speed Accuracy

Since shutter speed is so important in relation to photograph quality, it is only to be expected that it is very important that the speeds of a shutter be

accurate. The accuracy of a shutter at any given speed is expressed as a percent of the elapsed time. It is customary at present to require that the top speed of a shutter be accurate within plus or minus 10%. The intermediate and low speeds are generally somewhat lower in importance than the high speed, and manufacturers take advantage of this fact by allowing an accuracy tolerance of plus or minus 15 percent on all but the high speed.

Shutter speed accuracy is also important in respect to the use of color film which is growing in popularity and use. Color film at present is rather critical

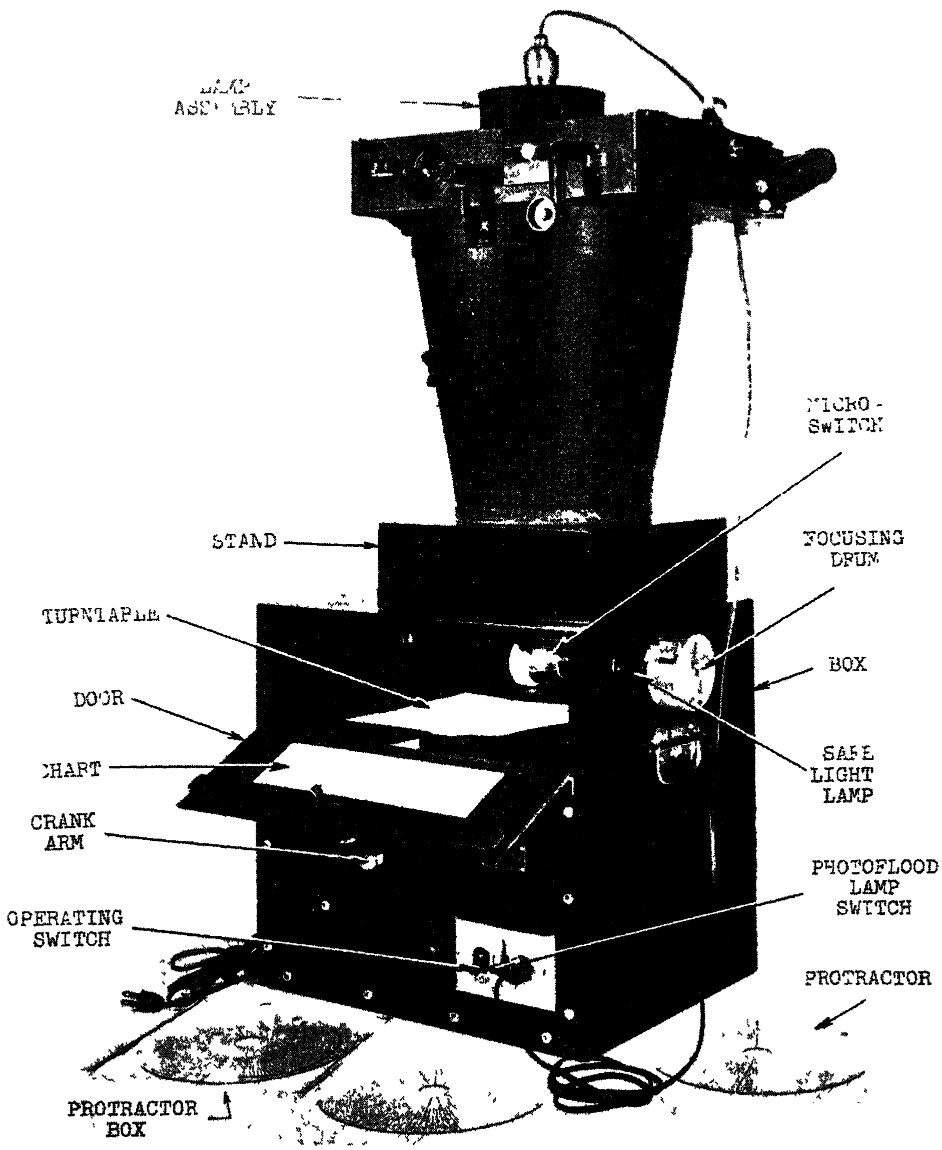


FIG. 1. Type F-82 shutter speed testing machine.

in regard to time of exposure and inaccurate shutter speeds could result in poor exposures

So important is it considered that shutter speed accuracy be maintained that aerial photographic organizations are frequently equipped with apparatus to measure the elapsed time for any given speed This apparatus is known by the very descriptive name of 'shutter speed tester.' A low cost, very portable shutter tester useful for medium range aerial camera shutters is the 'Mark Hurd Shutter Tester.' The Fairchild Model F-82 shutter tester (see Fig 1) covers a wide range of shutter speeds from that of slow speed ground cameras to the highest speed aerial camera shutter but is larger, heavier, and more costly. Aerial camera manufacturers make up special shutter speed testers for use in the manufacturing process. These are usually photo-electric or electronic devices

Shutter Efficiency

Although shutter speed and shutter efficiency are being treated separately here, they are nevertheless closely interrelated Considerable difference of opinion has existed in the United States among manufacturers of camera shutters of all kinds as to the proper method of rating the speed and efficiency of a shutter The following is a discussion of the method of computing speed and efficiency which has been applied to aerial camera shutters up to the present time

There are many ways of rating the speed and efficiency of shutters All are based on the two fundamental performance characteristics of a shutter; namely, the total time that the leaves are open (from the instant they start to open until they are fully closed), and the ratio between the actual light admitted and the total light which would have been admitted if the shutter had been open to full aperture during the above total time The diagram shown in Fig. 2 illustrates these two characteristics It may be applied to any of the three types of shutters used in aerial cameras, between-the-lens, focal plane, and louvre

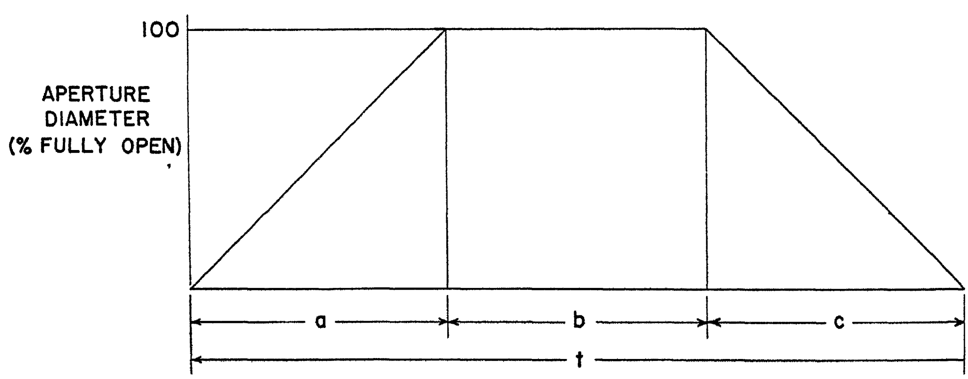


FIG. 2. Determination of shutter speed.

This figure is a plot of shutter opening versus time. The curve shown represents the approximate rate of change of light during the cycle. Actually the rate of change during opening and closing is not exactly linear but a straight line closely approximates the actual curve in most designs

The first fundamental characteristic of a shutter is represented by the distance (t) marked off on the time scale. This figure is the only 'true' measure of the speed of a shutter. It directly determines the amount of object motion which will be recorded in the photograph. For practical reasons a modifying factor is

usually applied to this time (t) by the manufacturer to get the 'effective' shutter speed. It is argued for instance that the light admitted before the leaves are 20% open (on the diameter), in the case of between-the-lens shutters, does not register on the negative at all and that it may therefore be disregarded Likewise, for the light passed during the final 20% of closure (This is a reasonable assumption when it is remembered that the light varies as the square of the diameter so that only about 4% of the light would have been admitted when the aperture reached 20% of full open) In such a case the effective speed (manufacturer's rating) is:

$$\text{Speed} = b + .8(a+c)$$

Where a = time to open
b = time fully open
c = time to close

When comparing shutters of various types and makes, however, the best standard for comparison is the 'true' speed (t)

Efficiency must also be known before the quality of a shutter can be fully evaluated, since two shutters having the same 'true' speed could admit radically different amounts of light through the same sized opening. The method of determining efficiency is illustrated in Fig. 3 The heavy line represents the rate of change of light striking the film during the cycle as in the preceding figure The area under this curve (double cross hatch) represents the quantity of light actually admitted by the shutter during the cycle, while the area under the horizontal dashed line (entire shaded area) represents the total light that would have been admitted had the shutter been fully (100%) open during the entire time, t.

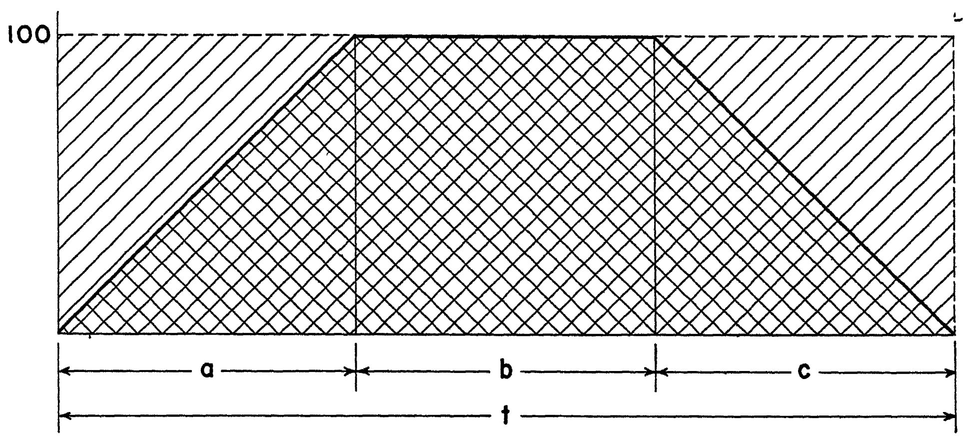


FIG. 3. Determination of shutter efficiency

The ratio of these two quantities of light is called the efficiency of the shutter It can be expressed in terms of the symbols on the above diagram as follows·

$$\text{Efficiency} = \frac{b + \dfrac{a+c}{2}}{t}$$

Here again liberties are taken by manufactures because of the ineffectiveness of the light at very small apertures. In the case of between-the-lens shut-

ters, for the method of speed rating derived above, the corresponding efficiency formula is

$$\text{Efficiency} = \frac{25b + 12(a + c)}{25b + 20(a + c)}$$

Speeds and efficiencies of shutters are measured by various optical and electrical methods. Usually a permanent record is obtained which is essentially a plot of the aperture versus time from which measurements of the values given in the above formulas can be made A rough check of the efficiency of a between-the-lens shutter can be made visually by tripping off the tension, turning the leaves through their cycle slowly by hand, and noting the portion of the cycle that the leaves are fully open, relative to the total cycle from actual start to open to just closed. Only the efficiency at the highest speed can be obtained in this way since at the retarded speeds the leaves are held wide open for a greater portion of the cycle

Efficiency of a focal plane shutter is determined by the design of the camera and the lens used It can be found by measuring the curtain slot width (W) and the distance between curtain and film (D), and substituting these values in the following formula

$$\text{Efficiency} = \frac{W \times f}{Wf + D}$$

where f = the lens stop

The efficiency of the louvre type shutter is best determined by measuring the image of a trace left on a piece of moving film or paper by light passing through a single louvre. Graphical analysis can also be used to get an approximation of the efficiency of this type of shutter

With the entrance of manufacturer's of amateur cameras into the aerial camera field, due to war conditions, some confusion has arisen as a result of different viewpoints with respect to the method of determining shutter speed. The United States Army Air Forces, the largest purchaser of aerial cameras, therefore undertook to settle the question. A civilian employee of the U S A A F., Mr. A. H. Katz, made a very thorough study and analysis of the problem and made recommendations which will tend to bring a very desirable standardization to the matter of shutter speeds.

Shutter Life

The life of a shutter is the period over which it can be depended upon to operate at the speeds marked on the adjusting dial with the specified accuracy and without failure. Reliable life is most important from a practical point of view. Unexpected failure of an aerial camera shutter brings a photographic mission to an abrupt end In a commercial enterprise this may involve a considerable monetary loss; in a military combat mission the failure may have exceedingly serious consequences.

Life of a shutter is usually expressed in number of cycles of operation rather than as a guarantee over a specified period of time. In using the 'life cycle' concept it is assumed that the shutter is maintained properly lubricated, cleaned, and free from corrosion.

Most military specifications require that shutters have a life of 10,000 cycles of operation. In stating the requirement the specification usually stipulates that in life tests a certain percentage of cycles shall be at each shutter speed. Contrary

to the common belief, between-the-lens shutter will not have longer life at slow shutter speeds The construction of the mechanism is such that the shutter will have longest life when operated at the maximum speed

Between-the-Lens Shutter

The type of aerial camera shutter most popular in the United States carries the very descriptive name of 'between-the-lens' (see Fig. 4) This name is derived from the fact that the shutter leaves operate in the fraction of an inch space which separates the front and rear elements of an aerial camera lens.

The reasons for the popularity of the 'between-the-lens' type shutter for aerial camera use are durability, mapping application, and absence of obstructions in the lens aperture

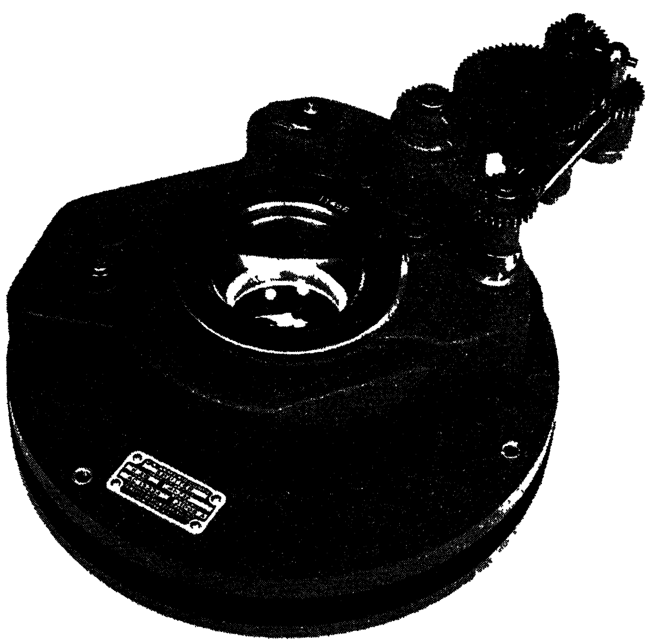


FIG 4. K-3B Camera shutter assembly with Metrogon lens Note partially opened shutter leaves

Durability—This characteristic is equally important in either military or commercial aerial photographic projects The shutter is the heart of an aerial camera which must function over long periods of time and under conditions which range from the humid, sultry heat of the tropic jungles to the dust laden heat of tropic deserts to the minus 70° Fahrenheit temperatures found in Arctic regions and at sub stratosphere altitudes Over a period of years, experience has shown that with proper maintenance and care, this type shutter can be depended upon to operate consistently whereas the other types of shutters under similar conditions do not perform as favorably.

It is this ability of the between-the-lens shutter to continue to operate with accuracy and high efficiency in the face of the abuse of extreme operating conditions that has been responsible for its extensive use in American aerial cameras

Mapping Application—Theoretically all rays of light entering an aerial camera lens pass through a small area usually located symmetrically between the elements of the lens Shutter leaves operating near this region have the effect of cutting off all light passing through the lens instantly. It is this feature of design that contributes to the high efficiency of between-the-lens shutters and, more important, to the use of this type of shutter in aerial photographic mapping

This characteristic of the between-the-lens shutter which enables it to admit light to all parts of the negative instantly upon opening and to cut off the light from all parts of the negative instantly at the end of the selected exposure time interval preserves in the negative the precise relationship of all object points photographed The resultant photographs can be used confidently for whatever type of mapping may be under consideration. No errors will be present as a result of shutter operation

The only other type of aerial camera shutter popular in the United States is the so called 'Focal Plane' type The fundamental principle of operation of the focal plane shutter introduces positional errors in the relationship of object points photographed of varying magnitudes Consequently, of the two shutter types, only the between-the-lens type can be used for any mapping purpose. Photographs taken through between-the-lens shutters are entirely suitable for all applications of aerial photography where shutter speeds faster than 1/500 of a second are not necessary Consequently, any person or organization faced with the possibility of having to take aerial photographs for any of the various applications and yet not having a special camera for each special purpose, will be best equipped with a camera having a between-the-lens shutter.

Absence of Obstructions—The shutter leaves of the between-the-lens shutter are pivoted in a ring outside of the lens aperture. Consequently, when the shutter leaves are open, the entire area of the aperture is open for the unobstructed passage of light Thus, when difficult light conditions are encountered the availability of the maximum amount of light is of importance. In this respect, the between-the-lens shutter is superior to the louvre type.

Shutter Construction.—The fundamental feature of an aerial camera shutter is the diameter of the clear aperture through which the light must pass. This feature is often referred to as the 'size of the shutter ' The diameter of the aperture has a very direct effect upon the speed of the shutter. Since the shutter leaves are pivoted outside of the circular aperture, the leaves must rotate to move out of the aperture to admit light and then swing back to close the aperture. The greater the diameter of the aperture the longer the distance the blades must travel and the longer will be the time involved. Furthermore, a small aperture requires only small blades to cover it whereas a large aperture requires larger blades. The greater weight of the larger blades results in greater inertia and hence requires greater power. Thus it can be said that as the shutter size increases the maximum shutter speed decreases.

Table II shows the Fairchild aerial camera shutter sizes available, speeds, and the lenses normally used in the shutters:

The principal components of an aerial camera between the lens shutter are:

a. Leaf center
b. Actuating cam assembly
c. Retard
d. Spring housing assembly
e. Trip mechanism
f. Shutter housing

The Leaf Center (see Fig. 5 and 6) consists of two metal plates in most cases

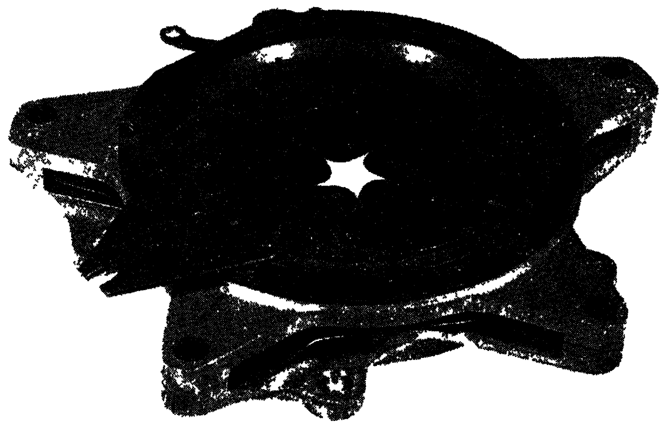


FIG 5. Leaf center assembly, showing partly opened shutter, and iris diaphragm.

between which the shutter leaves and diaphragm are sandwiched The shutter leaves or blades as they are sometimes called are four or five in number, and generally somewhat pear shaped The leaves are pivoted at their small end in the leaf center plates All leaves are connected by links which cause them all to rotate simultaneously when actuated Spacers cause the leaves to stack up one on top of the other and prevent one leaf from cutting into the other thereby causing a shutter failure. The diaphragm consists of 3 to 20 leaves about $\frac{1}{4}''$ wide which have a curvature similar to the circumference of the aperture circle

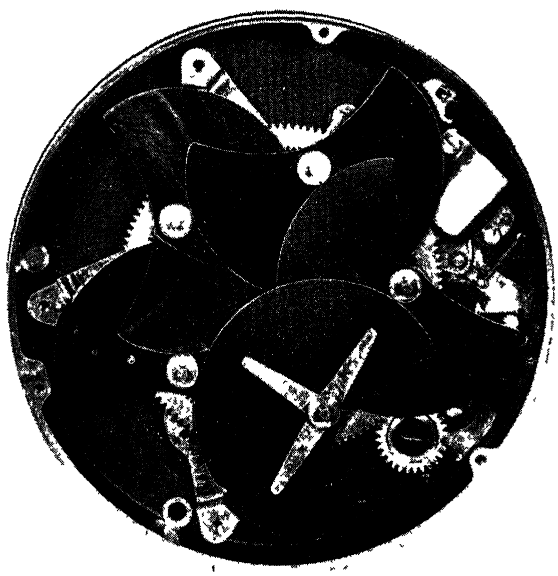


FIG. 6. Experimental Rotary-type shutter showing arrangement of shutter blades and partly obscured driving ring gear.

and are about 100 degrees in length. At each end of each diaphragm leaf is a stud One stud is pivoted in one of the leaf center plates while the other pivots in a slot in a rotatable diaphragm actuating plate. As the plate rotates, the diaphragm leaves are moved from a position entirely out of the clear aperture to some position within the aperture The multiplicity of diaphragm leaves causes a circular aperture to be maintained in spite of the varying diameter.

The variation in the diameter of the aperture caused by the adjustment of the diaphragm leaves gives rise to the series of lens *stops* found in aerial camera shutters. These stops are the *f* values which are a measure of the light gathering power of the aperture These *f* values are the ratio of focal length of the lens to the diameter of the aperture.

TABLE II AVAILABLE FAIRCHILD BETWEEN-THE-LENS SHUTTERS

Aperature Diameter in inches	Designation	Speed (Seconds)		Efficiency		Lens used with
		Max.	Min	Max	Min	
0.606	Unit 298	1/150	1/35	No Data		5¼″ *f*/6.3
0.709	Unit 335	1/300	1/50	90%	68%	6″ *f*/6.3
1.234	Unit 280	1/500	1/125	83%	64%	6⅜″ *f*/4.5
1.650	Unit 322	1/225	1/75	82%	65%	8¼″ *f*/4.0
2.079	Unit 152	1/225	1/75	82%	65%	12″ *f*/5.0
2.080	Unit 323	1/225	1/75	82%	65%	20″ *f*/5.6
3.464	Unit 156	1/150	1/50	79%	57%	24″ *f*/6.0
3.464	Unit 325	1/150	1/35	79%	57%	40″ *f*/8.0
3.560	Unit 132	1/100	1/25	81%	63%	13½″ *f*/3.5 12″ *f*/2.5

The Actuating Cam Assembly (see Fig. 7) is actuated by the shutter spring and transmits this rotational force to the shutter leaves. It causes the shutter leaves to open as instantaneously as possible, to stay open for the prescribed time interval, and then to close instantaneously. This assembly includes a mechanism which prevents the shutter leaves from bouncing open after an exposure which would impair the photographic quality.

The Retard assembly serves to slow down the operation of the shutter when required, to permit exposures of longer duration than that at the maximum speed of the shutter. At the maximum speed of the shutter, the retard mechanism is not used. Thus, the un-retarded speed is sometimes called the *free* speed of the shutter.

When the shutter speed knob is set for one of the low speeds of a shutter, a sector having a serrated periphery is made to contact a rocking pallet. After the shutter leaves have opened, the serrated sector must rub past the pallet until clear of the pallet, at which time the shutter leaves are free to close. The time during which the shutter leaves stay open depends upon the number of serrations which must rub past the pallet.

Spring Housing Assembly —The motive power which operates the shutter is stored in a coil spring. The spring in turn is enclosed in a housing which includes means for winding the shutter spring and for preventing the spring from unwinding until released by the trip mechanism. When the shutter is at rest, the leaves are held in the closed position by spring being slightly twisted. This is known as 'initial tension.' The spring is wound beyond this point one or more full turns depending on the design to provide power to operate the shutter. If, by accident, the shutter spring is wound much beyond this operating point, the spring

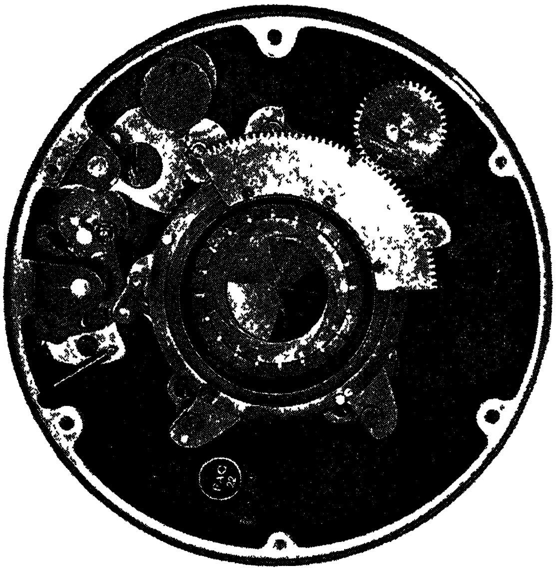


FIG. 7 Typical between the lens shutter mechanism

is said to be 'over wound,' which may result in serious damage to the shutter.

The Trip Mechanism serves to release the power stored in the spring thereby starting the exposure cycle. A twisting action is transmitted to the trip mechanism from the camera drive mechanism which causes a pawl to be withdrawn from the shutter actuating cam. The cam which is tied to the leaf center by a link is then free to turn with the full force of the spring behind it and the shutter leaves open and close in the interval of time desired The trip mechanism also is designed to prevent the shutter tripping twice in one cycle which would cause a double exposure and release the initial tension of the spring

The Shutter Housing consists of two parts; namely, the main housing and the cover. The main housing carries all of the shutter mechanism, leaf center, spring housing, and the rear element of the lens The shutter housing cover carries only the front element of the lens The size of the shutter housing depends upon the size of the shutter leaves The large leaves require space into which to swing when the aperture is uncovered. Thus some of the larger shutters require housings as much as ten inches in diameter

Focal Plane Shutter

The focal plane shutter is the only other type of shutter popularly used in American aerial cameras This shutter is so named because it operates close to the focal plane of the camera This type shutter is also known as a 'curtain' shutter because of the fact that it operates somewhat in the manner of a common window curtain.

The focal plane shutters are being used today primarily because they permit higher shutter speeds than can be had with the between-the-lens shutters. There are some specialized types of photography, almost entirely military, which require very high speed shutters. Some military photography calls for camera operation in aircraft flying only a few hundred feet above the ground

and at speeds of 300 m.p h and over As pointed out earlier in this chapter, object points on the ground will be stretched out into lines on the negative unless the shutter operation is fast enough to stop the motion.

An analysis of the image movement in a photograph which causes blurring resolves itself into the following factors

1. The shutter speed of the camera
2. The focal length of the camera
3. The speed of the aircraft carrying the camera
4. The altitude from which the photographs are made.

The following are the symbols used in making the necessary computations·

M = Image movement in inches
H = Altitude of flight in feet
F = Focal length of camera in inches
S = Photograph scale in feet per inch
V = Ground speed of aircraft in miles per hour
T = Camera shutter speed in seconds
Airplane movement over ground in feet per second $= 1\ 467 \times V$
Airplane movement during exposure $= 1\ 467 \times V \times T$

$$\text{Image movement during exposure } (M) = \frac{1.467 \times V \times T}{S}$$

$$S = \frac{H}{F}$$

The amount of image movement to be expected due to aircraft speed alone with given equipment and under given flying conditions can be found from the basic formula:

$$(M) = \frac{1.467 \times V \times T}{S}$$

It has been found experimentally that, exclusive of aircraft vibration, photographic quality will be quite usuable when the image movement does not exceed about .01 inch. Thus, the desirable minimum shutter speed can be found for any given vertical photographic project by solving for 'T' in the formula above. A few sample computations of hypothetical low altitude problems will show the limitations of the existing between-the-lens shutters for this specialized type of photography.

Focal plane shutters contribute to reducing the cost of aerial camera equipment which utilizes interchangeable lens cones Since the focal plane shutter is located in the camera body, the lens cones consist only of a simple metal truncated cone and a lens with a diaphragm. On the other hand, the between-the-lens shutter is a relatively costly item of equipment and each lens cone must be equipped with one, making a complement of between-the-lens camera equipment more expensive than the corresponding focal plane camera equipment.

Shutter Construction—A focal plane shutter in its simplest form consists of a black rubberized cloth curtain slightly wider than the film, and long enough to wind back and fourth on rollers which are located just beyond the ends of the negative area (see Fig. 8).

A focal plane type shutter consists of the following principal components:

a. The curtain

FIG 8 Type F 8 Focal plane shutter assembly.

b. The capping curtain or other means for covering film during rewind
c. The driving means

The curtain consists usually of a piece of closely woven, light, flexible cloth impregnated with crude rubber. The curtain is somewhat wider than the frame which determines the negative size and is a little longer than twice the length of the negative. The curtain is attached to rollers, one at each end of the negative area. This assembly of curtain and rollers is so located in an aerial camera that the direction of travel of the curtain, as it winds from one roller to the other, is parallel to the ground track of the aircraft (assuming correct crab).

Each curtain has a slit which is necessary in order that light may be admitted to the film. This slit is across the narrow dimension of the curtain and hence is parallel to the rollers. The edges of the slit are made rigid by a metal strip or wire rod reinforcing. The slit is at the center of the curtain and the two sections thus made by the slit are held together by very narrow strips of tape at the edges. The size of the slit depends on several factors and slits ordinarily range from $\frac{1}{8}$ to 1″ in width.

Before making an exposure, the curtain is wound on to one roller against a spring tension in the opposite roller until the slit is clear of the negative. In this position, the light is prevented from striking the film by the remainder of the curtain. When the shutter is tripped, the slit moves across the negative toward the opposite roller. Light passes through the slit and an image is formed on the negative in successive bands as the slit moves from one roller to the other. When the slit passes beyond the negative area the exposure is completed and the cur-

tain is ready to be wound back on to the other roller in preparation for the next exposure.

The capping curtain.—The exposure described above is only made by the shutter when it moves one direction. However, unless means are taken to avoid it, light would be admitted to the film while the curtain is being wound back to the starting position. Frequently aerial cameras are equipped with an auxiliary curtain known as a capping curtain located between the lens and the shutter curtain, usually as close as practical to the latter. The capping curtain is made of the same material as the shutter curtain and has its own set of rollers. Instead of a slit, it has an open area slightly larger than the negative area

When an exposure is about to be made, the capping curtain is rolled on the roller on the opposite side of the negative area from the shutter curtain slit In this position the open area of the capping curtain is in position in front

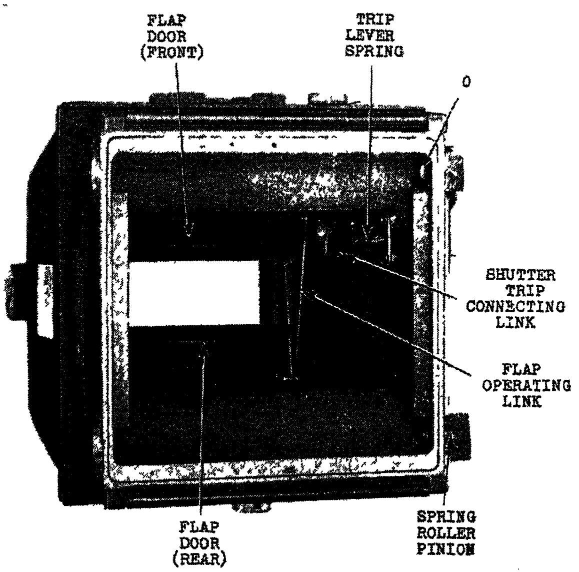


FIG. 9. Capping trap doors as used in Type K-15 camera.

of the film. As the shutter curtain slit moves across the negative area, nothing interferes with the light passing through the slit. However, when the rewind cycle starts, the capping curtain is first pulled across the negative area after which the shutter curtain slit moves back to the starting position again and thereby prevents light from striking the film. There are many variations of the foregoing procedure all of which achieve the same result—protection of the negative during the rewind cycle.

The same capping effect can be obtained mechanically thereby rendering a capping curtain unnecessary. Here again there are many variations according to the camera design. In the model K-15, for instance (see Fig. 9), trap doors behind the interior element of the lens are hinged and connected to the camera trip lever. As the trip lever is pressed, the doors automatically rotate out of the cone of the light rays. As soon as the lens cap is entirely away from the lens the shutter curtain slit moves across the negative area. As soon as the trip lever is

released, the doors again seal out the light and the shutter can be rewound

The Driving means—As in the between-the-lens shutter, the driving force for the focal plane shutter is a coil spring The spring is usually installed in one of the rollers.

For any given slit width, the shutter speed is a function of the speed of travel of the shutter curtain slit across the negative area Thus a means is provided for winding the spring several turns to a tension sufficient to provide a curtain travel which will give the desired shutter speed. After the spring is once wound to the desired tension, it is rewound by the turning of the roller as the shutter curtain is wound back to the starting point. Only when it is desired to change the shutter speed is it necessary to alter the shutter spring tension.

Only recently have any attempts been made to regulate or govern the movement of the slit across the film. When the shutter curtain is first released, it must accelerate from its static condition to a condition of maximum speed In most designs maximum speed is not attained within the width of the negative and hence the speed at the end of the exposure is much greater than at the beginning This variation in exposure due to changing speed of the curtain is called 'tapering.' 'Tapering' is always present unless a form of governor is used to maintain a constant speed of shutter curtain travel However, in well made aerial camera focal plane shutters without governors, the taper is kept to a minimum and does not greatly impair the photographic quality In focal plane shutters of faulty design or construction, the slit may travel in a jerky manner This gives rise to alternate bands of exposures of varying density. This condition is known as 'banding.' It should not be noticeable in high quality focal plane shutters.

Louvre Shutter

British made aerial cameras have in the past used a louvre type shutter (see Fig. 10) in preference to the between-the-lens shutter. The choice is a matter of opinion and the technical experts of both countries can marshall impressive arguments in favor of their particular choice.

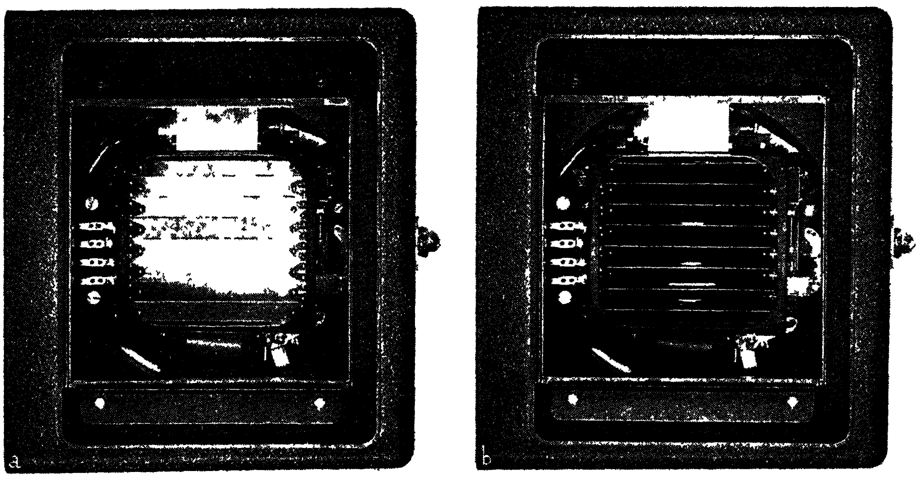


FIG. 10. Louvre shutter (a) closed; (b) open.

The louvre shutter interrupts the light passing through the lens in a similar manner to the between-the-lens shutter. It does not have the handicap from a mapping standpoint that a focal plane shutter has.

The louvre shutter is used by the British in connection with cameras having any focal length lens. However, in the United States, interest in louvre type shutters is confined to their application to long focal length wide aperture lenses. For this class of lenses, physical limitations preclude use of between-the-lens type shutters The mass of the shutter leaves and operating parts become too great for fast shutter speeds. Furthermore, in providing space for the leaves to rotate into, the entire shutter assembly becomes too large for practical use. One alternative is to utilize the focal plane type of shutter. However, this is not desirable because of the mapping limitation and because of life considerations.

The louvre shutter is usually placed in front of the lens—sometimes in back of the lens—but in either case, as close to the lens element as possible. In some designs for long focal length wide aperture lenses there is sufficient space for a louvre shutter assembly between the lens elements.

The louvre shutter assembly consists of a rectangular frame and associated mechanism. The frame supports a multiplicity of louvres—strips of metal in the neighborhood of $\frac{1}{4}''$ wide, and sufficiently long to project beyond the cone of light rays which enter, pass through, or leave the lens Each louvre is supported in the shutter frame at its ends by a bearing placed on the central longitudinal axis of the louvre. The bearing points of the louvres are so spaced that when the louvres are in a flat position, each one overlaps the next sufficiently to prevent the passage of any light through the lens

On one end of each louvre axis is a small gear or pinion. The pinions of all louvres are engaged by one rack The rack is spring actuated and by its motion causes the louvres to rotate from the flat closed position to a vertical open position. The closing is either caused by continuing the rotation of the louvres beyond the vertical to a flat position or by reversing from the vertical to the original flat position. When the louvres are in the vertical open position, the light is permitted to pass through the lens thereby making an exposure. Obviously, there must be a considerable number of louvres to completely cover the lens aperture. When these louvres are all in the vertical position their thickness totals an area which reduces the light gathering area of the lens aperture by an appreciable amount Furthermore, the width of the louvres in the vertical position tend to interrupt oblique rays entering the lens which further reduces the light reaching the film. The efficiency of louvre shutters is not always as favorable as that of between-the-lens shutters; and the mechanical durability has not always proved as great.

The great advantage of louvre shutters lies in their application to reasonably high speed shutters for large lens apertures—apertures of 5″ diameter and greater The mass of each louvre does not increase to the same extent as that of a between-the-lens shutter leaf for an increase in lens aperture Thus less power is required and greater speed is possible. Furthermore, while a large between-the-lens shutter might exceed 10″ in diameter, a louvre shutter need not be much larger than the lens with which it is used.

Camera Drive Mechanism

In any case, whether a camera is operated by hand or automatically, every aerial camera has its drive mechanism. This mechanism performs many functions and varies from a simple mechanism to a relatively complex unit.

In operating any aerial camera, two primary functions must be performed;

namely, wind the shutter and advance the film for the next exposure. The camera drive mechanism is located in the camera body which is usually located between the lens cone and the film magazine (see Fig. 11 and 12) Rods extend downward from the drive mechanism to the shutter called the wind rod and the trip rod One or two male couplings extend upward to connect with female couplings in the magazine. Power is put into the camera drive by manual turning of the hand crank or by electric motor. This power input is distributed to the shutter and to the magazine through the rods and couplings mentioned above. Thus, both the winding of the shutter and of the film take place simultaneously. Clutches

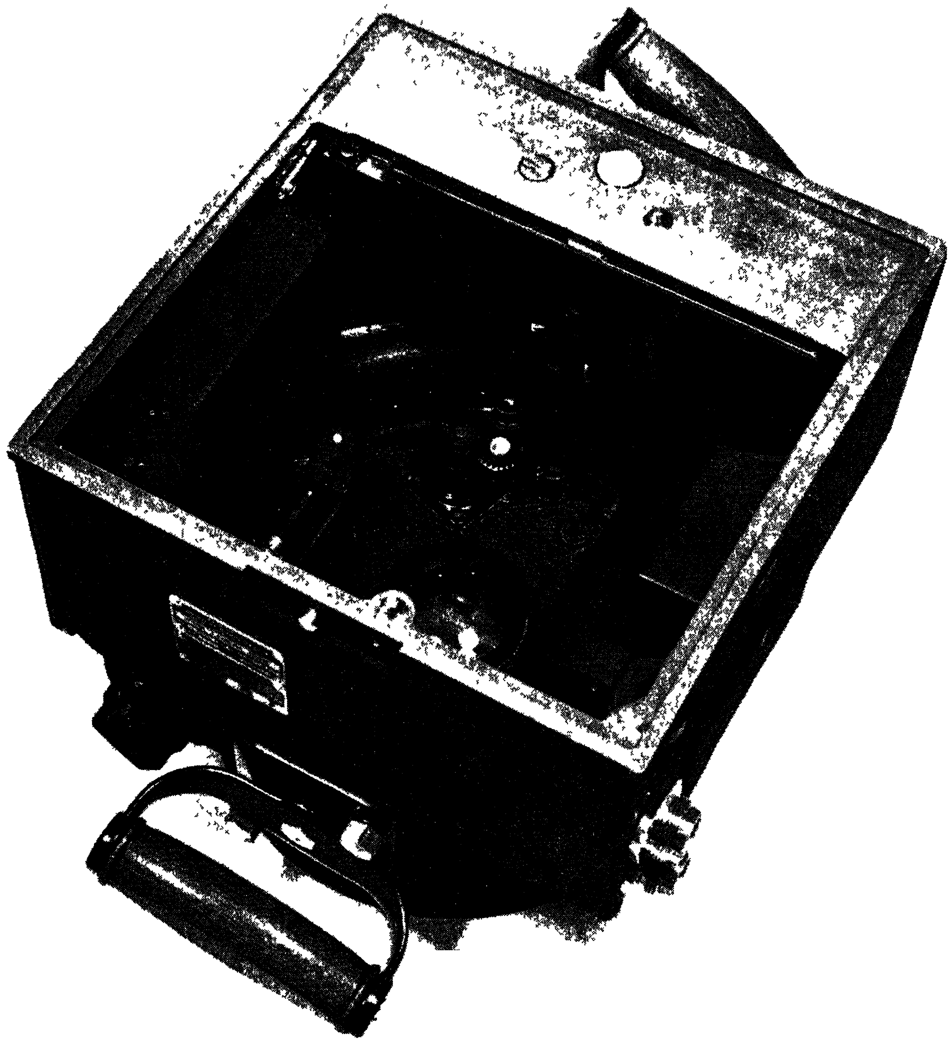


FIG. 11. Type F-56 cone showing drive mechanism in place, and also showing wind and trip rods connecting shutter, and wind and trip couplings for magazine.

are provided in the drive mechanism which disengage as soon as the shutter or the magazine is fully wound. In the hand winding of the camera the operator can readily tell when the camera is fully wound by the reduction of the winding load which occurs when the clutch disengages thus permitting the winding handle to turn freely.

The drive mechanism also serves to transmit the tripping force to the shutter and to the magazine. This force originates either by pressure of a manual trip lever or by an electrical impulse Here again the force is distributed by the drive

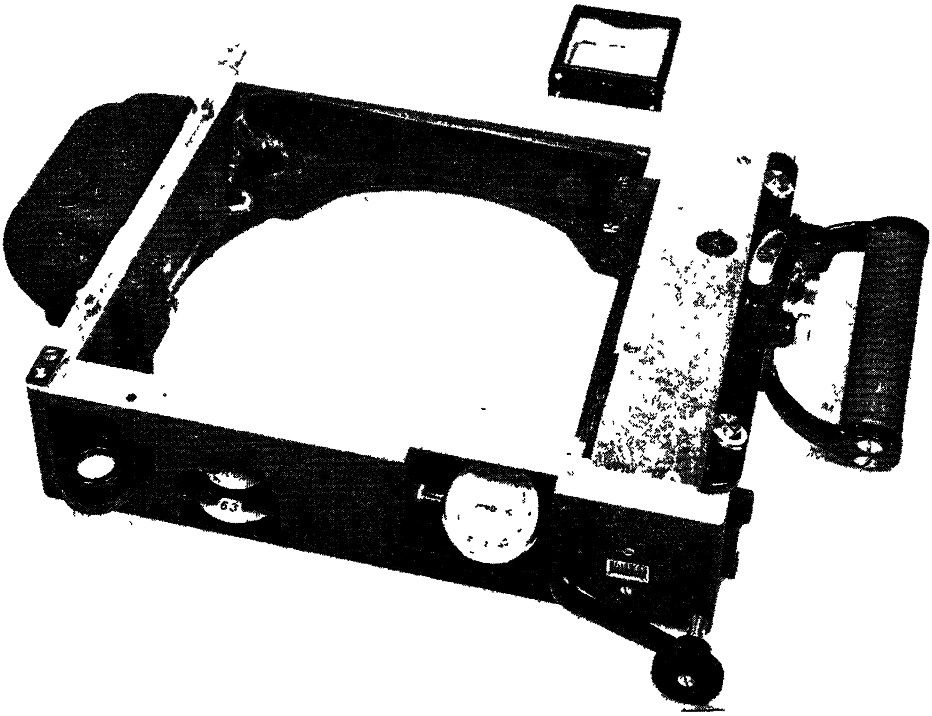


FIG. 12. K-17 camera body assembly complete showing camera drive mechanism on right and driving motor on left.

and travels to the shutter by way of the trip rod, and to the magazine by way of the trip coupling in some types of cameras. For electrical operation, an electrical solenoid is contained in the drive (see Fig. 13). When energized, the solenoid releases a tripping spring in the drive which results in an exposure being made.

In automatic aerial cameras, an electric motor is mounted in the camera body in close proximity to the drive mechanism. This motor operates on either 12 volt current or 24 volt according to the requirement. The latest trend of aircraft power is very definitely to 24 volt power supply and all modern aerial cameras are designed for that current. The power of the motor varies according to the size and design of the aerial camera. For small cameras such as the K-25, motors having a 15 watt output are sufficient while the larger cameras such as the K-17 require motor power in the neighborhood of 40 watts. An important characteristic of a camera drive mechanism is the time required for its complete operating cycle. One complete cycle consists of all the operations which must be per-

formed in order to trip a fully wound camera and restore it to the fully wound condition again Certain aerial camera uses require that photographs be taken at very close intervals. Thus, the use to which the camera is to be put determines the length of the operating cycle which must be allowed for in the design The shortest operating cycle in general present day use is approximately one third of a second per cycle as in the case of the K-25A camera A very generally used cycle is three seconds as in the K-17, while the longest cycle at present is 10 seconds in the case of the T-5 camera. The size of the negative area has considerable bearing on the duration of the operating cycle when short time intervals are required. It requires less time and/or power to move a four inch length of film and to wind a small shutter for instance than it takes to move nine inches of film and wind a large shutter.

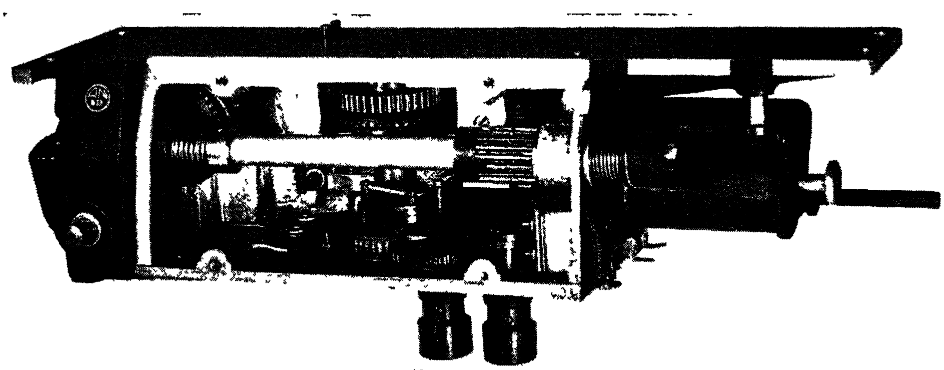


FIG 13. Typical camera drive mechanism showing tripping solenoid on right, shutter wind and trip couplings on bottom and magazine drive coupling on top.

FOCAL PLANE

The focal plane of an aerial camera is the plane in which all light rays passing through the lens come to a focus. The focal plane is bounded by a frame which determines the size of the negative

The focal plane may be either of two types; namely, the glass plate type or the open type. In the case of the former, a piece of very high quality plate glass equal in size to the negative is used. The glass is so located in the camera that it is perpendicular to the central axis of the lens. The distance from the lens is such that the surface away from the lens lies in the exact focal plane. In determining this distance it must be noted that the distance from the lens to the focal plane without a glass plate being interposed is less than the distance to the focal plane when the light rays must pass through a glass plate.

By means to be discussed later, the film is pressed flat against the glass plate when the camera shutter is tripped and the picture is taken. The focal plane glass presents a rather simple solution of the focal plane problem. However, it has several disadvantages.

Distortion.—Reference to any book on physics or optics will confirm that a ray of light is deflected as it passes through a glass plate by an amount proportional to the angle of incidence. Thus, a ray of light coming through the central axis of the lens perpendicular to the glass focal plane plate is not deflected but passes straight through as if no glass were present. Any other ray of light which strikes the glass plate at any other angle than perpendicular is bent

away from the center and strikes the film at some point other than where it would normally strike if there were no glass plate involved Thus, since glass plate distortion gives rise to erroneous positioning of points photographed, and since the errors are not constant but vary throughout the negative, glass plate focal plane cannot be used in cameras which may be utilized for mapping photography. A complete discussion of the errors caused by glass plates may be found on page 59

Static—Static electricity is created by the rubbing together of dissimilar materials, and frequently builds up to a point where an electrical discharge takes place. Not a great deal is known of the theory of static discharges in aerial cameras However, extensive research has been carried out by the Fairchild Camera and Instrument Corporation so that conditions under which static discharges can be expected are known. One of the main facts learned is that the generation of static electricity is augmented by the presence of a glass plate.

The discharge of static electricity results in white branching tree-like lines on the photograph. In serious cases it renders the photograph useless while in any case it detracts from the quality. This has been a large factor in the discontinuance of the glass type of focal plane

Scratching.—When the film is advanced after an exposure, it is pulled across the focal plane glass Unless the glass is scrupulously clean scratches will be made on the negative This detracts from the quality

Breakage.—While the aerial camera should not be subject to rough use, nevertheless the danger of breakage is an ever present possibility Since the glass plate has accurate dimensions and is of special quality, it is not easily replaced.

The Open Type Focal Plane, insofar as American aerial cameras are concerned, has replaced the glass type in all but very elementary types of aerial cameras in order to eliminate the objections cited previously With the focal plane glass removed from the camera, the focal plane becomes a plane in air. In order to provide a means for placing the emulsion of the film in the exact focal plane, a metal plate known as the 'locating back' is required which is located just above the focal plane on the side away from the lens The top surface of the focal plane frame is in the exact focal plane of the lens. Thus, when the film is stretched across the focal plane opening, and when the locating back presses the film against the focal plane frame, the edges of the negative, at least, lie in the focal plane. The central portion of the negative, for large negative sizes, at least, would sag out at the focal plane unless means to avoid this condition are resorted to. By means of air pressure or vacuum, the film is made to hold closely to the locating back thereby achieving the same result as would have been obtained with the glass focal plane but without any of the objectional features

Fiducial Marks.—Almost all aerial cameras, whether they have the glass type or open type of focal plane, have fiducial marks in the focal plane. Fiducial marks are only omitted in aerial cameras having no mapping, mosaic, or measurement application whose photographs are of value only for their pictorial quality.

Fiducial marks in all aerial cameras are four in number and are either located in the four corners of the negative opening or in the center of each of the four sides. In either case, the intent is to locate the marks so that lines joining opposite fiducial marks will intersect at the principal point of the negative. In cameras which are to be used for mapping of a low order of accuracy, the fiducial marks are located by the manufacturer to indicate the geometrical center of the focal plane opening. By reason of close manufacturing tolerances, the geometrical center of the opening does not differ greatly from the principal point. How-

ever, for mapping of a high order of accuracy, the fiducial marks must be so adjusted as to indicate the principal point of the negative within exceedingly close limits which are usually specified by the government agency or other organizations for which the mapping photography is being done

In the case of the glass type of focal plane, the fiducial marks are etched on the surface of the glass which is in contact with the film Thus, a clear cut image is obtained For purpose of adjustment, the entire focal plane glass can be moved. For the open type focal plane, the fiducial marks are usually triangularly shaped metal projections which extend into the negative area. The locating back presses the film tightly against the marks and their shapes are accurately silhouetted in the emulsion These marks are movable by virtue of slotted holes through which the screws pass to affix them to the focal plane frame After accurate adjustment the marks are dowelled permanently in place

Film Magazine

A very important component of any aerial camera is the film magazine The four main functions of this unit are

a To contain the film
b To wind or advance new film after each exposure
c To measure out or 'meter' exactly the correct amount of film for each new exposure
d To maintain the film perfectly flat in the focal plane

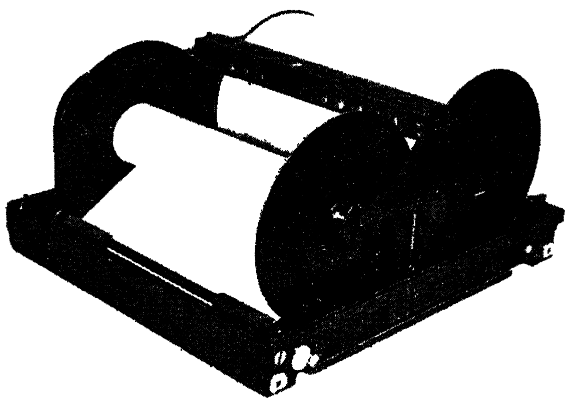


Fig. 14. Type A-9 large capacity detachable magazine for 500 exposures (9″×9″ negative size)

There are, in general, two types of film magazines; namely, *integral* and *detachable*. The integral type magazine is characterized by the fact that it is designed as part of the camera so that it cannot be considered a separate unit This has certain advantages in that it usually contributes to lightness and lower cost. This is helpful for hand held cameras However, several disadvantages overbalance the low cost and lightness for most types of aerial photography. The necessity of carrying the entire camera into the darkroom for loading and unloading is cumbersome while the alternative of daylight loading of the sensitive

films is not entirely satisfactory. Furthermore, if any part of the mechanism of the magazine fails, the entire camera is temporarily out of service

The detachable type magazine is by far the most popular and practical for aerial photography of almost all kinds (see Fig. 14 and 15). The main advantages are at once apparent:

1. Only the relatively small magazine need be taken into a darkroom for loading and unloading.
2. In case of magazine failure, the camera can still be used with a spare magazine.
3. For projects of long duration several loaded magazines can be carried and changed quickly when required.

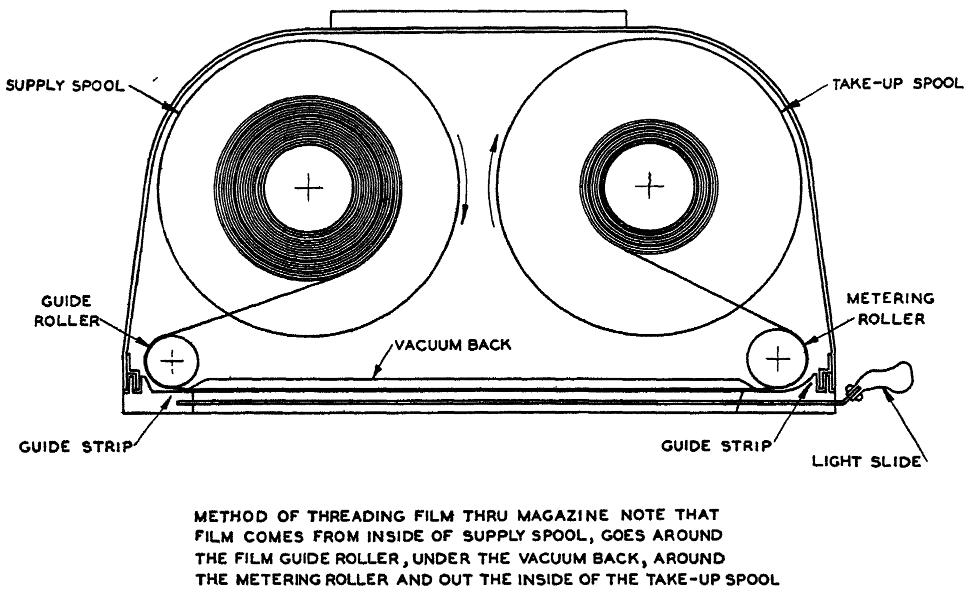


FIG. 15. Schematic drawing of typical magazine.

In order to perform functions, *b*, *c* and *d* mentioned previously, the film magazine includes appreciable mechanism. This mechanism is referred to as the *magazine drive*. The magazine drive receives its power from the camera drive through the wind coupling Thus, the *take-up* spool is made to turn, thereby pulling film from the *feed spool* across the focal plane. Winding of the film calls for considerable power in some instances. Some of the larger rolls of film used weigh close to twenty pounds, although this is an extreme not so commonly used. However, rolls of film weighing half this amount are common and present some problems. In the dry cold atmosphere of high altitudes the film becomes brittle and this imposes an added load upon the magazine drive at a time when the mechanism is being handicapped by a general stiffening due to contraction of the metal and congealing of the lubricants.

A further problem in film winding, particularly in cameras having a short operating cycle, is the necessity for overcoming the inertia of the stationary feed spool; and then after the film winding is complete, to check the momentum gained and prevent the film from unspooling and jamming the magazine. These

problems are partially solved by the use of slip clutches, over running brakes, etc , and further development of an extensive nature is still in process on this important matter.

Film metering is the term used to describe the function of the magazine drive which measures the exact amount of film required for each exposure.

The most elementary type of film metering is obtained when the take-up spool is caused to revolve through a specific number of turns or fraction thereof. The number of turns cannot be less than enough to pull through sufficient film for the exposure, or overlap of negatives will occur. Thus, the number of turns of the film spool which will cause successive negatives to be closely adjacent to each other without overlap, is selected However, as film is wound on to the take-up spool the diameter becomes greater and the same number of turns of the take-up spool draws more and more film through for each exposure. Even in a short length of film, the increase in waste space between adjacent negatives due to increasing spool diameter can become a matter of inches. The very elementary Model F-8 camera utilizes this simple form of metering Each exposure is 5″ wide and the maximum length of film is approximately 20 feet. Yet, the spacing toward the end of the roll becomes as great as $1\frac{1}{4}''$ Not only does this cause a waste of film but it reduces the photographic capacity of the magazine which is a serious matter.

The next form of film metering applies a correction for the increasing diameter of the take-up spool due to film winding onto it. A lever is forced outward by the increasing film on the spool. This lever is connected to a clutch mechanism which causes the take-up spool to make less turns in proportion to an increase in the spool diameter. Thus, the space between successive negatives is very closely the same throughout the entire roll of film.

The pressing of a lever against the film has some drawbacks and a better solution of the metering problem is by the use of a metering roller. The metering roller is usually located on the take-up spool side of the magazine. The film is bent around the metering roller to secure as many degrees of contact as possible up to a desirable total of 180°. The metering roller is usually about one inch in diameter. To prevent slippage of the film, a pressure roller is used which presses the film against the metering roller at the edges. The metering roller is so designed that after it has turned a specific amount, a clutch is caused to disengage which stops the take-up spool from turning Since the film does not wind up on the metering roller, the diameter remains constant and the same amount of film is advanced for each exposure. This system permits quite accurate spacing

Another form of metering consists of actually driving the film by means of the metering roller. In this case the take-up spool merely takes up the slack. An over-running clutch serves to prevent a pull being exerted on the film by the take-up spool.

Film Flattening.—In addition to mechanism to wind and meter the film, the magazine also contains means to hold the film flat in the focal plane at the instant of exposure. This is accomplished in one of four ways:

a Direct tension
b. Pressure plate against glass focal plane
c Air pressure
d. Vacuum.

In the case of cameras whose negative area does not exceed $4''\times 5''$ and where no mapping use for the photographs is intended, the film can be held reasonably flat by means of direct tension. During the winding of the film, it is stretched across the negative opening. Just before the exposure is to be made, a metal

plate clamps the film around the edges of the focal plane frame The clamping action, plus the slight tension on the film, plus the stiffness of the film, all combine to prevent film from sagging sufficiently to seriously impair the photographic quality No attempt should be made to apply this system to cameras having negative sizes greater than 4″×5″

When a focal plane glass is used, the matter of holding the film flat is relatively simple Just before an exposure is made, a metal pressure plate clamps the film against the focal plane glass over its entire area. After the exposure has been made, the pressure plate releases and the film can be advanced.

Where no focal plane glass is used and where the negative area is greater than 4″×5″ either air pressure or vacuum must be used to hold the film flat. In

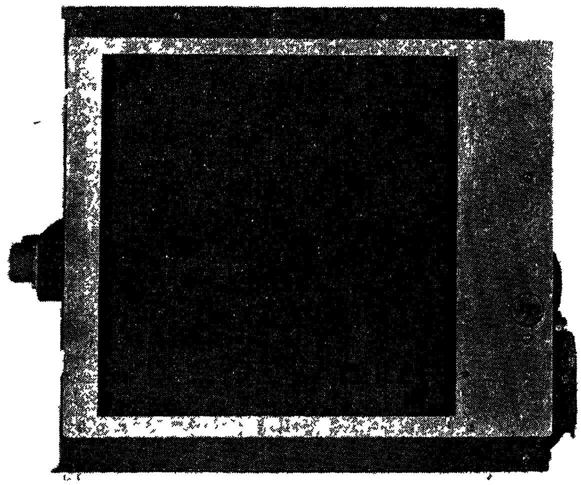


FIG. 16. Underside of magazine showing open focal plane frame, fiducial marks, and locating back with grooves for distributing the vacuum

the air pressure system the entire lens cone and camera body must be air tight. The film is pulled across the negative opening whereupon the locating back clamps the edges of the film against the focal plane frame The locating back is a metal plate pierced by a large number of small diameter holes. By means of a small hose connection on the camera cone, air under pressure is admitted to the cone. The air pressure is obtained either by a wind scoop or by a pump. The film stretched across the negative opening normally sags. However, the air pressure which builds up in the camera cone forces the film flat against the locating back This system has the disadvantage of requiring an air filter to clean the air before it enters the camera. Furthermore, paint chips and any minute particles loose in the camera cone are blown against the film emulsion with a possibility of scratches occurring. Also, not every camera cone and body can be made air tight without special design.

The method of flattening film most popular in American aerial cameras is the vacuum system (see Fig. 16). For this system, the locating back has its surface which contacts the film criss-crossed with grooves about 1/16″ deep and 1/16″ wide. Small holes at a number of intersections of the grid grooves lead to

a central vacuum connection As in the air pressure system, the locating back clamps the film to the shoulders of the focal plane frame. However, the film is drawn up against the locating back by the evacuation of air from the space between the film and the locating back Because the film only sags slightly, the amount of vacuum required to flatten the film is very small, being in the neighborhood of one half to two inches of mercury.

The vacuum is obtained by use of a conventional aircraft venturi, by means of a vacuum pump, or by a self-contained piston The venturi is a simple means

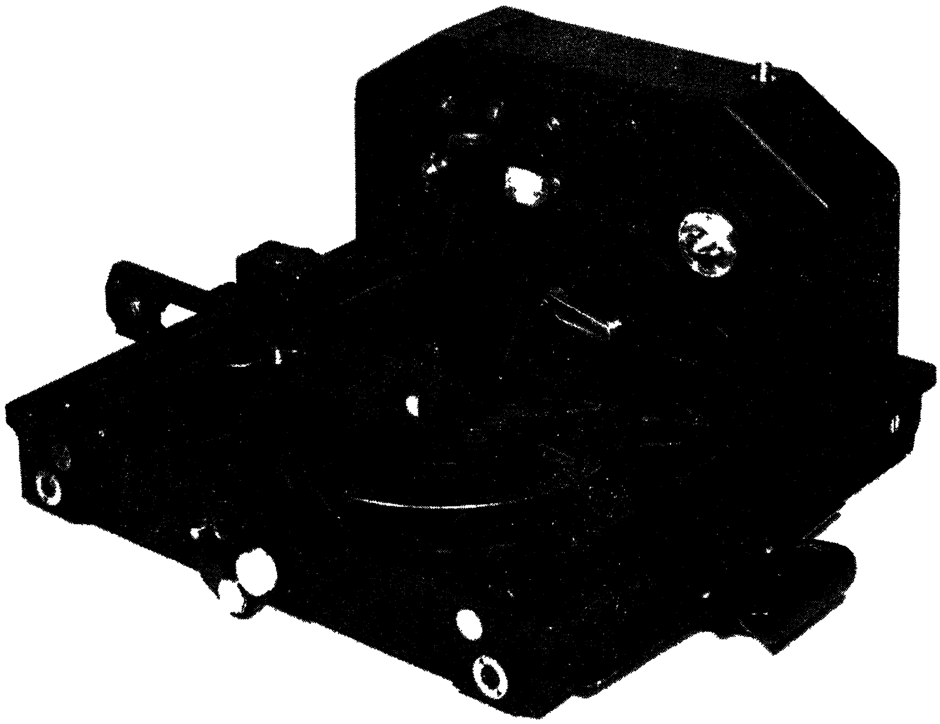


FIG 17. F-56 magazine showing internal vacuum pump assembly.

for planes operating at normal altitudes However, at high altitudes or in low temperatures, ice sometimes interferes Dirt picked up during take off from an airport can also clog the venturi and prevent sufficient vacuum from reaching the locating back. Some pumps fail as does the venturi, due to the rarified air at the high altitudes. Considerable success has been achieved with a cylinder and piston built into the locating back (see Fig. 17). The piston is so timed that an instant before exposure it is pulled sharply upward, thereby creating a vacuum which draws the film flat. This system avoids all of the shortcomings mentioned above and has the added advantage of making the camera independent of hose connections to the aircraft.

Film Capacity.—An important consideration with regard to any film magazine is its capacity. The capacity is expressed either in the maximum length of film which can be wound on the largest spool which the magazine will accom-

modate, or as the number of exposures which can be made on that length of film This information is best shown in the following table with the other data given.

Magazine Designation	*Negative Size*	*Type*	*Camera Used on*	*Focal Plane*	*Film Flattening Means*	*Maximum Film Length*	*Maximum Number Exposures*
K-20 Magazine	4″×5″	Integral	K-20	Open	Self Contained Vacuum	19½′	55
K-25 Magazine	4″×5″	Integral	K-25	Open	Direct Tension	19½′	55
F-8 Magazine	5″×7″	Integral	F-8	Glass Plate	Pressure Plate	20½′	40
F-56 Magazine	7″×7″	Detachable	F-56	Open	Self Contained Vacuum	125′	200
K-15A Magazine	5″×7″	Integral	K-15 K-15A K-16	Glass Plate	Pressure Plate	20½′	40
A-5 & A-5A	9″×9″	Detachable	K-17 K-17B K-19 K-19B K-22	Open	External Vacuum	205′	250
A-7	9″×18″	Detachable	K-18	Open	External Vacuum	75′	48
A-8	9″×18″	Detachable	K-18	Open	External Vacuum	410′	260
A-9	9″×9″	Detachable	K-17 K-17B K-19 K-19B K-22	Open	External Vacuum	410′	500
A-11	9″×9″	Detachable	K-17 K-17B K-19 K-19B K-22	Open	External Vacuum	205′	250

Cameras in Present Day Use

The K-15 aerial camera shown in Figure 18 was originally designed for high altitude oblique spotting photography for Military Intelligence purposes. However, after only a small quantity of cameras were put into service they were reworked for automatic operation and were given the designation of K-15A. The K-15 and the K-15A are ordinarily equipped with 40″ telephoto f/8.0 lenses. However, the camera is so designed that a 20″ f/5.6 lens cone can be interchanged on the camera without much difficulty. The camera provides negatives 5″×7″ in size. The magazine is integral with the camera and accommodates film 7″ wide and 20′ 6″ long, which is sufficient to produce approximately 40 negatives. The focal plane shutter has a maximum speed of 1/600th of a second and intermediate speeds of 1/200th and 1/400th of a second.

This camera is strictly an Army type camera and relatively few are in existence. The camera like all long focal length cameras does provide large image size and reasonably sharp definition of detail in spite of the high altitudes from which the photographs are usually taken.

The camera is unique in that a means is provided for focusing the camera for infra-red photography. This is the only aerial camera which includes this feature. When used for infra-red photography, special infra-red sensitive film must be used in conjunction with a type 89A filter.

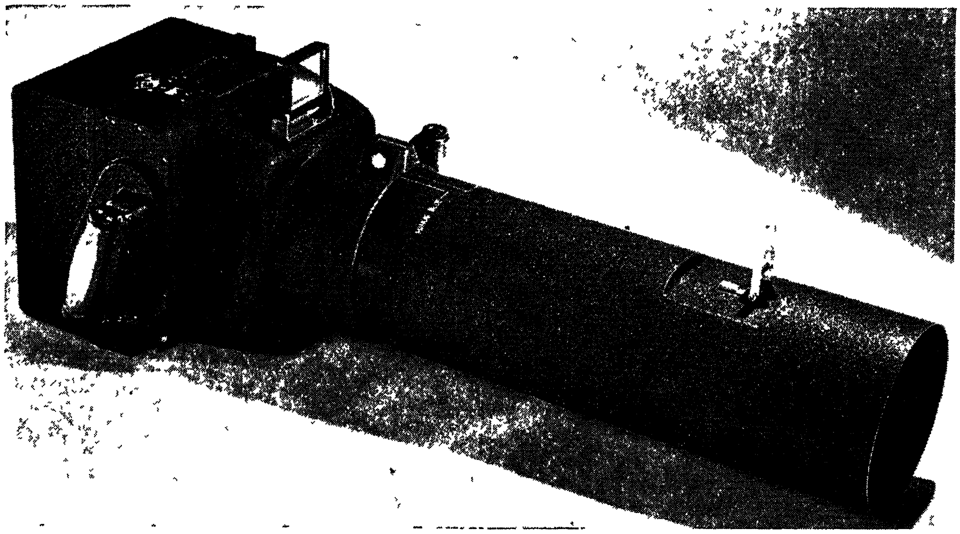


FIG. 18. Fairchild type K-15 camera for high altitude oblique spotting photography.

The designation of type K-16 was given to the type K-15 camera when equipped with a 20″ telephoto lens. However, the designation type K-16 has been eliminated, and the camera is referred to as K-15 and followed by the focal length of the lens which is used with it

The type K-17 camera, shown in Figure 19 and is the oldest of the standardized Fairchild cameras in use today. The design started years ago with the type K-3 camera which progressed through the types K-3A, K-3B, K-3C and eventually to the type K-17. To the long years of experience with the type K-3 and type K-3B camera has been added the many improvements learned by the Army Air Forces and by commercial aerial photographic organizations until today the type K-17 is probably the most rugged, dependable and generally useful aerial camera.

The type K-17 camera is used for various types of work ranging from wide angle photography using 6″ metrogon lens for multiplex mapping work, to use in conjunction with the tri-metrogon mapping system for aeronautical charting, to the construction of all classifications of mosaics and to intelligence spotting type of photography from high altitudes. This wide range of adaptability is made possible by the fact that all major units of the camera, such as shutter, lens, cone, camera body, camera drive and magazine are quickly and easily detachable and interchangeable.

The K-17 camera accommodates the type A-5 or the improved A-5A roll film

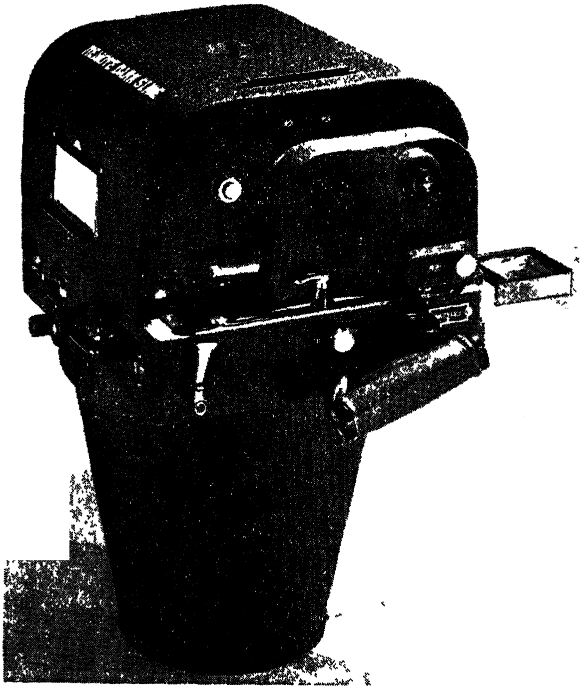


FIG. 19. Fairchild type K-17 all purpose camera (9″×9″ negative).

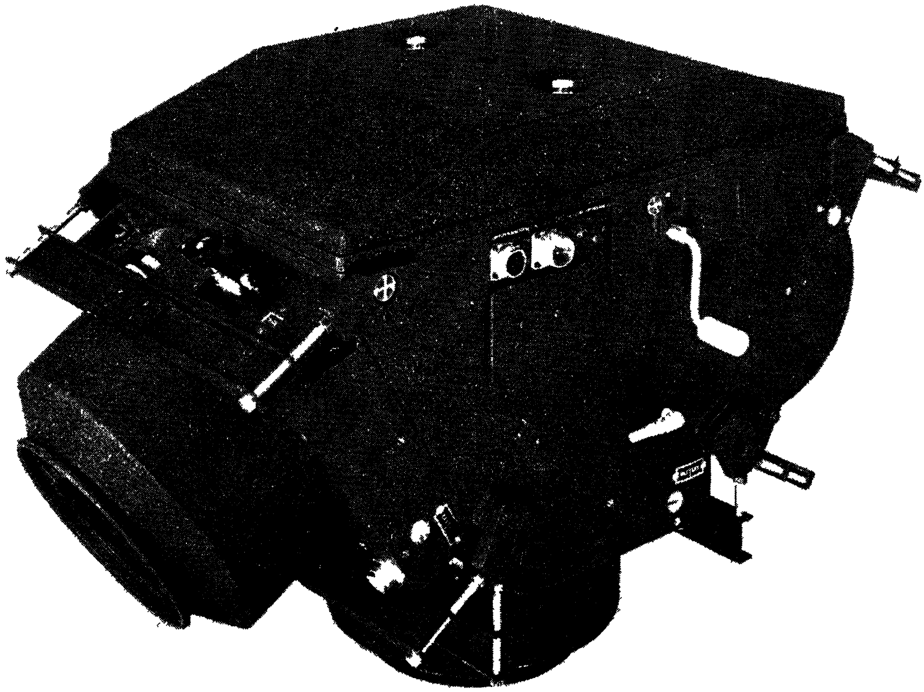


FIG. 20. Camera for tri-metrogon photography. Consists of the cone and body of three type K-17 cameras attached to a special magazine.

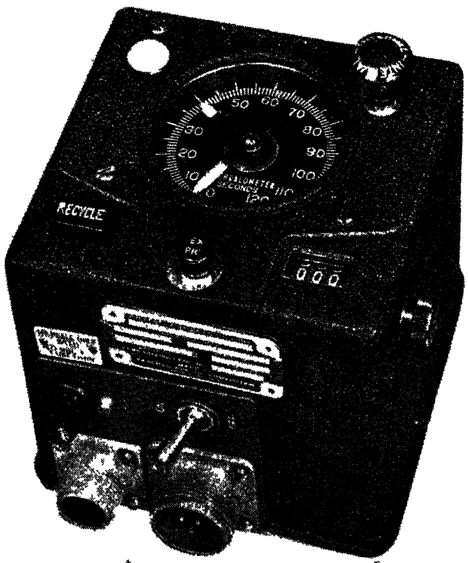


FIG. 21. Fairchild type B-3B intervalometer.

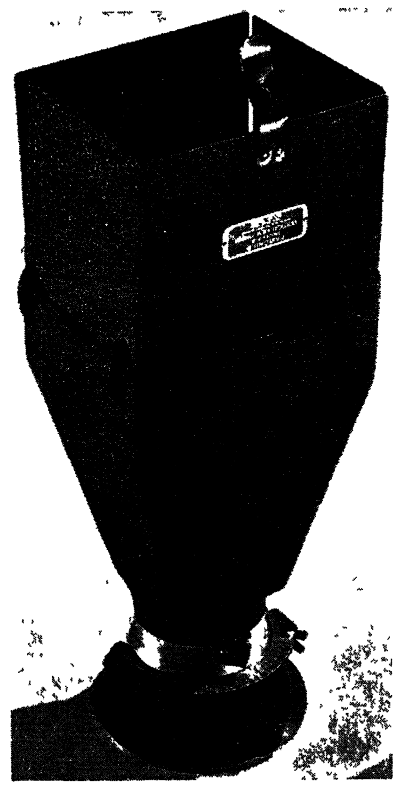


FIG. 22 Fairchild model F-5 vertical view finder (side view). Note crab circle

magazine customarily. The magazine provides negatives 9″×9″ in size and accommodates a maximum of 205 ft. of film which is sufficient for 250 negatives. It is also possible to use the type A-9 magazine on the K-17 camera which magazine likewise gives negatives 9″×9″ in size but which has a film capacity 410 ft. giving approximately 500 exposures.

Three quickly interchangeable lens cones can be used with the K-17 camera which are the 6″ f/6 3 metrogon, the 12″ f/5.0 lens and the 24″ f/6 0 lens.

When the K-17 camera is equipped with the 6″ metrogon lens cone it is frequently referred to as the K-17B The speeds for the between-the-lens type shutters are, for the 6″ cone, 1/50th, 1/100th, 1/200th and 1/300th; and for the 12″ cone are 1/75th, 1/150th and 1/225th; and for the 24″ cone are 1/50th, 1/100th and 1/150th

The K-17 camera 6″ metrogon cone and body can be attached to a special experimental magazine assembly in order to provide a camera for tri-metrogon photography Figure 20 shows the only camera made for this purpose.

The K-17 camera is usually used in conjunction with the type B-3B intervalometer which is shown in Figure 21. This intervalometer provides for intervals ranging from one second to one hundred and twenty seconds in one second intervals. A very useful feature of the intervalometer is that it can be started at any desired point by pressing the recycle button. An extra picture can be taken at any time desired without interfering with the cycle.

Also commonly used with the type K-17 camera is the vertical view-finder shown in Figures 22 and 23. The purpose of the view-finder is to give a view of the area being photographed by the camera and to provide a means for determining the angle of drift of the aircraft when flying in the face of a cross-wind and also

to provide a means for determining the exposure interval between successive pictures

The K-17 camera with its normal equipment weighs approximately 60 pounds and its approximate size is 15″ wide, 15″ long and 24″ high.

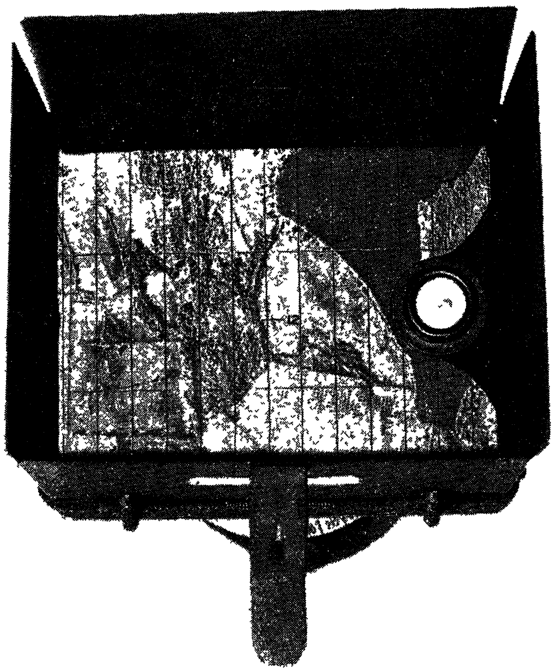


FIG. 23. Fairchild model F-5 vertical view finder (top view). Note the crab lines and the two exposure interval lines.

The Type K-18 Camera

The K-18 camera shown in Figure 24 is the outgrowth of type K-7 camera developed some years ago, which went through a series of improvements which brought about the designation of the K-7A, K-7B, and finally the K-7C, until the camera was finally converted to motor operation whereupon the designation of the type K-18 was applied.

The type K-18 camera is distinctive as was the type K-7C for the size of the negative, which is 9″×18″ long. This camera is equipped with a 24″ focal length lens which gives large scale photographs and at the same time fairly wide coverage. This makes the camera very useful for high altitude intelligence photography.

The camera is a well made camera and thus can be used for mapping where the order of accuracy is not the highest. The predecessor of the K-18 camera, the K-7C, was used successfully by the United States Coast and Geodetic Survey for constructing a map of New York Harbor. The United States Marine Corps has been quite successful in constructing mosaics with photographs made by the K-18 camera as has also been accomplished by other branches of

the armed forces The K-18 camera utilizes the type A-7 roll film magazine which accommodates a 75 foot length of film yielding approximately 48 negatives, 9″×18″ in size The new type A-8 magazine can be also used on the K-18 camera which has a film capacity of 410 ft. which results in some 260 exposures.

The K-18 camera utilizes a between the lens shutter which has a maximum speed of 1/150th of a second. The camera can likewise be used with the Type B-3B Intervalometer and with the view-finder explained previously.

The type K-18 camera is a relatively large camera weighing approximately sixty-five pounds and having an over all size of 14″×24″ 35″.

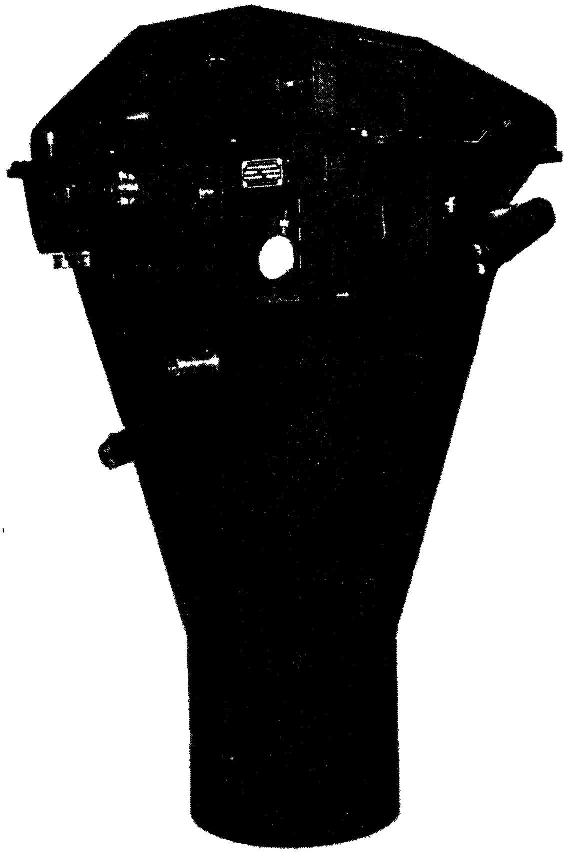


FIG. 24. Fairchild type K-18 camera (9″×18″ negative).

The Type K-19 Camera

The type K-19 aerial camera shown in Figure 25 is a very specialized aerial camera which is the outgrowth of the type K-12 camera designed for night aerial photography. The present day type is the K-19B which has evolved from the K-12 and the K-19 The type K-19 camera is principally for making single exposures from any desired altitude up to a maximum which at present is a secret. However, a series of overlapped photographs have been taken successfully

on several occasions and mosaics have been made there from The intent of photography with K-19B cameras is to photograph enemy positions at a time when they are uncovered due to the protection of darkness

The type K-19B camera utilizes the type A-5 or A-5A roll film magazine previously explained Part of the equipment of the camera is a photo-cell which is a detached unit containing a light sensitive tube Part of the equipment needed for a night photographic mission is a special flare which when released from the

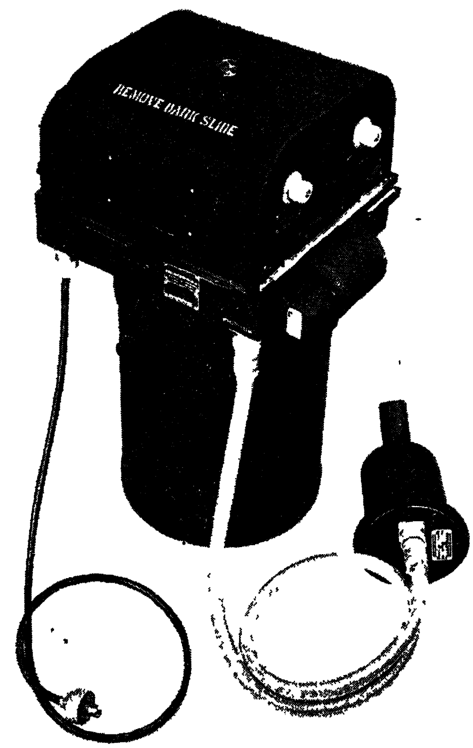


FIG. 25. Fairchild type K-19 camera for night photography. Note photo-cell unit at lower left.

aircraft explodes at any desired altitude. The light from the flare illuminates the ground and the light from the flare is likewise picked up by the light sensitive element in the photo-cell assembly The photo-cell by action of the light creates a minute electrical current which is magnified by an amplifier built into the photo-cell assembly. The amplified current is of sufficient strength to trip the shutter of the camera thereby taking a picture. All this occurs within a few milliseconds of the time when the flare first explodes. After the picture has been taken the camera automatically rewinds the shutter and the film so that it is ready for the next exposure.

The camera is equipped with a 12″ f/2.5 lens.

The shutter speeds are 1/25th, 1/50th and 1/100th of a second.

The camera is normally operated automatically but can be operated manually if desired.

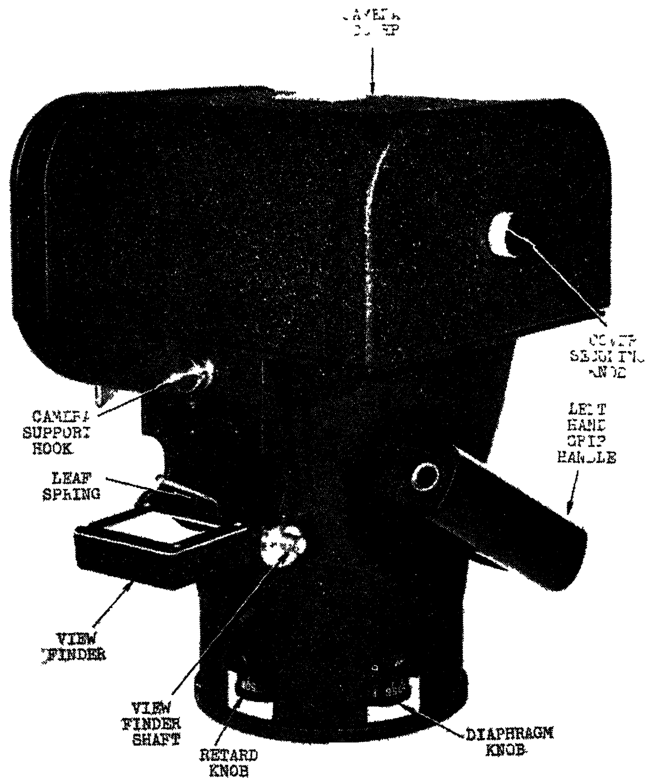


FIG. 26 Fairchild type K-20 hand held camera (negative size $4'' \times 5''$).

The Type K-20 Camera

The type K-20 camera as shown in Figure 26 was developed to meet the demands for a light rapid action hand held aerial camera The camera especially replaces the larger and obsolete K-10 camera. The K-20 camera was originally developed by the United States Navy for the purpose of taking identification photographs during the period of so-called 'armed neutrality' which was the policy of this country prior to the outbreak of the present war. The need was for aviators on neutrality patrol, who would fly over ships operating within the United States waters, to photograph the ships for record purposes. However, after the first few cameras were available for use a multitude of other applications were found and the type K-20 camera became one of the most popular cameras in use by both the Army and the Navy

The K-20 camera utilizes a $6\frac{5}{8}''$ f/4.5 lens mounted in a high speed between the lens shutter capable of speeds ranging from 1/125th of a second to 1/500th of a second. The magazine is built into the camera and is of sufficient size to accommodate a roll of film $19\frac{1}{2}$ ft. long and $5\frac{1}{4}''$ wide, which is sufficient for approximately 50 exposures $4'' \times 5''$ in size. The camera is operated by twisting the right hand handgrip and the exposure is made by pressing a trigger with the forefinger of the right hand. Twisting the right hand handle serves not only to wind the film but likewise the shutter.

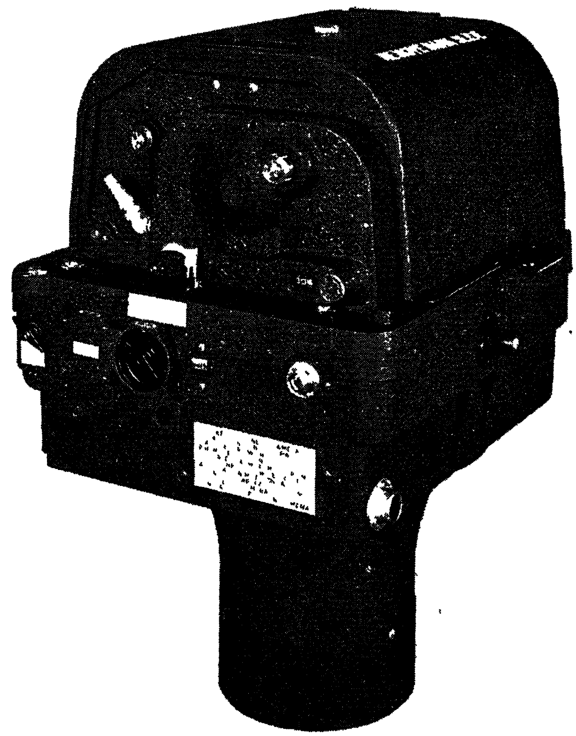


FIG. 27. Fairchild type K-22 military camera equipped with focal plane shutter

The Type K-22 Camera

The type K-22 camera is pictured in Figure 27. As mentioned previously in this chapter the between the lens shutters are limited for practical purposes to a maximum speed of approximately 1/500th of a second. Furthermore, the larger the aperture of the lens the lower is the maximum possible speed.

In the present war airplane speeds have increased to a high degree and also photographic tactics in many instances call for photography in these high speed aircraft at low altitudes This combination of high speed aircraft and low flying altitudes makes the relatively slow speeds of between the lens shutters not entirely suitable This condition gave rise to the development of the type K-22 camera which is equipped with a focal plane shutter capable of high speeds. The type K-22 camera is strictly an Army camera for general intelligence photography. The camera has no mapping applications due to the use of the focal plane shutter.

The camera is primarily for electrical operation and there is no provision made for manual operation in the ordinary manner. In practically all instances the K-22 is remotely controlled.

The type K-22 camera utilizes interchangeable focal plane shutter assemblies in order to get a range of speed from 1/150th of a second to 1/800th of a second. One shutter assembly gives speeds of 1/150th to 1/350th of a second, while the second shutter assembly gives speeds of 1/350th to 1/800th of a second. This

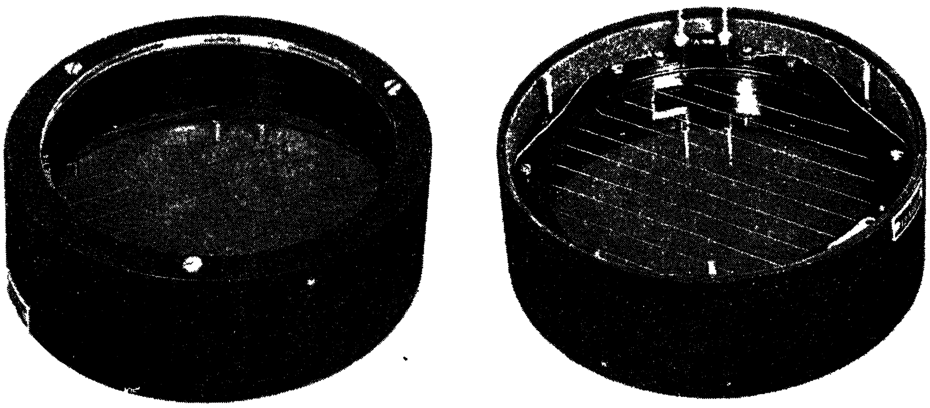


FIG. 28 Electrically heated filter.

removal of the shutter and replacement by the second shutter can be done very quickly and simply without the use of anything more than a screw-driver

The camera features interchangeable lens cones which are the 6″ f/6 3 metrogon lens, the 12″ f/5 0, the 24″ f/6.0 lens and the 40″ f/8.0 lens.

Like the K-17 camera, the K-22 utilizes the A-5, A-5A and A-9 magazines previously described Also used with this camera is the B-3B intervalometer. The vertical view-finder previously described can be used as an accessory but it is not customary.

One of the interesting accessories with which the K-22 camera is equipped is the filter heater assembly shown in Figure 28. The fine wires which can be seen across the filter are electrically heated which is to serve the purpose of preventing the condensation of moisture on the filter and lens during rapid descent from a high altitude to a lower altitude where the atmosphere contains more moisture at a higher temperature. The question of filter heaters is not entirely settled and there is some indication that filter heaters contribute to poor definition.

Another interesting item in connection with the K-22 camera is the heating jacket assembly for the lens cone shown in Figure 29. This jacket is designed to keep the camera cone warm to prevent contraction due to low temperatures which might change the length of the cone sufficiently to cause a change in the focal distance, thereby reducing the sharpness of the photographs.

FIG. 29. Camera heating jacket.

The remainder of the camera including the magazine is heated by internal heaters within the mechanism. The desire is to maintain the entire camera at a uniform temperature both for purposes of holding the focal length of the lens constant, for insuring proper functioning of the mechanism and for retaining the original sensitivity of the film.

The Type K-25 Camera

The type K-25 aerial camera was developed as an outgrowth of the type K-20 camera. The small size of the K-20 camera made it very attractive for installation in small aircraft. Consequently, an electric driving motor assembly was built into the type K-20 camera and other slight modifications made which resulted in the K-25 designation

The K-25 camera is normally mounted in an oblique position in the tail of an aircraft. It is primarily intended for low altitude photography. In order to obtain overlap of successive pictures when flying at low altitudes and at high speeds the mechanism is designed to operate at the rate of three cycles every two seconds. A special modification of the K-25 camera has recently been made whereby the camera operates at approximately three cycles per second.

The electrical system of the camera is so arranged that once the circuit is closed which causes pictures to be taken the camera will operate continually until the film is completely run out—at which time the camera automatically stops. The camera can also be operated by the conventional pickle switch which is held in the operator's hand. As long as the switch button is depressed the camera will take pictures and as soon as the switch button is released the camera will cease taking pictures. It is not possible to take individual exposures with the type K-25 camera nor to use it in conjunction with the type B-3B or any other intervalometer.

The type K-25 camera does not utilize the self contained vacuum piston pump which the K-20 camera features but relies solely upon the tension of the film and the pressure of the locating back to hold the film flat at the instant of exposure. This results in quite useable photographs but the photographs cannot be enlarged to the same degree as those of the K-20.

The film capacity, the lens, the shutter speeds, etc , are all identical to those of the type K-20 camera

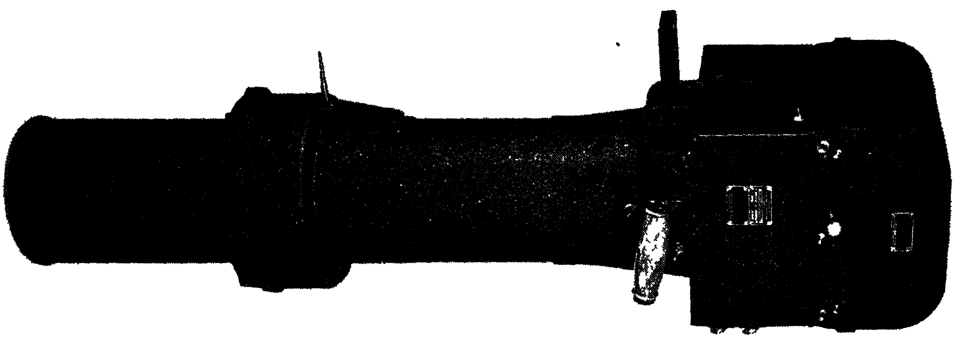


FIG. 30. Fairchild type F-56 camera with 40″ lens.

The Type F-56 Camera

Figure 30 shows the type F-56 camera which is strictly a Navy camera. This camera was developed a number of years ago and has been constantly improved and strengthened until today it is one of the most reliable cameras in use. The F-56 camera is a sturdy, rapid action camera suitable for either vertical photography or for oblique photography. The camera is built in four different sizes; namely, to carry lenses of $5\frac{1}{4}''$ f/6 3, $8\frac{1}{4}''$ f/4.0, 20″ f/5 6 and

40″ f/8 0. Unlike the K-17 the F-56 does not employ interchangeable lens cones and each camera is a unit in itself.

This camera utilized the type F-56 magazine which is specially designed for this camera and is not interchangeable with any other magazine. Neither can the magazine of any other camera be used on the type F-56 camera. The F-56 camera embodies the self contained vacuum piston pump which holds the film flat successfully at all altitudes and under all conditions. The negative size is 7″×7″ and the magazine normally accommodates 125 ft of film 7″ wide sufficient to give approximately 200 negatives

A warning device is built into the magazine which causes a light to flash while the film is winding to indicate that there is film in the magazine or that the film has not broken.

FIG. 31. Camera filter showing method of attachment

The $5\frac{1}{4}$″ metrogon lens version of the F-56 camera has not been made for sometime and only a few of those cameras are in existence They are primarily used for wide angle mapping such as that done by the United States Hydrographic Office of the Navy Department. The $8\frac{1}{4}$″ lens is mounted in a shutter having speeds ranging from 1/75th of a second to 1/225th of a second; the 20″ lens is mounted in a shutter having the same range of speeds while the 40″ lens is mounted in a shutter having speeds ranging from 1/50th to 1/150th of a second.

The camera can be operated in many ways. The camera can be wound by twisting the right hand handgrip somewhat similar to the manner of operating the K-20 camera If desired, the camera can be wound by winding a conventional

type of winding handle Thirdly, the camera can be operated entirely automatically by use of an electric motor Each time a photograph is taken the image of a watch is recorded on the film which gives the time of the day that the exposure was made. When the camera is being operated by motor, pictures are taken at the rate of one per second which is the fastest cycle of any camera of this size.

The camera like all other cameras is equipped with detachable haze cutting filters such as that shown in Figure 31. The F-56 camera is conventionally used with the B-3B intervalometer and with the vertical view-finder mentioned previously.

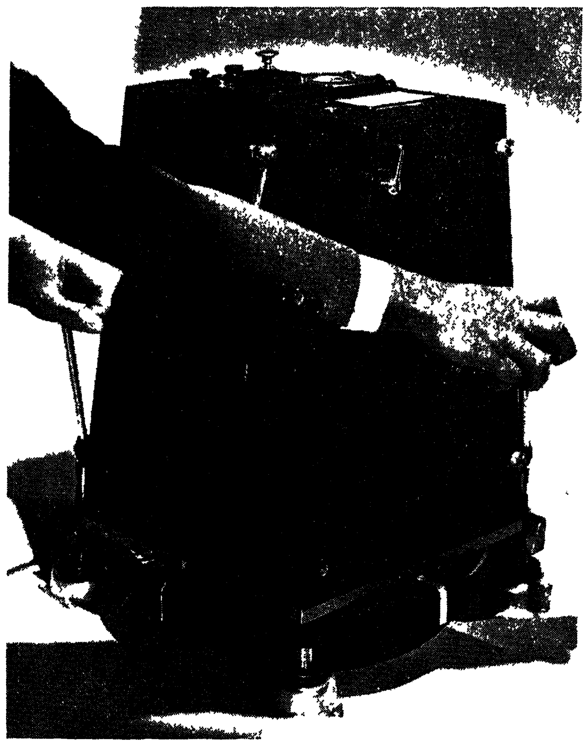


FIG 32. Fairchild type F-51 photogrammetric camera in screw-leveling vertical mount

The Type T-5 Camera

The original strictly photogrammetric camera developed by the Fairchild Camera and Instrument Corporation was the model F-51 camera shown in Figure 32. This camera fulfilled all the requirements set forth by the National Bureau of Standards in their report on the characteristics of an aerial camera for photogrammetric mapping photography.

The F-51 camera was strictly for commercial aerial photographic mapping. The existence of a camera for this specialized purpose contributed to the development of the model T-5 aerial camera which is the United States Army Air Forces' photogrammetric camera. This camera is characterized by the precise relationship which exists between the principle axis of the lens and the fiducial marks of the focal plane plate. However, in addition to this fundamental char-

acteristic the model T-5 camera shown in Figure 33 embodies other somewhat novel features.

The camera is an integral camera with no readily interchangeable parts, with the exception of the magazine assembly which can be removed from the camera. The lens used is the 6″ f/6 3 metrogon lens. The shutter speeds range from 1/50th to 1/300th of a second A minus blue filter is built into the lens assembly.

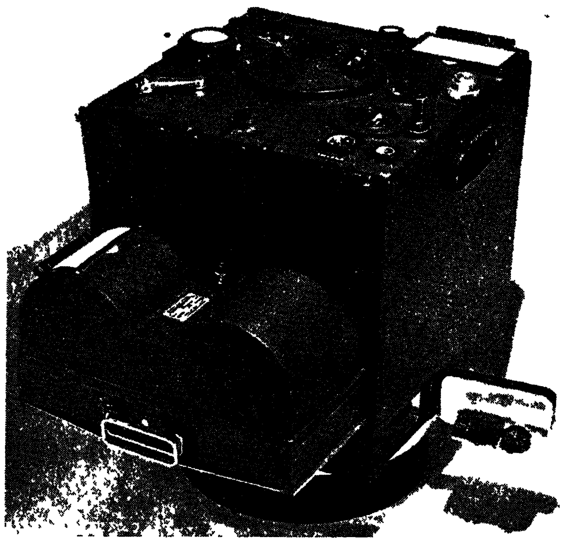


FIG. 33. Fairchild type T-5 photogrammetric camera showing film magazine partly removed.

The T-5 camera can be operated either electrically or manually and is almost entirely self contained It contains a built-in intervalometer which operates in one second intervals from ten to one hundred and forty seconds between exposures. The vertical view-finder is likewise built into the camera and includes an adjustable mechanism for setting the overlap lines in steps of five percent from forty percent to seventy-five percent overlap of negatives A light meter is built into the camera and an adjustment is provided to take care of the various emulsion speeds of the different films that might be used in the T-5 camera The light meter gauge is inter-connected to the shutter speed and diaphragm adjusting dials and indicates by means of a lubber line when either the diaphragm or shutter speed has been adjusted properly for the light conditions which exist.

The T-5 camera features the recording of certain desirable information. A sensitive aircraft altimeter is built into the camera and records its reading on the film. Also recorded are a watch, a level bubble, a negative number and a data card which might carry the name of the operator, or a description of the mission.

The T-5 camera magazine makes negatives 9″×9″ in size. The film normally used is 150 ft. long and 9½″ wide which is suitable for 190 exposures. A novel light slide is used which prevents pictures being made when the light slide is in place due to over-sight and prevents the magazine from being withdrawn from the

camera when the light slide is not in place Every effort has been made to make the camera as fool-proof as possible

The T-5 camera is $14 \times 14 \times 21\frac{1}{4}''$ high and weighs approximately 106 pounds loaded

Model T-3A Camera

The T-3A camera (see Fig 34) is a multiple lens camera which at present is not being used except incidentally by the armed forces. However, the camera is an interesting and useful one

The camera embodies five lenses and is in effect five separate cameras all co-ordinated into one unit. One lens unit points vertically downward while the other four units point obliquely downward at an angle of 43° from the vertical and at 90° horizontally with respect to each other Thus, one vertical photograph

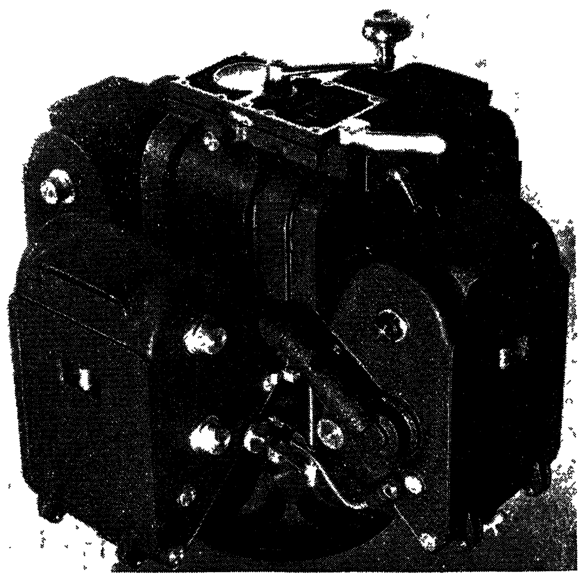


FIG 34 Fairchild type T-3A five lens camera.

and four oblique photographs are taken each time an exposure is made. Each camera unit has its own roll of film 6″ wide and 120 ft. long suitable for 200 exposures. All except the central vertical exposure must be put through a transforming printer and after transformation the five pictures are assembled in to one composite which is in the form of a Maltese cross.

The T-3A camera is wound by hand and is tripped manually. However, the action of the shutters is controlled by an electrical impulse which synchronizes the shutters so that all the exposures are made at the same instant. This camera was popular for mapping because of its wide angle of coverage which is in the neighborhood of 140°. For every thousand feet of altitude the camera covers an area one mile wide. Consequently, since the camera is normally used at twenty thousand feet or higher a strip twenty miles wide or wider is photographed which results in economy. Also, the wide angle of coverage gives greater strength for radial control plotting The non-use of the camera at the present time is due to its size, to the difficulty of assemblying the composites, and to the increase in popularity of the wide angle lenses of which the 6″ metrogon is the foremost.

Tandem T-3A

The T-3A camera had the shortcoming that the Maltese cross shaped photographs did not record any detail in the corners. This was overcome by the construction of the Tandem T-3A camera which consisted of two T-3A cameras mounted in a single mount and placed at 45° with respect to each other as shown in Figure 35. Except for the fact that an octagonal photograph resulted the Tandem T-3A camera is similar to the individual T-3 cameras which comprised it.

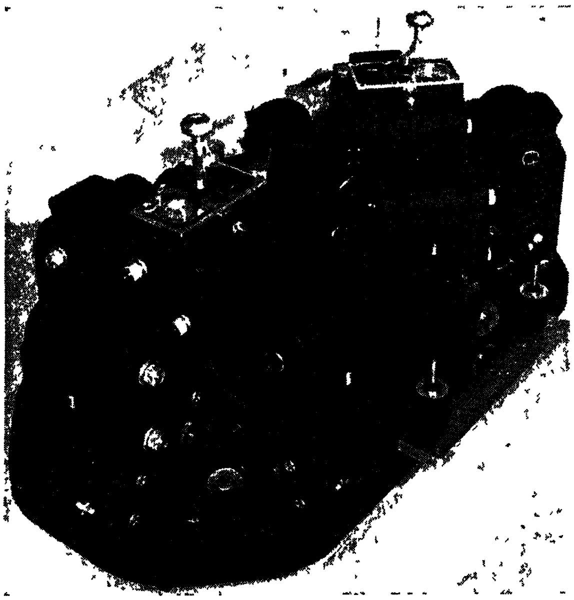


FIG. 35. Fairchild tandem T-3A camera

9-Lens Aerial Camera

Both the T-3A and the Tandem T-3A cameras utilize a separate film for each of the lenses in those cameras. They gave trouble from the standpoint of filing and storing the negatives and the composite prints. The United States Coast and Geodetic Survey, for this and other reasons, sponsored the development of the 9-lens camera which is shown in Figure 36. The camera utilizes nine lenses all of which point vertically downward. One lens is in the center while the others are located in a circle about the center The central lens photographs the ground below without obstruction whereas the other lenses photograph the ground as reflected from eight mirrors The lenses are of $8\frac{1}{4}''$ focal length and all nine lenses project their images on to a single piece of film approximately 23″ square. After rectification a single print results which is some 36″ square.

The camera can be operated either by hand or electrically through the use of an intervalometer which is governed by a special telescopic view-finder embodying the traveling grid principle.

The camera is the only one of its type in existence and has seen extensive service. A considerable amount of mapping has been accomplished. A year ago the airplane carrying this camera crashed, killing most of the crew and tossing the camera some fifty feet and subjecting it to the heat of a gasoline explosion

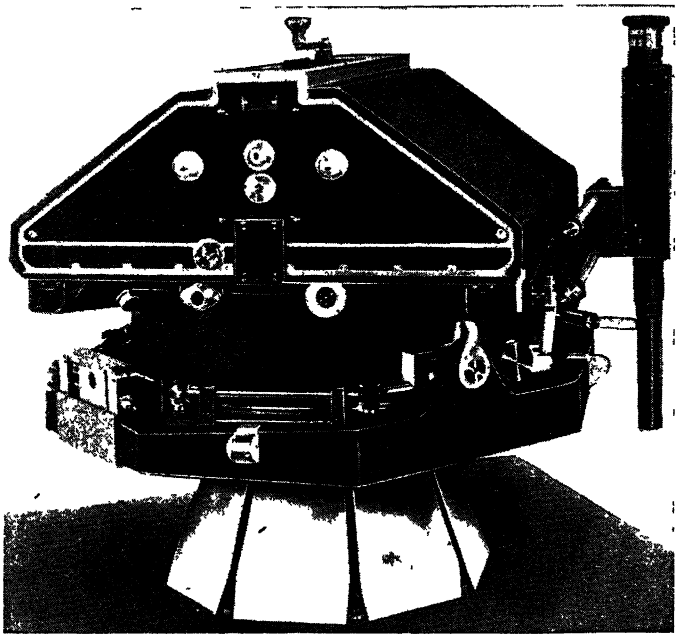


FIG. 36. Nine-lens camera of the U. S. Coast and Geodetic Survey.

and fire In spite of its ordeal the camera was found to be in surprisingly good condition, and is being restored to operating condition once again

Three Color Camera

An interesting aerial camera development is shown in Figure 37. The use of kodachrome film in aerial cameras has proved most popular and quite successful wherever there is a desire for aerial color photography. However, the color

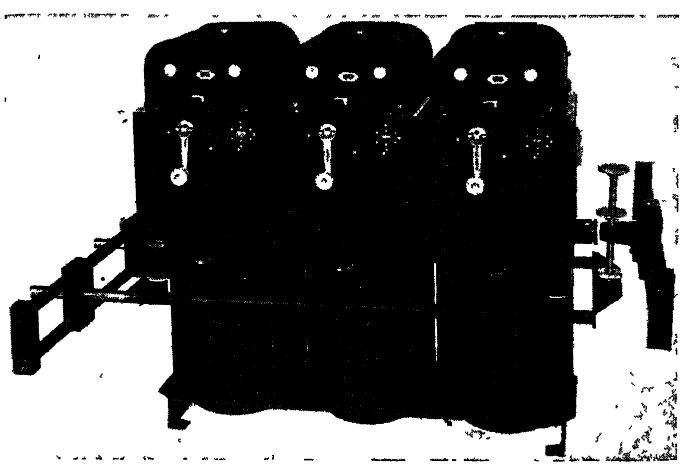
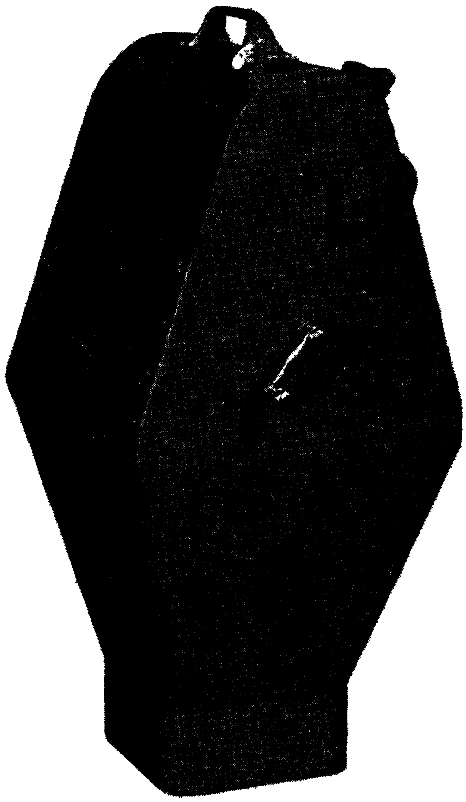


FIG. 37. Fairchild three-color camera (experimental).

separation process has some advantage over the use of kodacrome and an experimental camera was made to enable color separation negatives to be made from the air.

Aero Service Corporation's Cameras

The camera shown in Figures 38 and 39 is equipped with a between-the-lens shutter, which may be serviced without disturbing the accurate calibration setting of the lens. The focal plane and lens housing are fixed in a special unit metal cone. The model illustrated contains a lens of 8¼ inch focal length. Other models of similar construction have focal lengths of 4 and 5 inches.



Aero Service Corporation, Philadelphia, Pa

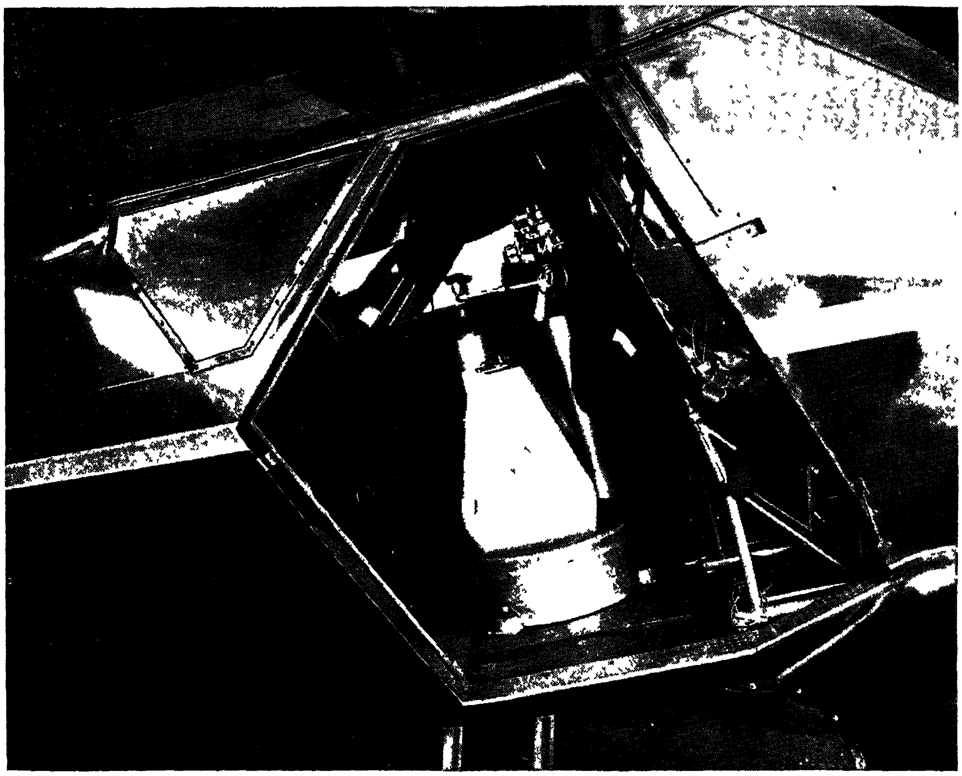
FIG. 38. Aero service precision aerial camera using film 9 inches wide in lengths up to 180 feet long.

The Brock precision camera shown in Figure 40 is one of the few aerial cameras using glass plates It is fully automatic in operation. Rotation of the handcrank accomplishes the following: (1) Returns the exposed plate to its proper section in the magazine; (2) Moves the magazine into position and draws an unexposed plate over the focal plane; (3) Sets the shutter ready for exposure. This entire operation may be completed in 6 seconds. Each plate magazine holds 48 glass plates 6½×8½ inches. The magazine may be loaded into the camera in daylight. The camera is equipped with a between-the-lens shutter. Interchangeable cones permit the use of lenses of various focal lengths, although 6.7 inch focal length is most commonly used.

Abrams Precision "Explorer" Aerial Camera

To eliminate inherent errors in aerial photography, the Abrams Instrument Company has developed special cameras, that greatly increase the photographic precision of aerial survey work.

One very important source of error has been the expansion and contraction of vital parts due to sudden temperature changes, thus causing a change in alignment of focal plane, lens, and fiducial marks.



Aero Service Corporation, Philadelphia, Pa.

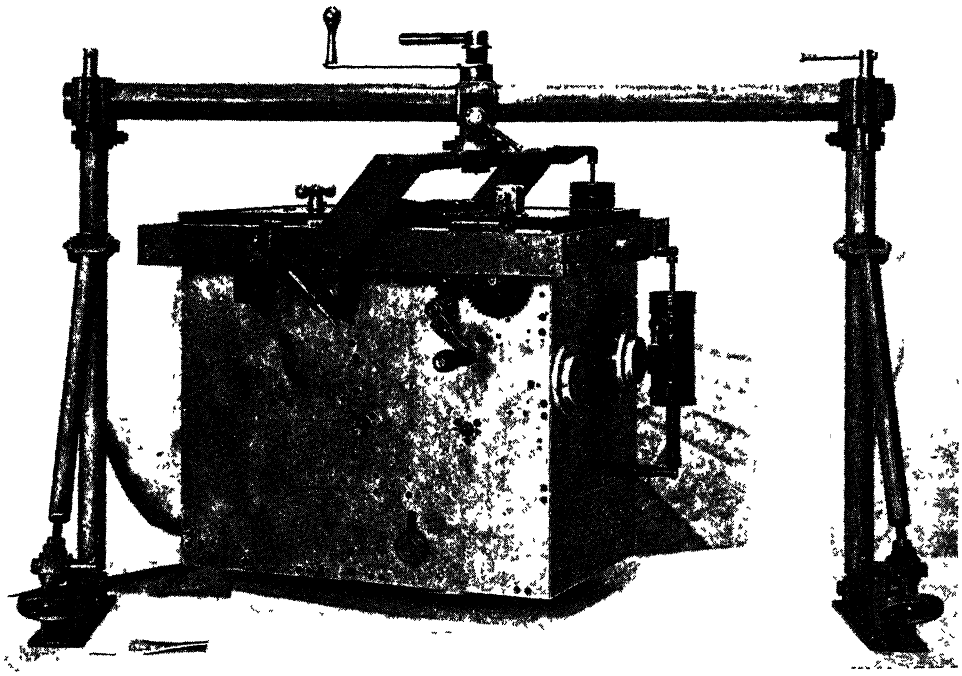
FIG. 39. Aero service precision aerial film mapping camera installed in an airplane showing shock mount which also permits rotation of camera to compensate for crab of airplane and leveling.

To reduce this error, the Abrams Instrument Company has substituted Invar Steel parts wherever a change of dimension due to temperature would cause an alignment change.

The focal plane is permanently sealed in position and does not move between exposures In other words, the "heart of the camera" floats in rubber and is free from dimensional changes of the aluminum castings used in the chassis.

Convenience and accuracy of operation are obtained by means of special built-in features such as level bubbles, stop watch, number recorder, and film gauge, all located close to the crank on top of the camera.

This "Explorer" camera, figure 41, has a very low center of gravity which



Aero Service Corporation, Philadelphia, Pa

FIG 40 Brock precision aerial camera

increases its stability. A choice of 3½", 4¾", 6", 8¼", or 10" lens is possible without changing the exterior of the camera.

A magazine capacity of 500 feet of film, sufficient to make 650 9"×9" photographs greatly adds to the general overall efficiency of the camera by eliminating many changes of a film magazine on a "photo mission," thus cutting down flying time required.

Abrams Precision "Baby Explorer" Oblique Camera

A smaller camera known as the "Baby Explorer" Hand Held camera, Figure 42, designed and manufactured by the Abrams Instrument Company, fills a long felt want in oblique photography

This completely streamlined, light-weight camera will produce 50 4"×5" negatives with one loading of its magazine

When held over the edge of the cockpit as in oblique photography, its convenient size, weight and general compactness increases efficiency and ease of operation.

The Mark Hurd Precision Camera

The Mark Hurd Precision Camera, Figure 43, is made up of two major units: (1) the magazine which contains the film and the mechanism for spacing, securing, and advancing of the film; and (2) the lens cone which contains the lens, shutter, focusing ring, fiducial marks, and light shield.

The magazine is built up with two side plates of cast aluminum alloy, pressed sheet cover, built-up section plate door, gear box cover, and instrument tray.

FIG. 41 Abrams Precision "Explorer" Camera and Abrams Universal Mount. Specifications: Height—31″; Diameter—16″; Capacity—650, 9″×9″ exposures; Weight (Empty)—48 lbs.; Weight (Loaded with 500 ft. of film—71 lbs.

The mechanism for operating the shutter, the pressure plate, and the film compensator is assembled as a unit to the gear box cover. All gears, wherever possible are of standard make. Levers, shafts, and pinions are of hardened steel. These shaft parts run in oilless bearings.

The instrument tray was designed to provide a convenient and serviceable location directly under the eyes of the operator for the level bubbles, stop watch, and two counters.

The cone is cast from a special alloy and is designed to have a uniform flow of metal to prevent the building up of internal stresses which might cause distortion or mis-alignment of the lens assembly when subjected to extreme ranges of temperature. Government approved fiducial marks are placed in the center of each side of the cone in the focal plane surface so that their images appear sharply on the negative, The top of the cone, which determines the focal plane, is brought to a flatness of plus or minus .0005 of an inch.

The lens mounting and focal plane surfaces are accurately machined so as to be parallel to each other in order to prevent distortion. Unusual precautions are taken to center the lens assembly and keep the axis of the lens perpendicular to the focal plane. For additional precision, all castings are aged and treated to maintain accuracy.

The pressure plate, or platen, is specially designed and cross ribbed to prevent

FIG 42. Mr Talbert Abrams, designer of the "Explorer" type precision cameras, inspecting the Hand Held "Baby Explorer" oblique camera

any warping caused by age. This plate is also brought to a flatness of plus or minus .0005 of an inch, which falls within the limits set by government specifications for precision cameras.

Film is held flat and secure in the focal plane by two means: (1) positive air pressure and (2) locking the pressure platen against the back of the film. There are numerous holes and slots forming a definite pattern in the face of the platen to permit the proper egress of air The platen releases when the film is advanced

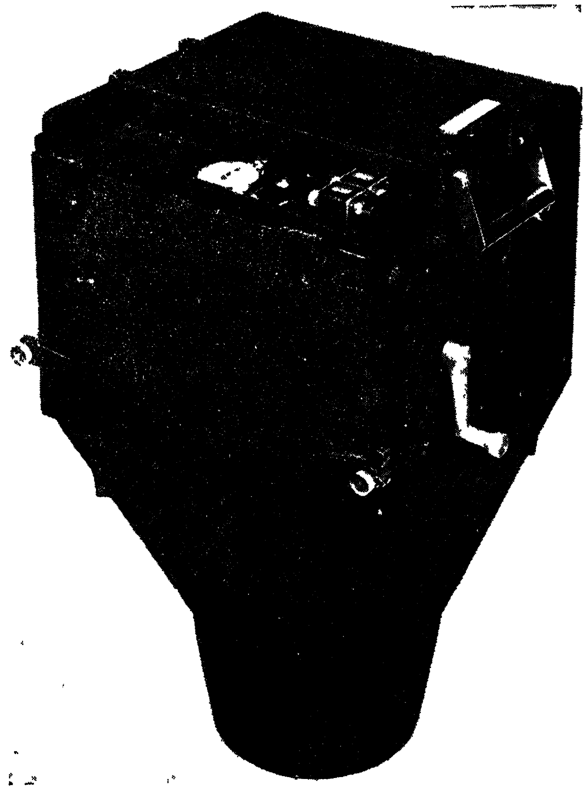


FIG 43. The Mark Hurd Precision Camera.

to permit easy, smooth transference of the film without the possibility of scratching. Rollers are placed at each end of the pressure plate to guide the film and at the same time prevent scoring. At the conclusion of the film advancement, the platen moves down and is firmly locked into position for the exposure.

The lens and shutter assembly are placed in the lower end of the cone in such a manner as to always maintain perfect alignment with the focal plane and are provided with a simple means of focusing and locking. Alignment of the lens and shutter is dependent upon the broad seat of the focusing ring against the machined end of the cone and not upon the threads of the rings. The lens and shutter may be removed without disturbing the focus. A light hood which fits the bottom of the cone serves to shield the lens from oblique extraneous light rays and also to protect the lens from damage during installation and take-off.

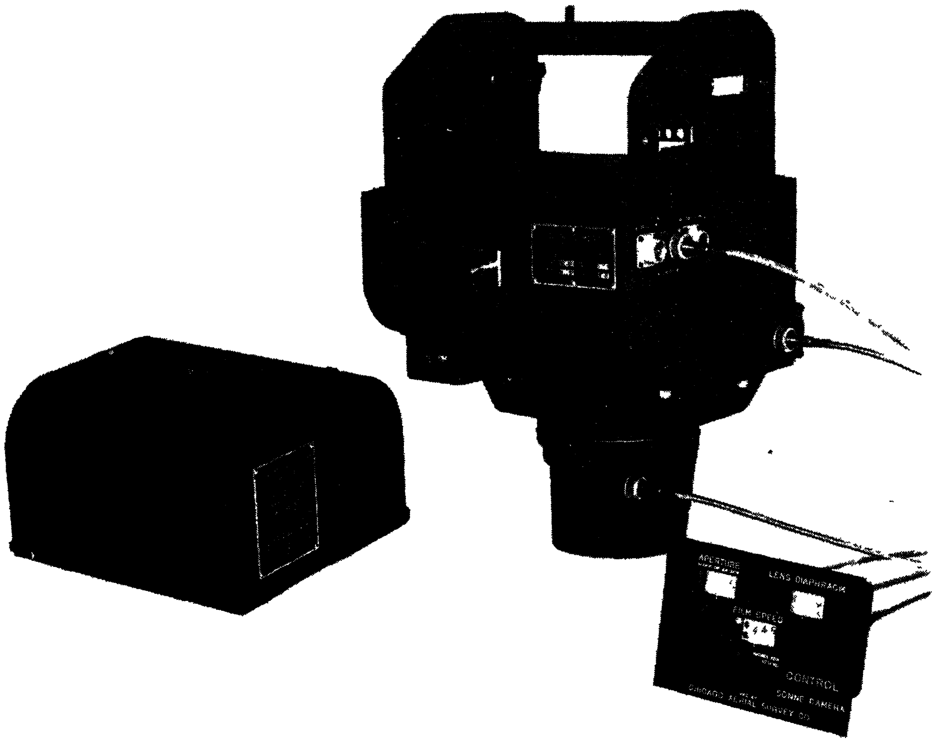
As a further means toward precision photography, a temperature adjustment correction has been developed. This adjustment was planned to keep the lens at the set focus while flying under extreme temperature variations The correction itself is easily made by turning a ring on the lens mount to various machined points which are calibrated to correspond with Centigrade or Fahrenheit readings.

Sonne Continuous Strip Camera

The newest mechanical spy equipment in the army air forces is the Sonne shutterless strip camera. Without any shutter, the intricate camera photographs

on a continuous strip of film. Either color or black and white film can be used, and the resulting print looks like a long narrow painting. How it's done, what area can be covered, are still military secrets.

The Sonne Continuous Strip Cameras produce a continuous image negative instead of a series of negatives as do the orthodox type cameras. The cameras are now built in four models: the model S2A accommodating film $18'' \times 150$ feet; the model S5A (See Figure 44) film $9\frac{1}{2}''$ wide $\times 150$ feet long; model S6A (See

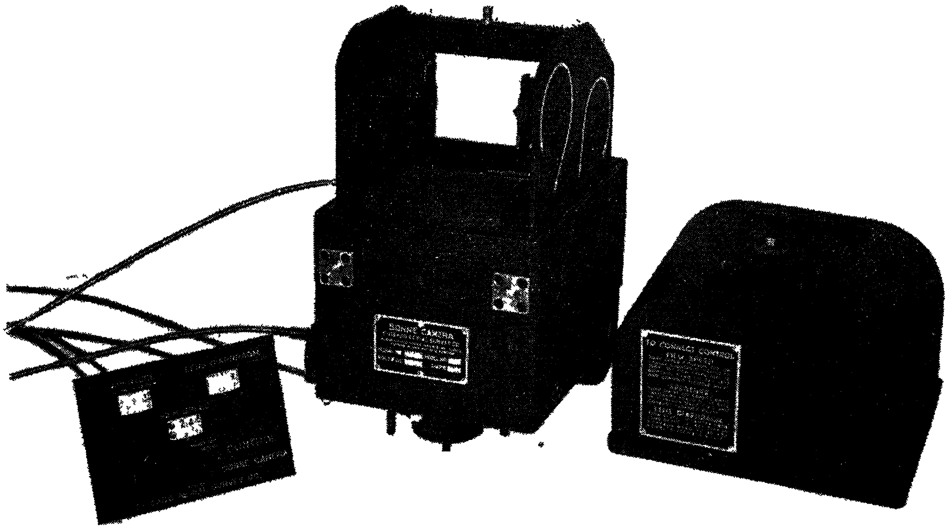


Chicago Aerial Survey Company

FIG. 44. Model S5A. This camera is normally furnished with 24″ lens and may be operated at altitudes from 4000 to 5000 feet, at air speeds from 200 miles to 450 miles per hour. The camera is designed to be operated manually and is equipped with a finder and means for visual synchronizing of film speed and image speed

Figure 45) film 7″ wide $\times 150$ feet long; and model S5B Stereo accommodating $9\frac{1}{2}''$ film $\times 150$ feet long These cameras can accommodate any desired focal length lens.

In continuous strip mapping, the film is in continuous motion and very closely synchronized with the image speed, resulting in a very small relative motion between film and image. This allows for comparatively long exposures and makes possible photography under very poor lighting conditions and the use of color film at very low altitudes and high speeds. It also makes possible extremely sharp photographs at low altitudes and high speeds. Section of a strip photography is shown in Figure 46.



Chicago Aerial Survey Company

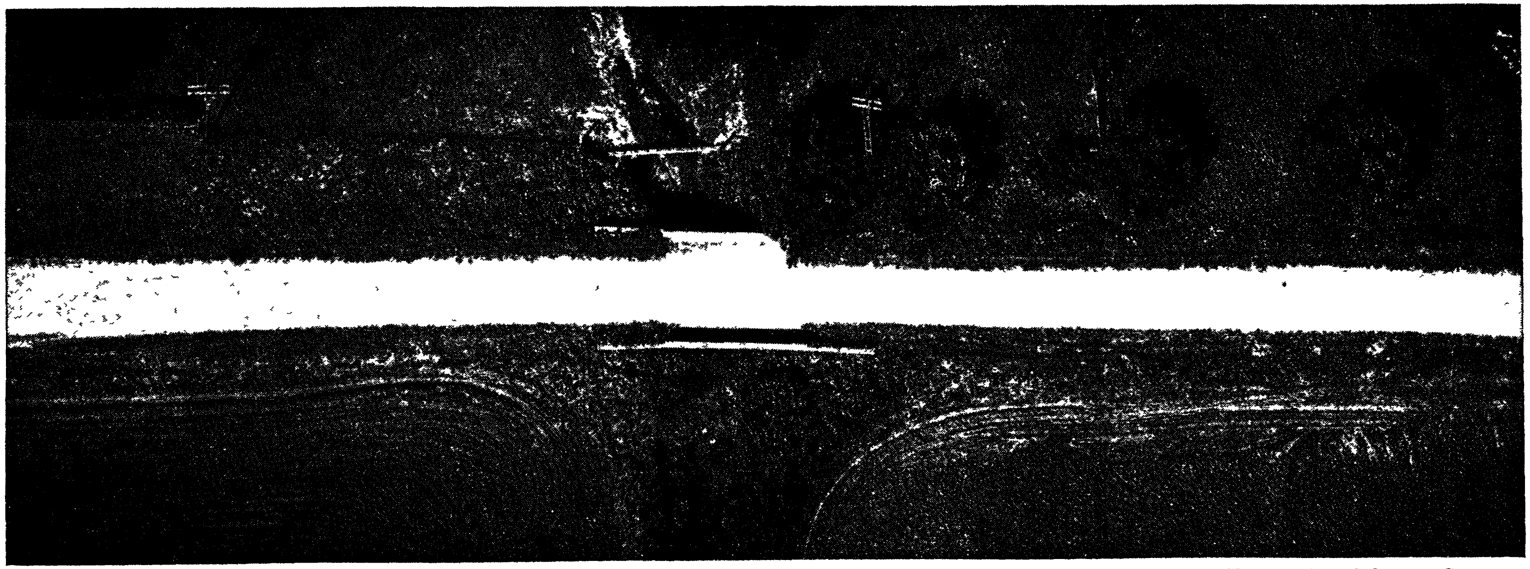
FIG 45. Model S6A. This camera is designed for pursuit ship installation and may be operated by remote control. It is designed to operate at altitudes from 100 feet to 900 feet, and air speeds from 200 miles per hour to 400 miles per hour

In continuous strip photography, it is necessary that the film speed and image speed be synchronized as close as possible over the entire range of the camera. With the speeds and altitudes mentioned, this amounts to a range of slightly over twenty to one, a difficult range for orthodox type of speed control, considering that it must be held constant under extreme temperature variations and fluctuations in line voltage. This is accomplished by means of a governor controlled 24 volt motor running at a constant speed, and a unique variable speed transmission consisting of hardened and ground tapered rollers and a movable ring, the position of which is controlled remotely. This unit has proven extremely reliable under all conditions and gives the necessary exactitudes in the variations of speeds

The cameras are small and compact, and are simple to load and adjust for operation. They have mounting lugs so designed that they may be quickly put into or taken out of the mount The mount itself is equipped with shock absorbers and fits into the standard camera mount pads of 20×12″ centers, it is so made that it may be readily adapted to any special installation. Both the camera and mount are of rugged design throughout, and should require very little service over a long period of time.

PRECISION CAMERA SPECIFICATIONS

The United States Geological Survey of the Department of the Interior and the various branches of the United States Department of Agriculture, as well as other agencies of the government do an extensive amount of mapping from aerial photographs. Since the aerial photograph gets its basic accuracy from the camera in which it was made both the Geological Survey and the Department of



Chicago Aerial Survey Company

Fig. 46. Section of strip photography made with Sonne Continuous Strip Camera. Altitude 100 feet. Focal length of lens $3\frac{1}{2}''$.

Agriculture are careful to specify the characteristics of the cameras to be used in conjunction with mapping photography done for them. Because these specifications are basic as well as informative, they are included as follows:

Department of the Interior
Geological Survey
July 1940

SPECIFICATIONS

AERIAL CAMERAS (TOPOGRAPHIC)

For taking photographs satisfactory for use with Geological Survey Multiplex Instruments

A. *General Statement*

In preparing topographic maps of the highest quality by means of Multiplex photogrammetric instruments, the Geological Survey must use aerial photographs that meet rigid requirements and that are taken with precise aerial cameras For the information of commercial aerial photographers and manufacturers of aerial cameras, the Geological Survey from its experience has compiled and here presents specifications for types of cameras suitable for taking photographs that will meet the Survey's requirements

The focal length, angle of field of view, film width, construction, tolerances and other specifications of the cameras described herein are such as to permit ready employment of the negatives in the Multiplex instruments and to assure determination of position and differences of elevation with a high degree of precision Cameras not complying with these specifications cannot be readily and economically employed.

B. *Camera Types*

The general characteristics of cameras suitable for taking photographs that will satisfy the needs of the Geological Survey are listed in Table I. Cameras must comply with these characteristics as to equivalent focal length, negative size, film width, type and placement of fiducial marks and minimum field of lens and with the general specifications that follow The type of camera to be employed on any particular project will be stated in the invitation to bid.

TABLE I. AERIAL CAMERAS FOR MULTIPLEX MAPPING

Type	Equivalent Focal Length (mm)	Angle of Field of Lens (°)	Size of Negative Image	Film Width (in cm.)	Fiducial Marks (drawing No)	Minimum Resolving Power (lines per mm)	*Maximum Distortion (mm.)
1	100±2.0	93	radius 105 mm.	24	105	14	±0 20
2	100±2 0	93	18×18 cms	19	106	14	±0 20
3	131±2 5	93	23×23 cms.	24	108	10	±0 30
4	131±2 5	93	radius 140 mm	29	109	10	±0.30

* Distortion indicated refers to the equivalent focal length and the diaphragm aperture recommended by the manufacturer of the lens.

C. *Materials, Workmanship, and Finish*

1. The materials used in the construction of the camera shall be entirely suitable for the construction of a precision mapping camera.

2. The workmanship shall be in accordance with that accepted as standard in the manufacture of high-grade cameras.

3. The finish of all exterior surfaces other than brass, bronze, or steel parts shall be of baked lacquer Exterior parts of brass, bronze, or steel, other than bearing surfaces, shall be chromium-plated. All bearing surfaces shall be clean

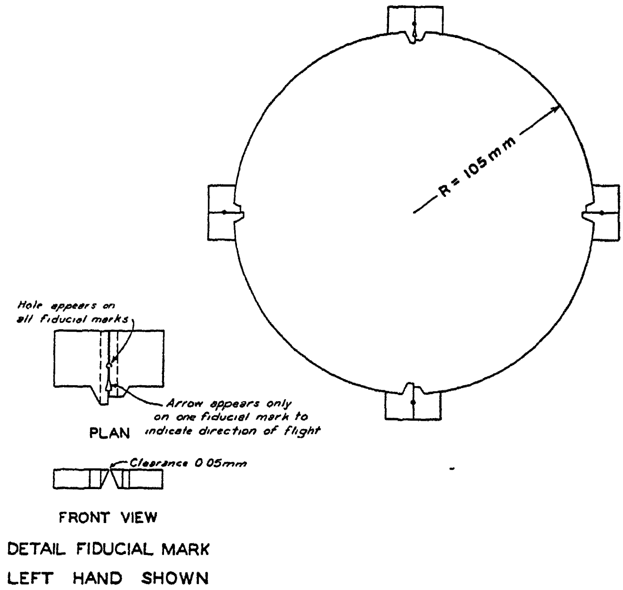
The finish of all interior surfaces, except rollers, the upper surface of the focal-plane frame, and bearing surfaces shall be finished with Egyptian optical black lacquer or equivalent All bearing surfaces shall be clean

D *Detail Requirements*

1 *Camera Cone Assembly*—The camera-cone assembly shall be so constructed that the lens and shutter, the camera cone, the surface that determines the location of the focal plane, and the fiducial marks shall be constructed as a unit This cone assembly shall be so designed and manufactured that all parts, should it be necessary for any reason to disassemble it, will return precisely to their original positions Precaution shall be taken to heat-treat the cone itself carefully to prevent secular changes The cone shall be shaped in such manner that the metal is uniformly disposed about the axis of the cone without large projecting lugs or abrupt changes in wall thickness. The cone shall be constructed of material that will maintain the alignment of its component parts under the great and sudden changes in temperature to which it will be subjected. The dimensional changes arising from temperature or other variations likely to be encountered shall not be inconsistent with the determination of the principal point with a probable error not exceeding 0 025 millimeter The dimensional changes of the camera-cone assembly arising from causes other than temperature shall be sufficiently small to permit the determination of the calibrated focal length with a probable error not exceeding ± 0 05 millimeter The opening in the focal-plane frame that fixes the size and shape of the negative shall be of the size mentioned in Table I for the specific camera and if this opening is rectangular it shall be used so that its major axis is in the line of flight unless specific instructions to the contrary are given

2 *Fiducial Marks*—The collimation index marks (fiducial marks) shall be as shown in the drawing that is mentioned in Table I and that forms a part of these specifications Lines joining opposite members of the two pairs of index markers shall intersect at an angle of $90° \pm 30''$ and their intersection shall indicate the position of the principal point with a probable error not exceeding ± 0 025 millimeters The design and position of each marker shall be such as to yield sharp shadowgraphs of a 0 05 millimeter slot, of a small drill hole whose center is on the line of the slot, and of those straight edges of each marker that are normal to the slot. Other edges of the markers shall be so shaped as to avoid scratching the emulsion surface of the film during the film winding operation The distance from the center of the aperture to the fiducial marks, in cameras having circular apertures, should be as great as possible without detracting from the usefulness of the shadowgraphs Provision shall be made for fastening the fiducial markers in position by dowels after their proper location has been determined by a National Bureau of Standards test. Two well-spaced fixed or removable stops shall be provided on a line approximately parallel to the line defined by one pair of opposite fiducial marks, and one fixed or removable stop shall be provided on a line perpendicular to this pair of stops, these to be used to position properly the photographic plates used by the National Bureau of Standards in recording the relative positions of the principal point and the four fiducial marks

3. *The Platen.*—The platen against which the film is pressed to insure its planeness at the moment of exposure shall be of rigid construction and shall not depart more than ± 0 0125 millimeters from a plane surface. Suitable provision shall be made for flattening the film against the platen by air pressure or suction, or both, at the instant of exposure, caution being taken to strengthen the platen sufficiently to prevent its deformation by this pressure.



DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

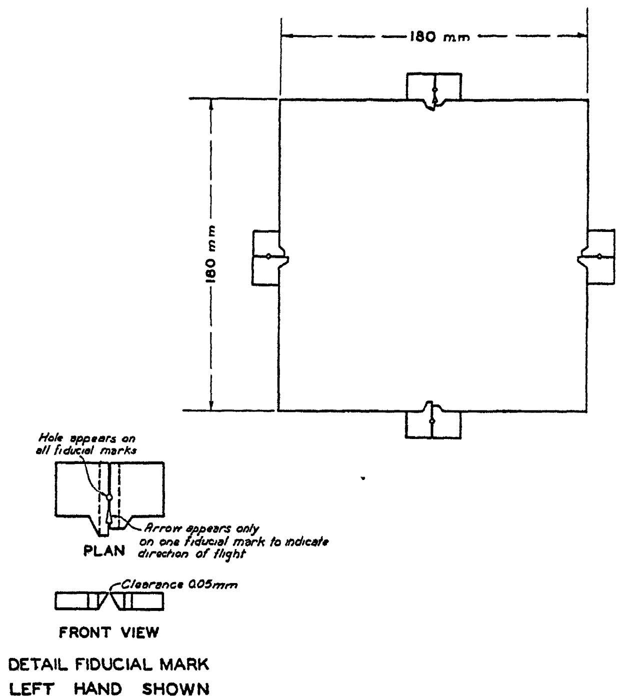
DESIGN OF FIDUCIAL MARKS
FOR A TOPOGRAPHIC AERIAL MAPPING CAMERA

DRAWING NO 105

4. *The Shutter.*—Shutters shall be of a type in which the shutter curtain or blades operate between the lens components. The efficiency of the shutter shall be such as to reduce the over-all duration of the exposure to such an extent that a minimum loss of definition of image will result from the movement of the airplane or vibration of the camera. The range of shutter speeds shall be such as to assure proper exposure at the diaphragm aperture recommended by the manufacturer with any aerial film that may be employed.

5. *The Lens.*—The lens employed in the camera cone shall be used at the diaphragm aperture recommended by the manufacturer of the lens. Equivalent focal lengths longer or shorter than that stated for the type of camera to be employed introduce serious and costly delays in the necessary adjustments of the Geological Survey photogrammetric instruments with which the negatives made with the camera will be employed The tangential and radial resolving power of the lens and its distortion shall be not less than that stated in Table I.

6. *Lens and Shutter Assembly.*—The lens elements and shutter shall be so mounted in the camera cone that they may be disassembled for cleaning the inner lens surfaces of the front and rear lens cells, or repairing the shutter, and reassembled without introduction of changes in the calibrated constants of the camera greater than their probable errors. Disassembly of the lens beyond that required for cleaning the inner surfaces of the front and rear lens cells shall be prevented by lock screws or equivalent devices. The assembly of the lens elements and the shutter shall be conducted in such manner



DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

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FOR A TOPOGRAPHIC AERIAL MAPPING CAMERA

DRAWING NO 106

as to maintain the alignment of the lens axes to the same precision that existed when the lens elements were mounted in the lens barrel by the manufacturer.

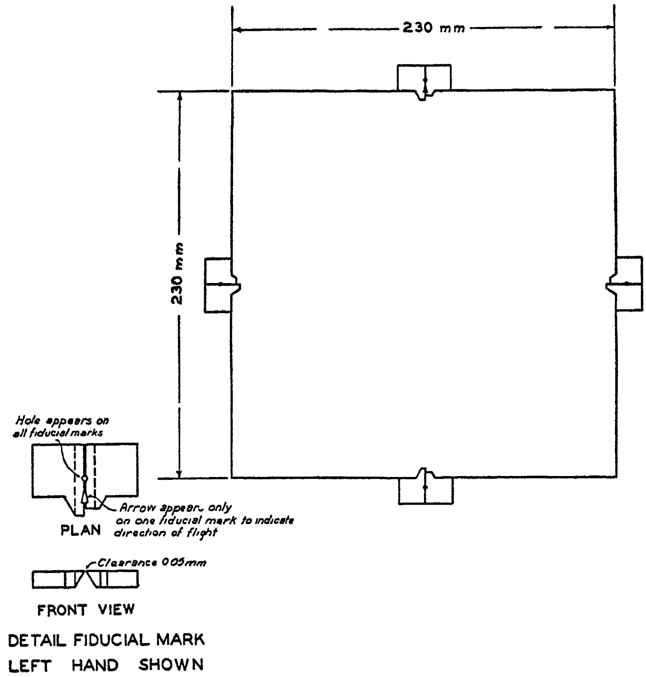
7. *The Camera Mount*—The mount in which the camera is suspended in the airplane shall be designed to absorb a maximum of vibration of the airplane.

8. *Light Filter.*—The filter to be employed with the aerial camera shall be used in front of the lens and shall be held firmly in place but not so tightly as to induce strain The type of filter to be used is a colored-all-through glass, ground and polished plane and of the homogeneity indicated below:

a. The maximum difference in edge-thickness at two diametrally opposite points shall not exceed 0.10 millimeter per 50 millimeters of diameter

b. The departure of either surface from planeness shall not exceed that corresponding to one interference fringe per centimeter of diameter.

c. The homogeneity of the glass composing the filter shall be such that there is no perceptible difference in the visible detail or sharpness when a resolution chart is viewed through a telescope and the filter introduced into the light path immediately in front of the resolution chart and immediately in front of the telescope objective. The resolving-power chart should be transparent, well-illuminated by transmitted light, mounted at such a distance from the telescope that the separation of adjacent lines of the finest pattern subtends an angle of 10″. A telescope (such as is found on a surveyor's transit of good quality) with an objective 30 or more millimeters in diameter and magnifying 20 or more diameters is suitable for this test. When the filter is immediately



DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

DESIGN OF FIDUCIAL MARKS
FOR A TOPOGRAPHIC AERIAL MAPPING CAMERA

DRAWING NO 108

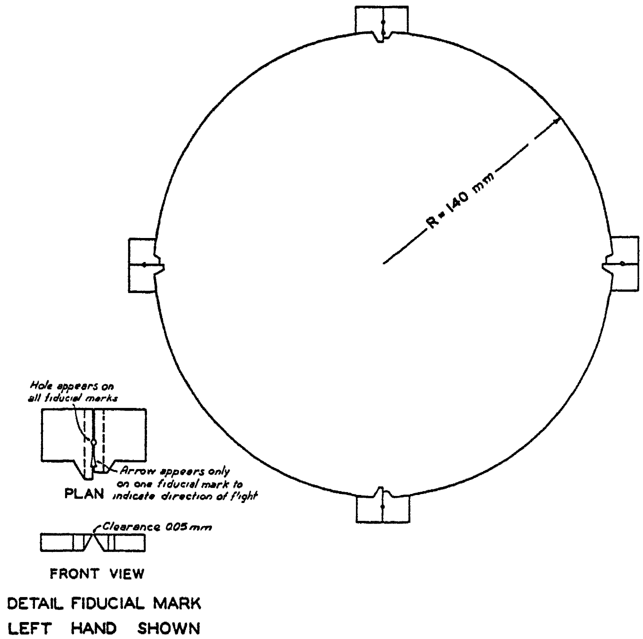
in front of the objective it should be moved about so that the chart is successively viewed through all portions of the filter. When the filter is immediately in front of the resolving-power chart care should be taken that stray light reflected from the filter does not enter the telescope and cause a decrease in the visible detail.

9. *Intervalometer.*—The use of an intervalometer of a type that will control the automatic exposure of the camera at specified distance intervals along the flight line, or to indicate the moment the exposures should be made in hand operated cameras, yields air bases that are much more uniform in mountainous regions than are attained without this device. The use of photographs made with the aid of such a device results in greater map accuracy and reduces the cost of ground control. Other things being equal, cameras so equipped will be preferred.

E. *Camera Calibration*

A report of the National Bureau of Standards shall be provided for each camera to aid the Geological Survey in determining the suitability of the camera for use with stereophotogrammetric mapping instruments now on hand. The report shall include a lens test, a metrical test of the camera as a mapping instrument, and a glass-plate negative showing the principal point of the camera and the four fiducial marks.

1. *Scope of the Report.*—The report shall be based on tests and measurements made after final assembly of all parts of the camera-cone unit, with the light filter to be employed in place on the lens. The report shall be based on tests made just before



DEPARTMENT OF THE INTERIOR
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DESIGN OF FIDUCIAL MARKS
FOR A TOPOGRAPHIC AERIAL MAPPING CAMERA

DRAWING NO 109

undertaking the photographic contract or proof be supplied that the adjustments of the camera have not been altered since the last test.

The report shall supply the following information:

a. The equivalent focal length of the lens.

b. The distortion, referred to the equivalent focal length, from the lens axis to the edge of the field, by 5° intervals

c The calibrated focal length.

d The distortion, referred to the calibrated focal length, from the lens axis to the edge of the field, by 5° intervals.

e. The radial and tangential resolving power of the lens from 0° to the edge of the field, by 5° intervals.

f. A precise measurement of the distance from the vertex of the last surface of the lens to the focal-plane frame.

g. A measurement of the angle between lines joining opposite fiducial marks.

h A photographic plate exposed in the camera showing the relation of the lines joining opposite fiducial marks to the principal point This negative shall be marked with the date of test, the test number, the camera and lens numbers, and the points where the plate contacted the three removable stops mentioned in Paragraph D-4

i. Measurements of the distance between opposite index marks shown on the photographic plate mentioned in Paragraph E-1-h with a probable error not exceeding 0.01 millimeter.

2. *Constructional Details Necessary to Permit Testing*—To permit the necessary tests for the determination of the calibrated focal length, distortion, resolution, and

position of the principal point without the introduction of errors resulting from film shrinkage, the camera shall be constructed in accordance with the following requirements

a It shall be possible to insert a photographic plate between the frame that determines the position of the focal plane and the platen that normally flattens the film in the focal plane.

b The focal plane shall be accessible from the rear so that a telescope placed behind the camera may view objects through the camera lens at any angle limited only by the size of the focal-plane opening. The removal of parts to secure this vision shall not include the removal of the collimation index markers or of the frame that determines the location of the focal plane

c. Provision shall be made for holding the shutter leaves open during the test period.

Spec. No. A-APC-1102
Approved March 12, 1940

UNITED STATES DEPARTMENT OF AGRICULTURE SPECIFICATIONS FOR A PRECISION AIRPLANE MAPPING CAMERA

D. Specifications

In order to secure the widest range of competition from manufacturers of, and dealers in, aerial precision cameras, and to insure that photography procured under contract, or directly, is of such a high degree of accuracy as to make it suitable for stereophotogrammetric plotting of topography, the following specifications are submitted

D-1. *Constructional Details Necessary to Permit Testing.* In order to permit the necessary tests for the determination of the calibrated focal length, distortion and position of principal point without the introduction of errors resulting from film shrinkage, the camera shall be constructed in accordance with the following requirements

It shall be possible to insert a photographic plate between the frame that determines the position of the focal plane and the platen that normally flattens the film in the focal plane

D-1a. *Focal Plane*—The focal plane shall be accessible from the rear so that a telescope, placed behind the camera and directed along the axis of the lens, may view objects through the camera lens. The opening providing this unobstructed vision shall be not less than an inch in diameter and may be provided by removable screw caps or by the partial disassembly of the magazine and the removal of the platen that presses the film against the focal plane. The removal of parts to secure this vision shall not include the removal of the collimation index markers (fiducial marks) or of the frame that determines the location of the focal plane.

D-2. *Constructional Details Required for Metrical Precision.*

D-2a. *Platen.*—The portion of the surface of the platen against which the film is held to insure its planeness shall not depart more than ± 0.0005 inch from a plane.

D-2b. *Lens Assembly*—The lens barrel shall be so constructed that the disassembly for cleaning the inner lens surfaces of the front and rear lens cells or repairing the shutter and reassembly shall introduce no changes in the calibration constants greater than their probable errors. Disassembly of the

lens beyond that required for cleaning the inner surfaces of the front and rear lens cells or repairing the shutter shall be prevented by lock screws or equivalent devices

D-2c *Duration of Exposure.*—In cameras in which the shutter curtain or blades do not operate between the lens components, the operation shall be such that the total duration of exposure for the entire plate is not greater than 1/50 second for an exposure of 1/100 second

D-2d *Collimation Markers*—The collimation index markers (fiducial marks) shall be as shown on drawing No. 1 which forms a part of these specifications. The lines joining the opposite members of the two pairs of index markers shall intersect at an angle of 90° ±1 minute and shall indicate the location of the principal point with a probable error not exceeding ±0.03 mm.

D-2e *Dimensional Tolerances*—The dimensional changes of the camera arising from causes other than temperature shall be sufficiently small to permit the determination of the calibrated focal length with a probable error not exceeding ±0 05 mm

The dimensional changes arising from temperature or other variations likely to be encountered in use shall not be inconsistent with the determination of the location of the principal point with a probable errors not exceeding ±0 03 mm

D-3. *Lens Specifications*—All tests of the lens contributing to the determination of the calibration constants of the camera will be made with the lens mounted in the camera

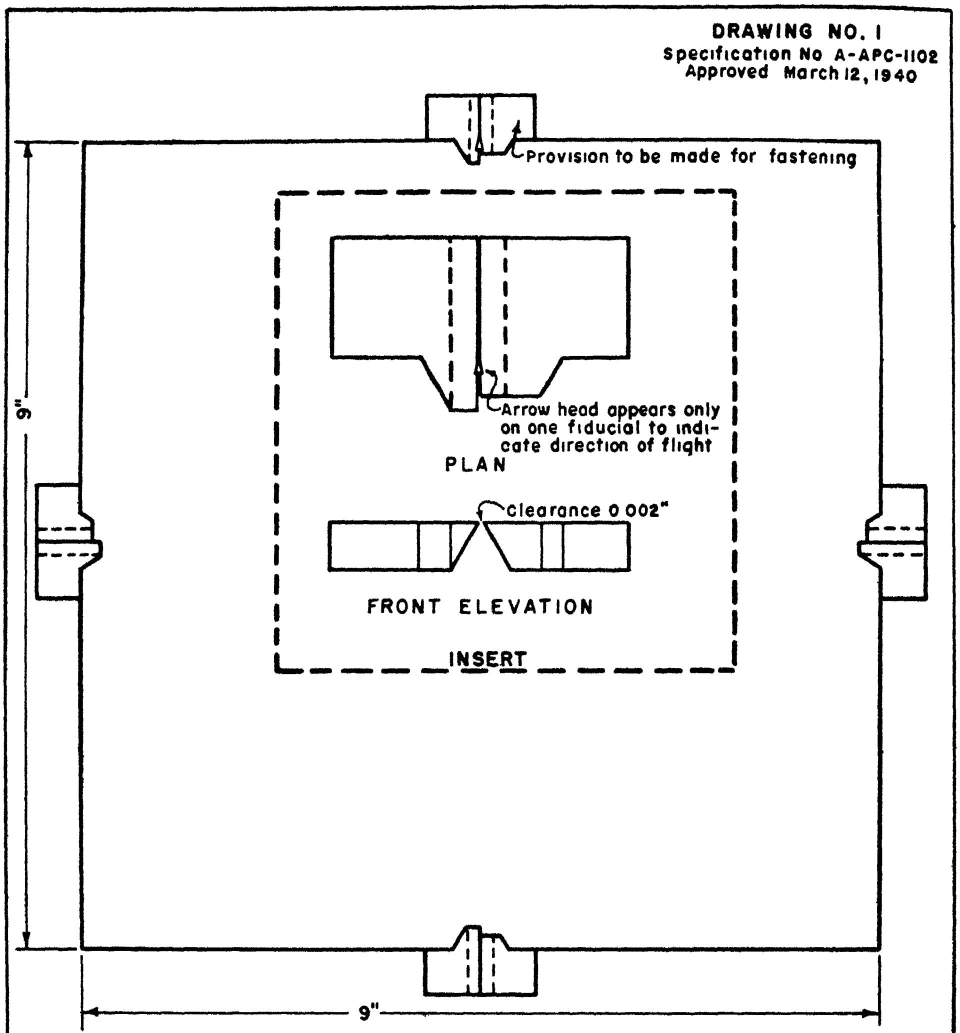
D-3a *Resolution.*—No lens shall be used which, at the maximum stop opening, fails to resolve lines in any orientation spaced 20 to the millimeter at the center of the field, or which fails to resolve lines in any orientation spaced 7 to the millimeter at all 5° intervals lying between the center of the field and the center of the shorter side of the negative; or which fails to resolve lines in at least one orientation spaced 5 to the millimeter at the angular distance from the center of the field which is the multiple of 5° falling within the field and nearest the corner of the negative. The resolution test shall be made in accordance with the standard practice of the National Bureau of Standards.

D-3b *Distortion.*—For a camera using 9 by 9 or 7 by 9 inch films or plates and having a lens of approximately 8 inch focal length, the distortion, referred to the calibrated focal length, shall not exceed ±0.03 mm.

D-3c. *Tolerance Exceptions*—The tolerances for resolution and distortion embodied in these specifications do not apply to cameras equipped with lenses of equivalent focal length of 6½ inches or less, nor to cameras making negatives larger than 10 by 10 inches. For such cameras the appropriate tolerances shall be specified by the contracting agency

E. TEST REQUIREMENTS

All precision cameras made in accordance with this specification shall be submitted to the National Bureau of Standards for test before field use under contract, or before purchase, and must be accompanied by a Certificate therefrom that it fully complies therewith.



U. S DEPARTMENT OF AGRICULTURE
DESIGN OF COLLIMATION MARKS
FOR
A PRECISION AIRPLANE MAPPING CAMERA

AIRCRAFT CAMERA MOUNTS

C S. Robinson

FOR many years there have been available to aerial survey engineers, aircraft cameras suitably designed and sturdily constructed for their purpose.

In utilizing these cameras, however, it has been necessary to mount them in supporting cradles known as vertical mounts. Throughout the past decade there have been available several types and makes of serviceable mounts, of simple design and well built, and with these, a tremendous amount of high quality mapping has been completed

It is the feeling of the writer, however, that camera mount design has not quite kept pace with the development of aircraft cameras and that this mapping would have been of even better quality, and flown in less time with fewer re-flights, had more efficient camera mounts been available.

The functions of an aircraft camera mount are: 1. To provide an adequate but light weight structural support for the camera. 2 To effectively insulate the camera from airplane and engine vibration 3. To provide means for leveling the camera and making azimuth adjustments.

Most camera mounts have met these requirements in part, but not to the extent now attainable

The structural materials used for camera mounts are metal, including steel, aluminum, magnesium and wood, principally plywood.

The light metals have constituted the principal materials used and in some cases in order to minimize weight, long slender attachment arms have been designed which have not always outlasted hard service. The concentric rings ordinarily used have been in most cases adequate and in a few cases unnecessarily heavy. Trunnion clamps and attachment fittings have usually been constructed of aluminum or magnesium, and sometimes these have been of such design that excessive clamping pressures have been developed with consequent fracture of the trunnion clamp parts.

Camera mounts of British design have recently been constructed almost entirely of plywood utilizing sponge rubber insulating pads It is understood that this plywood construction has proven to be durable and tough, and that in addition it is felt by the British that the vibration absorption characteristics of plywood are desirable Further improvement in these wood mounts may be expected.

Undoubtedly the further development and accumulation of experience in the use of magnesium will result in mounts fabricated from this material.

For many years the vital factor of vibration absorption in camera mounts has been dependent upon conventional design, and it is believed that comparatively little thought and constructive design have been applied to this problem.

Vibration absorbers as a rule, have consisted of rubber pads, either solid or molded of rubber sponge, and these pads have been placed as supports around the periphery of the frame. This system of mounting, while capable of absorbing a certain percentage of vibration within limited frequency ranges, often amplifies airplane and propeller vibration at other frequencies. The torsional, or oscillating type of vibration present in most airplanes is not effectively absorbed through this conventional arrangement, since the cushions are widely spaced, and vibration picked up is transmitted through a long couple to the camera. Furthermore, nearly all conventional systems utilize one layer or stage of flexible material. Vibration, after passing this single layer, is directly applied to the camera. In some cases, flat rubber biscuits have been utilized in which a narrow ring of

rubber has been considered to be in shear. Actually, an analysis will show that as soon as deflection under load occurs, the rubber in these units is in tension. Rubber in tension is not particularly long lived and oscillation which may occur is undamped. Furthermore such units are ordinarily not flexible in any horizontal direction and therefore have little or no absorption horizontally

Many measurements and readings taken in single and twin-motored airplanes have indicated that the predominant type of vibration encountered is lateral combined with a smaller amount of vertical motion. Superimposed on this is the torsional or oscillation vibration previously mentioned

A recently developed design which appears to have overcome most of the foregoing difficulties will be described in the latter portion of this paper.

In aerial photography whether it be extended precision type mapping or merely the taking of so called tactical or spot photographs, the matter of leveling the camera accurately and instantaneously is of vital importance. The variable scale resulting from tilted photographs is undesirable both for civilian use and military interpretation

Various leveling devices for camera mounts have been in use The provision most commonly made for leveling consists of a gimbal mount equipped with mechanical pivots and these pivots in turn equipped with some kind of a take-up mechanism such as an adjusting screw Such pivots must be kept sufficiently tight so that the camera will remain fixed in whatever position it may be placed and this degree of tightness ordinarily prohibits the smooth type of motion necessary for quick and accurate leveling.

Another system which is said to produce acceptable results consists of concentric rings ground to spherical surfaces, with a lubricant or small steel balls interposed between. This furnishes a universal type of support which is understood to be easily adjusted, and with proper bearing pressures, to remain in adjustment

Screws of various types have also been used with some success but it is believed that this system provides action somewhat too slow for instantaneous action and further requires accurate coordination in the manipulation of two or more screws by the cameraman.

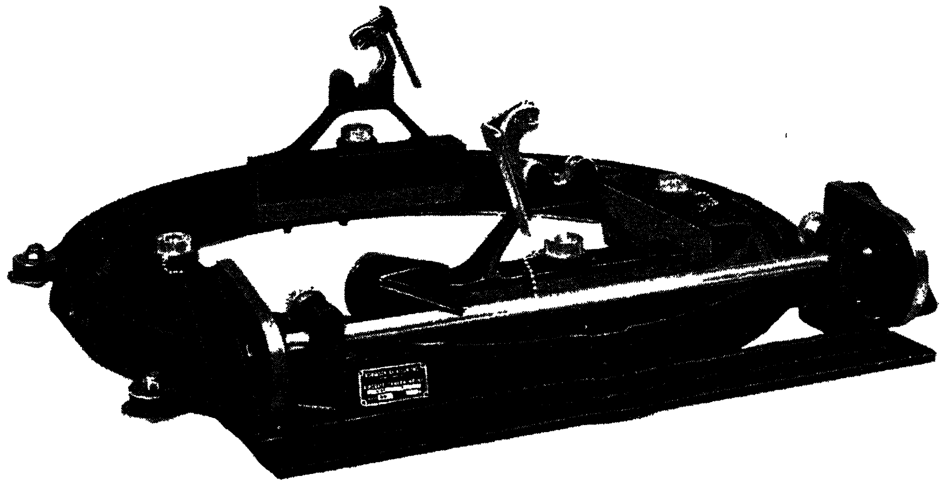
A camera mount leveled by automatic electrical means has been developed in this country. It is understood that this mount is very satisfactory

Photographs of the types of mounts described are not available upon short notice to the writer, but it is believed that most of the readers of this paper have utilized and are familiar with the types described in the foregoing paragraphs.

The writer's company has been actively engaged during the last few years in the development of camera mounts designed to overcome certain of the objectionable features which have been listed and since this equipment has been adopted as standard by the U. S. Army and Naval Air Forces, it is believed that it may be of interest to describe the manner in which these improvements have been accomplished.

Two types of camera mounts were developed for the Services, one being a manually adjustable type designed for precision mapping, and the other a fixed type designed to support the camera and absorb aircraft vibration but incorporating no manual adjustments. The adjustable type mount, illustrated in Figure 1, will be described.

The structural members are aluminum castings, of a strong and tough alloy. The main frame carries the attachment fittings by which the mount is secured to the airplane and concentric rings are enclosed within this frame. A mechanical



Robinson Aviation Inc

FIG. 1. U S. Army A -11-A Camera Mount.

gimbal suspension is incorporated in the mount, utilizing fiber lined bearings to provide tightness with a minimum of friction This gimbal suspension is not designed to operate directly through pressure on the camera, but is operated by means of cams These cams provide a smooth and easy leveling action covering a 10 degree range fore and aft and an 8 degree range laterally. The cams are instantaneous in operation, a single motion sufficing for any leveling adjustment and the adjustment is self-locking when released.

This cam leveling system is designed for a semi-permanent adjustment. The camera is leveled with the cams after the airplane reaches the altitude at which the mission will be undertaken With the camera thus set to compensate for the normal flying position of the airplane, it is ordinarily necessary only to set the proper azimuth angle on the rotating ring. The shock suspension is so designed that an additional 8 degrees of leveling action are available in any direction and this leveling action is obtained by direct motion of the camera in a desired direction. This motion is free from all mechanical friction and the leveling cushions exert a graduated resistance which assists in a smooth and accurate centering of the level bubble. When the camera is released after leveling for an individual shot, it will return automatically to the pre-set position for a normal flight.

The vibration absorbing suspension which is identical with the rubber cushions used for leveling is of a radically new design. It was found that by removing the shock absorbing cushions from the outside of the mount and locating them on the fore and aft neutral axis, a large amount of airplane vibration approximating 95% plus was easily absorbed. An intermediate ring was then superimposed on this first pair of cushions and the load carried around to a pair of secondary cushions placed on the lateral axis. These secondary cushions have a dual function of absorbing any fore and aft pitching vibration of the airplane and absorbing any residual, vertical, or lateral vibration which may have passed through the primary cushions.

The vertical loading is carried uniformly over the entire area of the cushions with no localized concentration. The cushions as stated operate in series and are in so-called shear in all horizontal directions. Mounts of this type have been in

Service with the Army and Navy throughout the war and the cushions have been shown to have an exceptionally long life under all climatic conditions, no replacements whatever having been required up to date. The composition of the cushions is such that they will continue to operate effectively at temperatures as low as 60 degrees below zero Fahrenheit.

While most of the structural material in the mount is cast aluminum as stated, the trunnion clamps are of thin steel so designed that a single motion will lock or unlock the camera trunnion No excessive loading is possible on these trunnion clamps, the locking pressure being predetermined in the design of these units.

An additional feature not shown in the illustration, consisting of a synchronized viewfinder, has been incorporated in camera mounts of this type designed and built for the U. S. Naval Air Forces This viewfinder, operated by a small gear train, turns about its own axis through any azimuth turned by the main ring on the camera mount This feature has eliminated the necessity of reading an azimuth angle on a separate viewfinder and then setting the camera ring to correspond

Undoubtedly camera mount design will continue to advance throughout the war and in post-war years, and it is the writer's opinion that a proportionately larger share of design effort applied to the camera mount problem will result in greatly improved overall accuracy and performance on aerial mapping projects.

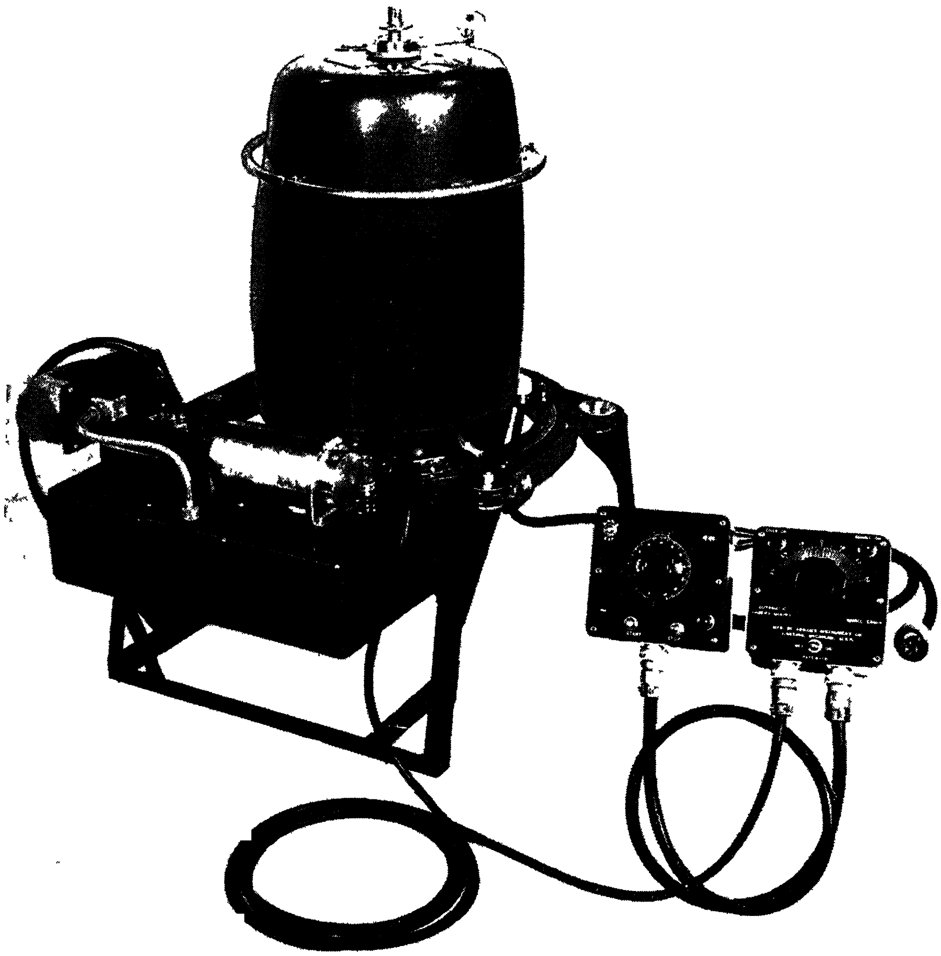
AUTOMATIC AIRCRAFT CAMERA MOUNT

J. E Meyer

Description

THE Automatic Camera Mount, Figure 1, is a device for automatically leveling a camera and for providing remote control of the camera about the vertical axis (crab control). Special attention has been given to simplicity of design, utility, and ease of installation and operation

The mount including cable and remote control box weighs only 70 pounds and is very compact, exceeding the clearance dimensions of present standard mounts in only one dimension, namely, that of depth. The leveling system is extremely accurate, as it is adjusted at 26 V D C. for any out of level condition exceeding 12 minutes from the true horizontal. The rate of change correction



Abram's Instrument Company

FIG. 1. Abram's Automatic-self-leveling Camera Mount, Model 2 MA-1 showing cable connection with remote control and intervalometer.

about either of the leveling axes (roll and pitch) has been increased to more than 3° per second which will assure a more rapid correction of out of level conditions Special motors with built-in limit switches and brakes assure adequate protection of the mount and greatly increase the overall accuracy

A special feature of this camera mount is that it does not require any additional cable installation in an airplane, providing that said airplane is wired for use of standard photographic camera of the automatic type. This is made possible by utilizing the camera as a power source and by virtue of the fact that the remote control box operates on a wireless principle.

All electrical parts are grouped together at one end of the mount, thus keeping to an absolute minimum the number of external moving cables, and greatly enhancing the appearance of the mount.

The camera mount itself conforms in shape and camera clearance dimensions to the Army Air Corps A-11 manual mount. Vibration absorption is accomplished by using single "Lord Mounts" mounted on each of the four main supports The camera is held tightly in position by use of trunnion pins and a tight clamp.

Another special feature of this mount is the special gimbal mount within the main mount. This gimbal is adjustable on both the roll and pitch axis and makes it possible to quickly and accurately level any camera to the mount.

Leveling System

The leveling system is built around four internally lighted level bubbles and four sensitive photo electric cells, which operate four tetrodes and relays, driving two reversible motors and will correct at 26 V.D C. any deviation greater than 12 minutes from a horizontal plane.

A unique feature of the level blocks is an automatic system which, as soon as a motor starts to correct tilt, speeds up the level bubble action to keep up with the motor speed and at the same time slows up the action of the opposite bubble thus lessening any tendency to over correct

When the camera mount is level, the internally lighted level bubbles let no light strike the sensitive portion of the photo cell but when tilted over 12 minutes light strikes the cell, which immediately passes current—thus swinging the grid of one of the tetrodes toward positive and causing an immediate flow of plate current thru the relay, which then closes, starting the motor to correct the tilt Also connected to this relay is the system for speeding up the bubble action insuring a fact, positive correction of over 3° per second. As soon as the camera again reaches a level position, the motor stops with little or no tendency to over correct.

Crab Control

The remote control box contains an extremely low powered transmitter using a beam power tetrode as a variable frequency oscillator, frequency of which is controlled by a knob on the front panel. The frequency range is approximately 400 to 550 KC and the output of the oscillator is coupled into the power wiring of the camera system.

In the motor control unit there is a pentode detector with variable tuned grid circuit coupled to the power line. The plate circuit of the pentode is inductively coupled to a discriminator and bias control which controls the grid bias on two beam power tetrodes, the tetrodes operating two relays connected to a reversible quick-stop motor.

When in operation (considering the control knob and crab ring at zero crab) the oscillator signal travels thru the power circuit to the detector, from the

detector to the discriminator whose tuned circuits are staggereed, one slightly above and one slightly below, the desired frequency. When at rest, enough voltage is applied to the bias control plates to swing both relay tube grids toward positive and hold both relays closed. When the control knob is turned, the oscillator frequency is changed, and in moving away from one discriminator coil frequency it causes a grid voltage drop in the corresponding relay control tube, allowing the relay to drop out. As soon as this relay drops out, it locks the other relay in place, running the reversible motor, which drives the crab ring and also the variable condenser in the detector and discriminator circuits. When these condensers have restored balance in the circuits, the relay picks up again, stopping the motor.

For the purpose of supplying a visual indication of crab position, a *green* light has been put on the panel of the remote control box and will light only when all circuits are balanced, insuring that the crab ring is in the position indicated by the knob on control box A red light is supplied to indicate that power has been supplied to the mount.

CHAPTER IV

PHOTOGRAPHIC MISSION

PLANNING AND EXECUTING AN AERIAL PHOTOGRAPHIC JOB

John E. Meyer

THE execution of the photographic mission may be considered the first "action" step in the chain of events which begins with the conception of a photogrammetric project, and finally ends when the map, mosaic, or other end product is completed. The engineering planning which has preceded, and the mapping or other use which will follow can be no more successful than the outcome of this mission. In the past few years new methods and better equipment for compiling maps or charts from aerial photographs have been developed, but the basic source of all the map data has not changed. We must still fly with the camera, view-finder, and intervalometer as standard equipment for obtaining photographs.

Preliminary Work

The photographic mission must be preceded by certain preliminary work. Each mission will have its own particular type of detail, but there are certain fundamental things that must always be considered The mapping crew should study all available maps of the area to be flown and in case no maps are available a reconnaissance photographic job must be run off preliminary to the flight planning. The information to be gained from the study of existing maps or the reconnaissance photography should answer certain questions

1 Starting point and direction of each flight line, in relation to identifiable features on the ground.
2. The method of approach by the crew

Cooperation between pilot and photographer can easily overcome this problem which more or less involves the personal question. This is very important because the photographer must correct for crab. He must have time to estimate and make his adjustments based on information observed through the view-finder, and determine the time between exposures to give the required overlap and setting of the intervalometer. A division of responsibility between the pilot and photographer must be fully understood.

3. Communications between pilot and photographer.

Some definite set of signs like that used between surveyor and rodman must be predetermined, or if it is available some kind of telephonic communication can be used.

A good crew will never fail to inspect its complete list of equipment, including photographic, before making a flight. Several cases have been recorded where, after the plane was in the air, it was found that the camera was not loaded and no film was available The photographic mechanics must be known before the actual flight, so enough film will be obtained, and reflights will be avoided as much as possible The following are considered as necessary data:

1. Focal length of camera to be used.
2. Altitude to fly to obtain desired scale.
3. Overlap in line of flight—usually 60%.
4 Number of exposures required for 1 strip.
5. Number of strips.
6. Overlap between strips and end overlap beyond project boundaries—usually 25%.
7. Total number of exposures for entire job.
8. Amount of film required.

Specifications usually prescribe items 1, 2, 3, and 6. The others are calculated therefrom.

The Flight Map

If prepared flight maps are not furnished by the contractee, the flight map can be prepared from U S Geological Survey maps which usually are to a scale of either 1:62,500 or 1:125,000 These maps are obtainable for most of the United States and its territorial possessions; however, as a last resort state and local county highway maps would serve the purpose. However, if, after an exhaustive search, no maps are found available from any of the above sources, the reconnaissance flight method must be resorted to This increases the cost of the job considerably.

A reconnaissance flight entails photographing several strips across the area at any height, normal to the proposed lines of flight, and about fifteen miles apart, these to be crossed at intervals by other flights tying the whole area together. After developing these negatives, prints are laid up in the form of strip mosaics and the probable flight lines laid out at right angles to the lines of reconnaissance flights. No more time should be spent on reconnaissance than enough to give the crew the information it needs A good flight map is one on which all the important objects on the ground are identifiable.

Preparing the Flight Map for Vertical Photography

After the area to be mapped is delineated on the flight map, the flight lines are then drawn on, to be used by the pilot only as guides to better flying. These flight lines should be so plotted that they will allow the proper amount of side overlap in the potographs The direction of the flight lines will depend on several things

1. Size of area to be mapped
2. Shape of area to be mapped.
3. Pattern of topographic relief over the area.

Flight direction is usually prescribed by specifications.

To place the flight lines on a map in their correct inter-relation it is necessary to know the scale of the map being used, the focal length of the camera to be used, the side lap to be desired, and the altitude required to give the desired photo scale

For instance, let's take an actual case of available data.

Area width:	35 miles
Focal length	8 25 inches
Altitude	13,750 feet
Plate size.	7″×9″
Side lap·	25%
Map scale:	1.62,500

Wanted Scale of Photograph.

$$\frac{\text{Focal length}}{\text{Altitude}} = \text{Scale}.$$

$$\frac{8.\ 5}{13{,}750 \times 12} = \text{Scale} = \frac{1}{20{,}000}.$$

Wanted. Distance between flight lines.

$$9'' \times 25\% = 2\ 25''$$
$$9'' - 2.25'' = 6.75''$$

Then using a simple ratio·

$$\frac{\text{Photo dist.}}{\text{Map dist.}} = \frac{\text{Photo scale}}{\text{Map scale}}.$$

$$\frac{6.75}{X} = \frac{1}{20{,}000} \bigg/ \frac{1}{62{,}500}.$$

Wanted Distance of first flight line from edge of area

It is good practice to overlap the boundaries by 25%; therefore, we proceed as follows.

$$\frac{2.25''}{X} = \frac{1}{20,000} \bigg/ \frac{1}{62,500} \quad X = 72''.$$

Therefore, we place the first flight line 72″ from the boundary of the area.

$$X = 2.16'' \text{ on Map}$$

2.16″ is the distance between flight lines on the map whose scale is 1·62,500

Wanted· Distance of second flight line from boundary of area.

$72'' + 2\ 16'' = 2.88''$

Then each additional flight line is removed 2.16″ from the preceding one.

Wanted Total number of flight lines

$5,280' \times 35 = 184,800'$ wide − Area.

Scale 1/62,500 means 1″ = 62,500″ or

$$1'' = 5,208.3' \quad \therefore \quad \frac{184,800}{5,208\ 3} = 35\ 48'' \text{ wide}$$

Therefore, we would need 17 flight lines with somewhat less than 25% overlap at the far boundary.

Preparing the Flight Map for Oblique Photography

Oblique photography is that taken with the camera inclined at some angle to the vertical. Specifications for vertical photography usually limit the allowable tilt to a very low figure In oblique photography the position of the camera axis with respect to the plumb line is usually not the subject of very rigid requirements, other than that the earth's horizon should show near the upper edge of the picture Therefore, the cameraman does not have as exacting a job in taking oblique exposures as he does in vertical photography. The work should, however, be carefully done, and the best of cooperation between crew members is necessary. In a given period of time, a far greater territory can be covered with oblique photographs than with verticals. Two methods of making maps and charts from oblique photography are in current use in North America, the system developed by the Department of Interior, Canada, and the Tri-Metrogon system in use by branches of the United States Government

It is not necessary to show flight lines on a map for oblique photography unless a very large area is being flown. Then, they are often placed on a map at intervals representing six miles in an east-west direction This direction is usually flown to prevent sunglare on the print caused by pointing the camera in the direction of the sun.

Estimate of Film and Photographic Paper

Ordinary arithmetic only is necessary to aid one in estimating how much photographic supplies need be obtained to do any vertical mapping job.

It is of prime importance that enough film is obtained and carried in the plane to do any one day's flying. The day's flying can be planned on the basis of two flights per day or one continuous flight, depending on the type, size, and ability of the plane. It has been commonly decided that the best mapping hours are from about mid-morning to mid-afternoon, or about six hours' actual flying time, and we should, therefore, estimate on the amount required for a full day's trip or enough to map an entire smaller job requiring less time.

Referring to the problem previously discussed under Flight Lines it was

found we needed seventeen flight lines to cover an area 35 miles wide Let us assume that the flight lines are to be 40 miles long Our area to be mapped will thus be 35×40 miles

Length of flight $= 40$ miles $\times 5{,}280' = 211{,}200'$. Each print covers an area $\frac{7 \times 20{,}000}{12}$ by $\frac{9 \times 20{,}000}{12}$ or about $11{,}500'$ by $15{,}000'$ With 60% overlap, in line of flight, there is available only 40% of each print, or $11{,}500 \times 40\% = 4{,}600''$ $\frac{211{,}200}{4{,}600} = 45+$; use 46 prints per strip In order to provide for errors in starting and stopping the camera, we should add 2 exposures at both ends or a total of 4 extra Thus, $46+4=50$ negatives per flight line Seventeen flight lines will require 850 exposures In a 75 foot roll of 9″ film, there is usually enough material to take at least 100 photographs allowing for from $\frac{1}{2}$ to $\frac{3}{4}$ inch between negatives, depending on the timing mechanism of the camera This would be an ideal case because each loaded magazine would then be enough to complete 2 flight lines and the magazines could be changed in-between flights.

Using these figures we would need the following 1 magazine loaded with 75 feet of film $= 2$ flight lines $\frac{17}{2} = 8\frac{1}{2}$ magazines Therefore, we would actually start this job with 9 full 75 foot roll magazines This would leave about 50 feet of film that could be used for other purposes On contract work on large projects it is not safe to assume that all original photography will meet the specification requirements. Prudence dictates that provision be made for possible reflights. An estimate of the number of reflights is based on the experience of the crew, their past performance, and several allied intangible factors Oftentimes, an estimate will include a contingency figure of 15% of the total area for reflights. Thus, allowance should be made for about 128 additional exposures

These figures assume, of course, that we are using a single lens camera. Estimates, however, on the basis of a multi lens camera would be computed in the same way. The coverage with a multi lens camera would have to be figured; the flight lines would be further apart If a 2 camera tandem arrangement were used the same number of flight lines but less exposures would have to be made. In the case of the wide angle single lens camera, a few less flight lines would be needed, and considerably less pictures would be taken However, grouping all the different cameras together, the method of computing the number of negatives and rolls is substantially the same We must consider overlap, coverage, length of flight line, and number of flight lines

How to Estimate Flying Time

Let's consider the area to be photographed is sectionized; that is the section lines are fairly well identifiable on the ground by roads, fences, hedges, power lines, etc , so that the crew will have no difficulty orienting themselves in the air. This is another ideal condition, never existing in jungles, of course, but will be taken as a special example.

Total length of flight lines	$40 \times 17 = 680$ miles
Assume 5 miles each flight line for turning	$17 \times 5 = 85$ miles
Dry run to determine camera position, crab, intervalometer reading necessary, assuming a 2 direction run	$= 90$ miles
Total mileage for photographing only	855 miles

If the magazine had to be changed over a flight line, time must be allowed for an additional reflight In this case, the magazine is changed after every 2 flight lines and we already allow 5 miles turning time, so the camera could be reloaded during this interval.

A plane capable of flying 200 miles per hour would require 4 hours, 22 minutes, 30 seconds to actually do this job. This, of course, does not anticipate emergencies which may have to be considered.

Assuming an 8 hour day, this would leave 3 hours, 37½ minutes for necessary travel to and from the home field, which could be 1 hour and 48 minutes distant or 1½ hours distant, leaving 37½ minutes for a time margin

Thus, 1½ hours at 200 m p h. =	300 miles—to
	300 miles—from
Mapping =	855 miles
Total flying	1,455 miles

$$\frac{1{,}455}{200} = 7 \text{ hours, } 22\tfrac{1}{2} \text{ minutes flying time}$$

If a contingency figure of 15% is allowed for possible reflights, the total flying time should be increased by an amount equal to 15% of 4 hours, 22 minutes, 30 seconds, plus the time necessary for a complete round trip from the base airport to the project Thus the total increase would be 39 minutes, 23 seconds plus 3 hours to and from the airport.

The Photographic Airplane

The qualities most desired but in rare cases actually achieved in a photographic plane are visibility, comfort, economy, stability, and performance For vertical photography an opening in the floor of the plane should be provided and care must be taken in making this hole not to interfere with the structural framework. For oblique photography an open cockpit plane is very suitable A cabin plane with removable doors, so the camera can be inserted in the opening is also satisfactory (see Figure 1).

Commercial planes that can attain the desired ceiling are often converted into photographic planes by adding extra windows for pilot visibility.

The modern military photographic plane probably is a fast pursuit ship, stripped of all extra load, with only camera equipment installed. The speeds of these planes, of course, are much higher than ordinary commercial photographic planes. Besides maintaining a ceiling, the commercial photographic plane must be able to carry a load of 2 men, camera equipment, oxygen tanks, film magazines, and fuel. Any plane, that could meet these specifications, could be used.

Ceiling

With the exception of a very few places in this world, a ceiling of 25,000 feet would be ample Photographing the Himalayas or the South American Andes would probably require a higher ceiling and hence more elaborate plane equipment.

It is common knowledge that up to 5,000 feet no effects of altitude are felt. The atmospheric pressure is about 12 pounds per square inch and the temperature change ranges from 3 to 5 degrees per 1,000 feet depending upon the latitude and the time of the year.

At 10,000 feet, there is a definite rise in breathing and pulse rates. The atmospheric pressure is about 10 pounds per square inch and the temperature about

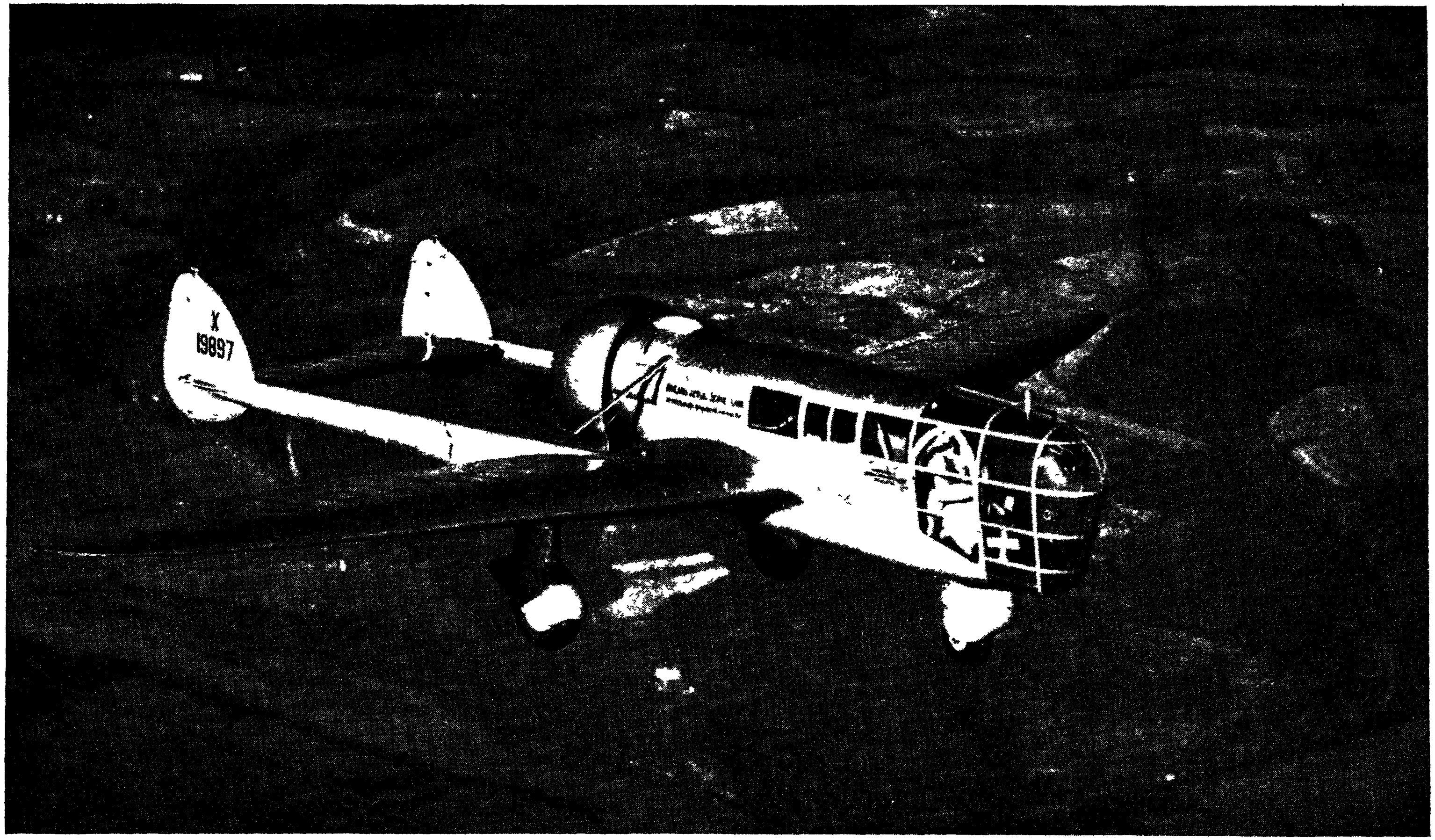


FIG. 1. The Abrams Strato-Plane "Explorer" in flight. It is the only purely Photographic Plane ever built.

33° less than on the ground This altitude is the beginning of the artificial breathing stage.

From 10,000 to 15,000 feet we are subject to a reduction in pressure from 10 to 8 pounds Without oxygen at this level, we would not be able to exercise regular judgment and would experience possible head discomfort and sleepiness.

At 15,000 to 25,000 feet, we are in the range of the sub-stratosphere and would experience a low pressure of 6 pounds per square inch. Physical reactions would range from loss of coordination, confusion of the mind to possible coma, and something akin to excessive alcoholism.

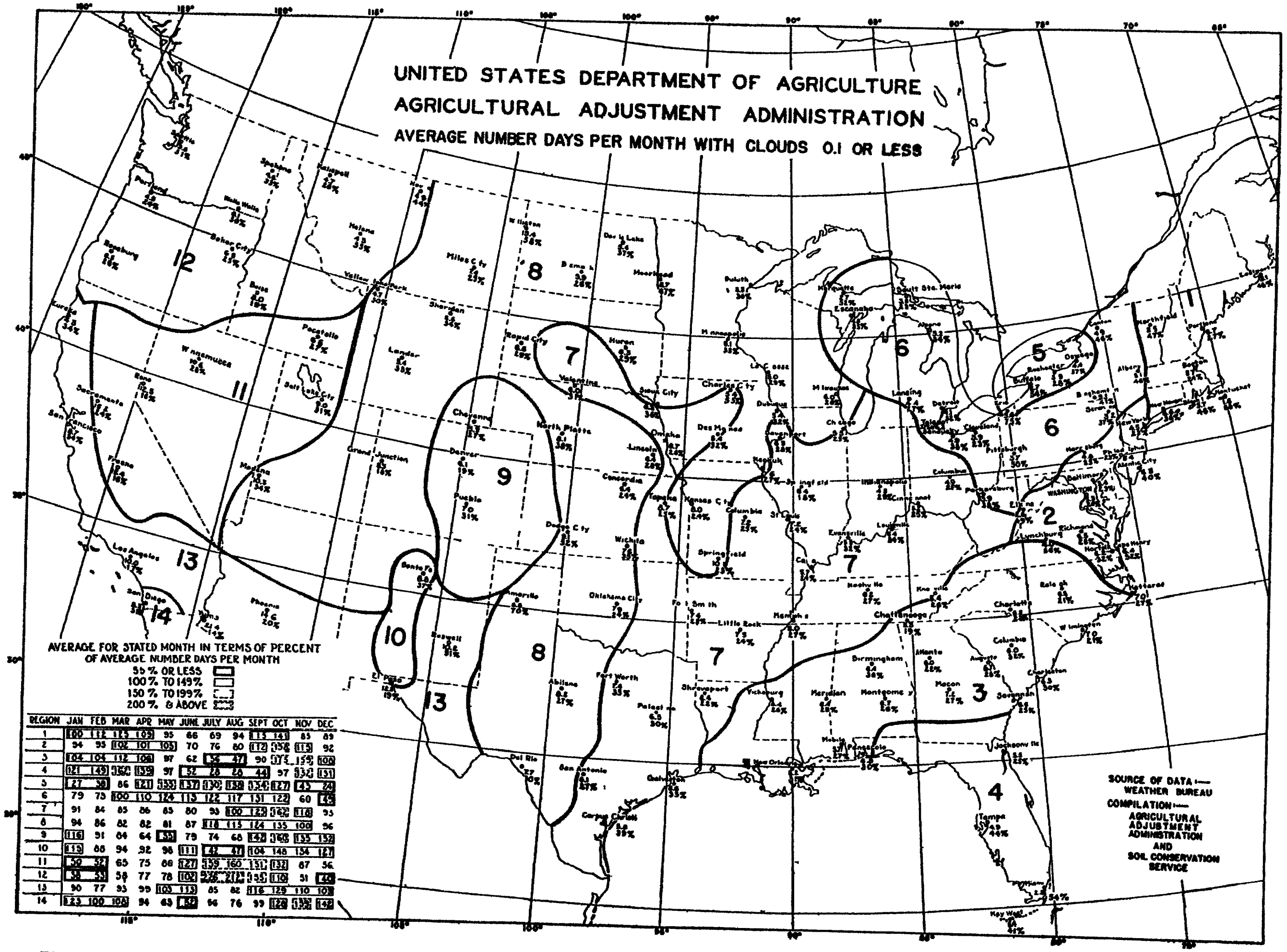
From this data we can readily see why oxygen equipment and extra warm clothing are required for most photographing jobs. The commonly prescribed photo scale and focal length of lens, make it necessary to operate at altitudes above 10,000 feet.

The most uncertain factor of aerial photography missions is the weather. The weather over any given area will tend to vary with the time of the year. Photographic crews must be on the alert to study the reports and forecasts which are normally available from U. S Weather Bureau Stations throughout the United States, and to take immediate advantage of any favorable weather conditions

In estimating the probable occurrence of photographic weather over the area of large projects, in the United States it is very helpful to use the results of a detailed study of weather occurrence probabilities which was made in 1939 under the direction of Mr. F. J. Settee of the U. S. Department of Agriculture Shown in Figure 2 is a map of the United States, with zone lines delineated thereon in such a manner that the expected number of days in any given month with the sky being only one-tenth obscured by clouds can be deduced for a given area The expectation is based on a study of the weather records for the 37 years prior to 1939. As an example suppose the area to be photographed lies between Little Rock, Arkansas, and Fort Smith, Arkansas, in region 7 on the map. Average number of days per month with clouds 0 1 or less are 7.3 for Little Rock, and 7.4 for Fort Smith. Therefore, take the average for the project area as 7.35. This figure is an average for a 12-month yearly period For some months the figure will be higher, and for some months less. Now refer to the tabulation in the lower left corner of the map. In the horizontal column for region 7 the percentage of the average for each month of the year is given. Thus, for June the figure is 80%. The number of days with 0.1 or less clouds in June would then be 80% of 7.35 or 5.88 days. Referring again to the map, for 1 year in every 10, the number of days at Little Rock and at Fort Smith can be expected to be 24% and 22% less, respectively. Thus, the worst expectation, as well as the average expectation is given. A day in which the clouds are 0 1 or less is considered a photographic day.

In using this probability map, an estimate of actual expected photographic hours would be the product of the probable photographic days in any given period multiplied by the number of photographic hours per day for the average latitude of the project. For example, at latitude 50° north, in winter-time, only about 2½ hours during the middle of the day are suitable for photographs intended for purposes which preclude appreciable shadows. At latitude 25° north in winter-time about 6 hours per day would be suitable. Conversely, if photographs are to be used for certain types of timber estimating studies, shadows are a distinct advantage, and the exposures would not be made when the sun is high in the heavens.

A quality of the atmosphere, commonly known as haze, often prevents a cloudless day from being suitable for photography. There are no data available



REGION	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1	100	112	125	109	95	66	69	94	115	141	85	89
2	94	95	102	101	105	70	76	80	112	158	115	92
3	104	104	112	108	97	62	56	47	90	175	159	108
4	121	149	162	159	97	52	28	26	44	97	152	131
5	27	38	86	121	153	137	130	138	154	127	45	24
6	79	73	100	110	124	115	122	117	131	122	60	45
7	91	84	85	86	85	80	93	100	125	162	118	93
8	94	86	82	82	81	87	118	115	124	135	100	96
9	116	91	84	64	53	79	74	68	142	168	135	152
10	115	88	94	92	98	111	42	47	104	148	154	127
11	50	52	65	75	86	127	159	160	151	132	87	56
12	38	53	58	77	78	102	236	212	155	110	51	40
13	90	77	93	99	105	113	85	82	116	129	110	105
14	123	100	108	94	65	52	96	76	99	128	155	148

FIG 2. Average number of days per month with clouds 0.1 or less, 37-year Weather Bureau record, 1900–1936. The first figure under each station denotes the number of days per month in which clouds were reported as 0.1 or less; the second figure, the percentage below average which can be expected 1 year in 10 Regions embrace stations having similar seasonal characteristics

on probable occurrence of haze. Its occurrence must be considered purely on a basis of knowledge of the area. Often, the use of certain filters in the camera will help to counteract the adverse effects of haze.

Camera Mechanics

Adjusting the camera for the proper exposure is one phase of the preliminary operations that requires much practical experience. The chief factors that must be considered—are the following:

1 Speed of film
2 Atmospheric conditions
   a. Amount of haze
   b. Cloud conditions
3 Color of subject
4 Aperture opening
5 Shutter speed
6. Distance between object and camera
7. Position of sun
   a. Time of year
   b. Time of day
   c. Locality

The speed of the particular type of film used is set by the manufacturer, and we must adjust our camera speed to that of the film.

All of the above items must be taken into account A light meter is particularly useful for testing purposes. It can be easily understood that a light colored subject like sand or snow will require less light than a dark forest or muck area, due to the amount of reflected light. As in portrait photography, a "close-up" requires more light, hence flight altitude enters into the picture to a certain extent. The position of the sun and locality exert an influence which effects the amount of actinic light available.

To a certain extent the speed of the plane controls the camera shutter setting. At 200 m p h air speed, and a photo scale of 1:20,000 the image will move across the sensitized film at the rate of .175 inch per second. From this it is easy to determine what the necessary speed will be to avoid objectionable blurring of images Generally a 1/100 of a second shutter speed, producing an image travel of .00175 inches, is fast enough. This shutter speed and the light conditions prevalent at the time of flight will govern the size of diaphragm opening required.

Placing Equipment

Every camera man has some particular way he likes his equipment arranged in the plane. However, some basic requirements must be met The relative position of the camera and view-finder are of prime importance. The view-finder must be so placed that it can be easily used without the camera man moving too much. The camera must be so placed that it is readily accessible for the operations of leveling, correcting for crab, reloading, and checking the exposure counter.

The arrangement most frequently used is for the view-finder to be mounted so it is between the operator's knees as he sits facing the camera. A number of experienced camera men have found this arrangement very good, particularly if the equipment is installed so the camera man faces the rear of the plane, and rides backward. With the image movement traveling toward the camera man it is much easier for him to check his position from the map and compare it to the

view-finder image. This speed attained in checking position has saved many a mile of costly reflight.

Many other items of equipment must be arranged for convenience in the plane The intervalometer must be readily accessible and so mounted that there is no parallax between pointer and graduations of the dial (see Figure 3). Spare

FIG. 3. The new Abrams B-7 Intervalometer

magazines should be placed within easy reach and provisions made so vibration will not cause them to shift position. A map holder should be provided and the maps folded in such a manner that a minimum of effort is required to find the area over which the mission is to be performed. The oxygen equipment must be thoroughly inspected and in fool proof condition. It has been shown in this chapter that flying above 10,000 feet altitude is quite dangerous with no oxygen equipment. Poor oxygen equipment is worse than none at all. Too much emphasis cannot be placed upon thorough preparation prior to the take off. Any detail that can be done on the ground and will lessen the amount of work to be per-

formed in the air will lend that much more to the assurance of accomplishing the photographic mission with satisfactory results.

SUMMARY

In this chapter we have discussed the planning and execution of the photographic mission. It can be clearly understood that aerial photographic work requires the utmost in teamwork, skill, and cooperation between camera man and pilot. Even with that, however, much depends on the element of chance As an example, the various operations performed by a ground survey crew are planned and systematized so that mistakes can be immediately detected and corrected at the time the work is done. With the present equipment this is not possible in aerial survey work. Mistakes, defects, and poor execution are only discovered after the flight has been completed, after the plane has returned to the home base, and after the film has been developed. It would seem that the greatest field for future development and progress lies in the sphere of perfecting new equipment and devices that will prevent mistakes from happening, and will assure the procurement of satisfactory photographs without resorting to costly and time consuming reflights Better plane control, better shutter performance, better timing apparatus, and better means of keeping the camera vertical at all times would all contribute greatly toward the desired end. The airplane has greatly improved along structural lines, and will continue to do so Film and emulsions are constantly being improved. Those advances must be augmented by the developments mentioned above if photogrammetry is to keep abreast of other sciences.

SPECIFICATIONS, VERTICAL AND OBLIQUE PHOTOGRAPHY

Louis A. Woodward and E J Schlatter

SPECIFICATIONS for aerial photographs are necessary for one prime reason, namely, to assure that the photographic materials being acquired for use in connection with a specific project or projects will be satisfactory and adequate to serve the purpose for which secured. Therefore, the preparation of specifications is a step in photogrammetric engineering which follows immediately after final decisions have been made as to the technical aspects of the map, chart, mosaic, enlargement, or other end product which is to result from the aerial photographs. The next step thereafter is, of course, the actual taking of the photographs.

The use of, and the demand for, aerial photographs was in its infancy in the late 1920's At that time the main uses for aerial photographs were in the preparation of mosaics, the extension of radial assemblies, and for various pictorial purposes The uses of stereoscopic plotting instruments for "bridging" a space of several exposures between existing control points of known positions was practically non-existent. Graphic means of taking data from photographs were almost universally used throughout the United States The main requirements for aerial photography at that time were good photographic quality, adequate coverage, fairly straight and uniform flight lines, and a fairly consistent scale For example, it was unnecessary to know the exact focal length of the lens used in taking the exposures. Today this information, together with other detailed information regarding the camera, is essential

The acquisition of vertical aerial photography by the United States Government began in the late 1920's and continued at a limited rate until 1934, at which time the acquisition rate began to increase tremendously, reaching its peak in 1938 when hundreds of thousands of square miles of coverage were obtained in one year alone This increase in the procurement of aerial photographs engendered and stimulated many developments in the science of photogrammetry which in turn indicated, as time passed, the need for development of general specifications In late 1935 the American Society of Photogrammetry, then in its infancy, appointed Colonel H. H. Blee as Chairman of the Committee to make a thorough study of the subject and to formulate specifications for consideration by the Society. This was very ably done by Colonel Blee and his committee, and through the sponsorship of the Society, the specifications promulgated were formally approved by the Secretary of the Treasury on May 27, 1937 and became the Standard Federal Specifications for Aerial Photography for General Map Work and Land Studies.

While stereoscopic plotting instruments had not yet come into wide-spread use when the Society's specifications were prepared, their advent was anticipated and the specifications were prepared in such a manner that they not only required suitable photography for needs at that time, but provided an excellent technical framework to which additional requirements could be added as the need arose. There follows a complete copy of these specifications

STANDARD SPECIFICATIONS FOR AERIAL PHOTOGRAPHY FOR GENERAL MAP WORK AND LAND STUDIES

SURVEY, VERTICAL NEGATIVES, CONTACT PRINTS

1 *Statement of Work and Areas To Be Photographed.*

(*a*) The contractor shall furnish all materials, superintendence, labor, equipment, and transportation, shall execute and finish the aerial photography of the areas hereinafter specified and shall deliver to the contracting officer, or to such official of the Government as he may designate, such sets of contact prints, index maps, ratioed prints, general enlargements, and oblique photographs as called for by the invitation and the schedule of the advertisement, together with the negatives herein required. All work shall be executed in an expeditious and workmanlike manner, to the satisfaction and acceptance of the contracting officer, in complete accord with these specifications and other conditions of bidding set forth in the invitation and in the schedule of the advertisement.

(*b*) Location, dimensions, and boundaries of the area to be photographed are set forth in the schedule of the advertisement and on the map or maps attached thereto. It is expressly understood and agreed that such dimensions are approximate only.

(*c*) The Government shall furnish the contractor at the time of the award with three copies of maps of the area to be photographed for use as flight maps.

2. *Camera To Be Used.*

(*a*) The photographs shall be made with single-lens standard aerial-mapping camera of type approved by the contracting officer and with a focal length of not less than 8 inches and an effective negative image area of not less than 60 square inches, unless other focal lengths and dimensions are specified in the schedule of the advertisement. Said camera shall be so equipped that negatives are held flat in the focal plane at the instant of exposure and the location of the principal point is directly shown or may be determined from collimation marks appearing on each negative. Only filters made from stained optical A glass shall be used.

(*b*) If specifically authorized in the schedule of the advertisement, alternate bids will be considered for use of a multiple-lens precision mapping camera of type approved by the contracting officer and of not less than 5-inch focal length, provided the negatives of each exposure are transformed by the contractor into a single composite negative, the several sections of which join perfectly. Said transformed composite negatives shall be approximately square, shall contain not less than 81 square inches of image; and shall conform in all other respects to the requirements of these specifications pertaining to single-lens photographs. The location of the principal point of each composite negative shall be shown directly thereon or there shall appear on each composite negative collimation marks from which the location of the principal point may be determined Said composite negatives shall give satisfactory color match, and image match discrepancies between component negatives shall not exceed 0 015 inch or show objectionable vertical displacement when viewed stereoscopically.

(*c*) No lens-camera combination shall be used which produces negatives with the definition of any portion, as determined by visual inspection, less than the definition of the corners or better than the definition of the center.

(*d*) Each bidder shall certify as to the make and model of the camera, or cameras; size of negative, the make, serial number, focal length, and aperture of the lens, or lenses; and the maximum stop opening he proposes to use; and shall also certify that the sample photographs submitted with his bid in accordance with the provisions of paragraph 13(*b*) hereof were taken with this type of equipment at said stop opening.

(*e*) Upon receipt of the award, the successful bidder shall furnish a master glass negative showing clearly the collimation marks of each camera-magazine combination to be used. Collimation marks shall be sharp in outline, so shaped as to facilitate accurate measurements, and shall not be altered. Said master negatives shall in each case be

made with their emulsions in the position occupied by the emulsions of the aerial negatives at instant of exposure and shall be neatly marked with the model numbers and serial numbers of the corresponding camera-magazine combinations and with the number of the contract

3. *Scale of Photographs and Method of Computing Same.*

The negatives of the entire area or each fraction thereof, as indicated on the map or maps attached to the schedule of the advertisement, shall be made at the proper altitude or altitudes above sea level to yield contact prints at the scale specified in said schedule. Said scale shall be computed from the mean altitude of the entire area, from the mean altitude of each fractional area, or from a specified datum plane—all as given in said schedule or as indicated on said maps. Where so specified in the schedule of the advertisement, negatives covering large areas having a reasonably uniform and gradual gradient shall be made at proper heights above the ground to yield the required scale In any event, photographs showing a departure from the specified scale of more than plus or minus 5 percent in excess of that causes by variations in relief within the areas covered by the individual photographs may be rejected by the contracting officer.

4. *Flight Lines.*

(*a*) Unless indicated to the contrary in the schedule of the advertisement or on the maps furnished the contractor, all photographic strips shall be flown northerly and southerly within 5 degrees of the true cardinal direction. The mean bearings of adjacent strips shall be within 5 degrees of parallel. Particular care shall be exercised to keep all flight lines as straight and as nearly parallel as possible In no case shall the lack of parallelism between adjacent photographic strips or sections thereof be such as to prevent the sidelap between photographs from conforming with the requirements of paragraph 6 hereof.

(*b*) Each flight line shall be continuous across the project or across such subdivision of the project as may be agreed upon between the contractor and the contracting officer, except in cases when it becomes necessary that the flight line be broken. The tolerances at any such breaks shall not exceed the tolerances permitted in paragraphs 3, 4, 6, and 7 hereof. The maximum overlap in line of flight at any such break will not be limited.

5. *Control Strips.*

When so indicated on the maps supplied to the bidders and/or specified in the schedule of the advertisement, the contractor shall furnish control strips of photographs following the lines shown on said maps or specified in said schedule and conforming in all respects to these specifications. Where flight control lines are made up of several sections of different bearings, each section shall extend at least two exposures beyond the points of intersection with adjoining sections. No photographs shall be taken on "banks" between successive courses, but each course shall be flown and photographed independently by turning back at the end of the preceding course and getting into position on a prolongation of the line of the new course at least one mile behind the point at which the first exposure is to be made.

6. *Overlap.*

(*a*) Overlap in the line of flight shall average approximately 60 percent and any overlap of less than 55 percent or more than 65 percent may be considered sufficient ground for rejecting all the photographs made on that particular flight. Except when the locations of the individual flight lines are indicated on the maps furnished the bidders or specifically described in the schedule of the advertisement, the sidelap beyond boundaries shall be not less than 25 percent, the sidelap between adjacent parallel flights shall average approximately 30 percent, and any sidelap of less than 15 percent or more than 45 percent may be considered sufficient ground for rejecting all the photographs

made on any flights which, in the opinion of the contracting officer, should be reflown in order to meet these requirements However, where flight lines are necessarily flown nonconsecutively, one strip with more than 45 percent sidelap will be permitted at each such juncture Each flight strip shall be so photographed that the principal points of the first two and last two negatives thereof fall outside the boundaries of the specified area in order to insure three-point intersections at ground control to be located at the edges of the area.

(*b*) In cases of extreme variations in elevation within any area, the contracting officer may permit deviation from the requirements of paragraph (*a*) *provided*, in his opinion, topographic features within the area warrant such deviation. Under such circumstances, deviations from the specifications will be limited to the amount actually caused by the extreme variation in elevation. In no event shall overlap in line of flight be less than 55 percent, nor shall sidelap between adjacent parallel flights be less than 15 percent.

7. *Crabbing.*

Any series of two or more consecutive photographs crabbed in excess of 10° as measured from the line of flight (the flight path, or "track," of the airplane as indicated by the principal points of the consecutive photographs) may be considered unsatisfactory and cause for rejection of that particular flight line or any portion thereof

8. *Tilt*

Vertical negatives (taken with the camera axis in a vertical position) are required. Particular care shall be exercised to reduce tilt of the negatives to a minimum. Tilt shall in no case exceed 5°, and shall not average more than 2° in any 10-mile section of a flight line nor more than 1° for the entire job.

9. *Film*

Only fresh, fine-grained, high-speed, panchromatic, aerial film shall be used, and no film shall be used, until there has been a determination by the contracting officer that it meets the requirements specified in the contract. Unless the schedule of the advertisement specifies standard aerial film, special low-shrinkage aerial film shall be used. For such low-shrinkage film to be approved by the contracting officer, the shrinkage in any direction on said film after developing and drying shall not exceed two and one-half parts per thousand when the film is dried in an oven for 7 days at a temperature of 120° F., the measurements before and after developing and drying being taken after the film has been conditioned in an atmosphere of 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F , plus or minus 2°, for at least 24 hours. The difference in shrinkage between measurements in any two directions after such developing and drying shall not exceed one-half part per thousand. For said standard aerial film to be approved by the contracting officer, the difference in shrinkage in any two directions after such developing and drying shall not average more than one part per thousand.

10 *Photographic Paper.*

No photographic paper shall be used until there has been a determination by the contracting officer that it meets the requirements specified in the contract.

11. *Contact Prints.*

(*a*) Unless otherwise specified in the schedule of the advertisement, contact prints from the vertical negatives shall be made without mask on double-weight, semimatte, standard commercial-grade photographic paper approved by the contracting officer and shall be trimmed with a uniform margin of one-eighth inch outside of the photographic image. For such photographic paper to be approved by the contracting officer, the

average difference in shrinkage, measured in any two directions after developing and drying, shall not exceed four parts per thousand, the measurements before and after developing and drying being taken after the paper has been conditioned in an atmosphere of 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F., plus or minus 2°, for at least 24 hours.

(*b*) On the back of each print shall be stamped, or neatly lettered with waterproof ink, the name of the Department, Bureau, or Agency, State and county, and the name and address of the contractor, with space left for insertion of the scale of the photograph.

12. *Processing and Drying Film and Prints*

(*a*) Special care shall be exercised to insure the proper development and the thorough fixation and washing of all film and prints and to avoid rolling film tightly on drums or in any way distorting it during processing and drying. Prints may be dried between blotters without the application of weights or by placing face down on cheese-cloth-covered frames. If the contractor desires to use any mechanical process for drying prints, he shall first demonstrate to the satisfaction of the contracting officer, with contact prints of suitable grids, that the maximum differential distortion in the resulting prints is not in excess of that which would occur if the prints were dried naturally.

(*b*) An adequate variety of grades of contrast shall be used in making prints in order to bring out all details of the negatives Prints shall be uniform in color and density and shall be of such a degree of contrast that all details of the negatives will show clearly, both in the shadows and the highlights as well as in the half tones between shadows and highlights.

(*c*) All prints shall be clean and free from chemical or other stains, blemishes, uneven spots, air bells, light fog, and finger marks, and shall be thoroughly washed to insure entire freedom from hypo or any other chemicals which would impair their permanency

13. *Quality of Photographs and Samples.*

(*a*) Photographs which are not clear and sharp in detail and of average and uniform density, and are not free from clouds and cloud shadows, light streaks, snow, static marks, and other blemishes which in the opinion of the contracting officer would interfere with their intended purpose, which are taken when streams are not within their normal banks or when the sun is less than 3 hours above the horizon, or which are not equal in quality to the representative samples submitted with the bid in accordance with the following paragraph may be considered unsatisfactory and may be rejected.

(*b*) Each bidder shall submit a sample contact print made without mask on double-weight semimatte paper, as specified in paragraph 11 hereof, from a vertical negative taken with the type of camera and lens at the maximum stop opening he proposes to use and at the approximate scale specified in the schedule of the advertisement, and, unless otherwise specified in the schedule of the advertisement, a sample single-weight semimatte, unmasked, two-and-one-half-diameter enlargement from one-quarter of the same negative, including the center and one corner of the negative. These samples shall show terrain having an average amount of clear-cut detail and shall represent the quality of work, including over-all definition, definition in the corners and photographic quality, the bidder proposes to furnish and will be used as criteria in judging the quality of the photographs the contractor delivers under the contract. Samples showing only heavily wooded areas or other types of terrain in which it is difficult to determine whether the photographic quality and definition meet the requirements herein specified, will not be accepted If specimen contact prints and enlargements of the quality the Government requires are attached to the schedule of the advertisement, the samples submitted by the bidder shall equal these specimens in all respects. The contracting

officer reserves the right to reject the bid of any bidder who, in his opinion, fails to submit samples of the quality required by these specifications and the attached schedule

(c) In case the use of a multiple-lens mapping camera is authorized in the schedule of the advertisement, as provided in paragraph 2(b) hereof, and it is proposed to use such a camera, a sample stereoscopic pair of contact prints from transformed composite negatives made from exposures taken with the camera and lenses and at the maximum stop opening it is proposed to use and at the approximate scale specified, and, unless otherwise specified in the schedule of the advertisement, a sample two-and-one-half-diameter enlargement from one-quarter of one of said composite negatives, including the center and one corner of the negative, shall be submitted with the bid in lieu of the sample single-lens contact print and enlargement required by paragraph 13(b) hereof Said contact prints shall be made without mask on double-weight semi-matte paper, as specified in paragraph 11 hereof, and said enlargement shall be made without mask on single-weight semi-matte paper.

14 *Indexing*

(a) Each vertical negative shall be marked clearly with a designating symbol of not to exceed three letters followed by the serial number of the roll and the serial number of the exposure on the roll (thus ABC–116–110) and also with a numerical abbreviation of the month, day, and year of exposure (thus 12–8–35). The rolls of film used in the performance of each contract shall be numbered in an unbroken series beginning with number 1, and the exposures on each roll shall be numbered in an unbroken series beginning in each case with number 1 The designating symbol and serial numbers shall be placed in the northeast corner of each negative for north and south flights and in the northwest corner of each negative for east and west flights, with the exception of the control strips, in which case said symbol and numbers shall be placed in the upper right-hand corner of each negative, progressing along the line of flight The abbreviation of the date shall in each case be placed in the adjacent corner in a counterclockwise direction, with the exception of the control strips, in which case it shall be placed in the adjacent corner in a clockwise direction In addition, on the first and last negative of each flight line and on the first and last negative of each roll the initials of the Bureau or Agency for which the work is being performed and the approximate scale of the negatives shall be placed immediately preceding the designating symbol and the serial numbers (thus. GS–1:20,000–ABC–116–110), and the numerical abbreviation of the approximate time of day of the exposure shall be placed immediately following the date (thus: 12–8–35–12:30) The characters used in marking negatives shall be three-sixteenths inch high and may be perforated, mechanically stamped with opaque ink, or neatly drafted with celluloid ink, in such manner as to print clearly in positive form on the photographs in the positions herein specified.

(b) The metal container for each roll of film shall become the property of the Government, shall be neatly labeled with the names of the Department and Bureau or Agency for which the work is being performed, the name or number and the location of the project, the designating symbol, the number of the contract, the name and address of the contractor, the date or dates and hours of exposure, the serial number of the roll, the serial numbers of the first and last exposures, the approximate scale of the negatives, and the model numbers and serial numbers of the camera-magazine combination used in making the exposures.

(c) Photo-index maps shall be prepared by photographing, to the approximate scale specified in the schedule of the advertisement, a stapled assembly of contact prints from each vertical negative carefully laid to match corresponding images and clearly showing the serial numbers of each negative. Said contact prints shall be made without mask and shall be trimmed to the edges of the photographic images In case serial

numbers are obscured by duplicate or overlapping exposures, they shall be neatly lettered in their proper positions on the completed assembly with white ink or with stick-up numbers and shall correspond in size and style to the unobscured numbers on the contact prints. The specified number of photographic copies of said photo-index maps shall be submitted on the type and size of paper and to the scale specified in the schedule. If specified in the schedule of the advertisement, the index maps may be prepared by drafting the outline and position of each vertical negative on drawing paper or tracing cloth, lettering the serial number of every fifth negative in each flight, and submitting the specified number of photographic copies of this index on the type and size of paper and to the scale specified in the schedule. Said index maps shall show the boundaries of the project and have neatly executed titles showing the names of the Department and Bureau or Agency for which the work is being performed, the name or number and the location of the project, the designating symbol, the number of the contract, the name and address of the contractor, the approximate scales of the photographs and of the index, and the date of completion of the work In case any portion or portions of the project is required to be reflown, the specified number of additional copies of the original index map or maps shall be delivered with the reflights included thereon

RATIOED PRINTS AND GENERAL ENLARGEMENTS

15 *Ratioed Prints.*

(*a*) If ratioed prints from the vertical negatives to be used for radial-line plotting of planimetric maps are specified in the schedule of the advertisement, they shall be made to the scale specified in said schedule by using the ratio factors supplied by the Bureau or Agency for which the work is being performed Such ratioed prints shall be sharp in detail, of uniform density, and equal in quality to the sample contact print submitted with the bid. They shall show no appreciable radial or other distortions of imagery or scale due to faulty optics or mechanics of the projection camera Before making the award, the contracting officer may, at his discretion, require the successful bidder to submit ratioed prints of suitable grids or to demonstrate otherwise to the satisfaction of the contracting officer the accuracy of the projection camera the bidder proposes to use. Said ratioed prints for radial-line plotting shall be made without mask on special, double-weight, semimatte, low-shrinkage photographic paper approved by the contracting officer, and shall be trimmed with a uniform margin of one-fourth inch outside of the photographic image. For such low-shrinkage paper to be approved by the contracting officer the average difference in shrinkage measured in any two directions after developing and drying shall not exceed one part per thousand, the measurements before and after developing and drying being taken after the paper has been conditioned in an atmosphere of 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F., plus or minus 2°, for at least 24 hours.

(*b*) If ratioed prints for general use, not requiring special low-shrinkage photographic paper, are specified in the schedule of the advertisement, they shall be made on standard commercial-grade photographic paper approved by the contracting officer, and with or without mask, on the weight and finish of paper and dimensions of trimming—all as specified in the schedule of the advertisement. Such ratioed prints shall conform in all other respects to the specifications given in paragraph 15(*a*) hereof.

16. *General Enlargements.*

If general enlargements from the vertical negatives are specified in the schedule of the advertisement, they shall be made to the number of diameters or to such scale as may be specified in said schedule, shall conform to the requirements of paragraph 15(*a*) hereof, pertaining to detail, density, and freedom from distortions of imagery and scale due to faulty optics or mechanics of the projection camera, and shall be equal in

quality to the sample enlargement submitted with the bid. Said enlargements shall be made on standard commercial-grade photographic paper approved by the contracting officer. The paper shall be of weight and finish, trimmed, and printed with or without mask—all as specified in the schedule of the advertisement

Oblique Photographs

17 *Areas To Be Photographed*

If oblique photographs are specified in the schedule of the advertisement, the approximate location from which each photograph is to be taken and the direction in which each is to be taken will be indicated on the flight map furnished the contractor by the Government Unless otherwise specified in the schedule of the advertisement, said photographs shall be taken between the hours of 9 a m and 3 p m from an elevation of approximately 1,000 feet above the highest point in the area being photographed.

18 *Camera To Be Used*

The photographs shall be made with a standard single-lens aerial camera having a focal length of not less than 8 inches with an effective negative image area of not less than 60 square inches, and indicating on each negative the location of the principal point or the collimation marks from which the principal point may be determined Each bidder shall specify the make and model of the camera, the size of the negative, and the focal length of the lens he proposes to use

19. *Quality and Numbering of Photographs*

(*a*) The oblique photographs shall be equal in quality to the vertical photographs hereinbefore specified Clouds above the horizon will not be objectionable providing they do not cast shadows on the terrain shown in the photographs, which, in the opinion of the contracting officer, would impair the value of the photographs for their intended purpose. The transverse axis of the camera shall be level at the instant of exposure and approximately one-fourth of the area of each negative shall show above the horizon.

(*b*) The negatives shall be numbered in consecutive order with three-sixteenths-inch figures placed in the lower right-hand corner thereof, with the numbers progressing around the area in the manner indicated on the flight map

20. *Film*

Unless otherwise specified in the schedule of the advertisement, oblique photographs shall be made with standard fine-grained high-speed panchromatic aerial film, approved by the contracting officer Only fresh film shall be used.

21. *Prints.*

The oblique photographs shall be masked and ferrotyped, using single-weight standard commercial-grade photographic paper approved by the contracting officer. Said photographs shall be trimmed with a uniform margin of one-half inch outside of the photographic image.

Special Conditions

22 *Conditions of Bidding*

(*a*) Each bid shall be accompanied by a certificate showing that the bidder has available and will use in the execution of the proposed contract the required number of airplanes, specified in the schedule of the advertisement, that will perform satisfactorily at the necessary altitude to yield the negative scale specified in said schedule and that are equipped with the necessary instruments and photographic apparatus to carry out the photography covered by the specifications, and that he has available and will use, for each of said airplanes, the services of a qualified pilot with at least 50 hours of map-flying experience, and a competent aerial photographer with at least 50 flying hours of

mapping experience. Said certificate shall also show the make and model of camera or cameras, size of negative; the make, serial number, focal length, and aperture of the lens or lenses, and the maximum stop opening proposed to be used; and shall certify that the sample photographs submitted with the bid in accordance with the provisions of paragraph 13(*b*) hereof were taken with this type of equipment at said stop opening.

(*b*) No bid will be considered unless it complies with the requirements set forth in the invitation and the schedule of the advertisement.

(*c*) Each bid shall be accompanied by a showing of the facts as to the business and technical organization of the bidder available for the contemplated work, including financial resources and experience on similar projects. The right is reserved to reject any bid where investigation of the business and technical organization of the bidder available for the contemplated work, including financial resources and experience on similar projects, does not satisfy the contracting officer that such bidder is qualified to perform the work.

(*d*) Bidders are expected to examine the specifications and maps, to visit the locality of the work if necessary, and to make their own estimates of the facilities needed and the difficulties attending the execution of the proposed contract, including local conditions, uncertainty of weather, availability of landing fields, and all other contingencies Any request for clarification or interpretation of any portion of the specifications or advertisement shall be submitted in writing or by telegraph to the officer issuing the invitation at least 1 week prior to the time fixed for the opening of the bids in order that he may notify all prospective bidders of such interpretation or clarification without the necessity of postponing said time of opening.

(*e*) Each bidder shall state where he proposes to base his flying operations and where he proposes to perform the work other than the flying.

(*f*) Each proposal (on U. S Standard Form No 33) will be received with the understanding that these specifications and special conditions form a part thereof, and that when accepted in writing within the time specified such accepted proposal shall constitute the contract between the bidder and the Government.

23 *Bid Guarantees.*

(*a*) Unless otherwise specified in the schedule of the advertisement, bids of $1,000 or less need not be accompanied by a bid guarantee.

(*b*) Each bid in excess of $1,000 shall be accompanied by a bid guarantee of not less than ten percent (10%) of the amount of the bid, which guarantee may be a money order, certified check, or cashier's check, made payable to the Treasurer of the United States; or, *where the bid is in excess of $2,000,* a bid bond on U. S. Standard Form No. 24 will be acceptable. Each such money order and check shall be submitted with the understanding that it shall guarantee that the bidder will not withdraw his bid within the period specified after the opening of the bids, that (if bid exceeds $2,000) he will, if his bid is accepted, give performance bond as may be required, and that in the event of the withdrawal of his bid within said period, or (where bid exceeds $2,000) the failure to give said bond within the time specified, the bidder shall be liable to the Government for the difference between the amount specified in his bid and the amount for which the Government may otherwise procure the required work, if the latter amount be in excess of the former, and the Government shall have the right to retain the proceeds of such money order or check to apply on account of such excess cost. Money orders, checks, and bid bonds of unsuccessful bidders will be returned when award is made.

(*c*) Where the bid is not in excess of $2,000, the guarantee of the successful bidder will be returned on acceptance of completed work, except that the Government shall have the right to retain the proceeds of said guarantee to such extent as may be required to reimburse the Government for any loss or damage resulting from default in the performance of the contract.

(*d*) Where bid exceeds $2,000, the money order or check of the successful bidder will be returned when performance bond is approved.

24. *Performance Bond.*

If the bid of the successful bidder is $2,000 or less, a performance bond will not be required, otherwise the bidder to whom the award is made shall furnish, within 10 days after receipt of notice of award, a performance bond on U S. Standard Form No 25, for the faithful performance of the contract. Such performance bond shall be in an amount representing such percentage of the contract price as may be specified in the schedule of the advertisement

25. *Award and Notice to Proceed*

(*a*) If the accepted bid is not in excess of $2,000, the notice of award by registered mail, or telegram, will be the notice to proceed.

(*b*) If the accepted bid is in excess of $2,000, the notice to proceed will be given, by registered mail or telegram, promptly after approval of the performance bond.

26. *Commencement and Prosecution of Work.*

(*a*) The contractor shall undertake the photography of the areas to be photographed within 10 days after the receipt of notice to proceed, or within such other period of time as may be specified in the schedule of the advertisement unless no substantial areas are free from snow or high water at that time, in which case the time for starting the work will be postponed until such areas are free from snow or high water. The contractor shall notify the contracting officer in writing the day his flying equipment and personnel arrive on the project and, unless otherwise authorized in writing by the contracting officer, shall keep said flying equipment and personnel continuously on the project until all the flying covered by the contract is completed and the contracting officer or his representative notifies the contractor by letter or by telegram that the photography is approved.

(*b*) If the area required to be photographed is made up of several separate areas of subprojects, the contractor shall photograph said subprojects in the order indicated in the schedule of the advertisement, and shall notify the contracting officer by letter or by telegram immediately upon completion of the photography of each such subproject. If any such subprojects or portions thereof are required to be rephotographed, the contracting officer shall indicate the order in which such work is to be performed and the contractor shall notify the contracting officer in each case immediately upon completion of said work.

27. *Time Allowance.*

(*a*) The maximum time allowance for delivery of contact prints, together with index maps, and oblique photographs will be the number of calendar days specified in the schedule of the advertisement, after the contractor has received the written notice of the acceptance of the flying.

(*b*) The maximum time allowance for delivery of ratioed prints and general enlargements will be the number of calendar days specified in the schedule of the advertisement after the contractor has received written notice of the acceptance of the flying, and has received the ratio and/or enlargement factors. *Provided, however,* That if the ratio and/or enlargement factors are to be determined by the contractor, the said time allowance shall be the number of calendar days specified in such schedule after receipt by the contractor of written notice of acceptance of the flying.

(*c*) The maximum area for which the contractor may be required to deliver contact prints, ratioed prints, and enlargements during any 30-day period shall in no case exceed the maximum 30-day delivery area specified in the schedule of the advertisement.

(*d*) The contractor shall not be liable for liquidated damages specified in paragraph 32, between the dates of delivery and dates of acceptance or rejection of contact prints

with index maps, oblique photographs, ratioed prints, or general enlargements on original flight or any reflight under the foregoing specifications.

28. *Specifications and Maps*

The contractor shall keep on the work a copy of the maps and specifications and shall at all times give the contracting officer access thereto. Anything mentioned in the specifications and not shown on the maps, or shown on the maps and not mentioned in the specifications, shall be of like effect as if shown or mentioned in both. In case of difference between maps and specifications, the specifications shall govern. In any case of discrepancy in the figures, maps, or specifications, the matter shall be immediately submitted to the contracting officer, without whose decision said discrepancy shall not be adjusted by the contractor, save only as his own risk and expense.

29. *Changes*

The contracting officer may at any time, by a written order, and without notice to the sureties, if any, make changes in the maps and/or specifications of the contract and within the general scope thereof. If such changes cause an increase or decrease in the amount due under the contract, or in the time required for its performance, an equitable adjustment shall be made and the contract shall be modified in writing accordingly No change involving an estimated increase or decrease of more than $500 if the contract price be in excess of $2,000, or of more than 25 percent of the contract price if the latter be $2,000 or less, shall be ordered unless approved in writing by the head of the department or his duly authorized representative Any claim for adjustment under this paragraph must be asserted within 10 days from the date the change is ordered *Provided, however*, That the contracting officer, if he determines that the facts justify such action, may receive and consider, and with the approval of the head of the department or his duly authorized representative, adjust any such claim asserted at any time prior to the date of final settlement of the contract If the parties fail to agree upon the adjustment to be made the dispute shall be determined as provided in paragraph 39 hereof. But nothing provided in this paragraph shall excuse the contractor from proceeding with the prosecution of the work so changed

30. *Inspection and Acceptance*

Immediately after the contact prints, index maps, and oblique photographs are received by the Government at the point designated in the schedule of the advertisement, they will be inspected, after which the contracting officer or his representative will notify the contractor in writing whether they are satisfactory and what areas, if any, shall be rephotographed because of nonconformity with the contract requirements Ratioed prints and general enlargements will be inspected promptly upon receipt thereof by the Government, and any found by the contracting officer to be unsatisfactory shall be reprinted immediately.

31. *Delays—Damages.*

If the contractor refuses or fails to prosecute the work, or any separable part thereof, with such diligence as will insure its completion within the time specified, or any extension thereof, or fails to complete said work within such time, the Government may, by written notice to the contractor, terminate his right to proceed with the work or such part of the work as to which there has been delay. In such event the Government may take over the work and prosecute the same to completion, by contract or otherwise, and the contractor and his sureties (if any) shall be liable to the Government for any excess cost occasioned the Government thereby. If the Government does not terminate the right of the contractor to proceed, the contractor shall continue the work, in which event the actual damages foi the delay will be impossible to determine and in lieu thereof

the contractor shall pay to the Government as fixed, agreed, and liquidated damages for each calendar day of delay until the work is completed or accepted the amounts as set forth in paragraph 32 hereof, or in the schedule of the advertisement, and the contractor and his sureties (if any) shall be liable for the amount thereof *Provided,* That the right of the contractor to proceed shall not be terminated or the contractor charged with liquidated damages because of any delays in the completion of the work due to unforseeable causes beyond the control and without the fault or negligence of the contractor including, but not restricted to, acts of God, or of the public enemy, acts of the Government, acts of another contractor in the performance of a contract with the Government, fires, floods, epidemics, quarantine restrictions, strikes, freight embargoes, and unusually severe weather or delays of subcontractors due to such causes, if the contractor shall within 10 days from the beginning of any such delay (unless the contracting officer, with the approval of the head of the department or his duly authorized representative, shall grant a further period of time prior to the date of final settlement of the contract) notify the contracting officer in writing of the causes of delay, who shall ascertain the facts and the extent of the delay and extend the time for completing the work when in his judgment the findings of fact justify such an extension, and his findings of fact thereon shall be final and conclusive on the parties hereto, subject only to appeal, within 30 days, by the contractor to the head of the department concerned, whose decision on such appeal as to the facts of delay and the extension of time for completing the work shall be final and conclusive on the parties hereto

32 *Liquidated Damages*

Unless otherwise specified in the schedule of the advertisement the fixed, agreed, and liquidated damages to be paid the Government by the contractor in accordance with paragraph 31 hereof, for each calendar day of delay in undertaking the work and in delivery after expiration of the time allowances specified in paragraphs 26(*a*) and 27 hereof, and in the schedule of the advertisement, shall be as follows:

Undertaking the work	$10
Contact prints with index maps	10
Oblique photographs	10
Ratioed prints	10
General enlargements	10

33. *Risk—Damages.*

The contractor shall assume all risks in connection with the performance of the contract; and shall be liable for and save the Government harmless on account of any damages to persons or property in connection with the prosecution of the work.

34 *Reports.*

The contractor shall submit to the contracting officer regular weekly reports showing progress of the work Forms for said reports will be supplied the contractor with the flight maps.

35. *Ownership of Negatives*

Unless otherwise specified in the schedule of the advertisement, all negatives shall become the property of the Government and shall be delivered in accordance with instructions from the contracting officer to the Bureau or Agency for which the work has been performed. During the period the negatives are in the possession of the contractor, he may make for commercial use such prints, enlargements, mosaics, and reproductions as he may desire from any such negatives which do not show fortifications, military or naval defenses, or other restricted areas, the photographing of which is prohibited by the Government.

36. *Subcontracts.*

The contractor shall not, without prior written approval of the contracting officer, enter into any subcontract covering any part of the work contemplated by his contract.

37. *Patents*

The contractor shall hold and save the Government, its officers, agents, servants, and employees, harmless from liability of any nature or kind, including costs and expenses, for or on account of any patented or unpatented invention, article, or appliance manufactured or used in the performance of the contract, including their use by the Government

38. *Permits and Care of Work.*

The contractor shall, without additional expense to the Government, obtain all required licenses and permits, and shall be responsible for the proper care and protection of all materials until delivery thereof in accordance with instructions from the contracting officer.

39. *Disputes*

Except as otherwise specifically provided in the contract, all disputes concerning questions of fact arising under the contract shall be decided by the contracting officer subject to written appeal by the contractor within 30 days to the head of the department concerned or his duly authorized representative, whose decision shall be final and conclusive upon the parties thereto In the meantime the contractor shall diligently proceed with the work as directed.

40. *Payments to Contractors.*

(*a*) If the contract price be $1,000 or less, payment in full will be made upon completion and acceptance of the work.

(*b*) If the contract price be in excess of $1,000, partial payments will be made as the work progresses at the end of each calendar month, or as soon thereafter as practicable, on estimates, made by the contractor and approved by the contracting officer, of the value of work delivered and accepted.

(*c*) In making partial payments there shall be retained 10 percent of the estimated amount until completion and final acceptance of all work covered by the contract.

(*d*) If subprojects are involved, and if so specified in the schedule of the advertisement, payment in full with respect to each subproject (in lieu of partial payments hereinbefore provided and without deduction of any retained percentage) will be made upon completion and final acceptance of all work in connection with any such subproject.

(*e*) All material and work covered by partial payments made shall thereupon become the sole property of the Government, but this provision shall not be construed as relieving the contractor from the sole responsibility (prior to delivery to the Government) for the care and protection of materials and work upon which payments have been made or the restoration of any damaged work, or as a waiver of the right of the Government to require the fulfillment of all of the terms of the contract.

(*f*) Upon completion and acceptance of all work required under the contract, the amount due the contractor under the contract will be paid upon the presentation of a properly executed and duly certified voucher therefor, after the contractor shall have furnished the Government with a release, if required, of all claims against the Government arising under and by virtue of the contract, other than such claims, if any, as may be specifically excepted by the contractor from the operation of the release in stated amounts to be set forth therein.

(*g*) Payment will be made for additional prints ordered under the contract upon delivery thereof.

41. *Covenant Against Contingent Fees.*

The contractor warrants that he has not employed any person to solicit or secure the contract upon any agreement for a commission, percentage, brokerage, or contingent fee. Breach of this warranty shall give the Government the right to terminate the contract, or, in its discretion, to deduct from the contract price or consideration the amount of such commission, percentage, brokerage, or contingent fees. This warranty shall not apply to commissions payable by contractors upon contracts or sales secured or made through bona fide established commercial or selling agencies maintained by the contractor for the purpose of securing business.

42. *Definitions.*

(*a*) The term "head of the department" as used herein shall mean the head or any assistant head of the executive department or independent establishment involved, and the term "his duly authorized representative" shall mean any person authorized to act for him other than the contracting officer

(*b*) The term "contracting officer" as used herein shall include his duly appointed successor or his authorized representative

Additional Instructions to Contracting Officers

(Supplementing those printed on the back of U. S. Standard Form No. 33)

1. Invitations for bids for aerial photography shall be issued on U S Standard Form 33 (Revised), Invitation, Bid and Acceptance (short-form contract), using Standard Form No. 36 for continuation schedule—and care should be taken to attach to Form 33 the representations and stipulations required by the Walsh-Healey Act, inserting in Form 33 a provision that such representations and stipulations are by reference made a part of the bid.

2 These general specifications for aerial photography are based upon a thorough canvass of the technical requirements of the mapping and economic agencies of the Government as pertaining to aerial photography, and have been developed to meet these two sets of requirements in a satisfactory manner. It is anticipated that they will be revised from time to time as changing conditions may dictate, in order to keep abreast of new developments. These specifications necessarily include material that may not be applicable to every project. Hence, in preparing specifications for any particular project, the contracting officer should indicate in the schedule of the advertisement those portions of the standard specifications which apply to the work desired, making any authorized modifications therein, or additions thereto, that may be necessary to meet the requirements of the work. In this connection, consideration should be given to the needs of other agencies of the Government, and the schedule of the advertisement should call for bids for such additional prints of photographs as other agencies of the Government may need to purchase, even though the contracting officer may not plan to purchase such extra copies for the specific project for which the invitation is issued, to the end that maximum returns may be obtained from the aerial surveys and that duplication may be avoided. In some cases, the season of the year in which the photographs are taken has a marked bearing on their general utility; for example, the value of photographs of wooded areas may be greatly enhanced, from the standpoint of soil and geological studies, if taken when the leaves are off the trees.

3. As indicated in the paragraphs of the specifications hereinafter noted, the schedule of the advertisement should designate specifically:

Location, dimensions, and boundaries of area or areas to be photographed	Par. 1(*b*)
Type of camera to be used, including other focal lengths and dimensions, if any	Par. 2

Scale of the contact prints desired and the method of computing same.	Par 3
Flight lines in other directions, if any	Par. 4
Control strips, if any, required	Par. 5.
Type of film to be used	Par. 9
Type of paper to be used.	Par. 11
Different enlargement scale of sample, if desired	Par. 13(*b*) and (*c*)
Type, size, and number of copies of index maps required	Par 14(*c*)
Type and scale of ratioed prints, if any, required	Par 15
Type and scale of general enlargements, if any, required	Par 16
Oblique photographs, if any, required	Par. 17–21
Number of airplanes required to be used	Par. 22(*a*)
Amount of bid guarantee or bid bond required	Par. 23
Amount of performance bond required	Par 24
Commencement of work ..	Par 26(*a*)
Order of photographing subprojects, if any .	Par. 26(*b*)
Time allowance, including time allowances on different subprojects, if any, and maximum delivery area	Par 27
Rate of liquidated damages, including rates on separate subprojects, if any .	Par. 32
Ownership of negatives	Par. 35
Payments to contractors for separate subprojects, if any	Par. 40(*d*)

4. Where any patent or patents are to be excepted from the operation of paragraph 37 of the specifications, such exceptions will be specifically stated, by reference to the patent number, date of issue, and name of patentee in a proviso to be added to the said paragraph 37.

5. The contracting officer should give careful consideration to the type and scale of photographs to be specified and should examine representative prints made at the contemplated contact scale showing terrain of the type to be photographed in order to determine their suitability for the intended purposes In cases where vertical photographs are to be used by agencies desiring clear, sharp, cultural detail and freedom from excessive obliquity, as in studies of built-up areas, geological studies, timberland studies, and land-use projects, it is recommended that single-lens cameras be used, with reasonably narrow-angle and long-focal-length lenses (see specification par. *2a*)). In cases where wide-angle photographs would not cause objectionable loss of pictorial detail due to obliquity in the outer portions, they afford advantages from the standpoint of economy and accuracy of radial-line plotting. In cases where use of wide-angle lenses of shorter focal length than 8 inches is desired, or where multiple-lens photographs transformed into single composite negatives are desired, the contracting officer should so specify in the schedule of the advertisement. Use of wide-angle, short-focal-length lenses may require some revision of the requirements as to quality of photographs which will be accepted, so far as definition in the corners is concerned (see specification par. 13(*b*)), in which event the contracting officer should so specify in the schedule of the advertisement.

6. It is recommended that the single-lens vertical negatives for use in connection with the proposed National Mapping Plan be made at a contact scale of 1·30,000 (1 in. = 2,500 ft), that the ratioed prints for the radial-line plotting of the planimetric maps be made at a scale of 1:20,000 (1 in. = 1,667 ft.), and that general enlargements for the use of the land-planning and other similar economic agencies be made to two and one-half diameters (1 in. = 1,000 ft.), or to a scale of 4 inches per mile (1 in. = 1,320 ft.), as may be desired. Aerial surveys made at scales smaller than 1·30,000 are of very little

value for specialized land studies because of insufficient detail. In agricultural and other special areas where considerable detail is desired, aerial photographs made with single-lens cameras of not less than 8-inch focal length at scales of either 1 20,000 or 1:15,840, with prints and enlargements for land studies made to scales of 1 15,840 and 1.7,920, respectively, are very satisfactory for the work of the land-use agencies and at the same time are well adapted to map compilation.

7 Items of definite-quantity materials which the Bureau or Agency, for which the work is to be performed, proposes to purchase should be listed separately in the schedule of the advertisement These items may be entered in such combinations of separate bids or lump-sum bids as may be desired As stated in paragraph 2 hereof, the schedule of the advertisement should include such additional prints of photographs, made from the definite-quantity negatives, as any agency of the Federal Government may need to purchase within 1 year after the date of contract. These indefinite-quantity items should be numbered consecutively after the item numbers of the definite-quantity materials and should be preceded by a statement similar to the following "Bids are required on one set each of the following items of additional prints of the photographs specified in the preceding items, it being understood that orders for such quantities as may be necessary are to be placed if and when such items are required by any Federal agency during the period of 1 year after the date of the contract " There should be no time allowance or liquidated damage provision inserted in the schedule of the advertisement for such indefinite-quantity items as are not covered in the purchase order issued on acceptance of the bid, but delivery of such indefinite-quantity items, if and when ordered, should be made as soon as possible.

8. A copy of Standard Government Instructions to Bidders (No. 22) should be attached to each invitation for bids

9 The schedule of the advertisement should be accompanied by a sufficient number of copies of a map or maps on which the location, dimensions, and boundaries of the area or areas to be photographed are outlined, to permit the purchasing officer to send one copy to each prospective bidder (see specification par. 1(*b*)). The notice of award to the successful bidder should be accompanied by three copies each (or more if required) of the best available map or maps of the area or areas to be photographed, for use by the contractor as flight maps State maps published by the Geological Survey, the Post Office Department, or the General Land Office will be suitable for submission with the schedule of the advertisement, and Geological Survey topographic maps (if available) will be suitable for flight maps

10. The notice of award to the successful bidder should be accompanied by notification of the designating symbols and initials of Bureau or Agency required by specification paragraph 14(*a*) to be placed on the negatives, and also by a supply of the forms required for weekly reports showing progress of work (see specification par. 34).

11. Any deviation from the specifications not authorized by these instructions shall be submitted to the Director of Procurement for prior approval.

At the present time there are two general catagories of vertical aerial photography; that needed for use in stereoscopic plotting instruments, and that needed for radial triangulation extension by slotted templets, hand templets, or metal templets for land use studies of various kinds, and for mosaics. A combination of two obliques and one vertical photograph, commonly called "Tri-Metrogon," is used for military charting and reconnaissance maps. That type is discussed in another chapter in this manual. In general, photographs for use in stereoscopic plotting instruments must be taken with a camera having a lens focal length which is conducive to obtaining sufficiently accurate elevations by optical means to yield the desired contour interval. In other words, the desired contour interval, when combined with the map compilation scale and with the focal lengths of the

lens in the stereoscopic plotting equipment, arbitrarily determine the most desirable aerial camera focal length within narrow limits. All metric characteristics of the camera and lens must be known and are limited. Where accurate planimetric maps are available for use as flight maps, requirements fixing the exposure station of successive pictures within narrow limits result in a very substantial saving in cost of ground control surveys. Where accurate flight maps are not available, it is impossible to fix by requirement the positions of successive exposure stations, and consequently endlap and sidelap requirements of image area must be resorted to In either case, photographic quality of the negatives must be very good Highlights and dense shadows are to be avoided.

Aerial photographs intended primarily for radial work, land use studies and acreage determination, and assembly of mosiacs, must have high photographic quality, a minimum of distortion due to relief, and must be printed on paper which has the most favorable resistance to change in dimension due to developing, washing, drying, temperature, and humidity For such photographs it is common practice to require a given scale, with a narrow tolerance plus or minus, and to attendently provide the minimum focal length that may be permitted For example, a photograph taken at a scale of 1:20,000 with a 12-inch focal length lens at an altitude of 20,000 feet will be better from a standpoint of making measurements on the photographs, or constructing a mosaic, then one taken at a scale of 1:20,000 with a 6-inch focal length at an altitude of 10,000 feet On the latter, ground relief would cause considerable more horizontal displacement of images Summarizing—for land use studies, mosaic work, and general planimetric radial work, long focal length lens and high flight altitudes are desirable For use in stereoscopic plotting instruments, a short focal length lens with attendant high displacement due to relief is desirable, because the vertical dimension is thereby accentuated.

For most purposes the U S Department of Agriculture is at present using specifications which represent a compromise between these two opposing considerations. The lens focal length is usually specified to be not less than eight inches. The required scale of the photographs is 1:20,000 plus or minus 5%, exclusive of the difference caused by relief These two requirements serve automatically to establish the flying altitude above a given terrain.

Thus, photography suitable for most agricultural uses is obtained, and in addition, by requiring the use of precision cameras and high quality lens, the photography is suitable for use in some types of stereoscopic plotting equipment at certain compilation scales.

The present U S. Department of Agriculture Specifications (A-APC-1101) for Aerial Photography for General Map Work and Land Studies used in conjunction with Specifications (A-APC-1102, shown in Chapter III) for a Precision Airplane Mapping Camera are as follows. (The requirements for aerial cameras and lenses which are set forth in Paragraph 2 of Specifications A-APC-1101 for Aerial Photography, were effective only until the date when the Camera Specifications A-APC-1102 were approved, viz , March 12, 1940).

Spec A-AP-1101
Approved Nov 18, 1939

UNITED STATES DEPARTMENT OF AGRICULTURE

STANDARD SPECIFICATIONS FOR AERIAL PHOTOGRAPHY FOR GENERAL MAP WORK AND LAND STUDIES APPROVED FOR FEDERAL USE ON MAY 27, 1937, WITH DEVIATIONS AUTHORIZED BY THE SECRETARY

OF THE TREASURY ON NOVEMBER 9, 1939, FOR THE EXCLUSIVE USE OF THE U. S DEPARTMENT OF AGRICULTURE

(For use in connection with U. S. Standard Forms 32 and 33, revised See Instruction No. 1, Page 24.)

SURVEY, VERTICAL NEGATIVES, CONTACT PRINTS

SURVEY, VERTICAL NEGATIVES, CONTACT PRINTS

1. *Statement of Work and Areas To Be Photographed*

(*a*) The contractor shall furnish all materials, superintendence, labor, equipment, and transportation, shall execute and finish the aerial photography of the areas hereinafter specified and shall deliver to the contracting officer, or to such official of the Government as he may designate, such sets of contact prints, index maps, ratioed prints, general enlargements, and oblique photographs as called for by the invitation and the schedule of the advertisement, together with the negatives herein required All work shall be executed in an expeditious and workmanlike manner, to the satisfaction and acceptance of the contracting officer, in complete accord with these specifications and other conditions of bidding set forth in the invitation and in the schedule of the advertisement

(*b*) Location, dimensions, and boundaries of the area to be photographed are set forth in the schedule of the advertisement and on the map or maps attached thereto. It is expressly understood and agreed that such dimensions are approximate only.

(*c*) The Government shall furnish the contractor at the time of the award with three copies of maps of the area to be photographed for use as flight maps

2 *Plane and Camera Crew.*

(*a*) The photographs shall be made with a single-lens standard aerial mapping camera with an effective image area of not less than sixty (60) square inches and an equivalent focal length of not less than eight (8) inches, provided that the focal length of the lens shall be such that the half-angle of field for the extreme corner of the image area shall not exceed thirty-eight and one-half ($38\frac{1}{2}$) degrees, unless other focal lengths and negative dimensions are specified in the schedule of the advertisement Said camera shall be so equipped that negatives are held flat in the focal plane at the instant of exposure and the location of the principal point or geometric center is directly shown or may be determined from collimation marks which shall appear clear and sharp on each negative. The camera shall be equipped with a lens which meets the following requirements as indicated by a report, or a photographic copy thereof, of a test made by the National Bureau of Standards For a camera using 7 by 9 inch film and having a lens of approximately 8 inch focal length, the distortion shall not exceed plus or minus 0.004 inch (plus or minus 0.10 mm.) at 30° from the center of the field. For a camera using 9 by 9 inch film and having a lens of approximately 8 inch focal length, the distortion shall not exceed plus or minus 0.004 inch (plus or minus 0.10 mm.) at 35° from the center of the field. (Exception In accordance with paragraph 22*c*, a lens which has been tested at the National Bureau of Standards prior to April 1, 1939 and which shows distortion not exceeding plus or minus 0 004 inch at 30° from the center of the field will be acceptable, as regards distortion, for use on a camera using either 7 by 9 or 9 by 9 inch film.) These tolerances for distortion are referred to the equivalent focal length and correspond to a tolerance of approximately plus or minus 0.03 mm. when referred to the calibrated focal length. At the maximum stop opening the bidder proposes to use, the lens shall resolve lines in any orientation spaced 20 to the millimeter at the center of the field; 7 to the millimeter in any orientation at the center of the shorter side of the negative; and in at least one orientation 5 lines to the millimeter at the angular distance from the center of the field which is the multiple of 5 degrees falling within the field and nearest the corner of the negative. (Exception. In accordance with paragraph 22*c* a lens which has been tested at the National Bureau of Standards prior to April 1, 1939 and which resolves 5 lines to the millimeter at a distance of 30° from the center of the field will be acceptable, as regards resolution, for use on a camera using either 7 by 9 or 9 by 9 inch film.) For the

resolution tests, the lines and spaces of the test object shall be of equal width and the separation of the lines will be measured on the image of the test target recorded on a Wratten and Wainwright process panchromatic plate, or the equivalent, by the lens being tested, and the wave lengths of the light used for the test shall not be less than 460 millimicrons In the event that other focal lengths are specified, lens tolerances will be stipulated in the schedule of the advertisement Only filters made from stained optical A glass shall be used Each camera used in the performance of the work covered by this advertisement shall be in proper adjustment and in good operating condition to the satisfaction of the contracting officer before being placed on the work and shall be so maintained throughout the period it is in use on said work. The inspection of the camera shall cover the following points The lens cells shall be screwed home and the joints between lens-barrel assembly and cone, and between cone and camera back shall be sufficiently rigid to prevent the introduction of relative motion between the respective members by vibration of the airplane The joint between the magazine and the camera shall be such that there is no visible lost motion leading to a variation in the distance between the lens and the film by the removal and replacement of the magazine or by vibration of the airplane, furthermore, dowels or other positioning members which are intended to prevent displacement of the magazine in a direction lying in the plane of the film shall be free from wear and in such condition that there is no visible lost motion If the film is held flat in the focal plane by means of a glass pressure plate, the actuating mechanism of the platen shall so function that the film is held firmly against the pressure plate during each exposure. If the film is held flat in the focal plane by suction or pressure, all air lines shall be clear and free from leaks and kinks and all vents and air passages shall be clean and free from foreign matter. All mechanical parts of the camera shall be free from excessive wear and in good operating condition

(*b*) If specifically authorized in the schedule of the advertisement, alternate bids will be considered for use of a multiple-lens precision mapping camera of type approved by the contracting officer and of not less than 5-inch focal length, provided the negatives of each exposure are transformed by the contractor into a single composite negative, the several sections of which join without lack of image. Said transformed composite negatives shall be approximately square; shall contain not less than 81 square inches of image, and shall conform in all other respects to the requirements of these specifications pertaining to single-lens photographs. The location of the principal point of each composite negative shall be shown directly thereon or there shall appear on each composite negative collimation marks from which the location of the principal point may be determined. Said composite negatives shall give satisfactory color match, and image match discrepancies between component negatives shall not exceed 0 015 inches or show objectionable vertical displacement when viewed stereoscopically.

(*c*) Unless otherwise specified in the schedule of the advertisement, the successful bidder shall furnish with the first shipment of photographic materials a master glass negative showing clearly the collimation marks of each camera magazine to be used. Collimation marks shall be sharp in outline, so shaped as to facilitate accurate measurements, and so constructed or outlined that they conform to the design shown on page 6 hereof, or to such other design as may be approved by the contracting officer. Said master negatives shall in each case be made with their emulsions in the position occupied by the emulsions of the aerial negatives at instant of exposure and shall be neatly marked with the model numbers and serial numbers of the corresponding camera magazine and with the number of the contract.

(*d*) The pilot and the aerial photographer shall be qualified and fully competent to secure photography conforming to the requirements of the specifications and schedule of advertisement. The contracting officer shall have the right to disapprove any change of pilot or photographer on the work under the contract upon evidence of inability of the proposed substituted person or persons to perform the work satisfactorily.

3 *Scale of Photographs and Method of Computing Same*

The negatives of the entire area or each fraction thereof, as indicated on the map or maps attached to the schedule of the advertisement, shall be made at the proper altitude or altitudes above sea level to yield contact prints at the scale specified in said schedule Said scale shall be computed from the mean altitude of the entire area, from the mean altitude of each fractional area, or from a specified datum plane—all as given in said schedule or as indicated on said maps. Where so specified in the schedule of the advertisement, negatives covering large areas having a reasonably uniform and gradual gradient shall be made at proper heights above the ground to yield the required scale If control strips are required, the scale shall be as specified in the schedule of the advertisement. In any event, photographs showing a departure from the specified scale of more than plus or minus 5 percent in excess of that caused by variations in relief within the areas covered by the individual photographs may be rejected by the contracting officer

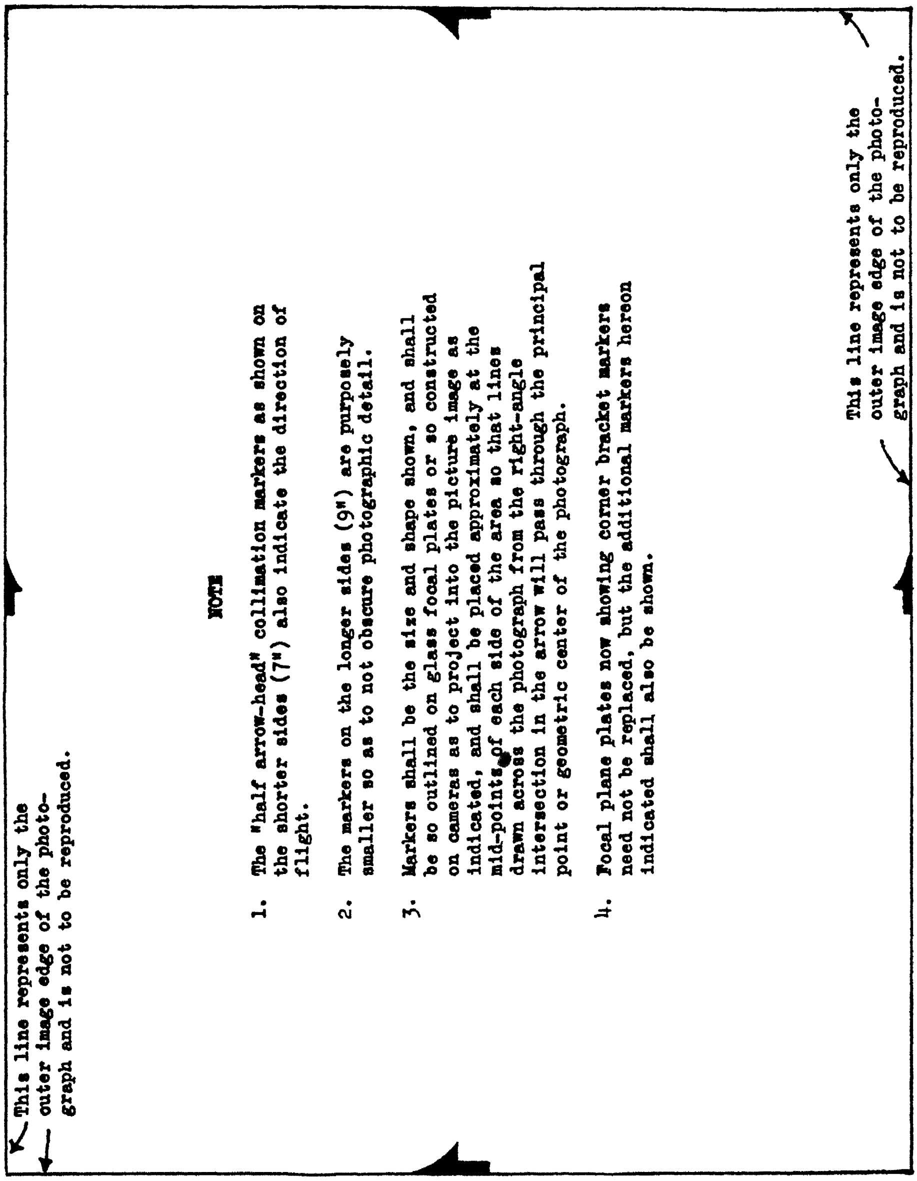
4. *Flight Lines and Flight Strips*

(*a*) Unless indicated to the contrary in the schedule of the advertisement or on the maps furnished the bidders, the direction of all flight lines shall be north and south, except that if mutually agreed upon in writing by the contracting officer and the contractor the lines may be flown in other directions. In all cases the resulting flight strips shall be within five degrees of the specified direction and the mean bearings of adjacent strips shall be within five degrees of parallel. If so indicated in the schedule of the advertisement, the desired flight lines may be shown on suitable maps furnished the contractor, or the contractor may be required to furnish the contracting agency with a set of flight maps (supplied to him for this purpose by the contracting agency) with the proposed locations of the flight lines indicated thereon Unless otherwise specified in the schedule of the advertisement, every effort should be made to assure that each flight strip follows as nearly as possible its plotted location, and no strip or section thereof shall depart from its plotted location by more than 50 percent of the specified mean sidelap distance. Particular care shall be exercised to keep all flight strips as straight and as nearly parallel as possible. Failure of any flight strip or section thereof to meet the above requirements within the tolerances specified may be cause for rejection of any and all flight strips the reflying of which are necessary in order to satisfy the above requirements and those of paragraph 6(*a*) hereof with respect to sidelap.

(*b*) Each flight strip shall be continuous across the project or across such subdivision of the project as specified by the contracting officer, except in cases where it becomes necessary that the flight be broken. The tolerances at any such breaks shall not exceed the tolerances permitted in paragraphs 3, 4, 6, and 7 hereof, except, that the minimum endlap at any such break in the flight strip shall not fall below 100 percent. The maximum endlap at any such break will not be limited. Unless otherwise specified in the schedule of the advertisement no flight strip whether original photography or rephotography shall consist of fewer than 8 exposures. Reflight strips shall be approximately centered over the area for which the reflight is required. Photography failing to meet the above requirements may be considered unsatisfactory and subject to rejection.

5. *Control Strips.*

When so indicated on the maps supplied to the bidders and/or specified in the schedule of the advertisement, the contractor shall furnish control strips of photographs following the lines shown on said maps or specified in said schedules, within the tolerances specified in said schedule, and conforming in all respects to these specifications. Where control flight lines are made up of several sections of different bearings, each section shall extend at least two exposures beyond the points of intersection with adjoining sections. No photographs shall be taken on "banks" between successive courses, but each course shall be flown and photographed independently by turning back at the end of the



COLLIMATION MARK DESIGN APPROVED FOR GENERAL USE OF THE DEPARTMENT OF AGRICULTURE

preceding course and getting into position on a prolongation of the line of the new course at least one mile behind the point at which the first exposure is to be made.

6. *Overlap.*

(*a*) Endlap (overlap in line of flight) shall approximate 60 percent and any endlap of less than 55 percent or more than 65 percent may be considered sufficient ground for rejecting any or all the photographs in the flight strip in which such defficiency occurs. Except when the locations of the individual flight lines are indicated on the maps furnished the bidders or specifically described in the schedule of the advertisement, the sidelap beyond boundaries shall be not less than 25 percent, the sidelap between adjacent

parallel flights shall approximate 30 percent, and any sidelap of less than 15 percent or more than 45 percent may be considered sufficient ground for rejecting any or all of the photographs made on the flight lines which, in the opinion of the contracting officer, should be reflown in order to meet these requirements, however, where flight lines necessarily are flown nonconsecutively, if so specified in the schedule of the advertisement, one strip with more than 45 percent sidelap will be permitted at each such juncture The first and last photographs on each flight line shall fall entirely outside of the project boundary, except, that if this requirement necessitates that the principal points of more than three photographs fall outside of the project boundary, coverage such as will permit securing three point radial intersections in the adjacent strip on the project boundary will be considered satisfactory. If so specified in the schedule of the advertisement, the minimum, average and maximum overlap may be numerically increased 5 percent.

(*b*) In cases of extreme variations in elevation within any area, the contracting officer may permit deviation from the requirements of paragraph (*a*) *above provided*, in his opinion, topographic features within the area warrant such deviation. Under such circumstances, deviations from the specifications will be limited to the amount actually caused by the extreme variation in elevation, except that any endlap of less than 55 percent, and/or any sidelap between adjacent parallel flight strips of less than 10 percent may be considered sufficient grounds for rejecting any or all the photographs in any flight strips which, in the opinion of the contracting officer should be reflown to meet these requirements.

7. *Alignment.*

Any series of three or more consecutive photographs in which the effective image area in the endlap area common to three photographs is reduced in any one photograph to less than ninety percent of the lateral dimension of the image area (as a result of any cause) may be considered unsatisfactory and cause for rejection of that particular flight strip or any portion thereof.

8 *Tilt*

Negatives made with the optical axis of the camera in a vertical position are required and tilt (departure from the vertical) of any negative exceeding 4° or averaging more than 2° in any 10-mile section of a flight line or more than 1° for the entire project, or for each entire subproject in the event the project is composed of subprojects, or relative tilt between any two successive negatives exceeding 6°, may be cause for rejection

9. *Film.*

Only fresh, fine-grained, high-speed, low-shrink base aerial film shall be used, and no film shall be used, until there has been a determination by the contracting officer that it meets the following requirements. The shrinkage in any direction on said film after developing and drying shall not exceed two and one-half parts per thousand when the film is dried in an oven for 7 days at a temperature of 120° F., the measurements before and after developing and drying being taken after the film has been conditioned in an atmosphere if 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F , plus or minus 2°, for at least 24 hours. The difference in shrinkage between measurements in any two directions after such developing and drying shall not exceed one-half part per thousand.

10. *Photographic Paper.*

(*a*) No photographic paper shall be used until there has been a determination by the contracting officer that it meets the specified requirements. In order to minimize the effects of shrinkage, the grain of the paper in the resulting prints shall be perpendicular to the length of the negative roll.

(*b*) For standard commercial-grade, double-weight photographic paper to be approved by the contracting officer, the average difference in shrinkage measured in any two directions after developing and drying, shall not exceed four parts per thousand, the measurements before and after developing and drying being taken after the paper has been conditioned in an atmosphere of 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F , plus or minus 2°, for at least 24 hours

(*c*) For special aero-mapping (water-resisting), double-weight photographic paper to be approved by the contracting officer, the shrinkage in any direction after developing and drying shall not exceed four (4) parts per ten thousand and the average difference in shrinkage measured in any two directions after developing and drying shall not exceed two (2) parts per ten thousand, the measurements before and after developing and drying being taken after the paper has been conditioned in an atmosphere of 65 percent relative humidity, plus or minus 3 percent, and at a temperature of 72° F , plus or minus 2°, for at least 24 hours

11. *Contact Prints*

(*a*) Contact prints from the vertical negatives shall be printed on the kind, weight, and finish of paper, with or without mask and trimmed as specified in the schedule of the advertisement

(*b*) Unless otherwise specified in the schedule of the advertisement, there shall be stamped or neatly lettered with waterproof ink, on the back of each print, in the same corner and position as the photograph number in a space not to exceed 3¼ inches by 1¼ inches, the name of the Department, Bureau, or Agency, State and county, and the name and address of the contractor, with space left for insertion of the scale of the photograph

12 *Processing and Drying Film and Prints*

Special care shall be exercised in exposing and processing, including the development, fixation, and washing of all film and prints, to insure freedom from chemicals or other stains and to insure normal and uniform density and fine-grain quality of the resulting negatives and prints Care also shall be taken to avoid rolling film tightly on drums or in any way distorting it during processing and drying Prints may be dried between blotters without the application of weights or by placing them face down on cheesecloth-covered frames If the contractor desires to use any mechanical process for drying prints, he first shall demonstrate to the satisfaction of the contracting officer, with contact prints and enlargements of suitable grids, that the maximum differential distortion in the resulting prints is not in excess of that which would occur if the prints were dried naturally.

13. *Quality of Negatives and Prints.*

(*a*) Negatives shall be free from stains, scratches, finger marks, dirt and blemishes of any kind Exposure and development shall be such as to yield negatives of high quality The film shall be developed in a non-staining developer to a gamma or contrast which will yield negatives showing clearly the demarkation of boundaries of fields, roads, woods, etc , when printed or enlarged on semi-matte surface photographic paper The exposure of the negatives shall be such that printable detail is obtained in the thinnest part of each negative The density of the thinnest parts of the negatives shall not be excessive, that is, negatives of medium density requiring moderate exposure time in making enlarged prints without the sacrifice of detail and contrast are desired Very thin negatives or very dense negatives shall be rejected Each bidder shall accompany his bid with two vertical aerial negatives as samples of the quality and density of the negatives he proposes to furnished. One of said samples shall represent the thinnest negatives and the other the densest negatives he proposes to furnish. These sample negatives shall be made at the maximum stop opening the bidder proposes to use and at the approximate

scale specified in the schedule of the advertisement and shall show terrain having an average amount of clear-cut detail and shall represent the quality of work, including over-all definition, definition in the corners and photographic quality the bidder proposes to furnish. In case these sample negatives are not acceptable as to quality and density, the contracting officer may request the bidder to furnish acceptable sample negatives before the contract is awarded. The acceptable sample negatives will be used as criteria for the acceptance or rejection of any negatives delivered under the contract The contracting officer reserves the right to reject the bid of any bidder, who, in his opinion, fails to submit samples of the quality required by these specifications and the attached schedule

(*b*) Prints shall be of uniform color and density and shall be of such a degree of contrast that all details of the negatives will show clearly, both in the shadows and the highlights as well as in the half tones between shadows and highlights An adequate variety of grades of contrast paper shall be used in making prints to accomplish this purpose. All prints shall be clean and free from chemicals, stains, blemishes, uneven spots, air bells, light fog or streaks, snow, clouds or cloud shadows, static marks and other blemishes which in the opinion of the contracting officer would interfere with their intended purpose, and shall be delivered in a smooth and flat condition Prints from exposures made when the streams are not within their normal banks or when the sun's altitude is less than that specified in the schedule of the advertisement, or which do not meet the above requirements or are not equal in quality to the representative samples submitted with the bid may be considered unsatisfactory and may be rejected.

14. *Indexing and Editing.*

(*a*) Each vertical negative shall be marked clearly with a designating symbol furnished by the contracting agency of not to exceed four letters followed by the serial number of the roll and the serial number of the exposure on the roll (thus ABC-116-110) and also with a numerical abbreviation of the month, day, and year of exposure (thus 12-8-39). The rolls of film used in the performance of each contract, unless otherwise designated by the contracting officer, shall be numbered in an unbroken series beginning with number 1, and the exposures on each roll shall be numbered in an unbroken series beginning on each roll with number 1 The designating symbol, roll and serial numbers shall be placed in the northeast corner of each negative for north and south flights and in the northwest corner of each negative for east and west flights, with the exception of the control strips, in which case said symbol and numbers shall be placed in the upper right-hand corner of each negative, progressing along the line of flight The abbreviation of the date shall in each case be placed in the adjacent corner in a counterclockwise direction, with the exception of the control strips, in which case, unless otherwise specified, it shall be placed in the adjacent corner in a clockwise direction. In addition, the first and last negative of each flight strip, at each break in each flight line, on the first and last exposure of each flight strip as shown on each index sheet, and on the first and last negative of each roll the initials of the Bureau or Agency for which the work is being performed and the approximate scale of the negatives shall be placed immediately preceding the designating symbol and the serial numbers (thus SCS-1 20,000-ABC-116-110) and the numerical abrevіation of the approximate time of day of the exposure shall be placed immediately following the date (thus· 12-8-39-12·30). The characters used in marking negatives shall be approximately three-sixteenths inch high and may be mechanically stamped with opaque ink, or neatly drafted with celluloid ink, in such manner as to print clearly in positive form on the image area of the photograph in the position specified. The top of the characters shall not be less than $\frac{1}{8}$, nor more than $\frac{1}{4}$ of an inch from the image edges of the negatives

(*b*) Unless otherwise specified in the schedule of the advertisement each roll of film shall be edited by the contractor and, before delivery, all rejected negatives shall

be marked "Rejected" outside the margin of the image area, but shall not be removed from the roll except by authority of the contracting officer

(*c*) There shall be at least eighteen inches of film leader beyond the first and last exposure of each roll of film Said leader shall be a part of the film or shall be permanently spliced to the film In the event the flight strips are photographed nonconsecutively there shall be at least nine inches of blank film or one unrequired exposure between flight strips Any splicing of film shall be made by thoroughly cleaning the portions to be spliced and cementing them together with nitrate dope or acetone in which cellulose nitrate has been dissolved, or by other adhesive which will give a permanent splice and will not be affected by heat or moisture. Particular care shall be given to alignment of film in splicing in order that, after splicing, the film will be perfectly straight Said splice within the roll shall in no case be closer than 3 inches from the image edge of any accepted exposure and shall overlap approximately $\frac{3}{8}$ inch In no case shall any type of adhesive tape be used on the film. If splicing of film becomes necessary, each composite roll shall consist of exposures made with the same camera-magazine combination All film shall be cleaned thoroughly before shipment and shall not be delivered in lengths greater than can be wound, without strain, on a $5\frac{1}{4}$ inch diameter film spool unless otherwise specified Said film shall be delivered in cans of size corresponding to the film spools

(*d*) The metal container and spool for each roll of film shall become the property of the Government. The container shall be neatly labeled with the names of the Department and Bureau or Agency for which the work is performed, the name or number and the location of the project, the invitation number, item number and number of the contract, the type, model and/or serial numbers of the camera, lens and magazine used in making the exposures, the serial number of the roll, the dates and times of exposure and the serial numbers of the first and last exposures made on each date, the project symbol (or in the event the project is comprised of subprojects the subproject names and symbols) together with the exposures relating to each project or subproject, the name and address of the contractor and the approximate scale of the negatives

(*e*) Unless otherwise specified in the schedule of the advertisement indices to the photography shall be prepared by photographing an assembly of contact prints from each vertical negative after such negatives have been properly designated Such contact prints shall be made without mask and shall be trimmed to the image edge. Every effort shall be made to insure uniformity of tone throughout the assembly The assembly shall be carefully laid to match corresponding images and clearly show the roll and serial number of each negative Before reproductions are made there shall be placed over the roll and serial number indicated on the first and last print in each flight strip in the subproject, on every fifth print and at every change in roll number, stick-up numbers approximately three-eighths inches high indicating the roll and serial number on the roll The photo-index negatives shall be approximately 8″×10″ and shall be made on safety base film The quality of such negatives shall conform to the requirements of paragraph 13(*a*) The photo-index prints made from such negatives shall be to the scale of approximately 1 inch equals 1 mile on 20″×24″ sheets of double-weight fine grain surface semi-matte finish standard commercial grade photographic paper The copying process shall not be such as to require enlargement of more than two and one-half diameters in projecting the index negative on such 20″×24″ sheets. All prints shall conform to the requirements of paragraph 16 hereof, and every effort shall be made to secure uniformity of tone between sheets In the event the project or subproject is too large for the index to be placed on one sheet, it shall be made in sections, each section of which shall be on approximately 8″×10″ negatives and 20″×24″ sheets with sufficient overlap to facilitate working from sheet to sheet Each sheet shall be made with white margins, and shall have a neatly executed title, which shall appear on the negative, showing the names of the Department and Bureau or Agency for which the work is being performed, a key to the index sheets; the name or number and the location of the project, the designating

symbol, the number of the contract, the name and address of the contractor, the approximate scales of the photographs, a graphic bar-scale of the index and the date of completion of the index The boundaries of the project and/or subproject shall be clearly shown on such sheets In case any portion or portions of the project is required to be reflown, the specified number of additional copies of the corrected sheet or sheets on which the reflown areas appear shall be delivered.

Ratioed Prints and General Enlargements

15. *Ratioed Prints*

Ratioed prints shall be made with or without mask, on the kind, weight, and finish of paper and to the dimensions of trimming as specified in the schedule of the advertisement and shall be clear and sharp in detail, of uniform normal density and contrast and shall equal in quality the sample enlargement submitted with the bid They shall show no appreciable radial or other distortion of imagery or scale due to faulty optics or mechanics of the projection camera Before making the award, the contracting officer may, at his discretion, require the bidder to whom it is proposed to make award to submit ratioed prints of suitable grids or demonstrate otherwise to the satisfaction of the contracting officer the accuracy of the projection camera the bidder proposes the use Unless otherwise specified in the schedule of the advertisement the ratios or the dimensions to which the prints are to be made and the collimation or other points between which measurements are to be made, will be furnished by the contracting agency and the prints shall be of the correct dimensions between such points, within two-tenths of one percent (0 20%) in the case of prints on special aero-mapping (water-resisting) paper and within one-half of one percent (0.50%) in the case of prints on standard commercial grade paper, *provided, however*, that the above tolerance for prints on standard commercial grade paper shall apply after the prints have been subjected for twenty-four hours to prevailing atmospheric conditions and the scaled dimensions have been adjusted to 50% relative humidity by compensating for prevailing humidity in an amount of one-tenth of one percent (0 10%), of the required dimension, for each ten percent (10%) change in relative humidity above or below fifty percent (50%) relative humidity Prints failing to meet the above requirements may be rejected.

16 *General Enlargements.*

General enlargements shall be made to the number of diameters or to such approximate scale as may be specified in the schedule of the advertisement, shall conform to the requirements of paragraph 15(*a*) hereof, pertaining to detail, density, contrast and freedom from distortions of imagery and scale due to faulty optics or mechanics of the projection camera, and shall be equal in quality to the sample enlargement submitted with the bid Said enlargements shall be made on the kind, weight and finish of paper trimmed and printed with or without mask as specified in the schedule of the advertisement.

Oblique Photographs

17 *Areas To Be Photographed.*

If oblique photographs are specified in the schedule of the advertisement, the approximate location from which each photograph is to be taken and the direction in which each is to be taken will be indicated on the flight map furnished the contractor by the Government Unless otherwise specified in the schedule of the advertisement, said photographs shall be taken between the hours of 9 a. m. and 3 p m. from an elevation of approximately 1,000 feet above the highest point in the area being photographed.

18. *Camera To Be Used.*

The photographs shall be made with a standard single-lens aerial camera having a focal length of not less than 8 inches with an effective negative image area of not less

than 60 square inches, and indicating on each negative the location of the principal point or the collimation marks from which the principal point may be determined. Each bidder shall specify the make and model of the camera, the size of the negative, and the focal length of the lens he proposes to use

19 *Quality and Numbering of Photographs*

(a) The oblique photographs shall be equal in quality to the vertical photographs hereinbefore specified Clouds above the horizon will not be objectionable providing they do not cast shadows on the terrain shown in the photographs, which, in the opinion of the contracting officer, would impair the value of the photographs for their intended purpose. The transverse axis of the camera shall be level at the instant of exposure and approximately one-fourth of the area of each negative shall show above the horizon.

(*b*) The negatives shall be numbered in consecutive order with three-sixteenths-inch figures placed in the lower right-hand corner thereof, with the numbers progressing around the area in the manner indicated on the flight map.

20 *Film.*

Unless otherwise specified in the schedule of the advertisement, oblique photographs shall be made with standard fine-grained high-speed aerial film, approved by the contracting officer Only fresh film shall be used.

21. *Prints.*

The oblique photographs shall be masked and ferrotyped, using single-weight standard commercial-grade photographic paper approved by the contracting officer Said photographs shall be trimmed with a uniform margin of approximately one-half inch outside of the photographic image.

Special Conditions

22 *Conditions of Bidding*

(*a*) No bid will be considered unless it complies with the requirements set forth in the invitation and the schedule of the advertisement.

(*b*) Each bid shall be accompanied by a certificate (in addition to any other(s) required by these specifications or by the schedule of the advertisement) showing. (1) that the bidder has available and will use in the execution of the proposed contract the required number of airplanes, specified in the schedule of the advertisement, that will perform satisfactorily at the necessary altitudes to yield the negative scale specified in said schedule and that are equipped with the necessary instruments and photographic apparatus to complete the required photography in full accordance with the specifications; (2) that he has available and will use for each of said airplanes the services of a qualified pilot, competent to perform the work required under the proposed contract and a competent aerial photographer, and (3) the organization, equipment, capacity, and experience of the bidder on similar projects Each bidder shall submit to the contracting officer with the first bid submitted by him in each calendar year a certified statement (in the form prescribed by the Department) of his current financial status. The right also is reserved to the contracting officer to require, in connection with the submission of any bid, evidence of the competency of the pilot(s) and aerial photographer(s) proposed to be used The right also is reserved to the contracting officer to require a current financial statement in connection with the submission of any bid and to reject any bid where investigation of the business and/or technical organization of the bidder available for the contemplated work, including financial resources, equipment, or experience on similar projects, does not satisfy him that such bidder is qualified to perform the work

(*c*) Unless otherwise specified in the schedule of the advertisement, a photographic copy of a report from the National Bureau of Standards showing the equivalent focal

length and the distortion and resolution values of the lens or lenses the bidder proposes to use should accompany the bid and will be required before an award is made Reports dated prior to April 1, 1939 showing distortion and resolution values at 30 degrees from the center of the plate will be considered satisfactory Reports dated subsequent to April 1, 1939 shall indicate the resolution and distortion values at 35 degrees from the center of the plate

(*d*) Each bidder shall submit with his bid a sample contact print made from each of the sample vertical negatives required under paragraph 13(*a*), without mask on double-weight, semi-matte paper and, unless otherwise specified in the schedule of the advertisement, a sample single-weight, semi-matte, unmasked, two and one-half diameter enlargement from one-quarter of each of the same negatives, including in each case the center and one corner of the negative, and also furnish such other samples as are specified in the schedule of the advertisement The bidder shall mark on the back of each sample print and enlargement the trade name of the paper and the contrast grade used These samples shall show the photographic quality the bidder proposes to furnish and will be used as criteria in judging the quality of the photographs the contractor delivers under the contract Samples showing heavily wooded areas or other types of terrain in which it is difficult to determine whether the photographic quality and definition meet the requirements herein specified, will not be accepted If specimen contact prints and enlargements of the quality the Government requires are attached to the schedule of the advertisement, the samples submitted by the bidder shall equal these specimens in all respects The contracting officer reserves the right to reject the bid of any bidder, who, in his opinion, fails to submit samples of the quality required by these specifications and the attached schedule

(*e*) In case the use of a multiple-lens mapping camera is authorized in the schedule of the advertisement, as provided in paragraph 2(*b*) hereof, and it is proposed to use such a camera, a sample stereoscopic pair of contact prints from transformed composite negatives made from exposures taken with the camera and lenses and at the maximum stop opening it is proposed to use and at the approximate scale specified, and, unless otherwise specified in the schedule of the advertisement, a sample two-and-one-half-diameter enlargement from one-quarter of one of said composite negatives, including the center and one corner of the negative, shall be submitted with the bid in lieu of the sample single-lens contact print and enlargement requirement by paragraph 22(*d*) hereof Said contact print shall be made without mask on double-weight semi-matte paper, and said enlargement shall be made without mask on single-weight semi-matte paper.

(*f*) Bidders are expected to examine the specifications and maps, to visit the locality of the work if necessary, and to make their own estimates of the facilities needed and the difficulties attending the execution of the proposed contract, including local conditions, uncertainty of weather, availability of landing fields, and all other contingencies. Any request for clarification or interpretation of any portion of the specifications or advertisement should be received in writing or by telegraph by the officer issuing the invitation at least 1 week prior to the time fixed for the opening of the bids in order that he may notify all prospective bidders of such interpretation or clarification without the necessity of postponing said time of opening.

(*g*) Each proposal (on U. S. Standard Form No. 33) will be received with the understanding that these specifications and special conditions form a part thereof, and that when accepted in writing within the time specified by the contractor, which in no event should be less than 15 calendar days (excluding the day the bid is opened), such accepted proposal shall constitute the contract between the bidder and the Government, unless the execution of a formal contract is required by paragraph 24 of the specifications or by the schedule of the advertisement.

23 *Bid Guarantees.*

(a) Unless otherwise specified in the schedule of the advertisement, bids of $1,000 or less need not be accompanied by a bid guarantee

(*b*) Each bid in excess of $1,000 shall be accompanied by a bid guarantee of not less than ten percent (10%) of the amount of the bid, which guarantee may be a money order, certified check, or cashier's check, made payable to the Treasurer of the United States, or, *where the bid is in excess of $2,000,* a bid bond on U. S Standard Form No. 24 will be acceptable Each such money order and check shall be submitted with the understanding that it shall guarantee that the bidder will not withdraw his bid within the period specified after the opening of the bids, that (if bid exceeds $2,000) he will, if his bid is accepted, enter into a formal contract and give performance bond as may be required, and that in the event of the withdrawal of his bid within said period, or (where bid exceeds $2,000) the failure to enter into said formal contract and to give said bond within the time specified, the bidder shall be liable to the Government for the difference between the amount specified in his bid and the amount for which the Government may otherwise procure the required work, if the latter amount be in excess of the former, and the Government shall have the right to retain the proceeds of such money order or check to apply on account of such excess cost Guarantees of unsuccessful bidders will be returned after award is made

(*c*) Where the bid is not in excess of $2,000, the guarantee of the successful bidder will be returned on acceptance of completed work, except that the Government shall have the right to retain the proceeds of said guarantee to such extent as may be required to reimburse the Government for any loss or damage resulting from default in the performance of the contract.

(*d*) Where bid exceeds $2,000, the guarantee of the successful bidder will be returned after the performance bond is approved.

24 *Contract and Performance Bond*

Unless otherwise specified in the schedule of the advertisement, the execution of a formal contract and the furnishing of a performance bond will not be required where the bid of the successful bidder is $,2000 or less If such bid exceeds $2,000, and if not otherwise specified in the schedule of the advertisement, the bidder to whom the award is made shall enter into a formal contract on U S Standard Form No 32, Revised (except that Article 5 thereof, entitled "Delays—Damages," is superseded by paragraph 31 of these specifications and shall be deleted) and shall furnish a performance bond on U S. Standard Form No 25, Revised, for the faithful performance of the contract Such performance bond shall be in an amount representing such percentage of the contract price as may be specified in the schedule of the advertisement and shall be mailed, together with the formal contract, if required, within ten calendar days after receipt thereof

25 *Award and Notice to Proceed.*

(*a*) If the accepted bid is not in excess of $2,000, the notice of award by registered mail, or telegram, will be the notice to proceed

(*b*) If the accepted bid is in excess of $2,000, the notice to proceed will be given, by registered mail or telegram, promptly after approval of the contract and performance bond.

26 *Commencement and Prosecution of Work.*

(*a*) Upon receipt of the award, the contractor shall notify the contracting officer in writing where he proposes to base his flying operations and where he proposes to perform the work other than the flying and shall keep the contracting officer currently advised of any changes in those bases of operation

(*b*) The contractor shall undertake from the base or bases of field operations the

photography of the areas to be photographed within 10 calendar days after the receipt of notice to proceed (excluding the day of receipt) or within such other period of time as may be specified in the schedule of the advertisement The time for starting the work may be postponed by the contracting officer if snow or high water renders photography of the area impractical Not later than the date on which he is required to commence operations, the contractor shall notify the contracting officer of the location or locations of his base or bases of laboratory and field operations and on the date of arrival of each plane, pilot and/or aerial photographer at any base of field operations shall notify the contracting officer by telegram, confirmed in writing, of such arrival Unless otherwise authorized in writing by the contracting officer, the contractor shall keep all required equipment and personnel continuously on the project and shall take full advantage of suitable photographic weather until all the original photography is completed

(*c*) If the project is made up of two or more subprojects, the contractor shall photograph said subprojects in the order indicated in the schedule of the advertisement, subject to such provisions or changes as specified in said schedule, and shall notify the contracting officer by letter or by telegram immediately upon completion of the original photography of each such subproject

(*d*) If any portion of portions of the project are required to be rephotographed, the contractor shall, weather conditions permitting, undertake such work in the order specified by the contracting officer, within the number of calendar days after receipt by the contractor of written notice from the contracting officer to proceed with the required reflights, as specified in the schedule of the advertisement The contractor shall notify the contracting officer immediately upon completion of said work and shall ship the materials within the number of calendar days specified in the schedule of the advertisement

27. *Time Allowance*

(*a*) The maximum time allowance for shipment of negatives, contact prints, index sheets, and oblique photographs, whether original or replacement materials, will be that specified in the schedule of the advertisement.

(*b*) The maximum time allowance for shipment of ratioed prints and general enlargements, whether original or replacement material, will be the number of calendar days specified in the schedule of the advertisement after the contractor has received written notice of the acceptance of the photography, and has received the ratio and/or enlargement factors· *Provided, however,* That if the ratio and/or enlargement factors are to be determined by the contractor, the said time allowance shall be the number of calendar days specified in such schedule after receipt by the contractor of written notice of acceptance of the photography.

(*c*) The maximum area for which the contractor may be required to make shipment of contact prints, ratioed prints, and enlargements during any 30-calendar-day period shall in no case exceed the maximum 30-calendar-day delivery area specified in the schedule of the advertisement.

(*d*) In the event inspection is to be made in the plant of the contractor the maximum time allowance for notification that materials are ready for inspection shall be the number of calendar days specified in the schedule of the advertisement.

(*e*) The contractor will be required to furnish the contracting officer with a receipt from the common carrier, or a photographic copy thereof, clearly showing the date of each separate shipment.

28. *Specifications and Maps.*

The contractor shall keep on the work a copy of the maps, schedule and specifications and shall at all times give the contracting officer access thereto Anything mentioned in the schedule or shown on the maps and not mentioned in the specifications, or mentioned in the specifications and not mentioned in the schedule or shown on the maps,

shall be of like effect as if shown or mentioned in all three In cases of difference between maps and specifications, or between maps and the schedule, the specifications or the schedule shall govern, the schedule shall supplement the specifications, but the specifications shall govern if necessarily inconsistent therewith. In any case of discrepancy in or between the maps, schedule or specifications, the matter shall be submitted inmediately to the contracting officer, without whose decision said discrepancy shall not be adjusted by the contractor, save only at his own risk and expense

29 *Changes*

The contracting officer may at any time, by a written order, and without notice to the sureties, if any, make changes in the maps and/or specifications of the contract and within the general scope thereof If such changes cause an increase or decrease in the amount due under the contract, or in the time required for its performance, an equitable adjustment shall be made and the contract shall be modified in writing accordingly No change involving an estimated increase or decrease of more than $500 if the contract price be in excess of $2,000 or of more than 25 per cent of the contract price if the latter be $2,000 or less, shall be ordered unless approved in writing by the head of the department or his duly authorized representative Any claim for adjustment under this paragraph must be asserted within 10 days from the date the change is ordered. *Provided, however,* That the contracting officer, if he determines that the facts justify such action, may receive and consider, and with the approval of the head of the department or his duly authorized representative, adjust any such claim asserted at any time prior to the date of final settlement of the contract If the parties fail to agree upon the adjustment to be made the dispute shall be determined as provided in paragraph 39 hereof But nothing provided in this paragraph shall excuse the contractor from proceeding with the prosecution of the work so changed

30 *Inspection and Acceptance*

An effort will be made to inspect all materials specified in the schedule of the advertisement within 21 calendar days after they are received by the Government at the point designated after which the contracting officer will notify the contractor in writing whether they are satisfactory and what areas, if any, shall be rephotographed and what materials, if any, shall be remade because of nonconformity with the contract requirements

31. *Delays—Damages*

If the contractor refuses or fails to prosecute the work, or any separable part thereof, with such diligence as will insure its completion within the time specified, or any extension thereof, or fails to complete said work within such time, the Government may, by written notice to the contractor, terminate his right to proceed with the work or such part of the work as to which there has been delay. In such event the Government may take over the work and prosecute the same to completion, by contract or otherwise, and the contractor and his sureties (if any) shall be liable to the Government for any excess cost occasioned the Government thereby If the Government does not terminate the right of the contractor to proceed, the contractor shall continue the work, in which event the actual damages for the delay will be impossible to determine and in lieu thereof the contractor shall pay to the Government as fixed, agreed, and liquidated damages for each calendar day of delay until the work is completed or accepted the amounts as set forth in paragraph 32 hereof, or in the schedule of the advertisement, and the contractor and his sureties (if any) shall be liable for the amount thereof. *Provided,* That the contractor shall not be charged with liquidated damages or any excess cost when the delay in completion of the work is due to unforeseeable causes beyond the control and without the fault or negligence of the contractor, including, but not restricted to, acts of God, or of the public enemy, acts of the Government, acts of another contractor in the perform-

ance of a contract with the Government, fires, floods, epidemics, quarantine restrictions, strikes, freight embargoes, and unusually severe weather or delays of subcontractors due to such causes, if the contractor shall within 10 days from the beginning of any such delay (unless the contracting officer, with the approval of the head of the department or his duly authorized representative, shall grant a further period of time prior to the date of final settlement of the contract) notify the contracting officer in writing of the causes of delay, who shall ascertain the facts and the extent of the delay and extend the time for completing the work when in his judgment the findings of fact justify such an extension, and his findings of fact thereon shall be final and conclusive on the parties hereto, subject only to appeal, within 30 days, by the contractor to the head of the department concerned, whose decision on such appeal as to the facts of delay and the extension of time for completing the work shall be final and conclusive on the parties hereto

32 *Liquidated Damages*

Unless otherwise specified in the schedule of the advertisement, the fixed, agreed, and liquidated damages to be paid the Government by the contractor in accordance with paragraph 31 hereof, for each calendar day of delay after expiration of the time allowances specified in paragraphs 2(*c*), 26, 27, and 34 hereof and in the schedule of the advertisement, and for each calendar day the flying equipment and personnel are not on the project, as required under paragraph 26(*b*), shall be as follows

Undertaking the work, maintenance of flying equipment and personnel continuously on the project, per plane camera crew (See Par 26(*b*))	25
Undertaking reflights, per plane camera crew (See Par 26(*d*)) .	10
Negatives (See Par. 27(*a*))	10
Contact prints with index sheets (See Par 27(*a*))	10
Oblique photographs (See Par 27(*a*))	10
Ratioed prints and/or general enlargements, per order (See Par 27(*b*))	10
Failure to notify that materials are ready for inspection (See Par 27(*d*))	10
Periodical reports (See Par. 34)	5

33 *Risk—Damages*

The contractor shall assume all risks in connection with the performance of the contract, and shall be liable for and save the Government harmless on account of any damages to persons or property in connection with the prosecution of the work

34 *Reports*

The contractor shall submit to the contracting officer regular periodical reports showing progress of the work as required in the schedule of the advertisement Forms for said reports will be supplied the contractor before commencement of the work.

35 *Ownership of Negatives.*

Unless otherwise specified in the schedule of the advertisement, all negatives shall become the property of the Government and shall be delivered in accordance with instructions from the contracting officer to the Bureau or Agency for which the work has been performed During the period the negatives are in the possession of the contractor. he may make for commercial use such prints, enlargements, mosaics, and reproductions as he may desire from any such negatives which do not show fortifications, military or naval defences or other restricted areas, the photographing of which is prohibited by the Government

36. *Subcontracts.*

The contractor shall not, without prior written approval of the contracting officer, enter into any subcontract covering any part of the work contemplated by his contract.

37. *Patents.*

The contractor shall hold and save the Government, its officers, agents, servants, and employees, harmless from liability of any nature or kind, including costs and expenses, for or on account of any patented or unpatented invention, article, or appliance manufactured or used in the performance of the contract, including their use by the Government

38. *Permits and Care of Work*

The contractor shall, without additional expense to the Government, obtain all required licenses and permits, and shall be responsible for the proper care and protection of all materials until delivery thereof in accordance with instructions from the contracting officer

39 *Disputes*

Except as otherwise specifically provided in the contract, all disputes concerning questions of fact arising under the contract shall be decided by the contracting officer subject to written appeal by the contractor within 30 days to the head of the department concerned or his duly authorized representative, whose decision shall be final and conclusive upon the parties thereto In the meantime the contractor shall diligently proceed with the work as directed

40 *Payments to Contractors*

(*a*) If the contract price be $1,000 or less, payment in full will be made upon completion and acceptance of the work.

(*b*) If the contract price be in excess of $1,000, partial payments may be made, at the discretion of the contracting officer, as the work progresses.

(*c*) In making partial payments there shall be retained 10 percent of the estimated amount until completion and final acceptance of all work covered by the contract

(*d*) If subprojects are involved, and if so specified in the schedule of the advertisement, payment in full with respect to each subproject (in lieu of partial payments hereinbefore provided and without deduction of any retained percentage) will be made upon completion and final acceptance of all work in connection with any such subproject.

(*e*) All material and work covered by partial payments made shall thereupon become the sole property of the Government, but this provision shall not be construed as relieving the contractor from the sole responsibility (prior to delivery to the Government) for the care and protection of materials and work upon which payments have been made or the restoration of any damaged work, or as a waiver of the right of the Government to require the fulfillment of all of the terms of the contract.

(*f*) Upon completion and acceptance of all work required under the contract, the amount due the contractor under the contract will be paid upon the presentation of a properly executed and duly certified voucher therefor, after the contractor shall have furnished the Government with a release, if required, of all claims against the Government arising under and by virtue of the contract, other than such claims, if any, as may be specifically excepted by the contractor from the operation of the release in stated amounts to be set forth therein.

(*g*) Payments will be made for additional prints ordered under the contract upon delivery thereof.

41. *Covenant Against Contingent Fees.*

The contractor warrants that he has not employed any person to solicit or secure the contract upon any agreement for a commission, percentage, brokerage, or contingent fee. Breach of this warranty shall give the Government the right to terminate the contract, or, in its discretion, to deduct from the contract price or consideration the amount of

such commission, percentage, brokerage, or contingent fees. This warranty shall not apply to commissions payable by contractors upon contracts or sales secured or made through bona fide established commercial or selling agencies maintained by the contractor for the purpose of securing business.

42 *Definitions*

(*a*) The term "head of the department" as used herein shall mean the head or any assistant head of the executive department or independent establishment involved, and the term "his duly authorized representative" shall mean any person authorized to act for him other than the contracting officer

(*b*) The term "contracting officer" as used herein shall include his duly appointed successor or his authorized representative

ADDITIONAL INSTRUCTIONS TO CONTRACTING OFFICERS

(*Supplementing those printed on the back of U S Standard Form No 33*)

1 Invitations for bids for aerial photography shall be issued on U S. Standard Form 33 (Revised), Invitation, Bid and Acceptance (short-form contract), using Standard Form No 36 for continuation schedule—and care should be taken to attach to Form 33 the representations and stipulations required by the Walsh-Healey Act, inserting in Form 33 a provision that such representations and stipulations are by reference made a part of the bid

2. These general specifications for aerial photography are based upon a thorough canvass of the technical requirements of the mapping and economic agencies of the Government as pertaining to aerial photography, and have been developed to meet these two sets of requirements in a satisfactory manner. It is anticipated that they will be revised from time to time as changing conditions may dictate, in order to keep abreast of new developments These specifications necessarily include material that may not be applicable to every project Hence, in preparing specifications for any particular project, the contracting officer should indicate in the schedule of the advertisement those portions of the standard specifications which apply to the work desired, making any authorized modifications therein, or additions thereto, that may be necessary to meet the requirements of the work. In this connection, consideration should be given to the needs of other agencies of the Government, and the schedule of the advertisement should call for bids for such additional prints of photographs as other agencies of the Government may need to purchase, even though the contracting officer may not plan to purchase such extra copies for the specific project for which the invitation is issued, to the end that maximum returns may be obtained from the aerial surveys and that duplication may be avoided. In some cases, the season of the year in which the photographs are taken has a marked bearing on their general utility, for example, the value of photographs of wooded areas may be greatly enhanced, from the standpoint of soil and geological studies if taken when the leaves are off the trees.

3. As indicated in the paragraphs of the specifications hereinafter noted, the schedule of the advertisement should designate specifically

Locations, dimensions, and boundaries of areas to be photographed	Par 1(*b*)
Type of camera approved, including other focal lengths if any, with lens tolerances and dimensions, if any.	Par 2
Scale of the contact prints desired and the method of computing same	Par. 3
Flight lines in other directions, if any, other tolerances for accuracy in following plotted flight lines, if any	Par. 4(*a*)
Minimum number of exposures in any flight strip	Par. 4(*b*)
Control strips and tolerances for accuracy in following plotted flight lines, if any control strips required	Par. 5
Overlap, other than specified, excessive sidelap, if specified	Par. 6(*a*)

Kind, weight, and finish of paper to be used for contact prints and whether masking and trimming required	Par 11(*a*)
Marking on back of contact prints, if other than stamped or lettered	Par 11(*b*)
Designation to be used to identify film	Par 14(*a*)
Whether removal of rejected negatives from roll is desired	Par 14(*b*)
Maximum lengths of film rolls delivered, of other than as specified	Par. 14(*c*)
Number, if any, of copies of indices required and number of additional copies of reflown areas	Par. 14(*e*)
Kind, weight, finish of paper, scale and dimensions of trimming, masking if any, of ratioed prints required	Par. 15
Kind, weight, finish of paper, number of diameters or approximate scale, dimensions of trimming, masking if any, of general enlargements required	Par 16
Oblique photographs, if any, required	Par 17–21
Kind of film to be used for oblique photography if other than as specified	Par 20
Number of airplanes required to be used, furnish financial statement, competency of pilot and photographer .	Par. 22(*b*)
Whether Bureau of Standards report shall accompany bid	Par. 22(*c*)
Other samples if desired	Par. 22(*d*)(*e*)
Amount of bid guarantee or bid bond required	Par. 23
Amount of performance bond required	Par. 24
Commencement of work	Par. 26(*b*)
Order of photographing subprojects, if any	Par. 26(*c*)
Time allowance, undertaking reflights	Par 26(*d*)
Time allowance for shipment of negatives, contact prints, index sheets, and oblique photographs	Par 27(*a*)
Time allowance for shipment of ratioed prints and general enlargements	Par 27(*b*)
Maximum delivery area in any 30-calendar-day period	Par. 27(*c*)
Time allowance for notification that materials are ready for plant inspection	Par 27(*d*)
Rate of liquidated damages, including rates on different subprojects, if any	Par 32
Time allowance, reports	Par 34
Ownership of negatives .	Par. 35
Payments to contractors for separate subprojects, if any	Par 40(*d*)

4 Where any patent or patents are to be excepted from the operation of paragraph 37 of the specifications, such exceptions will be specifically stated, by reference to the patent number, date of issue, and name of patentee in a proviso to be added to the said paragraph 37.

5. The contracting officer should give careful consideration to the type and scale of photographs to be specified and should examine representative prints made at the contemplated contact scale showing terrain of the type to be photographed in order to determine their suitability for the intended purposes. In cases where vertical photographs are to be used by agencies desiring clear, sharp, cultural detail and freedom from excessive obliquity, as in studies of built-up areas, geological studies, timberland studies, and land-use projects, it is recommended that single-lens cameras be used, with reasonably narrow-angle and long-focal-length lenses (see specification par. 2(*a*)). In cases where wide-angle photographs would not cause objectionable loss of pictorial detail due to obliquity in the outer portions, they afford advantages from the standpoint of economy and accuracy of radial-line plotting. In cases where use of wide-angle lenses of shorter focal length than 8 inches is desired, or where multiple-lens photographs transformed into single composite negatives are desired, the contracting officer should so specify in the

schedule of the advertisement. Use of wide-angle, short-focal-length lenses may require some revision of the requirements as to quality of photographs which will be accepted, so far as definition in the corners is concerned (see specification par 13(*b*)), in which event the contracting officer should so specify in the schedule of the advertisement

6 It is recommended that the single-lens vertical negatives for use in connection with the proposed National Mapping Plan be made at a contact scale of 1 30,000 (1 in =2,500 ft), that the ratioed prints for the radial-line plotting of the planimetric maps be made at a scale of 1 20,000 (1 in =1,667 ft), and that general enlargements for the use of the land-planning and other similar economic agencies be made to two and one-half diameters (1 in =1,000 ft), or to a scale of 4 inches per mile (1 in =1,320 ft), as may be desired Aerial surveys made at scales smaller than 1 30,000 are of very little value for specialized land studies because of insufficient detail In agricultural and other special areas where considerable detail is desired, aerial photographs made with single-lens cameras of not less than 8-inch focal length at scales of either 1 20,000 or 1:15,840, with prints and enlargements for land studies made to scales of 1 15,840 and 1 7,920, respectively, are very satisfactory for the work of the land-use agencies and at the same time are well adapted to map compilation.

7 Items of definite-quantity materials which the Bureau or Agency, for which the work is to be performed, proposes to purchase should be listed separately in the schedule of the advertisement These items may be entered in such combinations of separate bids or lump-sum bids as may be desired As stated in paragraph 2 hereof, the schedule of the advertisement should include such additional prints of photographs, made from the definite-quantity negatives, as any agency of the Federal Government may need to purchase within 1 year after the date of contract. These indefinite-quantity items should be numbered consecutively after the item numbers of the definite-quantity materials and should be preceded by a statement similar to the following. "Bids are required on one set each of the following items of additional prints of the photographs specified in the preceding items, it being understood that orders for such quantities as may be necessary are to be placed if and when such items are required by any Federal agency during the period of 1 year after the date of the contract " There should be no time allowance or liquidated damage provision inserted in the schedule of the advertisement for such indefinite-quantity items as are not covered in the purchase order issued on acceptance of the bid, but delivery of such indefinite-quantity items, if and when ordered, should be made as soon as possible.

8. A copy of Standard Government Instructions to Bidders (No 22) should be attached to each invitation for bids

9. The schedule of the advertisement should be accompanied by a sufficient number of copies of a map or maps on which the location, dimensions, and boundaries of the area or areas to be photographed are outlined, to permit the purchasing officer to send one copy to each prospective bidder (see specification par. 1(*b*)). The notice of award to the successful bidder should be accompanied by three copies each (or more if required) of the best available map or maps of the area or areas to be photographed, for use by the contractor as flight maps. State maps published by the Geological Survey, the Post Office Department, or the General Land Office will be suitable for submission with the schedule of the advertisement, and Geological Survey topographic maps (if available) will be suitable for flight maps.

10. The notice of award to the successful bidder should be accompanied by notification of the designating symbols and initials of Bureau or Agency required by specification paragraph (14(*a*)) to be placed on the negatives, and also by a supply of the forms required for periodical reports showing progress of work (see specification par 34)

11. Any deviation from the specifications not authorized by these instructions shall be submitted to the Director of Procurement for prior approval.

It will be noted that Specifications A-APC-1101 consists of the basic fundamentals taken from the Federal Specifications of 1937 with new requirements added which were the result of the advance in science and experience in the next few years. The camera specifications, shown in Chapter III, were promulgated through sponsorship of the American Society of Photogrammetry whose Committee on Precision Cameras was under the chairmanship of Dr. Irwine C Gardner.

As mentioned before, in mapping with stereoscopic plotting instruments it is highly desirable from the standpoint of economy in extending ground control and facility of "bridging" over as many models as possible to specify within narrow limits the position in space of the exposure station for each photograph This can only be done in areas where accurate planimetric maps are already in existence and available for use by the flight crew. Also, when photography is obtained to be used in stereoscopic plotting equipment, the metric camera requirements are fixed to suit that equipment.

A recent specification for aerial photography prepared by the Tennessee Valley Authority, which is engaged in extensive mapping operations using multiplex equipment, included an area where accurate planimetric maps were available. A copy of the specifications is given below. It will be noted that photography meeting these requirements represents the ideal for the multiplex equipment. The portions of the specifications relating to administration are couched in terms which permit the contractee to secure the highest type product for the intended use In this type of contract, payment is made to the contractor for time the airplane is in flight, and also for stand-by time Consequently the contractor really furnishes services rather than a product. Under certain conditions this method of procuring aerial photographs may be preferable to the direct contract method previously illustrated

TENNESSEE VALLEY AUTHORITY SPECIFICATIONS FOR AERIAL PHOTOGRAPHY IN TENNESSEE RIVER WATERSHED AND OTHER AREAS AS MAY BE DESIGNATED BY THE AUTHORITY

Section 1 *Areas To Be Photographed.* Photography is to be executed over portions of the Tennessee River watershed, and other areas as may be designated by the Authority. The general topography of the areas to be photographed ranges from rolling land to high mountainous areas. In general, the rolling land will be photographed from an altitude of 5,900 feet or 11,800 feet above the mean ground level; the mountainous areas will be photographed from an altitude of approximately 18,700 feet above mean ground level. This will necessitate photographing certain areas at altitudes of approximately 24,000 feet above mean sea level. The areas to be photographed will be indicated on Flight Maps which will be furnished by the Authority.

Section 2. *Order of Work.* The order and sequence of photographic operations shall be as requested by the Authority. Every effort will be made by the Authority to provide an orderly, systematic schedule of operations favorable both to its own program of work and to the operating efficiency of the Contractor's equipment and personnel.

Section 3 *Project and Flight Maps.* Flight lines will be laid out by the Authority on recently compiled planimetric maps for all portions of the Tennessee Basin. These maps will be provided by the Authority at a scale of 1/24,000 or reduced copies at smaller scales if the contractor prefers. In areas where planimetric maps are not available flight lines will be laid out on some type of map suitable for flight maps

Section 4. *Bases of Flying Operations.* Bases of flying operations will be specified by the Authority to conform with the program of aerial photography. The airports

selected will be such as will provide suitable hangar accommodations and fueling service Bidders desiring information on airports which may be utilized should communicate with the Maps and Surveys Division of the Authority at Chattanooga, Tennessee

Section 5 *Field Assistance* The Authority will arrange, if practicable, to procure for the photographic crew, daily reports of weather conditions in the areas to be photographed These reports will usually consist of telephone calls or telegrams directed to the crew chief.

Section 6 *Reports* A report in quadruplicate covering each day's operations and/or flying time shall be prepared on printed forms which the Authority will furnish. This report must be prepared and delivered to the Authority's flight inspector within 24 hours after the operations have been completed.

EQUIPMENT AND PERSONNEL

Section 7 *Airplane.*

(A) The airplane for immediate use must be of a make and type suitable for performing the type of aerial photography described in the specifications. The airplane must be so constructed as to provide ample visibility ahead, below, and through the sides, and accommodate the present mounting for the Zeiss RMK P-10 camera without structural changes Drawing No. 111 attached to these specifications, shows the overall dimensions of the camera and mount which will be used for work with this airplane The ship furnished shall have a service ceiling with full operating load (crew, camera, extra film, oxygen, parachutes, etc) of not less than 24,000 feet. It must be equipped with a modern type sensitive altimeter and auxiliary instruments which will aid in maintaining stable flight while photography is in progress. The normal cruising speed at service ceiling shall not be less than 120 miles per hour.

(B) It is planned to add a second airplane, crew, camera and photographic equipment, if and when needed at any time after date of contract. On notice in writing from the Authority, a faster airplane shall be placed in service within ten (10) days. The airplane must be so constructed as to provide ample visibility ahead, below, and through the sides The airplane furnished shall have a service ceiling with full operating load (crew, camera, extra film, oxygen, parachutes, etc.) of not less than 24,000 feet It must be equipped with a modern type sensitive altimeter and auxiliary instruments which will aid in maintaining stable flight while photography is in progress The normal cruising speed at service ceiling shall be not less than 180 miles per hour.

Section 8. *Cameras to be Used.* A Zeiss RMK-P-10 wide-angle camera owned by the U. S. Geological Survey will be furnished by the Authority. Arrangements for mounting the camera shall be made by the contractor. The camera to be provided by the contractor shall be a wide-angle type with an approximate focal length of 5 2 inches, and which is adaptable to the uses of a multiplex machine. Calibration data from the U. S. Bureau of Standards on the particular lens to be provided must be filed with the Authority at the time bids are made.

Section 9. *Personnel*

(A) *Personnel of Plane for Immediate Use.*

The pilot must be well qualified, possessing a minimum of 1,000 hours of photographic map flying experience, at least 50 hours of which have been in the execution of aerial photograph work at altitudes in excess of 19,000 feet above ground level The photographer shall possess a minimum of 1,000 hours of experience representing actual time spent in executing aerial photography. Not less than 200 hours of this time shall have been spent operating a Zeiss RMK-P-10 camera which the Authority proposes to furnish for this contract. Time spent by the aerial photographer must have been gained on full-time photographic assignment, oblique photography not being considered as qualifying experience

(B) *Personnel of Second Plane, if and when Needed.*

The pilot must be well qualified, possessing a minimum of 1,000 hours of photographic map flying experience, at least 50 hours of which have been in the execution of aerial photograph work at altitudes in excess of 19,000 feet above ground level. The photographer shall possess a minimum of 1,000 hours of experience representing actual time spent in executing aerial photography Not less than 100 hours of this time shall have been spent operating a similar type camera to that which the contractor proposes to furnish. Time spent by the aerial photographer must have been gained on full-time photographic assignment, oblique photography not being considered as qualifying experience

(C) No replacements whatsoever in equipment or personnel shall be made by the contractor during the term of the contract without express consent in writing by the Tennessee Valley Authority

Section 10 *Crew Chief.* The contractor shall designate one member of the crew (either pilot or photographer) as crew chief, and shall so notify the Authority. The crew chief shall act as the contractor's authorized representative in all matters pertaining to the execution of work under this contract All instructions, flight maps and weather reports which are to be furnished by the Authority will be delivered to the crew chief. He shall be expected to consult freely with the representatives of the Authority with the view of planning the most orderly and expedient methods of pursuing the work to be executed under this contract.

Section 11. *Maintenance of Operating Efficiency.*

(A) The contractor shall be required to fully maintain all flying and photographic equipment to avoid delay or lost time due to faulty mechanical equipment or similar failures.

(B) Absence of either the pilot or the photographer or both, or the inability of the crew to operate due to faulty mechanical equipment or lack of film or other supplies on any one photographic day will result in discontinuance of stand-by charges, such cancellation continuing by calender days until the next photographic day becomes available.

(C) Removal of the airplane or photographic equipment, or absence of necessary personnel, from the job during active prosecution of the work without express consent of the Authority shall be reason for cancellation of the contract.

(D) The contractor shall maintain at the base of operations a sufficient supply of aerial film, oxygen for high altitude flying, and all other incidentals necessary to continuous and uninterrupted photographic operations.

(E) Failure of the contractor to carry out these provisions or other evidence of neglect as regards mechanical efficiency, performance, or maintenance of supplies, shall be reason for cancellation of the contract.

Aerial Photography

Section 12. ***Acceptable Photography.*** In accordance with the description of work under Items 3 and 6, "acceptable" exposures shall consist only of those which are selected as effecting coverage and as meeting the specifications which follow. The rejection of any negative or negatives will also necessitate the rejection of the remaining negatives in that particular flight which fall within the limits of the $7\frac{1}{2}$-minute quadrangle affected.

Section 13. ***Aerial Film.*** A fine-grained emulsion of type and manufacture approved by the Authority shall be used for the aerial negatives, the base or carrier for the emulsion must be of low-shrinkage film, also of a type satisfactory to the Authority.

Section 14. ***Development and Drying of Film.***

(A) The aerial negatives secured under this contract will be used in the construction

of topographic maps by means of stereoscopic measurements made on the photographs so every effort must be taken to avoid stretching or otherwise deforming the negatives Special care shall be exercised to insure the proper development and the thorough fixation and washing of all film and to avoid rolling film tightly on drums or in any way distorting it during processing and drying All film must be developed within two weeks of its exposure date.

(B) The images on the aerial negatives will be subject to great reductions and enlargements in the mapping operation, so great care must be taken to secure negatives of the maximum sharpness and fineness of grain As the quality of the aerial negative has a great effect on its value for stereophotogrammetric use, the contractor shall confer with the Authority to determine the negative quality most satisfactory for the intended use

(C) The bidder shall submit with his bid two overlapping photograph negatives taken on the type of film he proposes to furnish. These negatives shall have been exposed and developed by the same personnel the contractor proposes to assign to this contract. The equipment used in developing and drying shall be of the type the contractor proposes to furnish.

Section 15 *Scale of Photographs* The Authority will specify the flight altitudes above mean sea level at which exposures are to be made These altitudes will be determined by the map compilation specifications and the topographic nature of the area to be photographed The flight altitude will be determined by adding 5,900, 11,800 or 18,700 feet as appropriate to the mean ground level. For this reason, it will be necessary to change the flight altitudes when the mean ground elevation changes sufficiently to endanger the proper overlap of the aerial photographs The proper flight altitude will be stated on each flight map. The departure from the specified flight altitude shall not exceed 2% lower nor 6% higher than the specified altitude, and flights made at lower or higher altitudes may be considered unsatisfactory and may be rejected The pilot must make proper allowance in his altimeter reading to compensate for the effect of differences in temperature on the ground and at the flight altitude The authority will provide accurate elevations of the airports that are used.

Section 16. *Flight Lines* The flight maps furnished the contractor will show flight lines to be followed, the desired altitude of flight above mean sea level and the camera to be used Departures not exceeding 250 feet from the indicated line will be allowed for flights at an altitude of 5,900 feet, 500 feet for flights at 11,800 feet, and 800 feet for flights at 18,700 feet above mean ground elevation respectively Particular care shall be exercised to keep all flight lines as straight and as nearly parallel as possible

Section 17. *Overlap.* Overlaps in the line of flight shall be such that the distance between exposure points (the air base) shall be as shown on the flight maps to be furnished by the Authority. The amount of departure in air base length which will be allowed will be indicated on each flight map. These amounts will vary from ±300 feet for photography at 5,900 feet above ground, ±500 feet for photography taken at 11,800 feet above ground, and 1,000 feet for photography taken at 18,700 feet above ground level.

Any air base exceeding the limits given above shall be considered sufficient grounds for rejecting all the photographs in the flight strips in which they occur The sidelaps will be governed by the permissible departures from the flight lines as stated in paragraph 18. Each flight must be planned so that the first exposure shall be within 500 feet immediately preceding the beginning limits of the area to be photographed and the last exposures shall be within 500 feet beyond the ending limits of the area Adjacent flights shall be planned so as to have all exposures immediately adjacent to respective exposures in the preceding flight.

Section 18. *Crabbing.* The effect of crabbing is very detrimental to the use of the photographs for use in stereoscopic mapping instruments. Any series of two or more

consecutive photographs crabbed in excess of 10° as measured from the mean flight path of the airplane, as indicated by the principal points of the consecutive photographs, may be considered cause for rejection of the flight line within the 7½-minute quadrangle in which it occurs.

Section 19. *Tilt* Particular care shall be exercised to reduce tilt to a minimum, as large tilts destroy the usefulness of the negatives for their intended purpose The average tilt for photographs in a single 7½-minute quadrangle shall not exceed one degree and a maximum tilt shall not exceed three degrees

Section 20 *Quality of Photographs* Negatives which are not clear and sharp in detail and of average and uniform density, and not free from clouds and cloud shadows, light streaks, snow, static marks, and other blemishes which in the opinion of the Authority would interfere with their intended purpose, may be rejected

Section 21 *Numbering the Negatives* The contractor will not be required to number the individual photographic exposure Identification numbers will be placed on the film by the Authority

Section 22. *Identification of Film Rolls* The metal container for each roll of film which becomes the property of the Authority shall be carefully labeled with identification data which shall be typed or carefully printed with ink. This information shall include roll number, commencing with "1" and continuing in an unbroken series, the flight line descriptions, the quadrangle number designated by the Authority, the date or dates of photography, the aerial number and focal length of the lens and an accurate count of the negatives in the container.

Section 23 *Delivery of Film* All cost of transportation, and all risk of loss on film, shall be borne by the contractor until final delivery of the developed film to the Authority, f o b Chattanooga, Tennessee.

Section 24 *Photographic Day* A photographic day shall be any day ***not*** excluding Sundays and holidays, containing a period of time of not less than two consecutive hours between the hours of 9 a m. and 3 p m , during which the sky is cloudless at the airport at which the crew is based

For topographic mapping, by use of stereoscopic plotting equipment over areas where accurate planimetric maps are not available for use by the flight crew, the specifications relating to the position of individual photographs must necessarily be broader than those given immediately above In such cases it is customary to use the original Federal Specifications accompanied by such modifying clauses as are necessary to make the photography suitable for use in the particular equipment. As an example, the Geological Survey of the U. S. Department of the Interior uses specifications similar to the following.

UNITED STATES GEOLOGICAL SURVEY
STANDARD SPECIFICATIONS FOR AERIAL PHOTOGRAPHY

(For use in connection with U S. Standard Form 31 or 33)

For furnishing single lens, vertical photographic negatives, contact prints, photoindexes at a scale of 1:125,000 and copy negatives of the same. The regions to be photographed are indicated on the ten (10) attached maps and are approximately 7103.82 square miles in extent. The area under each item, the project designation and the names of quadrangles involved are as listed below.

The Standard Specification for aerial photography for general map work and land studies shall apply to this Schedule of Advertisement except where specifically modified. Paragraph 27, Time Allowance, and paragraph 32, Liquidated Damages, of the Standard Specifications shall not be applicable to this schedule due to conditions beyond the control of the successful bidder at this time. Many of the modifications specified in this

schedule are necessary to the successful use of the aerial negatives with Geological Survey multiplex mapping instruments

The contractor shall regulate his camera exposure and develop the negatives in such manner as will yield the maximum of detail but avoid all extreme highlights or dense shadows, as these are detrimental to the intended use of the negatives

The contractor shall not make, nor allow to be made, any enlargements from these negatives The negatives shall be delivered with the contact prints but will be returned to the successful bidder for use should it be decided to purchase additional contact prints or other of the items listed in the Schedule of Indefinite Quantity Items in this contract

The photographs shall be made with a calibrated, wide-angle single, lens aerial camera of the precision type having an image size and lens characteristics such that the aerial photographs can be satisfactorily used with the Geological Survey's multiplex mapping instruments The Geological Survey shall determine whether or not a bidder's camera is suitable for this purpose and shall be guided in this respect by the sample negatives that are submitted and a report made by the National Bureau of Standards to which further reference will be made. The cameras that can be employed are restricted to those having the following characteristics

Equivalent focal length (m.m)	Angle of Field of Lens	Size of Negative Image (e.ms)	Width of Film (e m.)	Minimum Resolving Power (Lines per m m.)	Maximum Distortion* (m m)
131 ± 2.5	93°	23×23	24	10	$\pm 0\ 30$

Altitudes at which negatives shall be made will be indicated for each flight line on maps furnished the successful bidder. The approximate altitude of these flight lines will be 11,800 feet above mean ground level for projects AP, AR and AT, and 10,400 feet for projects AM, AN, AO, AQ, AS, AU and AV

Each negative shall be numbered in such manner that the date of exposure shall appear in the southwest corner and the project letters in the northwest corner, outside of the useful image area The film roll number and the consecutive negative number shall be stamped within the useful image area close to, but not obscuring the west index mark.

The photo indexes shall be delivered at the approximate scale of 1:125,000 for each of 15′ quadrangle areas. They shall be printed on single-weight glossy paper and mounted on cloth 9×10 inches in size. (See Spec. Par. 14(*c*)).

Contact prints shall be on double-weight semi-matte paper for items 4, 6 and 8, and on double-weight low uniform shrink paper for items 1, 2, 3, 5, 7, 9 and 10

The contact prints used in making the photo-index maps shall be so trimmed that all the dark area of each print outside the useful image area is trimmed away. The contact prints delivered with the other materials required by this contract must not be trimmed.

Flight lines shall be in an easterly or westerly direction, except for a part of project AP, and will be indicated to the successful bidder on maps furnished him on notification of award, but will otherwise conform with Spec. Par. 4(*a*). Each flight line shall be continuous across each subdivision of the project, except that breaks in flight lines shall be allowed providing photography is duplicated at each break so that each $7\frac{1}{2}'$ quadrangle is covered by an unbroken flight extending two exposures beyond its boundaries.

Overlaps in the line of flight shall be approximately 62 percent, and overlaps less than 55 percent or more than 66 percent shall be sufficient grounds for rejecting all the photographs in the strip in which they occur. The percentage overlaps in the line of

* Distortion indicated refers to the equivalent focal length and the diaphragm aperture recommended by the manufacturer of the lens.

flight refer to the overlap as measured in the plane of mean elevation adopted for the various quadrangles. Strict adherence by the contractor to the flight lines furnished will result in proper sidelap being attained Deviation from these flight lines shall be not more than 1,000 feet from due east-west lines at the latitude shown on the flight map to be furnished the successful bidder Otherwise Spec Par 6 shall be conformed with.

All island areas within the project shall be stereoscopically covered

The film used on this project shall be fresh, fine-grained, high speed, panchromatic aerial film on a nitrate base unless written permission is obtained from the contracting officer to substitute other film

The bidders shall supply the following

a Make and type of airplane which bidder proposes to use, including complete description and evidence of performance and suitability for use on this project.
b Two overlapping negatives made with the camera he proposes to use from an altitude of at least 10,000 feet above mean ground level. These negatives shall be used to determine the quality of the stereoscopic impression secured when they are projected in the normal manner through the multiplex mapping instruments Negatives that are not sufficiently sharp or are lacking in angular coverage as determined from this test will be considered sufficient evidence for the rejection of the bid The principal distance, the shutter speed and the stop opening employed must be supplied
c. A report of the National Bureau of Standards shall be provided for each camera to aid the Geological Survey in determining the suitability of the camera for use with stereophotogrammetric mapping instruments now on hand, The report shall include a lens test, a metrical test of the camera as a mapping instrument, and a glassplate negative showing the principal point of the camera and the four fiducial marks.

1. *Scope of the Report*—The report shall be based on tests and measurements made after final assembly of all parts of the camera-cone unit, with the light filter to be employed in place of the lens. The report shall be based on tests made just before undertaking the photographic contract or proof be supplied that the adjustments of the camera have not been altered since the last test.

The report shall supply the following information:

a. The equivalent focal length of the lens.
b The distortion, referred to the equivalent focal length, from the lens axis to the edge of the field, by 5° intervals.
c The calibrated focal length.
d. The distortion, referred to the calibrated focal length, from the lens axis to the edge of the field, by 5° intervals.
e. The radial and tangential resolving power of the lens from 0° to the edge of the field, by 5° intervals
f. A precise measurement of the distance from the vertex of the last surface of the lens to the focal-plane frame.
g. A measurement of the angle between lines joining opposite fiducial marks.
h A photographic plate exposed in the camera showing the relation of the lines joining opposite fiducial marks to the principal point. This negative shall be marked with the date of test, the test number, the camera and lens numbers, and the points where the plate contacted three removable stops used to record the relation of the plate to the focal plane of the camera.
i. Measurements of the distance between opposite index marks shown on the photographic plate mentioned in paragraph 1-h with a probable error not exceeding 0.01 millimeter.

2. *Constructional Details Necessary to Permit Testing.*—To permit the necessary tests for the determination of the calibrated focal length, distortion, resolution, and position of the principal point without the introduction of errors resulting from film shrinkage the camera shall be constructed in accordance with the following requirements.

a. It shall be possible to insert a photographic plate between the frame that determines the position of the focal plane and the platen that normally flattens the film in the focal plane.
b The focal plane shall be accessible from the rear so that a telescope placed behind the camera may view objects through the camera lens at an angle limited only by the size of the focal-plane opening The removal of parts to secure this vision shall not include the removal of the collimation index markers or of the frame that determines the location of the focal plane
c Provision shall be made for holding the shutter leaves open during the test period.
d The name and records of experience of the pilot and photographer the bidder proposes to use
e Method of developing and drying film the bidder proposes to use It is essential that this method be one that will avoid stretching of the film in drying, as this will destroy the value of the negatives for the intended use

The performance bond shall be 50% of the contract price (See Spec Par 24)
F.O.B. Washington, D. C.

In this particular example all applicable provisions of the Federal Specifications governed, except when modified by the clauses in the above specifications It will be noted that the contractor is required to furnish the camera, together with complete metric data regarding its optical and mechanical characteristics

An example of specifications for oblique aerial photography is contained in paragraphs 17, 18, 19, 20, and 21 of the Federal Specifications. As indicated therein, a coverage map showing the location from which each photograph is to be taken and showing the direction in which the camera is to be pointed should accompany those specifications. This coverage plan is usually prepared to meet the requirements of the individual project and the equipment which will be used in obtaining map data from the photograph

There are certain elements common to all of the foregoing examples, and which are generally essential to any complete specification These elements may be analyzed as follows.

Technical Requirements

1. *Area to be photographed*—This includes a definition of the project boundaries, together with a statement of required coverage outside the boundaries. It includes the requirements regarding coverage in the case of lakes or other water areas within the project area

2. *Scale of the photographs*—The scale of the photographs is a product of three factors; ground elevation of the general area, focal length of the camera lens, and flying height of the airplane. Scale may be fixed as a given ratio, i e, 1:20,000, or may be fixed by establishing the flying height and prescribing the focal length of lens. A plus or minus tolerance should be set forth.

3. *Flight lines.*—Except for special-purpose photography, either north-south or east-west flight lines are commonly required Special control flights may be specified, if needed. The positioning of the flight lines with respect to project boundaries should either be stated or indicated on maps. Sidelap between flights should be specified and endlap requirements both with respect to

individual pictures in a flight and with respect to breaks in flight lines should be stated The orientation of pictures in a flight is usually covered by either a statement regarding permissible "crabbing" or an "alignment" requirement

4 *Tilt*—Requirements regarding freedom from tilt commonly specify the maximum allowable for any one exposure, the maximum average allowable for a section of a flight line—for example, 10 miles, the maximum average allowable for the whole project, and the maximum allowable relative tilt between any two successive exposures

5 *Permissible time for obtaining photographs*—To avoid pictures containing objectional shadows, the photographic period during the day is usually specified either in terms of standard time, or in terms of the sun's angle above the horizon. As an example, it is common practice to limit the photographic hours to those when the sun is more than three hours above the horizon

It is common practice not to take aerial photographs when streams are flooded beyond their normal banks Except for certain purposes, in some of the far-north areas, photographs taken when snow is on the ground are not satisfactory Specifications may require that the photographs be obtained when trees have lost their foliage.

6. *Camera and lens requirements*—Except in the case of certain special-purpose photography, cameras and lenses permitted for use should conform to the requirements for precision cameras, so that the photography will be suitable for use in optical mapping instruments

7. *Film*—Film base of the highest grade, carrying a suitable emulsion is desirable Shrinkage characeristics of the base should be limited, with the attendant conditions of temperature, humidity, and processing fully defined. Safeguards to be observed in processing, developing, washing, drying, and packing the film are commonly specified Freedom from deleterious residual chemicals, mechanical injury, and defects of image is necessary.

The permissible density and contrast of the photographic images on the film are limited. The usual criteria for these qualifications consist of judging test prints, enlargements, or diapositives made from the negatives

8 *Paper*—Qualifications regarding shrinkage characteristics of paper due to processing and drying are specified Otherwise papers are usually classed as "water-resistant," "standard commercial grade, double-weight," and "single-weight " The type of finish should be specified, such as "glossy," "semi-matte," etc , and the thickness, "single-weight," or "double-weight," should be specified.

9. *Indexing and editing*—Requirements for editing and indexing individual exposures, and film rolls, are made to be consistent with the general film library systems established by large users of photographs. Photo-index sheets are usually required to permit the rapid and easy selection of photographs covering any given area. If they are not required, line indexes will suffice for some purposes. The system in use by most governmental agencies is such that no two exposures have the same index symbol and number.

10. *Special conditions*—These fall outside of the technical sphere. It may be said that they are intended to assure that the work will be accomplished with a minimum of delay, and that photographic crews will be continuously attempting to secure the photographs until the project has been satisfactorily covered. Under this category also fall the provisions regarding delivery of materials, inspection of material by the contracting officer, payments for work completed, and adjudication of disputes. The necessary legal provisions also come under this heading.

As pointed out previously, in the logical sequence of events, the preparation

of specifications must be accomplished immediately after the ultimate use for photographs has been determined The engineering requirements attendant to this use will govern the elements which became integral parts of the specifications. Once these are known final, satisfactory and adequate specifications can be prepared for any project, either of the types cited as illustrations herein or for special types, by using as a guide the outline of the ten elements given and selecting appropriate clauses, phraseology, and language from either of the four examples

The best record we can have of our basic national resource, the land, is good aerial photographs To preserve this record is of utmost, though often unrecognized importance. If those who have occasion to prepare specifications for aerial photographs will incorporate in them provisions which will insure that the photographs will have broadest possible utility for a multiplicity of uses, that the film will retain high photographic quality and resist chemical and physical deterioration for a maximum period of time, a very worthwhile accomplishment will have been attained

CHAPTER V

PHOTOGRAPHIC MATERIALS AND LABORATORY TECHNIQUE

CHARACTERISTICS OF PHOTOGRAPHIC MATERIALS

J L Tupper and Walter Clark

IN AN aerial photograph for photogrammetry, it is important to be able to recognize as much detail of the ground as possible, and to be able to use the photograph for accurate measurement There are several factors which influence the recognition of ground detail First of all, there are the characteristics of the ground and the objects on it, and the way they are illuminated by the natural daylight. The atmosphere between the camera in the aircraft and the ground influences this Then there is the movement of the aircraft itself Aerial photography differs from normal photography in one important respect, and that is that the camera is moving in relation to the object it is photographing. The characteristics of the lens and camera play an important role. Among the most important factors, however, are the nature of the photographic film, the way in which it is exposed, and the manner in which the negative is processed and printed. By proper use of the film, some of the factors which tend to decrease the effectiveness of visual observation of detail may be overcome. Contrasts may be exaggerated, for example, when they are visually lowered by atmospheric haze, or by the objects on the ground having colors and brightnesses which appear very much alike to the eye of the observer.

It is the purpose of this section of the chapter to describe the general structure and properties of photographic materials, and to deal in greater detail with those properties which make a material particularly suited for aerial photography.*

PHOTOGRAPHIC FILM

The film consists essentially of two parts· the sensitive layer, called the *emulsion*, which responds to light when the film is exposed in the camera, and in which the negative can be developed after exposure; and the *film base*, or *support*, which is the transparent, flexible material on which the emulsion is coated. The structure of film is shown in cross-section in Figure 1

The Emulsion. The emulsion, which responds to light when the film is exposed in the camera, has two essential parts One of these is gelatin, which is the medium which holds the sensitive material The other is the sensitive substance itself, which consists of compounds of silver in the form of separate minute crystals or grains, which are dispersed through the gelatin The silver compounds present in fast films for use in the camera are mainly silver bromide, the crystals of which contain a small proportion of silver iodide Other substances are present in small amounts to confer special properties, such as the ability to respond to colors, and to keep well when stored before being exposed

The function of exposure in the camera is to affect the crystals of silver bromide so that they will be changed to black metallic silver when the film is immersed in the solution called the *developer*. In Figure 2, there is shown a thin layer of emulsion photographed at high magnification in the microscope The left-hand photograph shows the silver bromide crystals before development, while the right-hand side shows the same crystals after development They are extremely small. In a film for use in the camera, the majority of them may be of the order of one ten thousandth to one twenty thousandth of an inch in width.

* In this chapter, specific products will be mentioned They are those with which the authors are most familiar. It is felt, however, that the loss of generalization in these instances will be compensated by the advantages to be gained from definite information. The general principles discussed should apply to other products of similar nature.

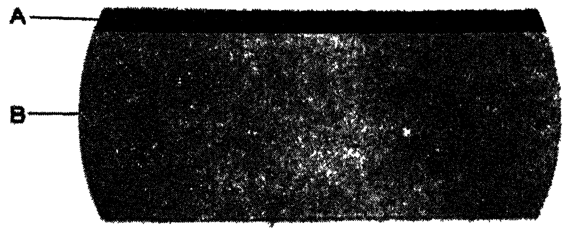
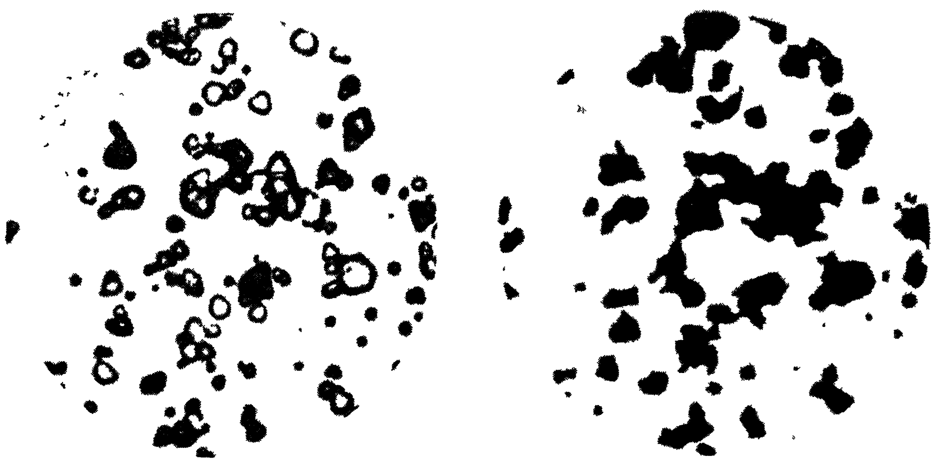


FIG 1 Cross section of film *A*, emulsion, *B*, support. In the case of film for aerial photography, the thickness is about 5/1000 inches

When a film is exposed to light, some of the crystals become affected in such a way that they will be changed to silver by the developer. This effect is not visible before development, and is known as the *latent image* The more the exposure, that is to say, the longer the time of exposure or the higher the intensity of the light, the greater is the proportion of the crystals which become developable As a result of this change of the crystals to black silver, the negative takes on its characteristic black appearance after it is developed, and the higher the exposure, the blacker is the negative. In any normal negative, the film has



A *B*

FIG. 2. Undeveloped (*A*) and developed (*B*) crystals in a photographic film.

been exposed to a subject which varies in brightness from one part to another, so that the exposure varies over the negative On development, therefore, the different parts of the negative have different degrees of blackness, or opacity. The brighter parts of the subject appear blacker in the negative, and the darker parts appear lighter There is thus in the negative an inversion of the tone values of the subject. That is why it is called a negative.

Density. The degree of darkening of the film on development is expressed as a number which is called the *density*. The higher the density, the darker is the film The density is evaluated on the basis of the proportion of light which the negative will let through. It is strictly defined as the logarithm of the opacity. If only one tenth of the light passes through, the opacity is said to be ten If only one hundredth passes, the opacity is one hundred The corresponding densities, which are the logarithms of the opacities, are therefore, one and two.

Density,

$$D = \log_{10} O = \log_{10} \frac{I_0}{I_t}$$

where I_t is the intensity of the transmitted light, and I_0 is the intensity of the incident light.

Characteristic Curve. The relationship between exposure, development, and the corresponding negative is shown by the so-called *characteristic curve*, sometimes called the H. and D curve, after the two English scientists, Hurter and Driffield, who first used. it. In the characteristic curve, densities are plotted against the logarithms of the exposures which resulted in them after development. A typical characteristic curve is shown in Figure 3

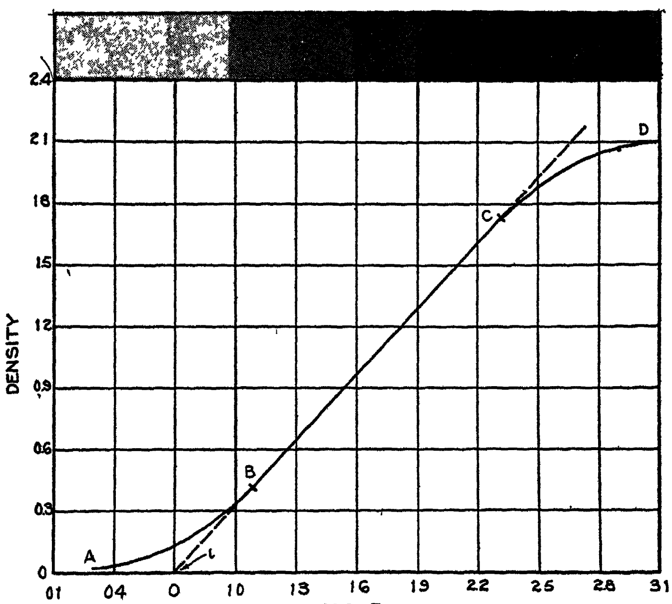


FIG. 3. Typical characteristic curve of a photographic emulsion.

The science of the measurement of the response of photographic materials to light is known as *sensitometry*, and the backbone of sensitometry is the characteristic curve. From it can be derived the speed and contrast of a film, the way in which its properties depend on the conditions of development, the fog value, and the way in which it will reproduce the tones of the subject photographed It should be possible to draw a characteristic curve by making a photograph of a scene of some kind, measuring the densities of various parts of the negative, measuring the brightness values of the corresponding parts of the subject, and relating the exposure values, in terms of exposure time and light intensity, to the density. It is very difficult in practice, however, to measure the brightnesses of the detailed parts of the scene Sensitometry is therefore done in the laboratory, and the exposure to light is done in a *sensitometer*, in which a series of known exposures increasing in known increments is given. The resulting negative resembles that in the upper part of Figure 3.

The densities of the steps in the negative are measured in a type of photom-

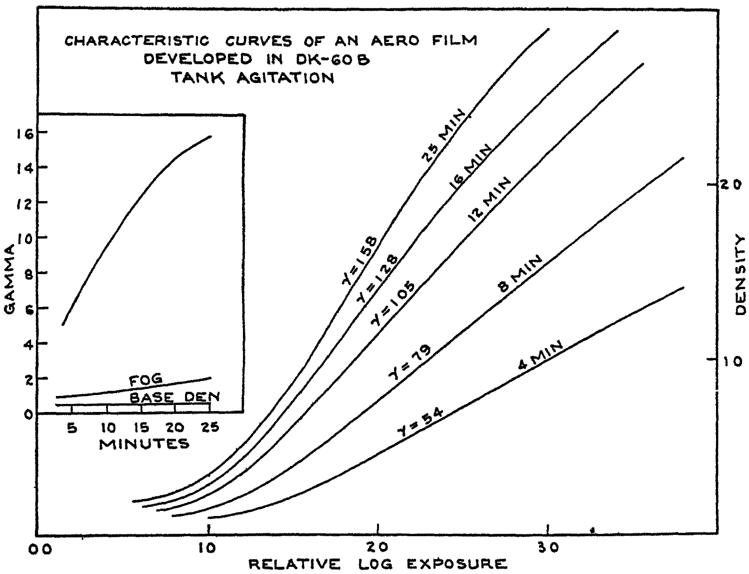


FIG. 4. Characteristic curves of an aerial film for different developing times. Also shown are curves relating developing time to gamma and fog, and the value of the film base density

eter known as a *densitometer*. If they are plotted against the logarithms of the exposures which produced them, the characteristic curve results Exposures, E, are given in terms of the product of the intensity, I, of the illumination on the film being tested, and the time, t of the exposure, usually in meter-candle-seconds *i e.*, $E = It$.

The characteristic curves of all photographic films and papers have the same general S-shape shown in Figure 3, although the form of the curve will vary with the material and the way in which it is developed. The lower part of the curve, AB, is concave upwards, and is the region of under-exposure. The top part of the curve, CD, is concave downwards, and is the region of over-exposure. Between the two is a part, BC, which is more or less straight, and which is called

the region of correct exposure, although in many cases for correct exposure the lower part of the curve may be also included.

Gamma. The slope of the straight-line part, or the tangent of the angle which its prolongation makes with the exposure axis, is known as *gamma* (γ) It is used as a measure of the contrast of the photographic material. The steeper the curve, *i.e* , the higher the gamma, the greater is the contrast of the negative. Gamma is also a measure of the extent of development of the material As the time of development is prolonged, the slope of the curve increases, as is shown in Figure 4, which gives the characteristic curves of a film for a number of different times of development. There is a limit, however, to the increase in gamma with development time. After a certain time, there is no more useful increase, as is shown in Figure 5, which is a typical gamma-time of development curve for a fast film for aerial photography. The shape of this curve, and the maximum value of gamma attainable, depends on the type of film and the nature of the developer

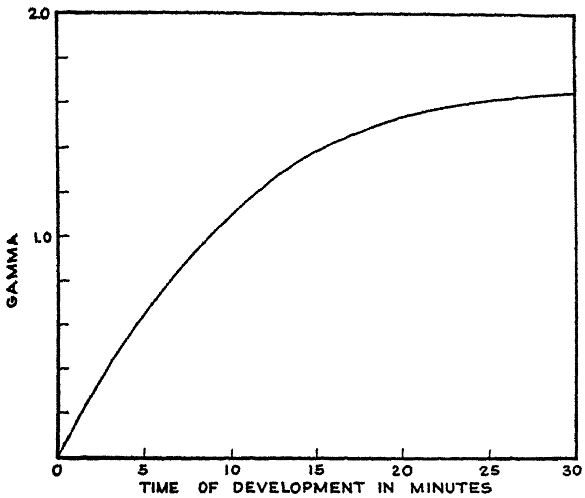


FIG 5. Typical curve relating gamma to time of development.

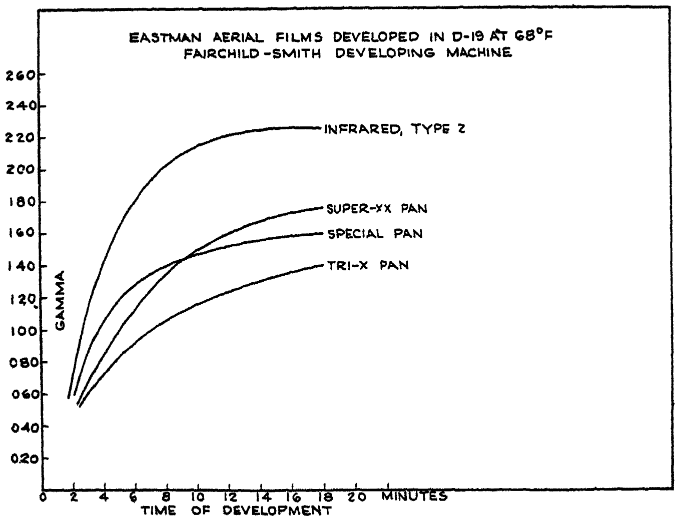


FIG. 6. Gamma-time of development curves for four types of aero film.

Figure 6 shows the gamma-time of development curves for four types of aero film developed identically in a common type of machine Figure 7 shows the

characteristic curves for the same four films developed for the same time, 12 minutes, in the same developer and machine It will be clear from these that the characteristics of a film vary considerably with its nature and the extent to which it is developed If a different developer had been used, the curves would probably have all been somewhat different in shape from those shown.

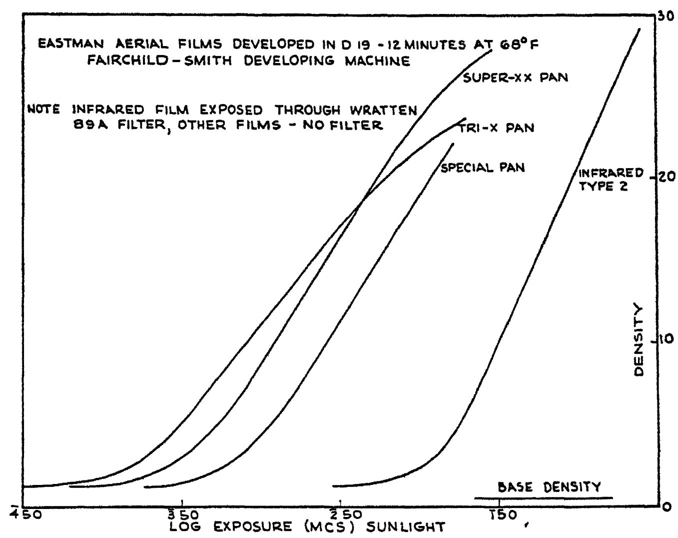


FIG. 7. Characteristic curves for the same films as in Figure 6, developed for one time.

The contrast of a film as indicated by its gamma is of great importance. In order to produce negatives from which excellent prints can be made, the negative density range should not be greater than the useful log exposure scale of the positive material on which the negative is to be printed In the case of a scene which has a very high brightness range, such as a combination of dark rock and snow, or for low altitude obliques with contrasty lighting, it may be necessary to make a negative having a low value of gamma (0.60 to 0.80). If the subject brightness range has been lowered by the veiling brightness of atmospheric haze, it may be necessary to compensate for this effect photographically by making a negative of high gamma, either by increasing development or by choosing a film having inherently high contrast Thus, a gamma value of about 0 80 to 0 90 is correct for low altitude verticals (2000 to 3000 feet), which on the average, have an image illumination scale of 34:1; but at 10,000 feet, where the average illumination scale is reduced to 8:1, and at 20,000 feet, where this ratio becomes 5:1 or less, negatives having gammas of 1 00 to 1.20 will give more useful photographs. Still higher gammas may be necessary when exposures are made under difficult conditions of heavy haze or poor light

The approximate developing times required to produce these gamma values are normally supplied by the film manufacturer for the various films and recommended developers. Similar data are also distributed by the photographic sections of the armed services. Sensitometric control of the gamma of aerial negatives is complicated by the necessity of placing the sensitometric exposure within the useful picture area of the roll. Anomalous development effects at the ends of the roll do not permit the placement of control exposures in these areas.

In aerial photographic practice, there has been a tendency to standardize on one type of film for the bulk of mapping purposes, and only to use other special films in less frequent cases where the subject has particular properties. There is also a tendency to use a limited number of developers. Not more than three should be required, one for low, one for medium and one for high contrast. In civilian mapping in peace time it is possible to select the fine days for photographic flying, and in general it is not necessary to provide for as many varying conditions as in wartime.

It is most important that aerial films for mapping purposes should be well exposed, for it is only by so doing that proper recording of all possible detail is ensured They should not be too heavily exposed, however, for this would increase the time required in making prints, and it is desirable to keep the camera exposure to the minimum to eliminate as much as possible the blurring due to movement of the aircraft.

Speed. The exposure of a film is based on what is called its *speed.* Many ways of measuring speed have been proposed, so that from a single number the photographer could tell what lens opening and shutter exposure time would be required for a certain type of film and lighting condition. The American Standards Association has recently agreed on a method of designating speeds for amateur negative films, and the principles of this method also apply to the measurement of the speeds of films for aerial photography. The speed is derived from the characteristic curve, and is based on the exposure corresponding to a certain gradient in the lower part of the curve. The actual point taken for amateur negative films is that at which the slope of the curve is 0.3 of the average gradient over a log exposure range of 1.5. For films intended for aerial photography, this fraction is somewhat higher. From these speed values, so-called *film exposure index* values have been derived for use with the American Standards Association Emergency Standard Photographic Exposure Computer. These values fall between the exposure ratings supplied for the Weston and General Electric exposure meters which are in common use. For the films whose characteristics are given in the illustrations, they are as follows:

TABLE I

Special Aero Panchromatic film	32
Super-XX Aero Panchromatic film	100
Tri-X Aero Panchromatic film	200
Infrared Aero film, Type 2	24*
Infrared Aero film, Type 2, Hyper	48*

* Through red filter.

Various other methods for designating speeds have been proposed in the past, and may be encountered by the photographer. The method proposed by Hurter and Driffield, and giving H. and D. speeds is based on the *inertia*, which is the point on the log E axis at which it is cut by the prolongation of the straight line part of the characteristic curve (see i, in Figure 3). The actual H. and D. speed was obtained by dividing the exposure at this point into 34. Scheiner speeds are based on the exposure corresponding to the first visible trace of image. The DIN speeds, standardized in Germany, are based on a density of 0 1 above the fog. Weston meter ratings, S, common in the United States, were originally given by

$$S = \frac{\log_{10} E_w}{4}$$

where $\text{Log}_{10}E_w$ is determined from the characteristic curve, and is chosen to be

numerically equal to the gamma General Electric meter ratings were derived in a similar manner More recently Weston and General Electric meter ratings have been determined by the sensitometric methods agreed upon by the American Standards Association.

Fog When a film is developed, a more or less uniform density may be produced over parts of the film which have not been exposed This is known as *fog.* It is usually of a low order of density, and may be due to inherent properties of the film itself, or to the nature of the development Film manufacturers generally keep the inherent fog to a low value, although it may be somewhat higher in the very fast films as compared with the slow films. Fog increases with the extent of development, and some developers, especially those which are very active, tend

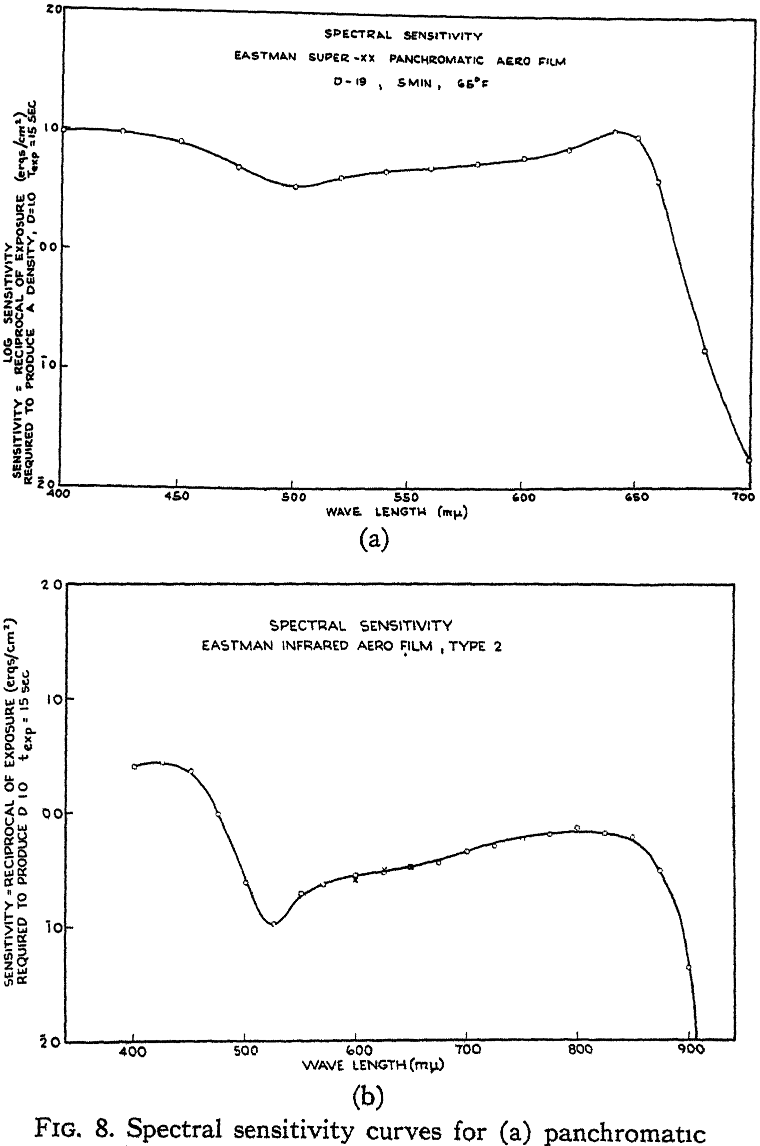


FIG. 8. Spectral sensitivity curves for (a) panchromatic and (b) infrared sensitive films for aerial photography.

to give enhanced values of fog In practice, it is best to operate so that the fog value is low In deriving quantitative data from the characteristic curve, it is customary to deduct the fog values from the densities being considered. The film base itself has a small density value, and this is usually included with that of fog in making corrections for fog.

Spectral Sensitivity. When a photographic emulsion is first made, it responds only to ultraviolet, violet and blue light. This means that if it were used unmodified, objects having colors other than those mentioned would not be recorded. For instance, in the print made from the negative, reds and greens would be black and indistinguishable. Moreover, as will be seen later, it would not be possible to use filters in making the photographs Films for aerial photography are therefore sensitized to colors other than blue There are two general types of sensitizing, known as *orthochromatic* and *panchromatic*. In the former, the film will respond to green light as well as blue, while in the latter it will respond also to red, *i e*, to all spectral colors. Many films for amateur photography are orthochromatic, and such films have been made in the past for aerial photography, but it is usual practice nowadays to use panchromatic film for aerial work. The color response is conferred on films by incorporating certain dyes during the manufacture of the emulsions, the dyes being selected so that among other things they absorb light of the wave lengths to which it is required to desensitize the emulsion

The spectral sensitivity of films is measured by determining their sensitivities at various wave lengths through the spectrum, and plotting the *spectral sensitivity curve* The true spectral sensitivity, independent of the spectral characteristics of the light source, daylight in the case of aerial photography, is given in the equal energy spectral sensitivity curve, examples of which for two films for aerial photography, are shown in Figure 8

If the true response to a light source of known spectral energy distribution is required, a curve is drawn in which the spectral sensitivity ordinates are multiplied at each wave length by the energy at corresponding wave lengths in the source. A common method of exhibiting such curves is in the so-called *wedge spectrogram*, which is obtained by photographing a spectrum of the light source on the particular film, using a graded exposure along the length of the slit of the spectrograph Examples of wedge spectrograms for four emulsion types are given in Figure 9.

Graininess To the eye, a developed negative appears to be homogeneous, but if it is examined under a moderate magnification, it will be seen to consist of clumps of black silver particles which give it a grainy appearance. This is known as *granularity*, and it gives the impression of *graininess*. It is of much importance in aerial photography, for it limits the magnification at which the photograph may be examined without an unpleasant structure becoming apparent Such a structure interferes with the ability to recognize detail in the subject photographed.

Graininess depends on a variety of factors It is dependent on the type of emulsion, being in general more apparent in the case of those of high speed. It varies with the developer, and, in fact, developers are known in which the effect of graininess can be markedly reduced This reduction is, however, accompanied by a reduction in the effective speed of the film. It increases with development time, or gamma, and depends on the density of the negative, and the conditions under which it is developed.

Sharpness. Another factor which determines the ability to recognize detail in a photograph is the *sharpness*. If the sharp edge of a bright object is photo-

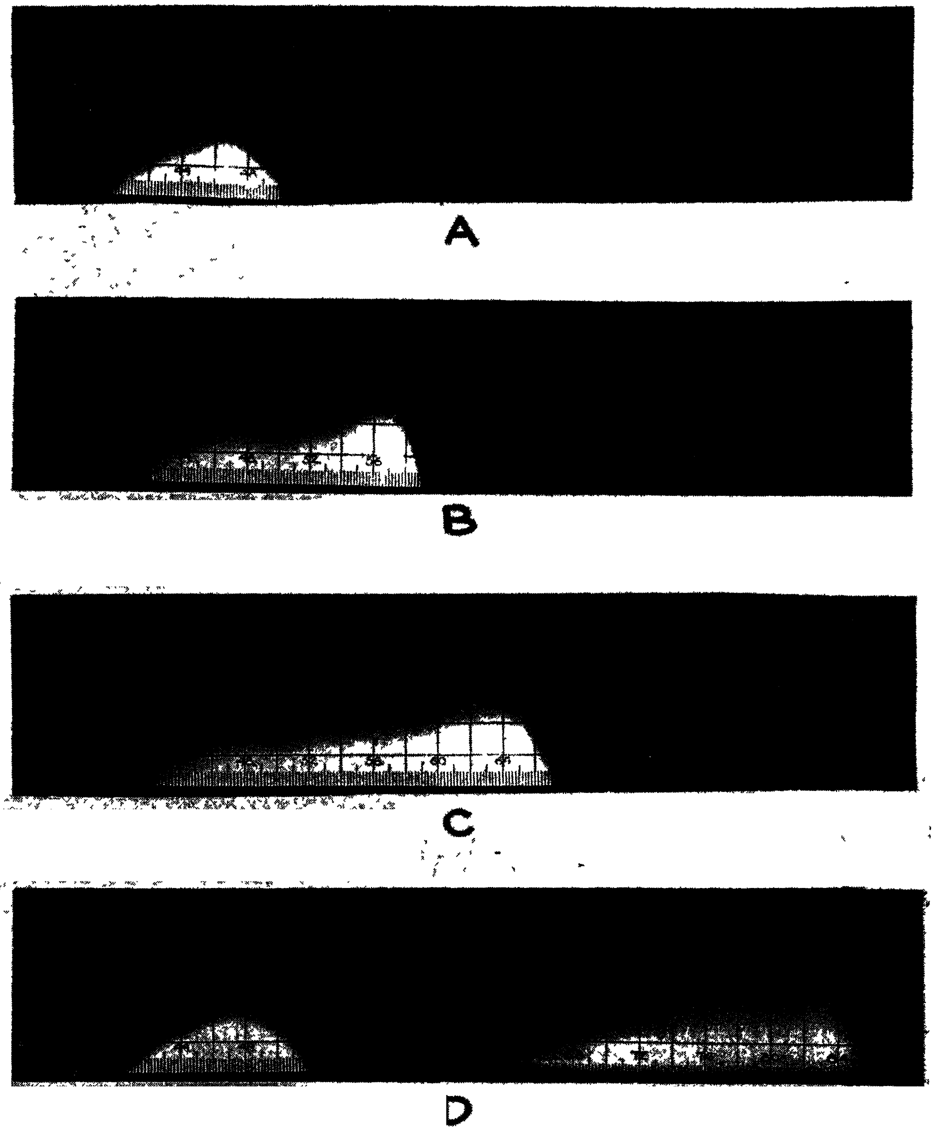


FIG. 9. Wedge spectrograms for four types of emulsion: A, ordinary; B, orthochromatic; C, panchromatic; D, infrared.

graphed against a dark background, the edge will not appear sharp in the negative, but will have a density gradient which is due to scattering of light in the emulsion near the edge being photographed. Its extent depends on the type of emulsion, in particular on its *turbidity* and *opacity*, on the wave length of the light, and on the developing conditions.

Resolving Power. A very important property of a film for photogrammetry is its ability to separate fine details. It has already been mentioned that graininess and sharpness play a part in this. There is a property of an emulsion known as

its *resolving power*, which takes into account the effect of both granularity and sharpness on the reproduction of the fine structure of a camera image, and which is usually measured as the number of lines per millimeter which can be separated by an emulsion Because of lens aberrations, line images formed by photographic objectives are surrounded by a halo of light, its character and extent depend upon the design of the lens The presence of this halo has less effect on visual resolving power than on the photographic resolving power Maximum photographic resolving power is obtained with a lens having a minimum amount of halo-forming aberrations. The following table shows the values of photographic resolving power obtained with such a lens for four of the films considered above

TABLE II

Film	Resolving Power Lines per mm.
Tri-X Aero Panchromatic film	40
Super-XX Aero Panchromatic film	50
Special Aero Panchromatic film	55
Infrared Aero film	45

(These values are for the common developer and development times, and for a low contrast test object.)

The value of resolving power depends on the contrast of the test object used in measuring it, increasing with the contrast; on the wave length of the light, being somewhat higher in the blue end of the spectrum; on the nature of the developer, although there seems to be no general rule about this; and on the development time, rising rapidly to reach a maximum as development time increases.

EFFECTIVE RESOLVING POWER OF LENS-FILM COMBINATION IN AERIAL CAMERAS

In an aerial camera, a film may be used having a rated resolving power of fifty lines per millimeter, and a lens capable of higher visual resolving power, and yet it may be found that the effective resolving power of the combination may be no higher than twelve lines per millimeter. This condition exists because the halo-forming aberrations of aerial camera lenses are generally much greater than those of the lens used in measuring the rated resolving power of the film and because the visual resolving power is not indicative of the extent and distribution of the halo. Emulsion turbidity causes the photographic image to spread, which tends to fill in the spaces between the image lines. Exposure produced by the halo adds to this filling-in and if it is of sufficient magnitude the lines may not be distinguishable in the photographic reproduction.

Effective resolving power may be further reduced by the use of optically imperfect filters and glass pressure plates and by plane motion and vibration.

Filters. All aerial photographs are made through filters, which are colored plates of glass (or dyed gelatin between glass) placed over the camera lens during exposure. The filters are generally yellow or red in color. The function of the yellow filters is to absorb blue and violet light, so that the photograph is made by green and red, while that of the red filters is to absorb blue, violet and green light, so confining the exposure to the long wave lengths of the visible spectrum.

In infrared photography, deep red filters are employed to confine the exposure to the infrared.

Filters serve two purposes The prime object is to penetrate atmospheric haze, which tends to flatten the contrast of the ground detail as seen from the air. This haze preferentially scatters short wave length light, especially the violet and blue These are the colors to which film is most sensitive, so that a photograph made without a means for eliminating their effect will appear flat, as if it were taken through a luminous veil The longer wave length light will more readily penetrate the haze, so that if the violet and blue are absorbed, by using a yellow filter, the negative will show the ground details much more clearly. Still better haze penetration is obtained if the photograph is made by red light, using a red filter, and the most effective penetration is produced by using film sensitive to infrared, and confining the exposure to the infrared by means of a deep red filter

It is frequently stated that the use of infrared sensitive film will permit the penetration of fog. This is not so, however, for the denser the atmospheric murk or the larger the size of the suspended particles, the less is the chance of penetrating it The infrared does, however, permit of a great increase in the rendering of detail on very hazy days, especially over long distances in oblique photography Infrared photographs distort the tone values of objects, particularly grass and foliage, which appears white, so that experience is required in the interpretation.

Since filters cut out some of the light which would reach the film if they were not on the lens, they require an increase in exposure This is denoted by the *filter factor*, which is the increase in exposure required when a filter is used

The filter factors for two aerial films in daylight for a number of the commonly used filters are given in the following table.

TABLE III. FOR SUPER-XX AND SPECIAL AERO PANCHROMATIC FILMS

Filter	Filter Factor
Wratten Aero No 1	1 5
Wratten Aero No 2	2 0
Wratten No. 12	2 0
Wratten No 15	2 5
Wratten No 25	4.0

In addition to permitting clearer rendition of ground detail by haze penetration, filters are used to modify the reproduction of the tones of the objects on the ground With a panchromatic film, a yellow filter will give a rendering of the relative tones of objects which closely resemble the tone values perceived by the eye. By proper selection of filters, however, it is possible to distort the tone values so that objects may be more readily distinguished. For instance, by using a green filter, scrub against a sandy background is more readily distinguished than if a yellow or a red filter is used.

Filters should be selected so that they do not increase the exposure of the film so much that the effects of ground motion or the aberrations of the lens destroy the ground detail being sought.

Proper selection of filters to enhance contrasts is specially important in geological and forest survey photographs. In these instances, it may even be desirable to resort to infrared photography. In general, conifers appear darker in an infrared photograph, and dead trees always appear darker, than do deciduous trees Infrared photography has been used to detect camouflage. Many green paints absorb infrared strongly, and therefore appear dark in an infrared picture, while green leaves and grass which the paint is intended to match visually ap-

pear light in tone by infrared. Naturally, if paints are selected to have the same spectral characteristics as natural chlorophyll in the infrared, they cannot be detected by infrared photography.

Characteristics of Film Support. Since aerial photographs for photogrammetry are intended for accurate measurement, it is important that they undergo the minimum of dimensional distortion during handling; or at least, if they are distorted, the change should be of a variety which can be easily corrected.

Photographic films are made on two types of support—*cellulose nitrate,* and *cellulose acetate* or *safety support.* Each of these is made in two forms—*topographic* and *nontopographic* The topographic film support is so prepared that the dimensional changes which it undergoes in developing or storing are reduced to a minimum, and the change is approximately equal in the length and width of the film. Most of the film used at the present time for mapping is of the safety topographic variety

There are two kinds of dimensional change which can take place in a film support. One of these is an irreversible shrinkage, which is due to the loss of minute amounts of volatile materials which remain in the film at the time of manufacture. The loss of these naturally results in a permanent and slight contraction of the film, since they cannot be replaced. In making topographic support, however, these are eliminated so that the irreversible shrinkage of this type is reduced to the minimum The second type of dimensional change is reversible, and is due to gain and loss of moisture

An interesting analogy with this type of reversible dimensional change is provided by a piece of wood When it is soaked in water it swells, but not equally in all directions. It shows a much greater swell across the grain than along it Ordinary film shows a somewhat similar behavior, but for topographic purposes it is necessary to reduce this effect to a minimum, and this is done in making topographic film support It shows a minimum of difference in behavior in the two directions.

In order to determine the suitability of a given type of film for aero mapping, several tests have been devised which apparently give a satisfactory indication as to how well the film will behave in actual use.[1]

The first of these tests is a Special Development Shrinkage Test which involves cutting strips of film both lengthwise and crosswise from the sheet, conditioning them to 50% relative humidity, processing, incubating at 125°F for one week, reconditioning at 50% relative humidity, and measuring This test measures both the amount of residual solvent and the residual strains left in the film. In actual practice, approximately four dozen strips are taken, half in one direction, and half in the other direction, from a wide strip of film in order to obtain accurate averages as well as deviations from the mean

Another test, devised to measure the degree of uniaxialism or the freedom from orientation in the lengthwise direction of the particles of cellulose ester, has been called the Swell and Shrink Amplitude Test. In this test, pieces of film cut similarly to those for the Special Development Shrinkage Test, are alternately wetted in water and dried in an oven, and the wet and dry dimensions taken. The difference between the wet and dry dimensions, expressed in per cent, is called the Swell and Shrink Amplitude, and if this value is the same in the two directions, the film is essentially uniaxial.

A further test which gives results similar to the above is called the Humidity Amplitude Test, and is made in the same way except that the dry samples are

[1] Carver, E. K. Properties of Safety Topographic Aero Film. Photogrammetric Engineering, 1938, *4,* 223–225.

brought to equilibrium with air at 20% relative humidity and the wet samples are brought to equilibrium with the air at 80% relative humidity and similar measurements taken.

The following table gives typical values for these three tests for ordinary nitrate aero film, Topographic Nitrate Aero film, ordinary acetate film, and Safety Topographic Aero film It will be observed that the safety aero film suffers in comparison with the best Topographic Nitrate Aero film only in one respect, viz., in the Special Development Shrinkage Test. Other tests run at room temperature indicate that this effect with safety aero film does not occur at room temperature and that, in ordinary use, we may expect the safety film to shrink no more than nitrate aero film.

TABLE IV

	Regular Aero Nitrate	Topographic Aero Nitrate	Ordinary Acetate	Safety Topographic Aero
Special Development Shrinkage				
Lengthwise	15%	11 %	55%	172%
Crosswise	21	135	70	200
Difference	06	025	15	028
Swell and Shrink Amplitude				
Lengthwise	50	62	1 55	55
Crosswise	70	64	1 75	57
Difference	20	.02	20	02
Humidity Amplitude				
Lengthwise	35	38	90	38
Crosswise	54	40	1 05	40
Difference	19	02	15	02

Glass Plates In the early days of aerial photography glass plates were used rather than film. They have the drawbacks of weight and fragility and difficulty in changing and exposing a long series in the aerial camera. For precision photogrammetry, however, such as by the Brock process, photographic plates are sometimes used in the aerial camera to ensure dimensional stability of the negative. The photographic characteristics of available plate emulsions in general are better suited to pictorial photography than aerial photography, but by proper development satisfactory results can be obtained.

Image Distortion The photographic emulsion itself, being made mostly of gelatin, undergoes changes in dimensions according as it is wetted or dried, and as it is subjected to the action of acids, alkalies and salts in processing. When it is coated on a glass plate or a film support, however, its expansion on wetting is confined almost entirely to a direction at right angles to the plane of the support

Under some circumstances, there may be very slight displacements or dimensional changes in the image developed on the negative. This is usually insufficient to be of significance in photogrammetry. Sometimes astronomers put their plates through what they call a "normalizing process" in order to reduce this

POSITIVE MATERIALS

The characteristics of the photographic printing materials which are used in making positive reproductions from aerial negatives have an important part in

determining the quality of the final result. For maximum usefulness the print should represent the brightnesses and dimensions of ground details in a manner which permits their easy recognition For photogrammetry, accurate dimensional representation is more important than exact tone reproduction, yet the recognition of detail may be seriously impaired if proper attention is not given to tonal rendering

Photographic Papers By far the largest amount of printing is done on photographic paper The characteristic of a paper printing material differs from that of a negative primarily because it is made on an opaque paper support and is viewed by reflected light. Surface reflection imposes a limitation on the total range of tones obtainable in a paper print and the density range and exposure scale of printing papers is always less than that of negative materials. Printing papers are available in a wide range of contrasts and it is usually possible to select a paper grade such that good brightness reproduction is obtained over most of the useful tonal region. In printing from a negative having a density range appreciably greater than that which can be accommodated by the exposure scale of the paper, detail in the extreme tonal regions will be lost In such cases it may be necessary to make two prints, one light and one dark, to obtain the maximum information from the negative. The importance of controlling negative contrast to restrict the density range to the limits dictated by the exposure scales of available papers is therefore apparent. This scale will vary with the contrast and density range of the paper; matte papers have less density range than glossy papers. Typical characteristic curves for glossy and matte papers are shown in Figure 10. In Figure 11 are shown curves for a series of glossy papers Indicated on these curves is the grade number which is here used to identify the contrast and exposure scale characteristics of the papers. Descriptive terms, such as "soft," "medium," and "hard" are also used for the same purpose. The characteristic curves of papers are analogous to those of films, with the exception that the densities are "reflection densities," measured by reflected rather than transmitted light.

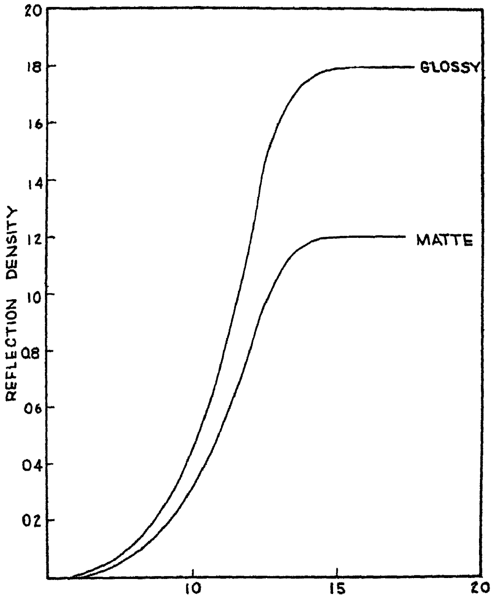


FIG. 10 Typical characteristic curves for glossy and matte papers.

Recently papers have been manufactured in which contrast can be varied by the use of filters in making the print. By this method, reasonable control of the effective curve shape is possible and a single paper can thereby be made to serve the purpose of several different grades. Papers of various speeds are also available The slow (chloride) papers are used for contact printing; the faster (bromide) papers, for projection printing; medium speed papers may be used for either contact or projection printing. The choice of surface texture and

paper thickness depends upon the application Single-weight, glossy prints are commonly used for making mosaics The double-weight, semi-matte papers are almost exclusively used in mapping

Characteristics of Paper Support Ordinary paper, which is made of a matting of cellulose fibers coated continuously in a web, undergoes dimensional changes when wet As in the case of film support, the expansion across the width may be greater than that in the length, *i e*, the direction of coating

When a wet sheet of paper is dried down, it shrinks back, and may even become smaller than it was originally This contraction of the paper is very much influenced by the method of drying For instance, if the paper is dried in a manner which tends to hold it and prevent it from shrinking, such as on a ferrotype sheet or a tight belt on a belt dryer, it will tend to stay wide and long If it is

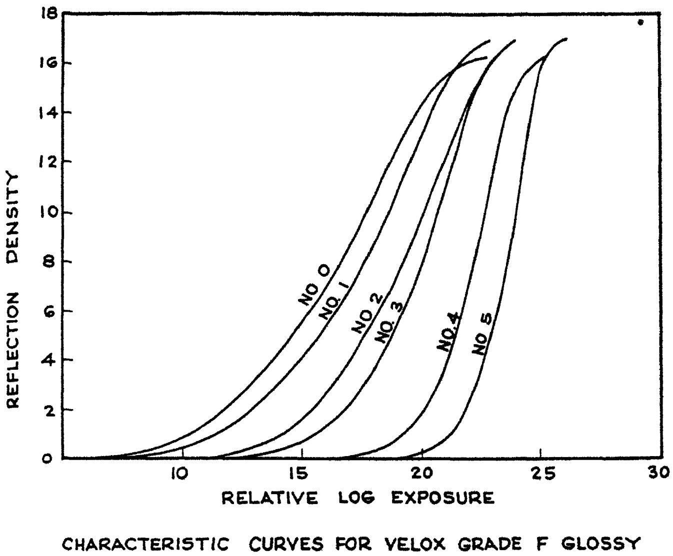


FIG 11 Characteristic curves for a series of six grades of glossy papers for contact printing

dried too hot, it will tend to greater contraction than if it is dried at lower temperature. In the case of very large sheets, particularly if they are dried by hanging, the actual weight of the paper may tend to expand it.

Since the dimensional changes undergone by paper are primarily due to the wetting of the cellulose fibers, improvement will obviously result if this wetting is reduced This is done in the case of some special papers, for instance, the so-called Aero Mapping papers, by waterproofing the paper itself According to some measurements made at the Bureau of Standards and published in 1938[2] and dealing with waterproof papers available at that time, the shrinkages were of the same order as for aero films. It is not possible to give figures which can be used quantitatively for various kinds of papers and all conditions at the present time. A rough idea may be obtained, however, from the following figures for one variety of paper: Single weight waterproof paper, 7 minutes complete

[2] Davis, R., Stovall, E J., and Pope, C. I. Dimensional Changes in Aerial Photographic Films and Papers. Photogrammetric Engineering, 1938, *4*, 195–197.

processing time, drying in free air, -0.25% lengthwise and -0.30% crosswise. Humidity changes in the air will also produce notable changes in the dimensions of finished prints. In the case of a single weight waterproof paper, over the range of 40%–80% relative humidity, this change may be of the order of 0.10% lengthwise and 0 20% crosswise, for each 10% change in humidity It is clearly desirable to control humidity for the most accurate work if paper prints are to be used for measurement.

Other devices than waterproofing have been proposed for reducing the changes in dimension of paper. One of these consists in laminating the paper to metal sheets. For the bulk of the work, however, the waterproof paper support has been found satisfactory.

The problems of a photographic raw paper manufacturer are not simple. He has to make a paper which will resist the action of strong alkalies and reducing agents in the developer and acids in the fixing bath, and which will remain strong enough to be handled easily even though saturated with water. At the same time, it must be completely inert towards the photographic emulsion coated on it.

Positive Plates and Films (Diapositives) When the positive is viewed by transmitted light or is projected onto the viewing surface, the limitations of density range and exposure scale associated with photographic papers is eliminated. Contrast can be controlled by development and consequently optimum reproduction quality is possible by proper adjustment of the processing factors Positive transparencies are capable of giving results of extremely high quality The photographic emulsions which are used for this purpose are normally fine grained and have relatively high inherent contrast. They are generally of the type used for making lantern slides. Diapositive plates find their principal application in the multiplex projector and in the Brock process. Diapositive films at present are not widely used in photogrammetry. Photographic plates are also used in copying maps and mosaics in order to obtain a reproduction free from dimensional distortion A blue sensitive emulsion of moderate contrast is generally preferred since the subject, being black and white, presents no color rendering problem. Similar emulsions on film support may also be used if slight dimensional changes are permissible.

Color Photography

The possibilities of color photography in photogrammetry have not yet been fully explored, largely because it was only during the present war that color films for aerial photography were made available. They are now made in rolls in the same manner as ordinary aerial film, they have adequate speed for aerial photography, and they can be processed in the equipment made for developing black-and-white film.

It would seem that color photographs, particularly in the form of positive transparencies, should have certain definite advantages Since they reproduce objects in the colors in which they appear to the eye, they should assist in the recognition of ground objects by virtue of the color contrasts Objects of different colors may appear of equal shades of gray in an ordinary photograph, whereas in a color photograph they should be distinguishable. It is not suggested that they will replace black-and-white film for all aerial photography, since interpreters have developed an extraordinary faculty for recognizing objects in this normal medium, particularly in the stereoscope. They might, however, well serve as a useful adjunct, particularly in such fields as forest survey and tree typing.

Storage of Photographic Materials

Unexposed photographic films, plates and papers are not particularly stable things They tend to undergo deterioration on keeping, and proper precautions should be taken in storing them The deterioration is accelerated by heat and moisture, and serious damage may result by exposure to various gases, X-rays or radium Nitrate films are dangerous if overheated or exposed to an open flame.

In correct storage, protection against heat should be provided by using refrigerated rooms if available Whenever possible, films should be kept below 60°F , and papers not above 70°F The ideal film storage temperature for periods up to one year is 50°F Sensitive photographic materials should never be placed near steampipes or other sources of heat, or on the top floor of uninsulated buildings. A hangar with iron roof exposed to the sun is one of the worst places in which to keep cans of film, for it provides an oven in which the film can be baked and rapidly spoiled. If refrigerated space is limited, preference should be given to high-speed films During transport, film cases should be kept out of direct sunlight In the field, if other cooled space is not available, film can be placed in properly sealed boxes and buried underground in a hole

In manufacture, photographic materials are usually packed under proper conditions of humidity, and if the package is of the right kind, *i e* , vaportight, the protection is usually adequate until the package is opened. Materials in other containers must be stored in dry places, never in damp basements, on damp ground, or in other humid locations The ideal relative humidity is between 40% and 60%.

Vaportight packages provide protection against harmful gases. Materials in ordinary packages may be spoiled by high concentrations of formaldehyde, hydrogen sulfide, illuminating gas, ammonia, mercury vapor, industrial gases, war gases, exhaust from motors, and vapors from solvents, cleaners and turpentine.

If sensitive materials must be stored near X-ray machines or radium, the storage cabinets should be lined with an adequate thickness of lead

Films marked NITRATE should be stored separately in fireproof vaults protected by sprinklers. These facilities should be in accordance with the regulations of National Fire Protection Association, 60 Batterymarch Street, Boston, Mass , or Army Regulations 850-65, *Storage and Handling of Nitrocellulose Film in U. S. Army Establishments* Other films do not require fireproof storage.

In using film from stock, the oldest should be issued first. If the film is in cold storage, it should be removed 24 hours before use to avoid condensation of moisture on the cold film when it is opened Individual packages of sheet film and paper should be placed on edge to avoid pressure marks.

In order to reduce deterioration of processed negatives and prints in storage, they should have been thoroughly fixed and washed, and they should be kept in a cool, dry atmosphere For short periods of storage, it is satisfactory to use a temperature of about 70°F. and a relative humidity of 50% to 70%. For so-called archival storage,[3] a temperature of about 50°F and a relative humidity of about 50% are satisfactory In the case of safety film there is a tendency to lose flexibility at low relative humidities, and it may be necessary to restore flexibility by conditioning the film at 70% relative humidity before use

[3] Scribner, B. W Summary Report of Research at the National Bureau of Standards on the Stability and Preservation of Records on Photographic Film. National Bureau of Standards Miscellaneous Publication M162 U S Government Printing Office, Washington; 1939.

PHOTOGRAPHIC LABORATORY EQUIPMENT AND TECHNIQUE

Reynold E Ask

ABSTRACT

The fiist section of this chapter was concerned with the characteristics of photographic materials The purpose of this section is to discuss the special equipment and technique used in processing aerial films and prints For a more detailed presentation of general photographic procedure the reader should consult the bibliography at the end of this chapter

CONTENTS

I PROCESSING OF AERIAL FILMS

A. General Theory

WHEN the minute grains of silver halide in an emulsion are exposed to light, an invisible latent image is formed which is converted to a visible image of black metallic silver when the film is placed in a developing solution Those grains which were not exposed to light are not affected by the developer and are still light sensitive The process of fixation (described below) removes the unexposed grains and makes the remaining silver image permanent

The composition of developing solutions* vary greatly, but most formulas contain the following essential constituents developing agent, preservative, accelerator and restrainer. The *developing agent* is a chemical reducer which converts the exposed silver halide grains into metallic silver, but does not affect the unexposed grains Only a few organic compounds have this power of selective reduction, the ones most used being metol, hydroquinone, and pyro A *preservative* (such as sodium sulfite) is needed to prevent the developing agent from reacting with the oxygen of the air. This oxidation of the developing agent not only lowers the developing power of the solution, but also causes the formation of dark colored compounds which may stain the emulsion. The term *accelerator* is given to the alkali which is added to increase the activity of the developing solution In the absence of an alkali the solution is practically inactive Strong alkalis such as sodium and potassium hydroxides give a fast-working high contrast developer, while the weak alkalis such as borax give a slow working low contrast developer. The addition of the alkali may make the solution so active that it will reduce some of the unexposed silver haloid grains, thereby forming a fog over the entire film. This fogging tendency is greatly reduced by the addition of a small amount of a *restrainer*, such as potassium bromide.

* See list of formulas at end of chapter, page 250.

The longer a film is left in a developer the denser the image becomes, also there is an increase in the contrast By *contrast* we mean the *difference in density* between the lightest (shadows) and darkest (highlights) portions of the negative. As previously mentioned in the first section of this chapter an increase in contrast means an increase in the slope (gamma) of the characteristic curve (see page 211). However, if development is carried too far, the developer may begin to act on the unexposed silver haloid grains, thereby causing developer fog over the entire negative. Other processing factors affecting the density of the developed image are: the temperature of the developing solution (65° to 70°F. recommended), the amount of agitation during development, and the degree of exhaustion of the developing solution. In order to take into consideration all of the above items, it is very desirable to run a test development on a single negative before proceeding with the entire roll. An examination of this test negative will tell whether a modification of the developing procedure will be necessary Recommended development time varies greatly, from about 3 minutes for the more active developers to 20 minutes for the slow fine grain types.

After removing the film from the developer it is desirable to place it for a few seconds in an acid rinse bath * This bath neutralizes the alkaline developer thereby stopping development instantly, it also prolongs acidity of the fixing bath into which the film is next placed

The usual constituents of a fixing bath* are: sodium thiosulfate (hypo), sodium sulfite (a preservative), acetic acid, and potassium alum. The purpose of a fixing bath is: (a) To remove the unexposed silver halide grains which are still light sensitive (the hypo in the bath converts these grains into soluble salts which can be washed from the emulsion); (b) to stop the action of any remaining developer (the acid in the bath neutralizes the alkaline developer); (c) to harden the emulsion (the action of the developer leaves the emulsion very soft and fragile, the alum in the bath hardens and toughens this emulsion thereby reducing the softening or swelling in the wash water, particularly during warm weather).

As a fixing bath ages it becomes slower in action. If the time it takes to clear a negative is twice that for a fresh bath the solution should be discarded. The usual time for fixing negatives ranges from 10 to 20 minutes.

Following fixation, the negative is washed in rapidly flowing water for at least 30 minutes. The purpose of this washing is to remove all the soluble silver salts created by the fixing bath, and also to remove all traces of hypo Any hypo remaining in the emulsion will cause fading or discoloration at a later date. Special indicating solutions may be used to check the thoroughness of hypo elimination.[1]

The last step in film processing is the drying operation. The special equipment and technique used in developing, fixing, washing and drying roll aerial film is discussed in the next section

B Equipment and Technique

Developing Outfit. The most satisfactory developing outfit for roll aerial film is the light weight portable unit shown in Fig. 1. The size illustrated accommodates film up to $9\frac{1}{2}$ inches in width and 250 feet in length. Each unit consists of three nesting tanks, reel assembly, loading plate, motor drive unit, and a carrying case The tank and reels are constructed of stainless steel to insure the best

* See list of formulas at end of chapter

[1] Evans, George H, "Testing Aerial Photographic Negatives for Residual Sodium Thiosulphate," Photogrammetric Engineering, Vol VIII (1942), page 121

possible resistance against the corrosive action of photographic chemicals. In using this equipment the following procedure is followed·

(1) Fasten the loading plate to a table top, place the exposed camera film spool on the plate and transfer the film to one of the reels.

(2) After the above transfer is completed, the outer end of the film is attached to the other reel and the knurled knobs twisted a few turns to check the film alignment It is obvious that the above operation, as well as those following, must be performed in total darkness.[2]

(3) Place the three nesting tanks side by side, the first tank containing the developing solution,* the second and smallest tank the wash water, and the third tank the fixing solution.* It is desirable that the tanks always be arranged in the same order to avoid placing the film in the wrong solution in the darkness The capacity of the tank illustrated is approximately 5 gallons

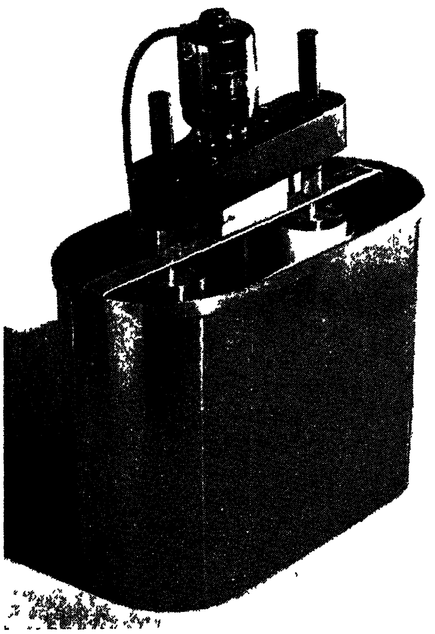


FIG. 1. Fairchild-Smith portable roll film developing outfit. (Courtesy Fairchild Camera and Instrument Corp)

(4) Check the film on the reel to see that it is wound snug but not tight, then lower the assembly into the wash tank. By using the knurled knobs the film is transferred from one reel to the other and then back. The purpose of this step is to wet the film, thus preventing sticking and uneven development later.

(5) Place the reel assembly in the developing tank and wind the film back and forth until it has been developed the time recommended for the formula used[3] In order to insure uniform development it is very important to keep the film moving at all times while in the solution. The rather tedious hand winding can be eliminated if a motor drive is available. This drive, which is placed on top of the reel assembly, is so constructed as to reverse the direction of rotation when the end of the film is reached. Before using the motor drive it is advisable to run the film through once by hand.

(6) At the completion of development remove the reel assembly from the solution and place it in the wash tank. The film is run through the water two or three times to remove surplus developer. In warm weather it is desirable to add a hardener* to this rinse bath to toughen the emulsion.

(7) Following the above rinse, place the reels in the tank containing the fixing bath for 10 to 20 minutes, depending on age of bath.

* See list of formulas at end of chapter

[2] A dark green Wratten series 3 safelight may be used if placed the proper distance away from the developing tank However, this is seldom used since the visual light transmitted is too low to aid the operator appreciably

[3] Run test development on single negative in a tray in order to determine the most desirable formula and development time.

(8) Transfer the reels to the washing tank A hose is inserted in the clip on the side of the tank for supplying running water. A thorough washing of at least 30 minutes is necessary to remove all traces of hypo from the emulsion Insufficient washing may result in a faded or discolored image at a later date.

Although very good results are obtained with the above equipment, the end negatives are often marked or distorted due to pressure against the reel core during the winding operation For this reason it is desirable to leave 2 or 3 feet of blank film at the ends of the roll, or to use the ends for test exposures

Drying of Roll Film. The last step in film processing is the drying operation. Illustrated in Fig 2 is a portable roll film drier widely used in aerial photo-

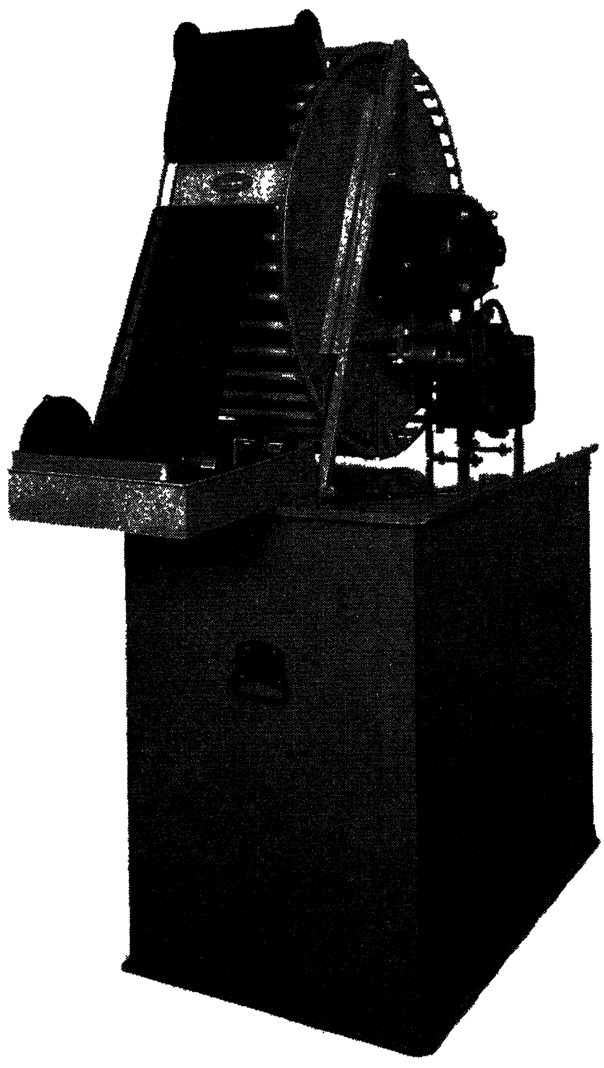


FIG 2. Smith portable roll film drier. Rate of film movement can be varied from ½ to 10 feet per minute.

graphic laboratories It consists of a cylindrical drum about 10 inches wide and 28 inches in diameter mounted on a base which also serves as a packing case. Located at the center of the drum is a motor driven centrifugal blower which forces air at high pressure through numerous slits on the periphery of the drum Also around the periphery of the drum (outside of the air slits) are a series of wood rollers attached to an endless sprocket chain and geared to the motor. After removal of the wet film from the washing tank the entire reel assembly is attached to the lower bracket of the drier. Threading of the film through the drier is accomplished by attaching a metal clip to its end and feeding it just in-

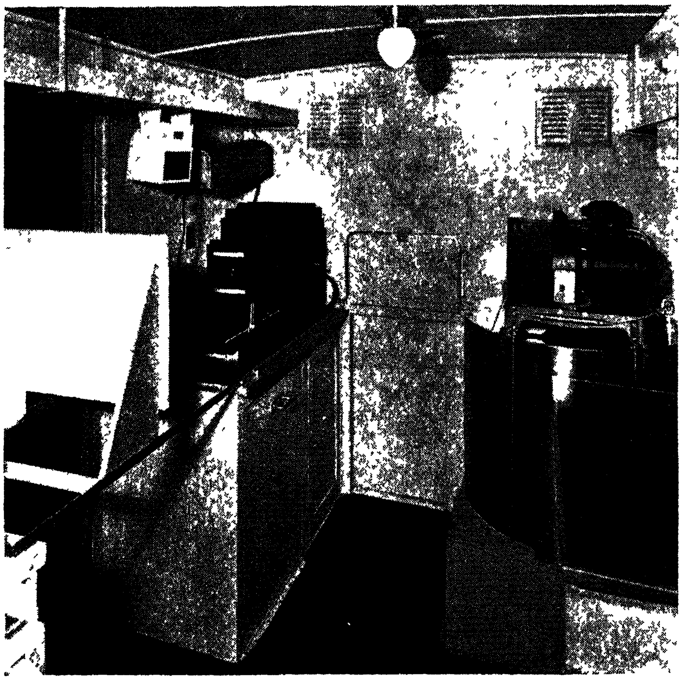


FIG 3. Interior view of mobile photographic laboratory. Water tank and refrigeration unit at right, and projection camera at left. Other end contains equipment for processing roll films and making contact prints. (Courtesy Fairchild Camera and Instrument Corp)

side the rollers, keeping the emulsion side toward the drum center. The force of the air jets pushes the film back against the rollers thereby avoiding contact with the delicate emulsion side. The rotation of the rollers carries the end of the film (which is now dry) around to the take-up spool to which it is then fastened. The driven take-up spool is provided with a friction clutch so as to wind the film without tension as fast as it dries The rate of film movement can be controlled by varying the speed of rotation of the rollers. Depending on humidity conditions the rate of drying varies from 2 to 5 feet per minute. For use in very humid regions a modification of the above drier has been developed. This type is completely enclosed and is so constructed that the moisture is removed from the air blast coming in contact with the film.

When the above type of equipment is not available, other methods of drying may be used One of these consists in hanging the edge of the film from long horizontal wires, using spring clothespins every foot or two Another method previously used was to wind the film spirally (emulsion out) on large reels constructed from buggy wheels and slats The ends of the film being attached to the reel by spring clips on elastic cords, thus reducing the tension on the film as it dries. When either of the above methods are used, some means must be provided for removing water drops Often used for this purpose are chamois skins or fine viscose sponges

Portable Field Laboratories Certain military activities require the processing of aerial films near the scene of action Several portable processing units have been developed for this purpose Shown in Fig 3 is an interior view of a mobile photographic laboratory which is mounted on a truck chassis The U. S. Army Air Forces have similar units mounted on trailers. The portable flight laboratory

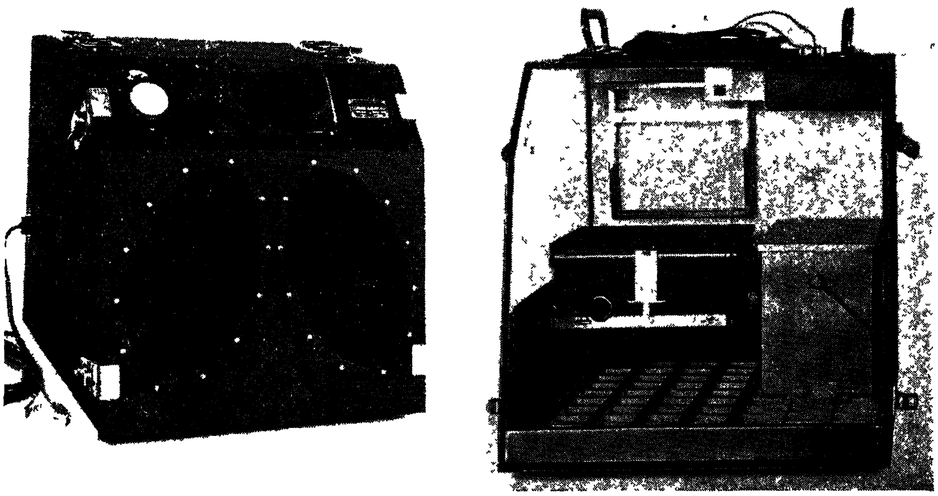


FIG 4. Portable flight laboratory. (a) Exterior view showing viewing window and arm holes. (b) Interior view showing processing tank and contact printer (Courtesy Fairchild Camera and Instrument Corp)

shown in Fig 4 was constructed for the U. S. Navy. It is used to develop sheet film and print photographs during actual flight in an aircraft. It is really a miniature self-contained darkroom.

C. *Inspection and Editing*

Inspection Tables. Inspection and editing of aerial negatives is best performed on equipment such as illustrated in Fig. 5. A rectangular opal glass illuminated by fluorescent tubes furnishes a diffuse light source, and convenient means are provided for mounting film spools at either end. Improvised inspection tables are sometimes made by attaching roll film holders to the usual "light-table" found in many drafting offices. One precaution in using such a set-up is that the light source be adequately ventilated to avoid excessive heating of the film. In an emergency the top of a contact printer may also be used for inspection work.

FIG. 5. Inspection of aerial negatives (Courtesy Agricultural Adjustment Agency).

Negative Editing. This operation is usually done with a rubber stamp, using a special stamp pad ink which adheres to the film base. The inspection surface of the above described tables should be long enough to permit several negatives to be viewed at once. Therefore, by the time the last negative is stamped the ink on the first one will be dry enough to permit winding the film on the take-up spool In order that the letters appear correct on the print, it is necessary that the rubber stamp impression be on the back of the film. Sometimes when hand lettering is resorted to, it is placed in reverse (right for left) on the emulsion side where the ink adheres better. Some organizations place the lettering on the negative in such a manner that it will appear on the forward edge of the photographic print. Other organizations prefer to have the lettering appear on the north edge of the photograph. The data placed on the negative often consists of the following date, project designation, scale, camera number, flight number, and photograph number. A few cameras are equipped with automatic recording devices which eliminates the need of hand stamping.

Negative Inspection.[4] The inspector should be able to recognize and report any visible defect on the completed negative caused either by conditions existing at the instant of exposure in the aircraft, or by faulty processing technique in the laboratory. The former will be discussed first and will be subdivided into: (1) atmospheric and ground conditions; (2) mechanical condition of the camera; (3) shutter and diaphragm settings

[4] Cultice, James M, "Method for Testing Photographic Quality of Aerial Negatives," PHOTOGRAMMETRIC ENGINEERING, Vol. VIII (1942), page 22.

Sometimes the aerial photographer has no choice in regards to atmospheric and ground conditions However, for peacetime mapping the optimum conditions are sought—that is, the following are avoided as much as possible; hazy atmosphere, clouds and cloud shadows, smoke due to brush fires, snow covered ground, ice-locked harbors, leaves on trees, and long shadows due to low altitude of sun.

Mechanical defects in the camera which affect the quality of the negative may be classified as improper film flattening, vibration of the camera in its mount, film static markings (sometimes occur in cameras using glass pressure plates), use of filters of poor optical quality, condensation of moisture on the filter or lens, camera lens not set at focal length which gives best average definition, obstruction of the field of view by the camera mount or some part of the aircraft.

If all the undesirable conditions mentioned above were eliminated, optimum photographic quality would still not be obtained unless the aerial photographer selects the proper shutter and diaphragm settings The shutter speed has to be fast enough to stop the motion on the film of images of ground objects (this was previously discussed in Chapter III). After the proper shutter speed has been selected, the lens diaphragm must be set to transmit the proper amount of light to give a satisfactory exposure. An underexposed negative is low in contrast and lacking in shadow detail, while an overexposed negative requires a long printing time Although there is considerable latitude in exposure it is desirable to have some sort of a guide to ensure acceptable results. Aerial photographers with considerable experience, and working under more or less constant conditions, seem to "hit it on the nose" most of the time In determining the correct exposure the following factors must be considered; film and filter combination, type of terrain, weather conditions, time of day, time of year, altitude, and degree of latitude on the earth's surface Exposure guides or computers of the dial calculator type have been developed which take all these factors into consideration, and which give an exposure value that is reasonably correct The best guide available at the present time is the "Photographic Exposure Computer" designed for the armed forces by a committee of outstanding technical experts of the American Standards Association.

Photoelectric exposure meters (such as the Weston or General Electric) are widely used in general photographic work. Their use in aerial photography has not been very successful since the meter not only indicated the intensity of light reflected from ground objects, but also that reflected from small dust and moisture particles in the atmosphere (aerial haze) The varying nature of this aerial haze (with altitude and weather conditions) makes it very difficult to apply a correction factor so as to obtain a satisfactory indication of correct exposure In the future, meters will probably be designed which are not sensitive to aerial haze, thereby indicating the intensity of light reflected from ground objects only The Fairchild type T-5 aerial camera, which is used extensively by the armed forces in all parts of the world, is equipped with a built-in photoelectric exposure meter.

Processing Defects. This list of defects is not all inclusive, but consideration will be given to the major ones encountered in the processing of roll aerial films.

(1) *Negative too thin*—Caused by underdevelopment or underexposure. If exposed properly but underdeveloped, shadow detail will be present but the contrast will be low.

(2) *Negative too dense.*—Caused by overdevelopment or overexposure. Over-

development results in excessive contrast, while overexposure (often combined with underdevelopment) results in low contrast In the former case the density may be reduced by using a proportional reducer* and in the latter case by using a cutting reducer

(3) *Improper contrast*—Low contrast may be caused by underdevelopment, various types of fog, atmospheric haze, or perhaps the terrain itself may be of low contrast. High contrast in aerial photographs is usually the result of overdevelopment, unless snow or white sand areas are present

(4) *Fog*—If in irregular patches it may be due to light leaks in the film container or perhaps in the aerial camera itself. If fog is uniform it may be due to such causes as too long development, use of old film, etc.

(5) *Yellow stain.*—Usually due to insufficient washing and is often accompanied by a fading of the image and the appearance of white crystalline areas. These defects are generally not visible until some time after the negative has dried.

(6) *Brown spots*—May be caused by using an oxidized developer or by using a wash water containing rust particles.

(7) *Abrasion streaks and scratches*—Fine streaks may be caused by the emulsion rubbing against a gritty surface either in the aerial camera or in some subsequent handling Also may be due to swabbing the emulsion with a sponge or chamois containing grit. Scratches may be caused by contact of the wet emulsion with finger nails or sharp edges in the processing equipment.

(8) *Finger prints*—If placed on the emulsion before or during processing they leave a permanent impression

(9) *Pin holes.*—May be due to dust on the film during exposure, hypo dust reaching the emulsion before development, or airbells adhering to the film during development.

(10) *Blistering.*—A separation of the emulsion from its support due to the use of too warm solutions. Wrinkling of the emulsion (reticulation) and frilling of the edges are caused by the same reason The temperature of all processing solutions should be kept between 65° to 70°F.

(11) *Drying marks*—Caused by water drops being left on either side of the film during drying.

(12) *Developer streaks.*—Caused by some portions of the film being developed longer than others. Can be avoided if film reels are always kept in motion while in the developing tank

II. Processing of Paper Prints

A. Paper Characteristics

In discussing the characteristics of papers used for printing aerial photographs the following items are considered; speed, contrast, surface, thickness or weight, and dimensional stability.

In regards to speed, papers used for contact printing are very slow compared to those used for projection printing (speed ratio roughly 1:200) Contact paper emulsions consist principally of silver chloride in gelatin. Typical papers of this type are Eastman's Azo, Ansco's Cykon, Defenders' Apex, and Haloid's Halo Projection paper emulsions consist principally of silver bromide and silver chloride in gelatin (sometimes only silver bromide). Typical projection papers are Eastman's Kodabromide and Royal Bromide, Ansco's Brovira and Cykora, Haloid's Halobrome, and Defender's Velour Black

* See list of formulas at end of chapter

In regard to contrast, photographic papers are classified as soft, medium, and hard. The soft papers (nos 0 and 1) are used for printing very contrasty negatives, the medium papers (nos 2 and 3) for average negatives, and the hard papers (nos. 4 and 5) for very flat negatives. It is very important that the correct contrast paper be used if optimum print quality is to be obtained When too soft a paper is used the print appears muddy, and when too hard a paper is used the print presents a glaring appearance, and detail is lost in the light and dark tones

Only two types of surface quality are of general interest in photogrammetry —glossy and semi-matte or velvet. The glossy surface is used almost exclusively for mosaic prints which later are to be copied photographically The semi-matte surface is preferred for most other work since it has a slight "tooth" which readily takes pencil or ink

Except for mosaic prints, aerial photographs are usually printed on a double weight paper stock Single weight stock is used for mosaic prints because it is more adaptable to feathering and can be stretched considerably when wet by paste, thereby permitting the prints to be adjusted to control points and to each other.

When using aerial photographs in any method of graphic plotting it is very important that they be dimensionally stable during processing and under varying atmospheric conditions Ordinary double weight paper sometimes has a differential shrinkage as high as 0.1 inch for a 9×9 inch print. This amount of distortion affects the direction of rays in radial plotting to such an extent that large triangles of error occur at interesected points Special paper types have been developed to combat this difficulty, among these are Positype's Air Map Special and Eastman's Aero Mapping Paper. Both of these papers have been treated with a waterproof coating which prevents moisture from reaching the paper stock The differential shrinkage of such papers is seldom more 0.02 inch on a 9×9 inch print. Another very successful method for reducing the

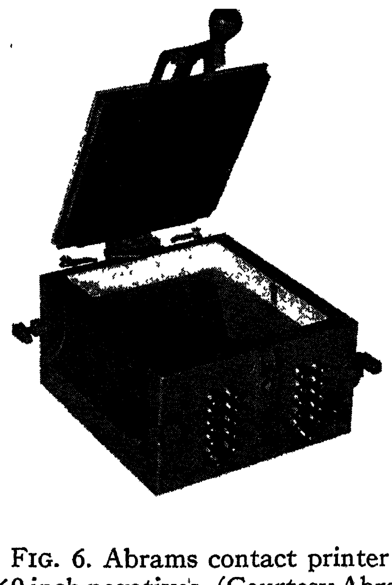


FIG. 6. Abrams contact printer for 9×9 inch negatives. (Courtesy Abrams Instrument Co.).

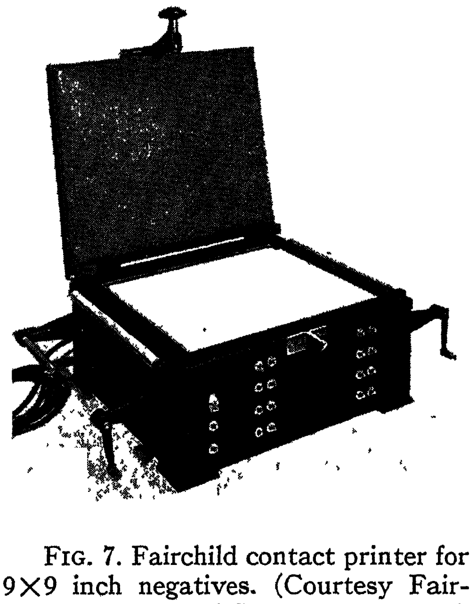


FIG. 7. Fairchild contact printer for 9×9 inch negatives. (Courtesy Fairchild Camera and Instrument Corp.)

shrinkage utilizes a thin sheet of aluminum foil laminated between two pieces of paper. Such a paper was developed in Europe a few years ago and was marketed under the name "Correctostat."

B Contact Printing

Types of Printers Ordinary contact printers found in the usual photographic laboratory are not suitable for use with roll aerial film for the following reasons: (1) They are not equipped to handle film in roll form; (2) they have no quick and convenient method of compensating for the tremendous variation in density of wide-angle aerial negatives For these reasons special equipment has been developed which will now be described

Shown in Fig 6 and 7 are two types of portable contact printers used extensively in photogrammetric laboratories Except for minor items both of these types are similar in construction and operation. The essential features of such printers are: (1) A light box containing a bank of small argon printing lamps (48—2½ watts in Fairchild and 20—1 watt in Abrams). These argon lamps consume little power, give a fast printing time and emit very little heat. Also mounted in the light box are white inspection lamps and amber safelights. (2) A series of toggle switches for controlling these lamps (in the Abrams model a separate switch controls each argon lamp, in the Fairchild model each switch controls three argon lamps (3) An opal or ground glass for diffusing the light and a clear glass for supporting the negative The Abrams printer has a 1½ inch space between the ground and clear glasses, thus permitting the placing of tissue paper on top of the ground glass for additional dodging if necessary. (5) A cover or lid for holding the sensitized paper in perfect contact with the negative The lid of the Abrams printer is lined with a heavy sheet of sponge rubber, while the Fairchild printer utilizes a rubber pad which can be inflated to obtain the desired pressure. In both models the argon printing lamps are turned on and off automatically when the lid is locked and unlocked respectively The Abrams model also has a built-in timer for turning off the printing lights after a set time interval (from ¼ to 15 sec.) Such a timer is very useful when a substantial quantity of prints are to be produced from one or more negatives (6) A bracket on either end of the light box for holding film spools up to 9½ inches in width.

Contact printers other than those described above are also available. Recently Ansco developed a printer for 9-inch film which differs from the above models chiefly in the method of illumination. Besides the usual bank of printing lamps, it has an argon lamp in the center mounted on a vertical rack which permits its movement to or from the negative This arrangement provides a very practical method for dodging wide-angle negatives. A recent Fairchild printer (model F-58D) has a similar lamp arrangement.

Although the most popular size contact printer is 9×9 inches, other sizes are also available A 9×18 inch size is used for printing negatives made by the Fairchild K-7C camera, and a 20×24 inch size is used for printing from mosaic negatives.

Illustrated in Fig. 8 is a special printer designed for making strip prints from negatives exposed in the Sonne Continuous Strip Camera (this camera is described on pages 126 to 129 incl) The printer is equipped with two sets of film spools—one set for holding the 9-inch aerial film, and the other set for holding a roll of sensitized paper (both rolls have a capacity of 250 feet) Printing is accomplished by moving the paper in contact with the film across an illuminated slit or printing aperture. This movement is actuated by an electric motor which permits a maximum printing speed of 40 feet per minute. Before the film comes

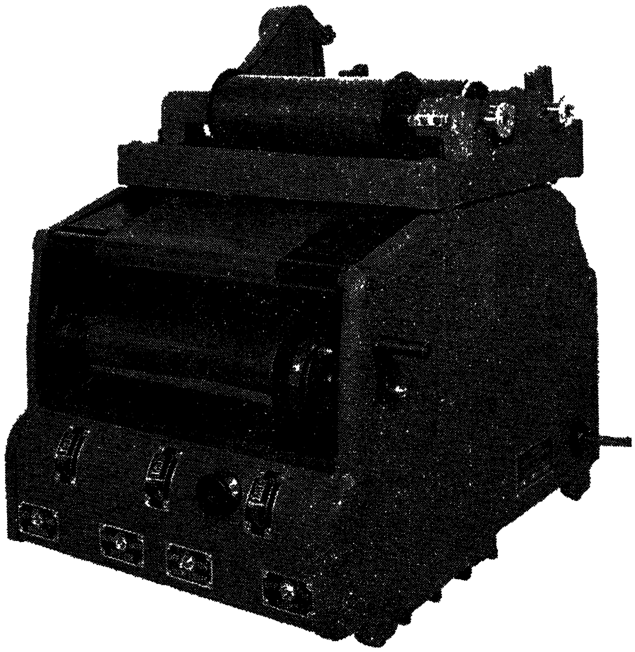


FIG. 8 Sonne continuous strip contact printer Film spool in foreground and paper spools on top

in contact with the sensitized paper it is drawn past an inspection window, thus permitting the operator to judge the variation in density of the negative strip By means of a control dial located on the front of the printer, the operator can change the width of the printing aperture to compensate for changes in negative density. Also located on the front of the printer are three "dodging" controls which enable the operator to narrow the ends of the printing aperture to compensate for the falling off in image density near the edges of the negative strip. The exposed paper strips are processed in the roll film developing outfit previously described (see Fig. 1). This printer has also been used recently for the rapid production of prints from negatives exposed in the conventional type aerial camera

Operation Procedure. The operation of an Abrams or Fairchild printer is very simple and a listing of the various steps follows:

(1) Place the printer on a table or bench and connect to 110 volt supply line.

(2) Check the operation of all switches and lamps Clean off all glass surfaces.

(3) Mount film supply spool in one bracket, draw film across the glass top (emulsion up) and attach to take-up spool mounted in the other bracket.

(4) Turn on clear lamps, estimate the required amount of dodging and turn off the proper toggle switches. In the Abrams model additional dodging can be accomplished if necessary by placing tissue paper or cellophane on top of the ground glass.

(5) Place a sheet of sensitized paper on the negative, emulsion down.

(6) Lower the cover and lock in position In the Fairchild model the air pad must be properly inflated before printing is started, a small bicycle pump being furnished for this purpose

(7) Locking the cover in position automatically turns on the printing lamps which remain on until the cover is unlocked (or until the red pilot light goes out on the Abrams model equipped with automatic timer). The timing of photographic exposures can be done visually with a sweep second timer or audibly with a metronome (see Fig. 9). The exposure for the first print is estimated, the print developed and corrections made if necessary to the dodging and exposure time

(8) After the exposure and dodging are determined for the first print in a flight, the remainder of the flight usually follows the same pattern. This statement will not be true when there is a marked change in the terrain, such as a transition from woodland to sand beaches

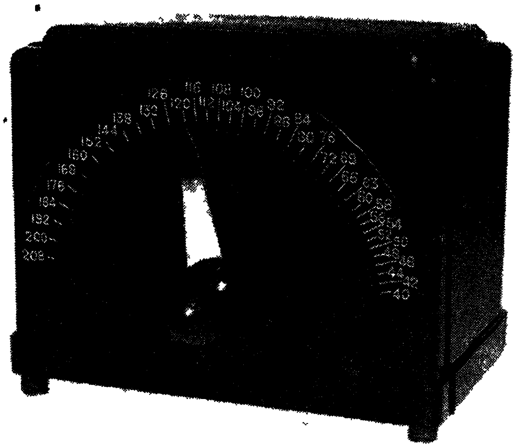


FIG 9. Franz Electronome—an electric metronome useful for timing printing exposures. The figures on dial indicate the number of beats per minute.

Print Processing. The required processing equipment consists of three trays containing the developer, rinse bath, and fixing bath, plus a washing tank. This equipment together with the contact printer, are arranged for convenient operation by one or more photographers. The exposed sensitive paper is slid into the developing solution* by a quick movement in order that the solution start acting on all parts at the same instant, thereby avoiding development streaks. When several prints are developed at one time they should be kept well separated and completely immersed. Since contact paper has a low sensitivity, a yellow safelight may be used (such as Wratten series 00). The print is removed from the developer when the desired density has been reached (usually 30 to 60 sec). Some laboratories mount what they consider an "ideal tone" print near the developing tray to enable the operator to make a quick comparison between it

* See list of formulas at end of chapter.

FIG. 10 Air base laboratory showing print washing tanks on the left and print drying racks on the right. (Courtesy U. S. Army Air Forces)

and the print being developed. Uniform tone is particularly necessary in prints to be used for mosaics

Immediately upon removal from the developer, the prints should be immersed for about 5 seconds in an acid rinse bath * This bath not only stops the developer action instantly, but also prolongs the life of the fixing bath into which the prints are now placed

While in the fixing bath* the prints should be kept separated by stirring with a wood or plastic paddle. The desired fixing time for ordinary paper prints is about 15 minutes An excessive fixation should be avoided since it causes a bleached image In the case of the special low shrink paper stocks previously mentioned, the fixing time should be only 1 or 2 minutes. After removal from the fixing bath the prints are placed in the wash tank (see Fig 10). Many types of washing devices are available, the chief requirements of any type being that it keeps the prints in motion and separated from each other, and that the prints not be damaged in any manner A thorough washing is necessary in order to remove all traces of hypo. Insufficient hypo elimination may cause a fading or discoloration of the print at a later date. Since paper is very porous compared to film, it must be washed longer. An hour is recommended for all papers except the low-shrink waterproof type. Washing the latter longer than 5 minutes may prove detrimental to the object desired, namely, minimum shrinkage

The washed prints are drained well, wiped with a sponge, and then dried by placing the emulsion side down on cloth covered frames (see Fig 10).[5] In some cases forced air circulation is used to speed up the drying. In the case of mapping prints, abrupt exposure to hot air blasts or the use of heated belt dryers should be avoided, as they cause a dimensional distortion of the paper. However, a gradual rise in circulated air temperature under controlled conditions is sometimes successfully used.

C. Projection Printing

When prints other than contact size are desired they are made in a projection or ratio printer. A few years ago the only printers available for roll aerial film

* See list of formulas at end of chapter.

[5] Cheese cloth may be used, but netting of about $\frac{1}{16}$ inch mesh is preferred

were those built for general photographic work These were unsatisfactory for aerial film because they were not equipped to handle roll film, and their usual wood construction made it practically impossible to keep them in accurate adjustment To overcome these difficulties metal printers of precise construction were developed A description of such a printer follows

Saltzman Ratio Printer. The model illustrated in Fig. 11 is one of several types manufactured by this firm The printer consists essentially of a camera unit, a supporting column, and a base

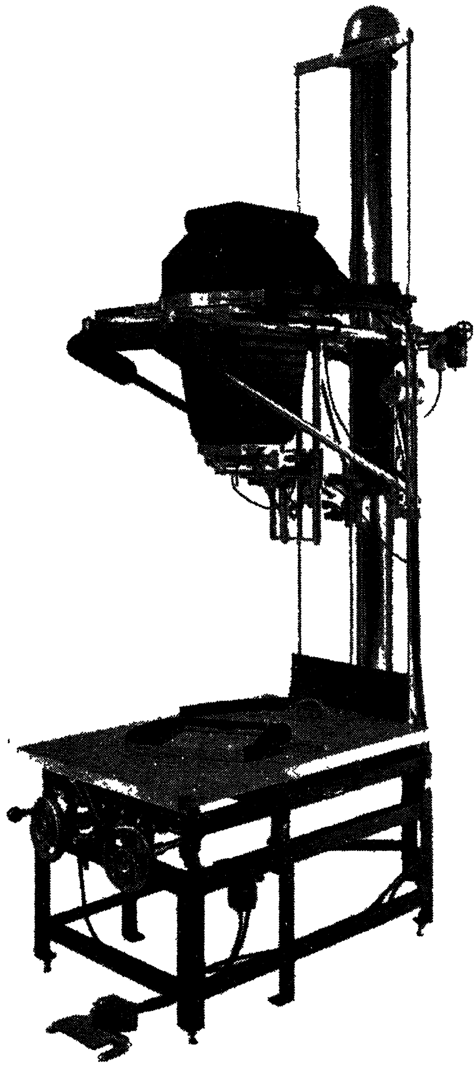


FIG 11. Saltzman projection or ratio printer When cut film is used the holder shown on projection board is substituted for the roll-film carrier (Courtesy J. G. Saltzman Inc.)

The camera unit is made up of the lamp house, roll-film carrier, bellows, and front board. The lamp house contains either a mercury vapor or a fluorescent lamp, neither one of which emits much heat. Diffusion of the light source is accomplished by a flashed opal glass The roll-film carrier has arms extending out on either end for mounting the film spools The film is held between two glass plates, the upper one of which is raised when one desires to move the film Controls for raising this plate and moving the film (from one negative to the next) are located in a convenient operating position on the front of the printer Immediately below the roll-film carrier are four sliding metal masks which prevent light from passing around the edges of the negative, thereby avoiding fogging of the sensitized paper. Leather bellows connect the lamp house and the front board. This board slides up and down on a pair of vertical guide rods. The actuating lead screw for this movement is connected to a dial counter (reading to 0 01″) which indicates the lens-negative distance The front board can be tilted about two axes, a feature which is desirable in rectification work (see pages 54 to 57 incl.). The projection lens is threaded directly into a circular metal lens board which is attached to the front board by four rotating clips. The lens board is interchangeable with a metal cone for increasing the enlarging range When the printer illustrated is equipped with a 12-inch lens the enlarging range varies from 1 0 to 5 0 The general requirements of a projection lens are similar to those used in copying work (see page 249). Mounted in front of the lens is an electric shutter actuated by a movable foot switch

The supporting column consists of a ground and polished seamless steel tube 4½ inches in diameter The camera unit moves up and down this tube, contacting it by eight roller bearings and guided by two keyways The camera unit is counterbalanced by means of a weight concealed within the column The raising and lowering of the camera is accomplished by a control wheel located in a convenient operating position in front of the printer. The vertical movement of the camera unit is registered on a counter which indicates the total distance from sensitized paper to negative The camera unit can also be moved horizontally fore and aft in order to accommodate various size enlargements In the model illustrated a motor drive is provided for this purpose.

The supporting column is mounted on a heavy cast iron base to insure maximum rigidity and constant alignment Seven adjusting screws are provided in the base for leveling the printer The projection board is fastened to the base at ten points, each point being adjustable for aligning purposes. Paper holders of several sizes (from 20 to 40 inches square) are available for use on top of the projection board or easel The use of tilting or "restitutional" easels was discussed on pages 54 to 57 inclusive

Adjustment and Calibration These operations are usually performed by the manufacturer when the printer is set up. However, an occasional check on the alignment is desirable. The best method of doing this is to use a glass plate on which is ruled a square grid of fine lines. This plate is placed in the negative carrier and the resulting image on the easel checked for squareness and size at various scale ratios. This checking may be done with either a beam compass or an accurate scale. A more precise method is to project the grid image on a sheet of sensitized metal[6] (or sensitized paper mounted on metal) and measure the resulting plate accurately under good conditions of illumination.

When we speak of "calibration" we mean the preparation of a chart which

[6] This material may be obtained from The Di-Noc Cramer Corp., 1700 London Road, Cleveland, Ohio.

indicates the proper counter settings for various scale ratios Persons attempting to calibrate a printer should be familiar with the optical aspects involved—such as, conjugate focal distances, lens aberrations, and errors caused by glass plates. (These items are discussed in Chapter II.) It should be obvious that the calibration chart furnished with the printer is correct only for the lens mounted in the printer at the time of calibration The use of any other lens will require another calibration.

Operation of Saltzman Printer New operators should become thoroughly familiar with the various mechanical motions on the printer before attempting to use it A listing of the various operational steps follows·

(1) Move central control lever (just below easel) to upper position, thus raising the top glass pressure plate in the film carrier

(2) On the negative carrier, pull out knob on right film spool holder and insert supply spool with film feeding off the top. Release knob and rotate spool until keys fit slot in spool

(3) Open light traps located on both sides of the lamp house and feed the film (emulsion down) between the two glass pressure plates.

(4) Insert empty take-up spool on left end of film carrier, attach the film to the spool and close light traps.

(5) Knowing the desired enlargement ratio (as 2 45 for instance), set both counters at the values obtained from the calibration chart. The right handwheel raises and lowers the entire camera unit. The left handwheel changes the distance between the lens and negative, but only when the central lever is *down*

(6) Turn on printing lamps, open lens shutter, and select the desired negative by turning handwheel at the left side of the base while the central lever is in the *upper* position. After centering the negative, adjust the film masks so as to cut out all light passing around the edges of the negative Also adjust the paper holder masks to give the desired border

(7) Push the central lever down—thus lowering the pressure plate in the film carrier. Check both counter readings and lock handwheels.

(8) Set the lens at an aperture which gives a convenient exposure time (determined by trial).

(9) Insert the proper contrast paper in the paper holder

(10) Start the exposure by stepping on the footswitch—thus opening the electric shutter in front of lens. Since aerial negatives are usually of uneven density it is necessary to resort to "dodging" in order to obtain a uniform tone print. This process consists in inserting a cardboard shield between the lens and paper, always keeping it in motion. The shape and size of the shield depends on the type of dodging necessary. In printing from negatives made in a wide-angle camera it will be necessary to use a shield with a hole in the center in order to give the center portion of the print a longer exposure than the edges. To aid them in determining the proper exposure and dodging pattern, some operators scan the easel image with a photoelectric meter (see Fig. 12). By means of previous test exposures, a constant is determined which is used for converting meter readings into seconds of exposure. This method eliminates waste of photographic paper caused by: (a) variation of negative density, (b) aging of the printing lamps, (c) fluctuations in the line voltage.

(11) After exposing the print, it is processed in a manner similar to that described under contact printing—with the following exceptions: (a) since projection paper is much faster than contact paper, a red safelight must be used instead of a yellow; (b) the developing solution is usually weaker and the development time longer (about $1\frac{1}{2}$ minutes); (c) this longer development time enables the

FIG 12 Photrix photometer used for determining print exposure By means of a selector switch, light intensities ranging from 0 05 to 250 foot-candles may be measured

operator to partly correct for errors in dodging—for instance, if part of the print is coming up light he can sponge on some concentrated developer to speed it up.

Miscellaneous Notes on Projection Printing In some printers the lens-negative and lens-paper distances are linked to a cam mechanism which causes these distances to vary according to the basic lens equation (see page 39, eq 2). It is therefore possible to obtain sharp focus at various magnifications merely by turning one control wheel. The term "auto-focus" is given to such an arrangement. In operating such a printer, the single control wheel (sometimes motor driven) is adjusted until the image size on the easel (as measured between opposite fiducial marks) agrees with a set distance on a special scale (see Fig 13). The scale illustrated has shrinkage correction graduations on one end, enabling the operator to allow for shrinkage of the paper in processing, thereby obtaining the desired scale in the dry print. In order to obtain sufficient illumination, the above scale settings are made with the projection lens at full aperture, and then stopping it down when the exposure is made. In printers not equipped with either a calibrated counter system or an auto-focus arrangement, proper image size and sharpness are obtained by manipulating two control wheels simultaneously.

In any ratio printer the operator must prevent stray light from reaching the sensitized paper. Such light may come from leaks in the lamphouse, the use of too bright safelights, or from small lamps used to illuminate the various dials and scales on the printer.

The use of the present high speed panchromatic films in aerial photography prohibits the making of satisfactory enlargements greater than four or five times the negative size. Any further increase in magnification will merely accentuate the grain size of the negative emulsion, without increasing the amount of detail that can be seen

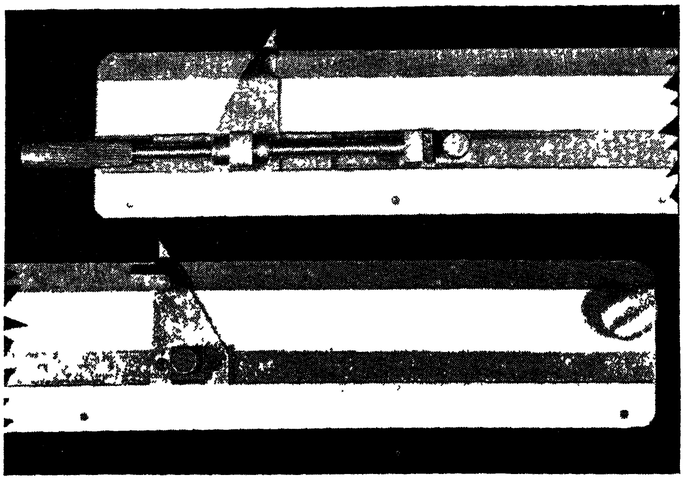


FIG 13 Metal scale used in projection printing. Upper view shows slow motion adjustment, lower view shows shrinkage correction scale (tab to left of point).

D. Print Defects

Listed below are several defects which may be found in paper prints The list is not all inclusive, but consideration is given to the major faults encountered in normal print production

(1) *Blurred areas* —In contact printing it is caused by poor contact between paper and negative. In projection printing, if the entire area is blurred it may be caused by use of too large a lens aperture in the case of a poor lens, by improper focusing, or by vibration of the printer during exposure

(2) *Finger prints* —When handling photographic paper always keep hands free of chemicals and moisture Persons whose hands perspire freely should avoid contact with the sensitive surface.

(3) *Abrasion streaks and scratches.*—Fine scratches may be due to friction in removing the paper from the box carelessly. Large scratches often due to contact with the finger nails when the emulsion is wet. Creases and cracks in the emulsion are due to careless handling.

(4) *Air bells* —Airbells adhering to the surface of the emulsion prevent the chemical solutions from acting Round white spots on print are caused by air bells present during development, and dark spots if present during fixation The remedy is to immerse the print, emulsion side up, with a quick sliding motion and thereafter keep solution agitated.

(5) *Fog.*— May be caused by improper safelights, light leaks in printer, old paper, or too long development.

(6) *Streaks.*—Caused by improper immersion in solutions, such that portions of the print are developed longer than others, thereby giving streaks of varying density.

(7) *Irregular white spots.*—Caused by dust on surface of negative or paper during exposure

(8) *Irregular dark spots.*—Caused by pinholes or scratches on negative

(9) *Flatness.*—A muddy appearance caused either by overexposure and underdevelopment, or by using a paper of too low contrast.

(10) *Excess contrast*—Use of paper of too high contrast.

(11) *Improper density.*—Cause obvious If too dark it is due to overexposure or overdevelopment (also see item 9). If too light it is due to underexposure or underdevelopment

(12) *Blisters.*—May be caused by a large difference in temperature existing between the various processing solutions, or by allowing a strong water spray to hit the print constantly in one spot

(13) *Brown spots.*—May be due to rust particles in wash water, to insufficient rinse after development, or to the use of exhausted developer

(14) *Yellow stain* —Such a stain may not appear until some time after the print has dried. It may be caused by using an exhausted developer or fixing solution, by insufficient washing, or by using old paper

(15) *Fading* —Usually due to excessive fixation or insufficient washing.

III. Special Printing Processes

A. Printing of Duplicate Negatives

Due to one of the following reasons a duplicate copy of the original aerial negative may be desired: (a) An aerial contractor is usually required to submit the original aerial film to the contracting agency, therefore he may find it desirable to make duplicate negatives of a certain portion of the project for anticipated future requests; (b) Instructors in photogrammetry, geology, and other sciences may be using certain aerial photographs which are ideal for instructional purposes, the securing of a set of duplicate negatives will enable them to make prints for future classes as needed, (c) If the aerial negatives are considered valuable from an archival viewpoint, a duplicate set can be made for routine laboratory use, thus minimizing wear on the original

If desired, a duplicate negative can be made by a two-step process which involves the making of a film positive by contact printing from original negative, and then making a negative by contact from the film positive. Such a method is wasteful of both time and materials.

Another method makes use of a reversal film, the emulsion of which is similar to a thin coated negative emulsion The film is exposed in contact with the original negative giving a positive transparency. This positive image is then bleached out and the remaining unexposed silver halide grains exposed to light and developed, thus giving the desired negative image The disadvantage of this method is that it requires considerable time and several chemical baths.

By far the most satisfactory method of obtaining a duplicate negative is by using special contact films developed for this purpose, such as Ansco's Direct Copy Film or Eastman's Autopositive film. These films are exposed, developed, fixed, and washed much like any other film, the chief difference being that *overexposure* makes the image *too light* and *underexposure* makes the image *too dark.* This phenomenon can best be explained by referring to the characteristic curve previously shown on page 210 For a very great exposure (about 1,000,000 times normal for ordinary film) the maximum density is reached slightly to the right of "*D* " Any further increase in exposure will produce a decreased density, therefore if the characteristic curve were extended it would curve downward toward the right. This is known as *solarization* and takes place in that section of the characteristic curve known as the *region of reversal* During the manufacture of direct copy film it is flashed to light of sufficient intensity to produce the maximum developable density, and any further increase in exposure will produce less density. Therefore when such a film is placed in contact with a negative, exposed and developed, a duplicate negative will be obtained.

The duplication of aerial negatives is performed in a contact printer so modified as to have two pairs of film spools for holding the original and duplicate negatives respectively The *back* side of the duplicating film is placed in contact with the emulsion of the aerial film in order that the former may be used in the normal manner when making prints—that is, emulsion side toward the sensitized paper For best results, the developer recommended by the film manufacturer should be used By proper manipulation during development it is possible to make the contrast of the duplicate negative greater than the original Also the usual dodging methods can be employed so that the duplicate negative can be printed straight, that is, with little or no dodging This is particularly advantageous in the case of negatives obtained from wide angle cameras

B Printing of Multiplex Diapositives[7]

A general description of the multiplex equipment is given in Chapter XI. The reader is urged to read this before proceeding with the following discussion

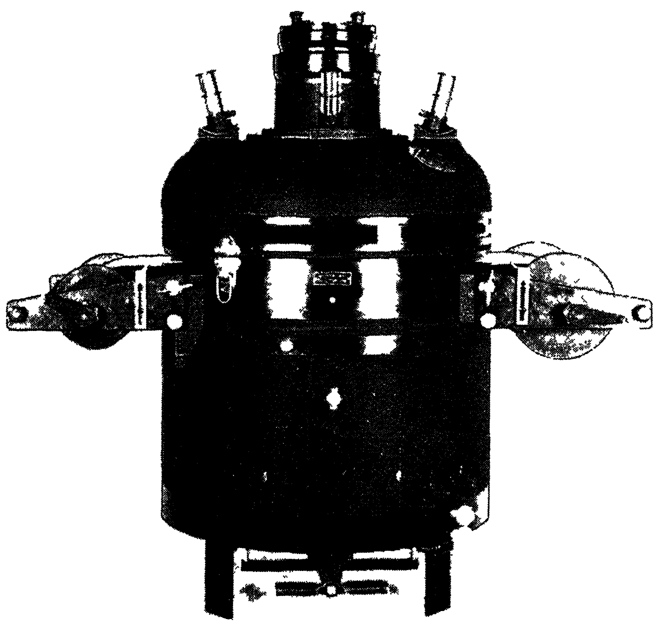


FIG 14 Wide-angle multiplex reduction printer This model is designed for use with air cameras of 100 mm. focal length and multiplex projectors of 22 mm. principal distance. (Courtesy U. S. Geological Survey)

The purpose of this section is to briefly list the necessary operational steps in making a satisfactory diapositive. In the following discussion we will assume that a wide-angle reduction printer such as illustrated in Fig. 14 is available. Briefly this type consists of a lower body and a upper head or dome. The body

[7] The writer is indebted to W. E Harman of the U S Geological Survey, Clarendon, Virginia, for help in preparing this section.

consists of: (1) A lamphouse containing a 60-watt lamp mounted in a parabolic reflector, also two diffusing glasses placed in front of lamp; (2) a fixed glass film supporting plate, and a movable upper pressure plate for holding the film flat; (3) brackets for holding the film spools.

The printer head consists of. (1) The projection lens, which is designed to compensate for the distortion of the air camera lens; (2) graduated rings for changing the conjugate focal distances so as to give the proper reduction ratio for the air camera lens used Each printer is designed for a very limited range in air camera focal lengths—for instance, a printer designed for an air camera focal length of 152 mm , can only accommodate focal lengths ranging from 150 to 154 mm ; (3) an adjustable illumination compensator for varying the intensity of light in the center of the image plane; (4) a platform for holding the diapositives; (5) viewing windows and microscopes for setting the negative in the printer

Before using any printer it is of course necessary that it be properly adjusted and calibrated. Items to be considered are (1) Plane of negative and diapositive must be perpendicular to lens axis; (2) calibration chart should be prepared which gives the correct settings on the graduated rings for air cameras of various focal lengths. High accuracy in these settings is absolutely necessary in order to obtain good definition and the correct reduction ratio (about one-fifth size); (3) the cross which indicates the intersection of the four fiducial marks on the film supporting plate should intersect on the lens axis.

The various steps in the operation of a multiplex reduction printer follows.

(1) Set both of the adjusting rings on the printer head to agree with the focal length of the air camera used.

(2) Remove printer dome and upper pressure plate.

(3) Mount film supply spool in one bracket, pull film (*emulsion down*) across film supporting plate and attach to empty take-up spool

(4) Replace upper pressure plate and dome.

(5) While looking through the viewing windows and microscopes on the dome, orient the negative by lining up its four fiducial marks to agree with those etched on the lower film supporting plate In making this adjustment the upper pressure plate is raised slightly by a foot treadle, thus permitting the film to move without strain.

(6) Place a ground glass on the diapositive support and adjust the illumination compensator (located between lens and diapositive) so the proper light distribution reaches the center of the negative As previously discussed in Chapter II, the density of a negative made in a wide-angle camera varies from 100 per cent at the center of the field to perhaps only 15 per cent at the edges In order to obtain a uniformly dense image in the multiplex projector it is necessary that the diapositive be dense in the center. The reason for this being that the projector (like the air camera) has the strongest illumination at the center. The above mentioned illumination compensator holds back the light around the edges of the field, thus permitting the center to receive the required amount Irregular densities in the negative caused by variation in the natural illumination or terrain are taken care of by: (1) placing dodging tissue (such as sheet cellophane) on top of the diffusing glass; (2) using hand held dodging devices (such as paper disk on end of wire) inserted through a slot in the dome between the lens and the negative. Some operators determine the exposure time and dodging pattern by scanning the negative with a photoelectric meter. When such a method is used it is necessary to remove the printer dome and place the meter cell on top of the upper glass pressure plate If the stereoscopic model formed by the multi-

plex projectors is of uneven density, the color balance of the anaglyphic filters will be disturbed In order to correct this the multiplex operator would have to continually change the relative intensity of the projector lamps.

(7) Turn out all white lights, place diapositive plate on its supporting platform and secure in position by spring clips. These plates are made from selected flat glass about 0 07 inch thick and 64 mm. square. They are coated with a contrasty lantern slide emulsion and backed with an anti-halation dye. The plate support consists of four bosses identical with those used in the multiplex projector thus avoiding trouble caused by slight warpage of the plates.

(8) Exposure and processing. It is very important that the diapositive have the required contrast to reproduce all the minute detail in the aerial negative, and to have a density thin enough for satisfactory projection. To persons unfamiliar with multiplex work a good diapositive appears to be of low contrast and rather thin. Since the diapositive is to be projected with a magnification of 12 times, it is absolutely essential that it be free of all scratches and blemishes Diapositive printing differs from other types in that only one contrast grade of emulsion is generally used to take care of negatives of various contrasts The contrast in the diapositive emulsion is controlled by altering the developing solutions The usual film developer formulas will give satisfactory results if the negative contrast range falls within certain limits However, a modification of these formulas is generally necessary to take care of the wide range in contrasts found in the average run of aerial negatives From considerable experience it has been found that the standard D-76 formula* is satisfactory providing glycin is substituted for hydroquinone (except for very thin negatives). The proper contrast is obtained by varying the amount of elon, and the desired development time by varying the amount of borax. It is necessary that the photographer familiarize himself with the action of each developer constituent, and to increase or decrease that constituent to yield the desired results After development the plates are fixed for 10 minutes in a standard acid hypo bath, washed 30 minutes in flowing water, wiped with a sponge and dried in plate racks Some operators hasten the drying process by giving the plates a final rinse of about 60 seconds in grain alcohol.

C *Copying*

Copying by photography is utilized to provide accurate reproductions of aerial mosaics, maps, etc. Aerial mosaics for instance, are sometimes constructed on boards as large as 8×12 feet in size. These are usually copied onto 20×24 inch film negatives, from which any desired number of prints can be made by contact printing.[8] Copying cameras (see Fig. 15) differ considerably from the previously described enlarging or ratio printers They are usually arranged with the lens axis horizontal, and the copy board and negative holder vertical In the model illustrated the copy board is fixed in position while the camera unit moves on a pair of horizontal tracks Such an arrangement requires the use of a plate carrying box for transferring the negative between camera and darkroom. The necessity of this plate carrying box is eliminated in some models by having the negative end of the camera project through the darkroom wall. When such an arrangement is used, scale change is accomplished by moving the lens and copy boards, usually on overhead tracks. Electric motor drives controlled by push-buttons in the darkroom are often used for motive power In order to

* See list of formulas at end of chapter

[8] When a large number of copies are desired a tone negative can be made by the contact screen process. From this negative a printing plate is prepared for use in an offset press.

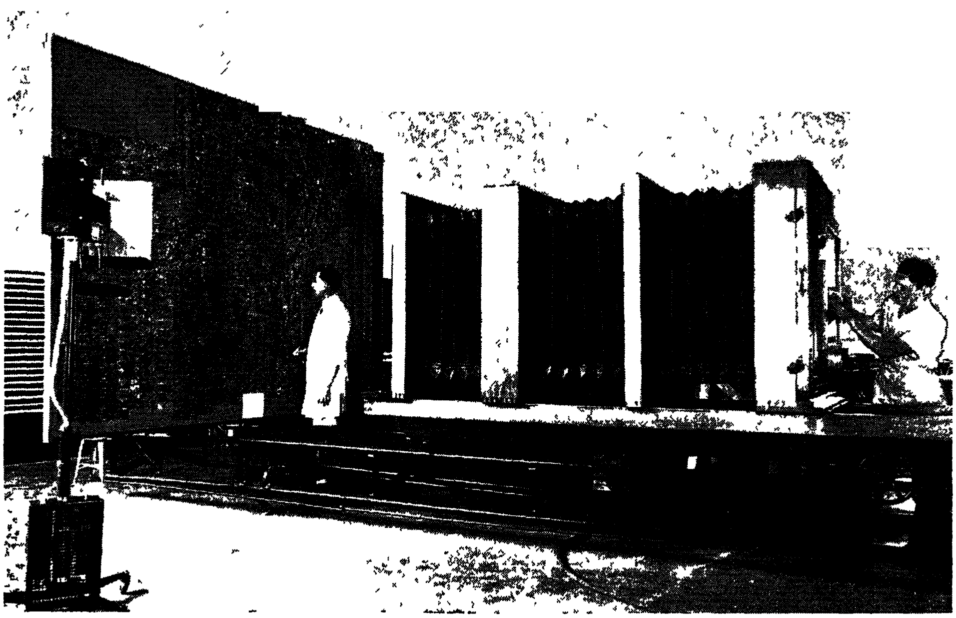


FIG. 15. Copying a photographic index. Operator at right is checking image size on ground glass screen. (Courtesy Abrams Aerial Survey Corp.)

illuminate the copy board, movable carbon arc lamps of high intensity are arranged in such a manner that no direct reflection of the arcs reaches the camera lens.

Lenses used in copying cameras (also in projection printers) are called "process" lenses and differ from those used in other photographic work—aerial photography for instance The more important characteristics of process lenses include the following.

(1) Lenses are designed for finite object distances.

(2) Large relative apertures are rather unusual, seldom being larger than f/8.

(3) Angular field is usually rather small, ranging from 30° to 50°.

(4) Focal lengths are rather large (24 to 72 inches) in order to satisfactorily cover the large copy size often used.

(5) Lenses must be free from distortion for the desired finite object distances. The allowable amount of distortion depends on the type of process work being performed, and each case must be considered individually. It is also desirable to mention that the distortion has a different value for each scale ratio.

(6) Lenses must give excellent definition. This is secured in the design of the lens by limitation to a small relative aperture and a small angular field. To obtain good definition over the entire field it is necessary that the image field be flat.

(7) Optical design usually is symmetrical, that is, corresponding optical elements in the front and rear components are identical. At a scale ratio of unity with a perfectly symmetrical lens there will be no distortion. In case an un-

symmetrical lens is used, it is desirable that the front of the lens (side with engraved figures) be on the side having the greater conjugate distance.

(8) Lenses must be highly corrected for color if multi-color work is to be done Lenses with such a correction are termed apochromats.

A complete discussion of the above lens terms, as well as the mathematical relationships between object and image are discussed in Chapter II, section I.

Negative emulsions used in copying may be coated either on film or glass. Films are used in mosaic copying for convenience in handling and because emulsions of a wide range of contrasts are available Glass plates are used in copying maps since dimensional stability is desired. The glass plates may be either the common "dry" plate prepared by various photographic manufacturers, or the "wet-collodion" plate prepared by the photographer himself. The preparation of the latter type consists in: (1) Flowing an albumen coating over the clean glass plate and allowing it to dry; (2) coating the plate with a collodion solution; (3) sensitizing plate by immersing for 5 minutes in a silver nitrate bath; (4) placing the plate (while still wet) in the camera and exposing from 2 to 4 minutes; (5) processing the plate, which includes developing, fixing, intensifying, and washing.

A brief listing of the various steps in the reproduction of a map[9] by photolithography follows (1) Photographing the original map drawing in a copying camera to the desired scale, one negative being made for each color; (2) preparing the negatives by painting out, retouching and hand engraving; (3) contact printing each negative onto a sensitized aluminum printing plate (see Fig. 16); (4) mounting of the printing plate in an offset press and running off the desired number of copies (see Fig 17)

IV. Formulas

For a complete listing of formulas used in photographic practice the reader should consult the references listed at the end of this chapter, or the booklets and instruction leaflets published by various manufacturers of photographic materials and chemicals However, it was thought desirable to include a few formulas here for emergency use when the more complete sources were not available. The aerial film by the time it reaches the photographic laboratory represents an investment of considerable magnitude, therefore it is false economy to use cheap impure chemicals in mixing the solutions. The various chemicals should be mixed in the order given in the formula, and care should be taken to see that each chemical is completely dissolved before adding the next. Unused solutions may be kept for a considerable length of time if placed in closed containers with little or no air space Prepared packages (in powder form) of most of the below formulas may be purchased These packaged forms furnish a quick convenient method of preparing the various solutions with assurance of purity and accuracy. They are especially convenient for field use.

In the following formulas the desiccated (or anhydrous) sodium carbonate may be replaced by the monohydrated variety if the listed quantities are increased by one-sixth. To make a 28% acetic acid solution use 3 parts of glacial acetic acid and 8 parts of water

(1) *Low contrast film developer (Eastman D-76)*[10]

Water (125°F.)	3 quarts
Metol	116 grains

[9] Birdseye, C H., "Map Reproduction," Photogrammetric Engineering, Vol. II (1936), page 9.

[10] Space does not permit the listing of comparable formulas published by other manufacturers.

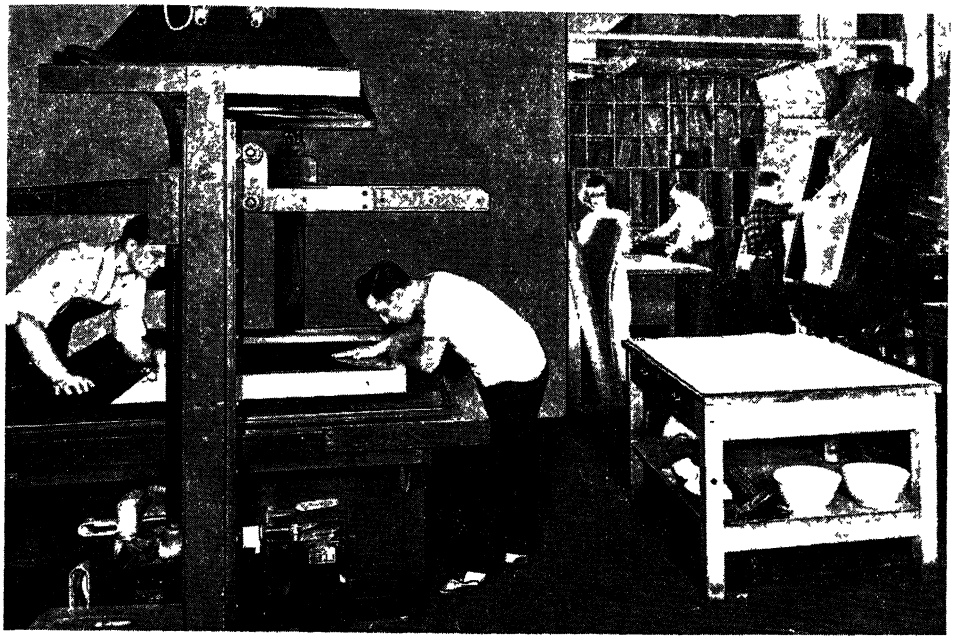


FIG 16 Preparation of printing plate—at the right is shown a *whirler* in which a grained aluminum plate receives a uniform coating of a light-sensitive emulsion. At the left is shown a vacuum printing frame with which a contact print is made from the negative onto the sensitized aluminum plate

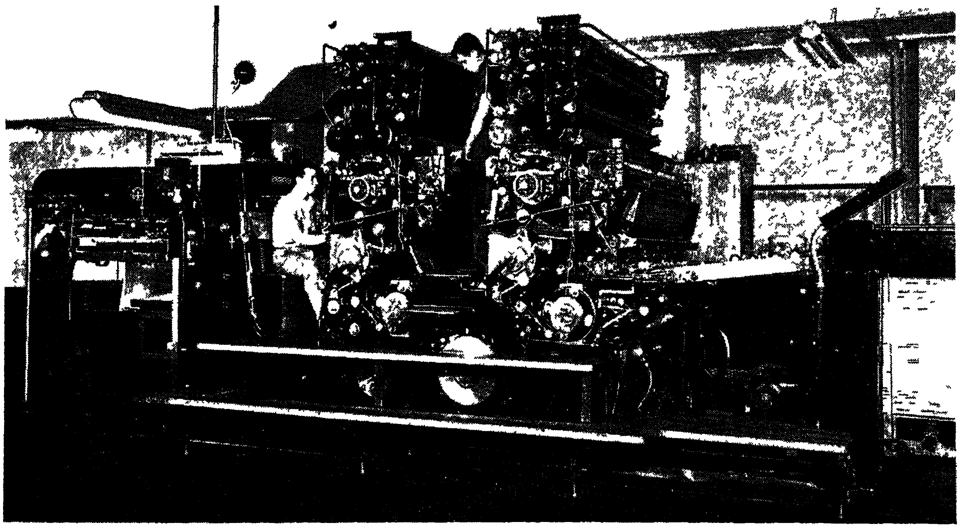


FIG 17 Two-color Harris offset press equipped with automatic feed and delivery. The paper stock pile is shown at extreme right and the delivery end at extreme left. The press has a capacity of about 5,000 impressions per hour. Above photographs courtesy U. S Coast and Geodetic Survey

Sodium sulfite, desiccated	13¼ ounces
Hydroquinone	290 grains
Borax, granular	116 grains
Water to make	1 gallon

Normal tank development time 20 minutes at 68°F. This time can be cut in half by increasing the borax quantity ten times (the same result can be secured by replacing the borax by Kodalk and using 2½ times the amount stated in formula).

(2) *Medium contrast film developer (Ansco 47)*

Water (125°F)	3 quarts
Metol	88 grains
Sodium sulfite, desiccated	6 ounces
Sodium bisulfite	60 grains
Hydroquinone	180 grains
Sodium carbonate, desiccated	300 grains
Potassium bromide	47 grains
Water to make	1 gallon

Normal tank development time 7 minutes at 68°F

(3) *High contrast film developer (Ansco 30)*

Water (125°F)	3 quarts
Metol	205 grains
Sodium sulfite, desiccated	8 ounces
Hydroquinone	1 ounce 80 grains
Sodium carbonate, desiccated	4½ ounces
Potassium bromide	120 grains
Water to make	1 gallon

Normal tank development time 10–15 minutes at 68°F.

(4) *Acid rinse bath for films and papers*

Acetic acid (28%)	6 ounces
Water	1 gallon

Immediately after development, rinse negatives or prints at least 5 seconds. The bath stops developer action instantly, thus preventing streaks and uneven spots and also prolonging the life of the fixing bath.

(5) *Chrome alum hardening bath*

Potassium chrome alum	4 ounces
Water	1 gallon

This bath is used in hot weather instead of the acid rinse bath when additional hardening of the film is desirable When first immersed in the bath, the film should be thoroughly agitated. Maximum hardening will be obtained in about 3 minutes The bath should be renewed whenever a green sludge forms

(6) *Hardening fixing bath for films and papers (Eastman F-5)*

Water (125°F.)	2 quarts
Sodium thiosulfate (hypo)	2 pounds

Sodium sulfite, desiccated	2 ounces
Acetic acid (28%)	6 ounces
Boric acid crystals	1 ounce
Potassium alum	2 ounces
Water to make	1 gallon

This bath keeps well and may be used repeatedly until exhausted. If the bath becomes cloudy, froths, or takes longer than 10 minutes to clear an emulsion, it should be discarded.

(7) *Farmer's cutting reducer for overexposed negatives*

Stock solution A	
Potassium ferricyanide	2½ ounces
Water to make	1 quart
Stock solution B	
Sodium thiosulfate (hypo)	2 pounds
Water to make	1 gallon

For use mix 4 ounces of A with 16 ounces of B and add water to make 1 gallon. Place the negatives in the solution, watch the action closely, and remove when reduction is sufficient and wash thoroughly

(8) *Farmer's proportional reducer for overdeveloped negatives*

Solution A	
Potassium ferricyanide	1 ounce
Water to make	1 gallon
Solution B	
Sodium thiosulfate (hypo)	27 ounces
Water to make	1 gallon

Immerse the negative in solution A from 1 to 4 minutes depending on degree of reduction desired Then immerse in solution B for 5 minutes and wash thoroughly Repeat process if more reduction is desired.

(9) *Paper developer (stock solution)*

Water (125°F.)	2 quarts
Metol	180 grains
Sodium sulfite, desiccated	6 ounces
Hydroquinone	1½ ounces 30 grains
Sodium carbonate, desiccated	9 ounces
Potassium bromide	¼ ounce
Water to make	1 gallon

For chloride papers dilute 1 part stock solution with 2 parts water and develop about 45 seconds. For bromide papers dilute 1 part stock solution with from 2 to 4 parts water (depending on contrast desired) and develop from 1 to 3 minutes.

The above formula can also be used for films when a fast high contrast developer is desired. Dilute 1 part stock solution with 2 parts water and develop 4 minutes at 68°F

Conversion table—avoirdupois to metric weight

Pounds	Ounces	Grains	Grams
1	16	7000	454
	1	438	28 3
	00229	1	0648
	0353	15 4	1

Conversion table—U S liquid to metric measure

Quarts	Ounces (Fluid)	Drams (Fluid)	Cubic Centimeters
1	32	256	946
	1	8	29 6
	125	1	3 70
	0338	270	1

Temperature conversion table—Centigrade to Fahrenheit

Centigrade	Fahrenheit	Centigrade	Fahrenheit	Centigrade	Fahrenheit
10	50	25	77	40	104
11	52	26	79	41	106
12	54	27	81	42	108
13	55	28	82	43	109
14	57	29	84	44	111
15	59	30	86	45	113
16	61	31	88	46	115
17	63	32	90	47	117
18	64	33	91	48	118
19	66	34	93	49	120
20	68	35	95	50	122
21	70	36	97	51	124
22	72	37	99	52	126
23	73	38	100	53	127
24	75	39	102	54	129

Bibliography

Clark, Walter, "Photography by Infrared," John Wiley, New York (1939).

Clerc, L P , "Photography—Theory and Practice," Pitman, New York (1937).

Crabtree, J I , and Matthews, G. E , "Photographic Chemicals and Solutions," American Photographic Publishing Co , Boston (1938).

Eastman Kodak Company, "Kodak Reference Handbook" (1941)

Henney, Keith, and Dudley, Beverly, "Handbook of Photography," McGraw-Hill, New York (1939).

Mack, J. E , and Martin, M. J., "The Photographic Process," McGraw-Hill (1939).

Mees, C E Kenneth, "The Theory of the Photographic Process," Macmillan, New York (1942).

Neblette, C. B , "Photography—Its Principles and Practice," Van Nostrand, New York, (1942)

War Department, "Basic Photography," TM 1–219 (1941) Copies may be obtained from Superintendent of Documents, Washington, D C. ($0.50).

CHAPTER VI

GEOMETRIC CHARACTERISTICS

GEOMETRY OF VERTICAL PHOTOGRAPHS

G C Tewinkel

IT HAS been pointed out previously that the vertical (or near vertical) aerial photograph is usually not a map of the area represented. A map is an orthographic projection while a photograph is a perspective one The result of this difference and others is to produce a photograph which has varying and irregular scale characteristics throughout its area while a map has a constant scale It may be said that a photograph is not a map because its images have been displaced with respect to one another relative to their respective map positions The total displacement is the resultant of several component displacements, each of which has a very definite magnitude and direction The different components may be enumerated as due to (1) the elevation of the object and incorrect flying height, (2) the tilt of the air camera, (3) uneven film and paper distortion, and (4) distortion produced optically by the lens. An understanding of each of these factors will aid the photogrammetrist in the application of methods and processes employed in the preparation of accurate maps from photographs which are not maps in themselves. The first two factors are discussed here in addition to other commonly used geometric relationships.

1. Scale

A photograph may not have one exact scale but rather it usually has many scales Nevertheless, the term is widely used to denote the average or approximate scale of a photograph in much the same manner as for a map. The Representative Fraction (R. F) is a universal term used to express the scale, as 1/10,000, 1:10,000, 1/62,500 or 1:62,500, which mean that the photograph is that fraction of the actual ground size, or that one linear unit of photograph distance is equivalent to 10,000 or 62,500 units of the same size on the ground. For example, if a given scale is $S=1/31{,}680$, it is further understood that one inch on the photograph represents on the ground 31,680 inches or 2640 feet or one half mile and that two inches represent one mile; and that one millimeter represents 31,680 millimeters or 31.68 meters on the ground

In any case, the scale may be stated algebraically as

$$S = \frac{d}{D} \qquad \text{Equation 1}$$

where S is the scale, d is a photograph distance and D is the corresponding ground distance, d and D being expressed in the same linear units To change the form of the relation to that of the representative fraction with a numerator of unity, the numerator and denominator of the fraction are divided by d:

$$S = \frac{1}{D/d}\cdot \qquad \text{Equation 2}$$

When a careful determination of the approximate scale of a photograph is desired, it is preferable to use the average scale obtained by measuring two rela-

tively long lines which intersect near their midpoints at about a right angle roughly in the center of the picture

2 Scale, Flying Height, and Focal Length

In Figure 1 is shown an elevation view of the geometric situation which occurs in the making of a truly vertical photograph Rays of light emanating from the objects A and B on the ground are considered as converging at the incident nodal point of the lens L and emerging from the emergent nodal point L' with their directions undeviated It is then obvious that the triangles $L'a'b'$ and LAB are similar and that the nodal separation does not alter the geometry of the figure Also, triangles Lab and $L'a'b'$ are congruent and the line ab represents the familiar photographic print. Hence, triangle Lab will be used hereafter in geometric figures and the upper triangle will not be considered Triangles Lab and LAB are then similar also and the following proportion may be written:

$$\frac{d}{D} = \frac{f}{H} \qquad \text{Equation 3}$$

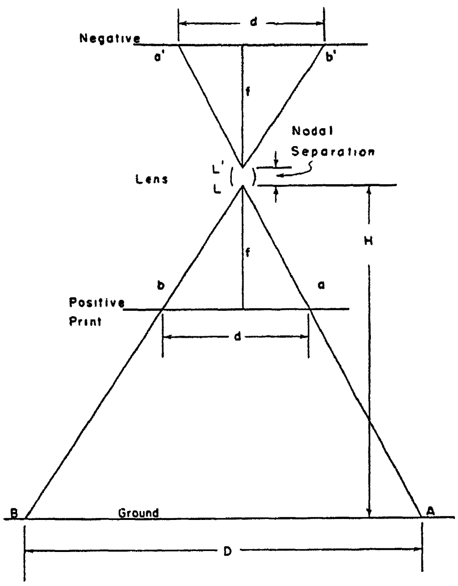


Fig. 1

where f is the focal length of the photograph and H is the elevation of the lens L above the line AB. It is customary to refer to the flying height H as the elevation of L above sea level. If the line AB had an average elevation of h above sea level, combining with Equation 1 we have.

$$S = \frac{d}{D} = \frac{f}{H - h} \qquad \text{Equation 4}$$

which expresses approximately the complete relation between scale, photograph distance, ground distance, focal length, and flying height. The equation is exactly true if the elevations of A and B are equal. The relation is close enough for many practical purposes if the elevation difference is small.

3 Exact Ground Length, Untilted Photograph[1]

The exact ground length D of a line may be determined from the photographic images, and also the exact flying height H, by the application of Equation 4 when the elevations of A and B are different. If x_a, y_a, x_b, and y_b are the photograph rectangular coordinates of the images a and b based upon the principal point as origin and one of the fiducial axes as the $+x$-axis with the usual convention as to algebraic signs, and if h_A and h_B are the elevations of the object points A and B, then by substituting x_a, etc , for d and X_A, etc., for D in Equation 4 and solving for X_A, etc , we have

[1] Church, Earl, *Determination of the Scale Data for Two Aerial Photographs from One Control Line and Three Additional Elevations*, Syracuse University, Syracuse, New York, December, 1942.

$$X_A = x_a \frac{H - h_A}{f}$$

$$Y_A = y_a \frac{H - h_A}{f}$$

$$X_B = x_b \frac{H - h_B}{f}$$

$$Y_B = y_b \frac{H - h_B}{f} \qquad \text{Equation 5}$$

where X_A, Y_A, X_B, and Y_B are ground rectangular coordinates of points A and B corresponding to the photograph coordinates The exact length of the line AB may then be determined from

$$D = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}. \qquad \text{Equation 6}$$

For example, a tabulation of given data and computations follow for which $f=210.06$ mm and $H=13,325$ feet

Given					Computed			
	x	y		h	$H-h$	$(H-h)\,f$	X	Y
	mm	mm		ft				
a	−79 83	−89 38	A	692	12,533	69 664	−4763	−5333
b	−20 33	+73 51	B	521	12,704	60 478	−1230	+4446

$$X_A - X_B = -\ 3533$$

$$Y_A - Y_B = -\ 9779$$

$$D_{AB} = 10,398 \text{ feet}$$

4. Exact Flying Height, Untilted Photograph[1]

The exact flying height for a truly vertical photograph may be determined by a similar computation when the distance between two points on the ground is known. If h' is the average elevation of the ends of a line AB, we may substitute it for h in Equation 4 and solve for a trial or approximate flying height H' to get

$$H' = h' + Df/d \qquad \text{Equation 7}$$

in which d may be measured directly on the photograph or computed from the photograph coordinates:

$$d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \qquad \text{Equation 8}$$

The procedure outlined in Section 3 above is then employed to give a ground length D' which is exactly consistent with the trial flying height. D' will be in error if the value used for H' is not the correct flying height. A better value for H is then found by

$$H = h' + (H' - h')\ D/D' \qquad \text{Equation 9}$$

This new value for H is then used as before for determining a new corresponding ground length D. The latter value is usually very nearly equal to the correct

value. By repeated use of Equation 9, finer adjustments to the value of H may be obtained, limited in accuracy only by the accuracy of measurements and indicated by the correspondence between the computed and correct values for D

The data employed in Section 3 is again used in the following illustration. The photograph coordinates of the two images, the focal length of the photograph, the elevations of the ends of the line, and the ground distance of $D = 10,398$ feet are given It is required to determine the exact flying height of the camera station. From Equation 8,

$$d = \sqrt{[(-79\ 83) - (-20\ 33)]^2 + [(-89\ 38) - (+73\ 51)]^2}$$

$$d = 173\ 42 \text{ mm}$$

From Equation 7,

$$H' = 607 + (10,398\ 210.06/173\ 42) = 13,202 \text{ feet.}$$

Then, as in Section 3, repeating only the computed quantities

	$H'-h$	$(H-h)/f$	X	Y	$X_A - X_B$	$Y_A - Y_B$	D'
A	12,510	59 554	−4754	−5323	−3527	−9761	10,379
B	12,681	60 368	−1227	+4438			

From Equation 9,

$$H = 607 + (13,202 - 607)\ 10,398/10,379 = 13,225$$

and

	$H-h$	$(H-h)/f$	X	Y	$X_A - X_B$	$Y_A - Y_B$	D
A	12,533	59 664	−4763	−5333	−3533	−9779	10,398
B	12,704	60,478	−1230	+4446			

Therefore, the exact flying height is 13,225 feet since that value gives the correct exact ground distance $D = 10,398$ feet.

5. Image Displacement Due to Relief[2]

It was pointed out in a previous chapter that images on a truly vertical photograph are displaced along lines radial from the principal point due to the elevations of the respective objects. The relation of the amount of displacement (e) to the flying height H, the elevation of the object h, and the radial distance r of the image from the principal point is discussed here The relation between any photograph distance, as r or d, and the focal length, f, is stated algebraically by Equation 4

In Figure 2, a represents the image on a truly vertical photograph of ground point A situated at an elevation h above the datum plane, a' represents the image of

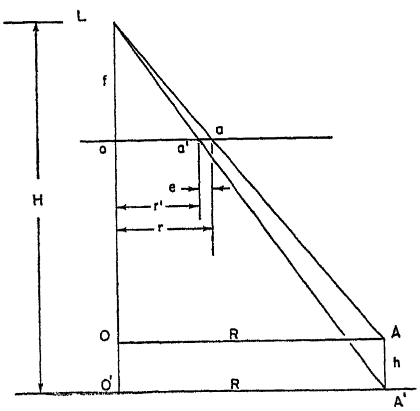


Fig 2

[2] Church, Earl, *Analytical Computations in Aerial Photogrammetry*, Edwards Brothers, Ann Arbor, Michigan, 1936

a point A' vertically beneath A at zero elevation; o represents the principal point of the photograph; and LO' or H the elevation of the camera station.

The distance $e = aa'$ is known as the displacement of the image of A caused by its elevation h.

Let $r = oa$, $r' = oa'$, and $R = OA = O'A'$ Then in the similar triangles Loa and LOA,

$$\frac{r}{R} = \frac{f}{H - h} \text{ or } \quad Rf = r(H - h)$$

and in the similar triangles Loa' and $LO'A'$,

$$\frac{r'}{R} = \frac{f}{H} \quad \text{or} \quad Rf = r'H$$

from which may be written

$$r(H - h) = r'H \quad \text{or} \quad \frac{r}{r'} = \frac{h}{H - h}$$

Then by principles of ratio and proportion

$$\frac{r - r'}{r} = \frac{H - (H - h)}{H}$$

and

$$\frac{r - r'}{r} = \frac{H - (H - h)}{H}$$

Solving for $(r - r')$ and substituting e for it,

$$e = rh/H \qquad \text{Equation 10}$$

and

$$e = r'h/(H - h). \qquad \text{Equation 11}$$

Then Equation 10 may be solved for h to give

$$h = eH/r. \qquad \text{Equation 12}$$

Equation 10 is used for computing the displacement on a photograph from an image to its datum position, while Equation 11 is useful in displacing the map position of a point to a photographic position in the preparation of a templet sometimes used in connection with rectification. The third form of the equation is useful for determining elevations of objects from measured displacements of images relative to their plotted or map positions. It is also significant that the displacement of images due to an incorrect flying height follows the relation of Equation 10 where h represents the actual flying height minus the correct or desired flying height.

Example 1. If the image of hilltop appears at a radial distance of 5 inches from the principal point of a photograph whose flying height is 14,000 feet, find the photograph position of a point 800 feet vertically beneath the hilltop assuming no tilt.

Solution.

$$e = rH/h = 5 \times 14{,}000/800 = 0.286 \text{ inches.}$$

The required point is therefore a distance of 0 286 inches from the image in the direction of the principal point

Example 2 A radial plot of multiple lens photographs shows the map positions of a mountain top and the photograph centers as well as other features The distance from a photograph center and the position of the mountain measures 9.88 inches on the plot and 12 17 inches on the photograph If the flying height were 10,000 feet, the photograph were truly vertical and at the same scale as the plot, find the height of the mountain.

Solution

$$e = r - r' = 12.17 - 9.88 = 2.29 \text{ inches}$$

$$h = eH/r = 2.29 \times 10{,}000/12.17 = 1882 \text{ feet, answer.}$$

6 Parallax* and Elevations‡

The subject of parallax and elevations is dealt with by Professor Church in a simple, thorough, and fundamental manner. The following quotation and figure are used with the symbols slightly changed so as to be consistent with foregoing material.[2]

Let us suppose that two photographs were exposed from points L and L', respectively, situated at exactly equal elevations H above sea level or some other definite datum plane, and that the photographs were exposed with the camera axis truly vertical

The distance LL', called the air base, is designated by B, as shown in Figure 3.

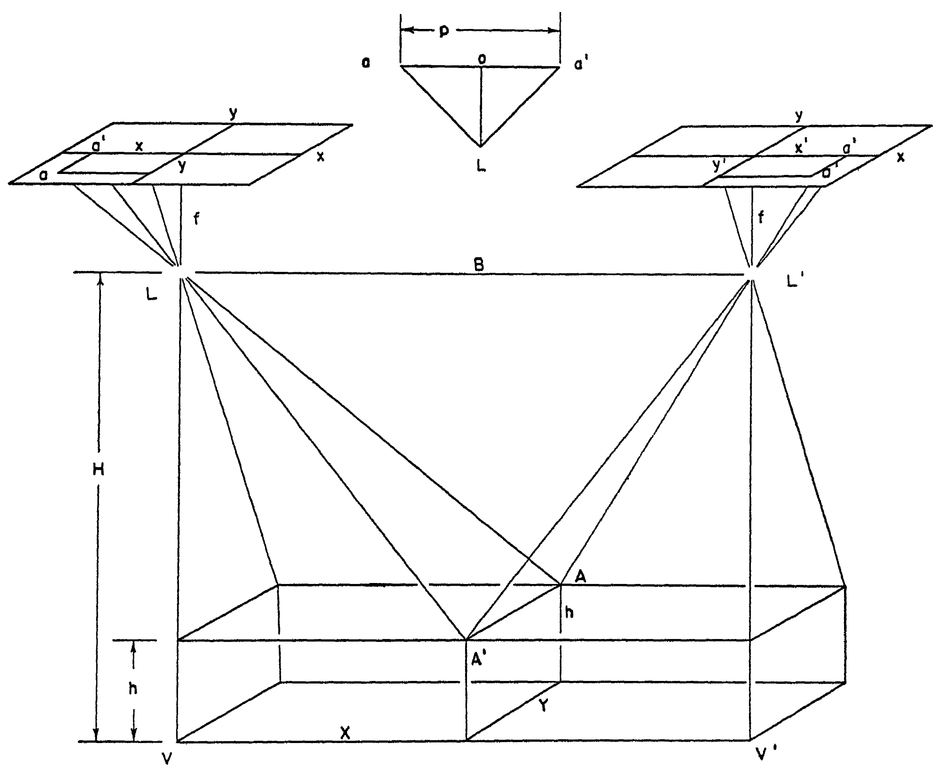


Fig. 3

* This is also discussed quite thoroughly in Chapter VII

‡ Elevation determination is also discussed in Chapter XI.

The height of flight H is the distance LV or $L'V'$. The space coordinates of some point A, based upon VV', the line of flight projected to the datum plane, as the x-axis, and the point V as the origin, are called X, Y, h, as shown To make the case a perfectly ideal one, the x-axes of the photographs are assumed to coincide with the line of flight The coordinates of the image a on the left-hand plate are called x, y, and those of the image a' on the right-hand plate are called x', y'.

By similar triangles it is clear that

$$\frac{y}{Y} = \frac{f}{H - h} \quad \text{and} \quad \frac{y'}{Y} = \frac{f}{H - h}.$$

Therefore, $y = y'$ This gives rise to the statement frequently heard, "There is no parallax perpendicular to the line of flight"

The algebraic difference p between the abscissas x and x' measured parallel to the line of flight, is called the parallax for the point A. That is

$$p = x - x' \qquad \text{Equation 13}$$

Now if the small triangle $a_1'L'o'$ is placed beside the triangle a_1Lo forming the triangle $a_1'a_1L$ as shown, the side a_1a_1' equals p Moreover, this triangle is similar to $LL'A_1$

Then $p/b = f/(H-h)$ But $x/X = f/(H-h)$ and $y/Y = f/(H-h)$ also Therefore, $x/X = y/Y = p/B$ Hence there follow what are known as the parallax formula.

$$X = \frac{B}{p}x \qquad \text{Equation 14a}$$

$$Y = \frac{B}{p}y \qquad \text{Equation 14b}$$

$$H - h = \frac{B}{p}f. \qquad \text{Equation 14c}$$

Thus, the ground coordinates with respect to origin V and x-axis VV', and the elevation of the point, can be found from these very simple relations.

If the photographs contain within the area of overlap a point A whose elevation h is known, the parallax can be calculated for this point from the relation

$$p = \frac{Bf}{H-h}. \qquad \text{Equation 15}$$

Then if the stereoscope measures parallax differences, the difference can be measured between the parallax for point A and some undetermined point This gives the parallax for the undetermined point and the elevation can be found at once from Equation 14c.

Let h_1 and h_2 represent the elevations of two points whose images appear on a pair of photographs, h_1 being the smaller. Substituting these values in Equation 14c, solving for them, and representing their respective parallax values by p_1 and p_2 we have

$$h_1 = H - \frac{Bf}{p_1} \quad \text{and} \quad h_2 = H - \frac{Bf}{p_2}. \qquad \text{Equation 16}$$

If dh represents the difference in elevation between the two points,

$$dh = \left(H - \frac{Bf}{p_2}\right) - \left(H - \frac{Bf}{p_1}\right)$$

$$dh = \frac{(p_2 - p_1)Bf}{p_1 p_2}. \qquad \text{Equation 17}$$

Let dp represent the difference in parallax, then

$$dp = p_2 - p_1 \quad \text{and} \quad p_2 = p_1 + dp$$

$$dh = \frac{dpBf}{p_1(p_1 + dp)}. \qquad \text{Equation 18}$$

Approximate elevation differences are sometimes desired from photographs when the values for B, H, h_1 and p_1 are not available from the data given. A useful relationship may be obtained by assuming $h_1 = O$. Then, from Equation 3,

$$b/B = f/H \quad \text{and} \quad B = bH/f$$

where b is the image of the ground distance B Also, by substitution in Equation 15 we find that $b = p_1$, whence Equation 18 may be written through substitution for B and p_1 and simplifying·

$$dh = \frac{dpH}{b + dp}. \qquad \text{Equation 19}$$

Equation 19 is exactly true when the lower point is at datum level. The quantity dp may be readily measured on a pair of photographs, but the value b, though apparently simple to measure, may be in large error because of relief displacement on one or both photographs Hence the term dp (which is a relatively small quantity) in the denominator is often omitted The flying height H is the elevation of the camera above the lower point but a nominal value is often the best that is available. Since neither b nor H are critical values, the relation is surprisingly accurate for small elevation differences where the two images are quite near each other to minimize the effect of tilt

Example 1. Find the elevation of a point whose images appear on two vertical photographs when the measured parallax difference is 0 121 inches, the focal length is 8 25 inches, the flying height is 14,000 feet, the base length is 5000 feet, and the elevation of the lower point is 700 feet above sea level

Solution. By Equation 15,

$$p_1 = (5000 \times 8.25)/(14{,}000 - 700) = 3.102 \text{ inches}$$

$$p_2 = 3\ 102 + 0\ 121 = 3\ 223 \text{ inches}$$

From Equation 14c,

$$h_2 = 14{,}000 - (5000 \times 8.25/3\ 223) = 1200 \text{ feet, answer.}$$

Example 2. Solve the above problem if the photograph base length measured 3 102 inches

Solution· By Equation 19,

$$dh = (0.121 \times 13{,}300)/(3.102 + 0.121) = 499 \text{ feet}$$

$$h_2 = 700 + 499 = 1199 \text{ feet, answer.}$$

7. Image Displacement Due to Tilt

Methods for producing many maps from vertical photographs are based upon the assumption that the axis of the camera was pointed vertically downward at the instant of exposure. Any departure from this ideal condition is commonly called tilt and is technically defined in either of two ways: (1) the angle at the camera station between the photograph perpendicular and a vertical line and (2) the dihedral angle between the plane of the photograph and a horizontal plane.

It is impossible at present to obtain truly vertical aerial photographs in practice The chief reason is the uneven characteristic of the atmosphere which affects the movement of an aircraft in flight. Not only is the aircraft disturbed from a constant level flight, but also inertia forces are set up by the irregular movement which affect the usual trustworthiness of level bubbles Also, in addition to the human error of the pilot encountered in maintaining the aircraft in steady flight, is added that of the photographer who must adjust the camera in its mount to maintain its vertical alignment as indicated by the level bubbles.

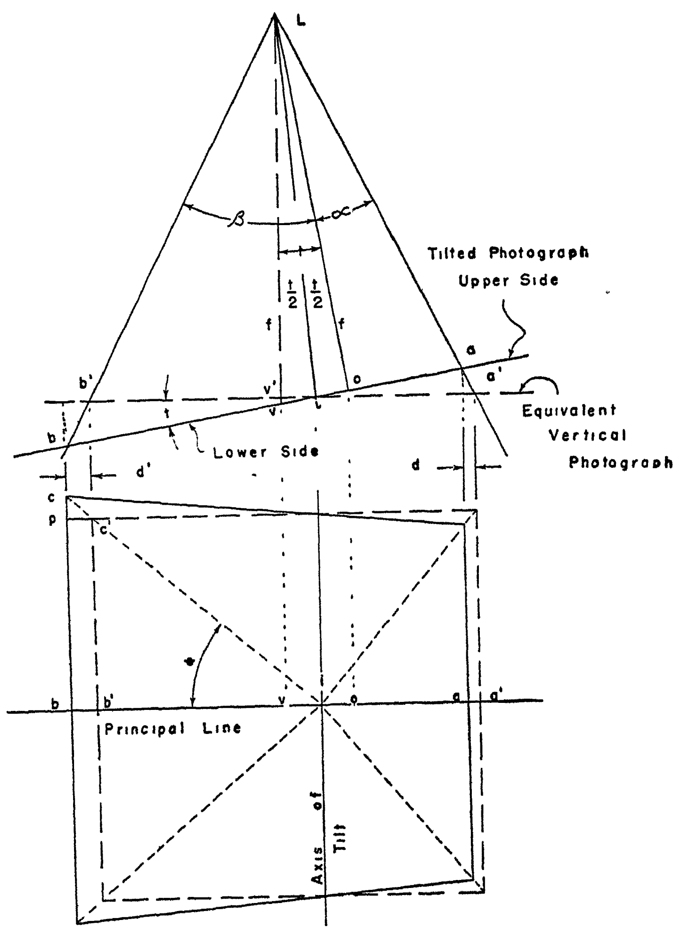


FIG. 4

The usual amount of tilt encountered is stated in the following quotation [3]

.. Under good air conditions and by skillful operation of both airplane and camera, it has been demonstrated that a set of photographs covering large areas can be taken with but small amounts of tilt Seldom is a vertical tilted more than 3°, usually of a lot taken for mapping, more than half will be tilted less than 1° In letting contracts for aerial photographs to be used for mapping it is common practice to limit the allowable tilt to 2°. ..

New devices and methods are steadily being discovered and improved for reducing the amount of tilt

Figure 4 shows a sectional view of the contact print of a tilted photograph in the principal plane, and beneath it a plan view, together with a corresponding equivalent vertical photograph which would have an equal focal length and would be exposed at practically the same point in space. The following notation is employed·

t—angle of tilt
o—principal point
v—nadir point
i—isocenter
a, *b*—images of ground points on the sides of a square field and in the principal plane
L—lens
Lo—photograph perpendicular for tilted photograph
Lv'—photograph perpendicular for equivalent vertical photograph
f—focal length of photograph
a', *b'*, *v'*—corresponding points on the equivalent vertical photograph.
d—displacement parallel to the principal line of an image lying on the *upper* side of the photograph and due to tilt
d'—displacement parallel to the principal line of an image lying on the *lower* side of a photograph and due to tilt

It is assumed that the tilt of the photograph and the position of the principal line are known. The determination of these quantities is discussed in later chapters.

The distances from the principal point to the nadir point and to the isocenter are important fundamental relationships.

$$ov = f \tan t$$
$$oi = f \tan t/2.$$

It is to be noted that the nadir point lies on the lower side of the tilted photograph in relation to the principal point

The change of position or displacement of any image a on the upper side of the tilted photograph as compared to the equivalent vertical photograph is:[3,4,5]

$$d = ia' - ia \qquad \text{Equation 20}$$
$$d = (v'a' - v'i) - (io + oa). \qquad \text{Equation 21}$$

From right triangle relationships, it is evident that

[3] Bagley, James W., *Aerophotography and Aerosurveying*, McGraw-Hill Book Company, Inc, New York and London, 1941, Chapter VII.
[4] Sharp, H Oakley, *Photogrammetry*, third edition, John Wiley and Sons, 1943
[5] Breed and Hosmer, *Higher Surveying*, fifth edition, John Wiley and Sons, 1940, pp. 392–397.

$$v'a' = f \tan (\alpha + t)$$
$$v'i = f \tan t/2$$
$$io = f \tan t/2$$
$$oa = f \tan \alpha$$

where oa is measured on the tilted photograph and which determines the value of the angle α Then by substitution in Equation 21 and simplifying,

$$d = f[\tan (\alpha + t) - \tan \alpha - 2 \tan t/2]. \qquad \text{Equation 22}$$

By a similar process of reasoning we may also obtain the displacement d' for any image b on the lower side of the photograph

$$d' = f[\tan \beta - \tan (\beta - t) - 2 \tan t/2] \qquad \text{Equation 23}$$

where ob is measured and

$$\tan \beta = ob/f.$$

It is to be noted without further proof that all images lying on a line parallel to the axis of tilt will have equal tilt displacements parallel to the principal line. Also, an image (i e its position if the photograph were not tilted), and the isocenter, lie on the same straight line. In other words, tilt displacements are radial from the isocenter. Further, images on the upper side of the tilted photograph have been displaced *inward*, and images on the lower side have been displaced *outward*

Let c be any image not on the principal line. Then according to the foregoing remarks, it will have been displaced to c' on a line radial from the isocenter and an amount $d = cp$ measured on the principal line and computed by Equations 22 or 23. If the line $ic'c$ makes the angle θ with the principal line, then in the right triangle cpc', the resultant radial tilt displacement is

$$d_r = cc' = d/\cos \theta. \qquad \text{Equation 25}$$

Tilt displacements may also be computed by the use of somewhat simpler equations originally given by Mr. R O. Anderson.[6] The following derivation is that of Mr. Jack Rihn In the oblique triangle iaa' of Figure 4, it is evident by the law of sines that

$$\frac{ia'}{\cos \alpha} = \frac{ia}{\cos (\alpha + t)}$$

$$ia' = \frac{ia \cos \alpha}{\cos \alpha \cos t - \sin \alpha \sin t}$$

$$ia' = \frac{ia}{\cos t - \sin t \tan \alpha}.$$

But

$$\tan \alpha = \frac{ia - io}{f} = \frac{ia - f \tan t/2}{f}.$$

Substituting for $\tan \alpha$ and re-arranging terms we may write

[6] Anderson, Ralph O., *Applied Photogrammetry*, Edwards Brothers, Inc., Ann Arbor Michigan, 1939, pp 90–92

$$ia' = \frac{(ia)f}{f(\tan t/2 \sin t + \cos t) - ia \sin t}.$$

And since

$$\tan t/2 \sin t + \cos t = 1,$$

$$ia' = \frac{(ia)f}{- ia \sin t} \qquad \text{Equation 26}$$

Substituting this value in Equation 20 and simplifying,

$$d = \frac{(ia)^2}{\dfrac{f}{\sin t} - ia}. \qquad \text{Equation 27}$$

And by similar logic, it may be shown that

$$d' = \frac{(ib)^2}{\dfrac{f}{\sin t} + ib} \qquad \text{Equation 28}$$

which are equal to Equations 22 and 23 respectively

The latter two expressions are combined by Mr. Anderson into the approximate form

$$d = (ia)^2(\sin t)/f \qquad \text{Equation 29}$$

designed for use on both upper and lower sides of the photograph by substituting ib for ia This equation may be sufficiently accurate for practical application with single lens photographs and moderate amounts of tilt

Example Compute the radial displacement for an image on the lower side of a photograph due to a tilt of 3° when the distance from the principal point to the image measured parallel to the principal line and the image is 53°, and the camera focal length is 200 mm

Solution Using Equation 28,

$$ib = ob - oi = ob - f \tan t/2 = 75 - 200 \tan 1°30'$$
$$ib = 69.762 \text{ mm}$$

$$d' = \frac{69\ 762 \times 69\ 762}{\dfrac{200}{.05234} + 69\ 762} = 1\ 251 \text{ mm}$$

Using Equation 23,

$$\tan \beta = 75/200 = \ 37500$$
$$\beta = 20°33.4'$$
$$\beta - t = 17°33.4', \tan(\beta - t) = .31638$$
$$2 \tan t/2 = 2 \times .02619 = .05238$$
$$d' = 200(.37500 - \ 31638 - \ 05238) = 1.248 \text{ mm.}$$

(When eight place tables are used with Equations 23 and 28 the results are identical $d' = 1\ 250694$ mm.)

Using Equation 29,

$$d' = (69.762 \times 69.762 \times 05234)/200 = 1\ 274 \text{ mm}$$

The component of the displacement which is parallel to the principal line is then 1.251 mm The total radial displacement is, from Equation 25,

$$d_r = 1\ 251/\cos 53° = 2\ 079 \text{ mm}$$

Hence, the image has been displaced because of tilt a distance of 2 079 mm *outward* on a line radial from the isocenter.

8. Effects of Tilt on Map Compilation

The previous section showed how images appearing on the upper side of a tilted photograph are moved toward the isocenter and those appearing on the lower side are moved away from the isocenter This of course tends to compress or decrease the photograph scale on the upper side, and expand or increase the scale of the lower side. The scale along the axis of tilt is equal to that of the equivalent vertical photograph, and for any line parallel to the axis of tilt the scale is constant throughout its length The scale changes constantly along any line not parallel to the axis of tilt and the rate of change in scale is maximum along all lines perpendicular to the axis of tilt.

There is no single point on a tilted photograph from which the resultant of tilt and relief displacements is radial. Images are displaced due to relief and scale error along lines which pass through the nadir point, while they are displaced from the isocenter, due to tilt. The comparative relation of the amounts of displacements is shown by Equations 10 and 29. displacements due to tilt vary directly as the *square* of the distance of the image from the axis of tilt; while displacements due to relief or incorrect flying height vary directly with the distance (on an untilted picture) of the image from the nadir point The following numerical example illustrates this feature:

Example. A photograph whose focal length is 8 inches is tilted 2°40′ in the direction indicated in Figure 5. At the point p appears the image of the top of a chimney, the base of which is obscured by an intervening hill If the chimney is 1000 feet high and the camera was at an elevation of 15,000 feet, indicate the relative position of the image of the base of the chimney on an equivalent untilted photograph

Solution: It is first necessary to find the location of the image of the top of the chimney on the untilted photograph by means of a tilt displacement determination. It is obvious that the image lies on the upper side of the photograph calling for the use of Equation 27 (or Equation 22)

$$oi = 8 \tan 1°20' = 0.186 \text{ in}$$
$$ov = 8 \tan 2°40' = 0.373 \text{ in.}$$
$$ia = 3 + 0.186 = 3\ 186 \text{ in}$$

$$d = \frac{3.186 \times 3.186}{\dfrac{8}{.04653} - 3.186} = 060 \text{ in.}$$

The component of the displacement is therefore .060 inches The angle θ for Equation 25 may be measured with a protractor on the photograph (or on a sketch drawn to scale) and will be found to be equal to 51°28′. The radial displacement is then

$$d_r = .060/\cos 51°28' = .097 \text{ in.}$$

To correct the image for tilt, it therefore must be displaced a distance of 097 in outward or away from the direction of the isocenter. The position of the base of the chimney

is toward the nadir point from the corrected top position and the amount is determined from Equation 10 The quantity r is the distance from the nadir point to the image of the top after it is corrected for its tilt displacement From the photograph (or the scaled drawing) the value r will be measured and found to be equal to 5 33 inches.

$$e = 5.33 \times 1000/15{,}000 = 0.355 \text{ in}$$

Therefore the equivalent vertical photograph position of the base of the chimney may be located by measuring (1) a distance of 097 in from the image of the top outward on a line through the isocenter, and thence (2) a distance of 355 inches toward the nadir point

Along with these discrepancies due to tilt follows the obvious characteristic that the angles between rays drawn from the principal point, nadir point, isocenter, or any other point, will not represent true ground horizontal angles. If there were no tilt, angles measured at the principal point would be true; if the ground were flat, angles measured at the isocenter would be correct on a tilted photograph; but when both relief and tilt are present there is no correct point. This feature may be disturbing to those working with the radial line process except for the fact that the errors are very small for moderately tilted photographs, and the discrepancy is minimized by wide angle cameras Furthermore, angular errors due to irregular film, paper shrinkage and drafting are usually larger than those caused by tilt. Large differences in elevation combined with tilt increase angular error, but moderate changes in relief have practically no effect upon success of the radial line system.

Perhaps the most noticeable effect of tilt upon the preparation of maps from near vertical photographs is that pertaining to the determination of elevations. Parallax equations are based upon the initial assumption of truly vertical photographs As shown by Equation 19, the difference in elevation between two points varies nearly in direct proportion to the difference in parallax measured from the two images. A determination of tilt is seldom made and is often unobtainable because of the lack of ground data However, images are displaced on the two photographs in unrelated amounts and directions and a parallax measurement assumes that all differences are due to elevation. Also, the line of flight as indicated by the principal points is usually the basis of orientation for parallax measurements. But the true line of flight is between the nadir points which are unknown and which may change the orientation of the photographs materially. The net result is that tilt may affect parallax differences, and since these differences are relatively small dimensions, the resulting elevation error may be alarming.

The use of precision stereoscopic plotting equipment for the compilation of maps is usually not hampered by the presence of tilt Means are usually provided for the effective removal of tilt in the process of photograph orientation.

On a pair of truly vertical photographs, common images are equal distances from the line of flight; that is, there is no y-parallax. However, a similar measurement on tilted photographs will indicate a noticeable presence of y-parallax. This fact is sometimes used in (1) the detection of tilt, (2) some analytical methods for tilt determination, and (3) as a criterion for the orientation of photographs in precise stereoscopic plotting instruments.

Tilt also affects the construction of mosaics from vertical aerial photographs. It is obvious from the above discussion that the scale along the edge of a photograph may be quite different from that of an adjoining picture. Tilt amounts of 1° or less are tolerable, but approximate rectification is frequently performed as a correcting measure for excessive amounts.

Another effect of tilt is the change of shapes and areas, as demonstrated in Figure 4 and in a previous chapter

9 Exact Ground Distance and Flying Height, Tilted Photograph

The above discussions regarding displacements of images due to relief and tilt are suitable for the illustration of the principles involved and for use where the limits of accuracy demand no more than that obtainable by graphic construction on paper prints It is a well known fact that the photographic process is capable of recording image positions to a higher grade of precision than one is able to maintain by graphic means Also, the end product of any compilation is the determination of distances between points Consequently, it is shown here how distances and flying heights may be computed accurately from the rectangular coordinates of images on a tilted photograph measured to the nearest 0 01 millimeter. The figure and system is that of Professor Church.[1]

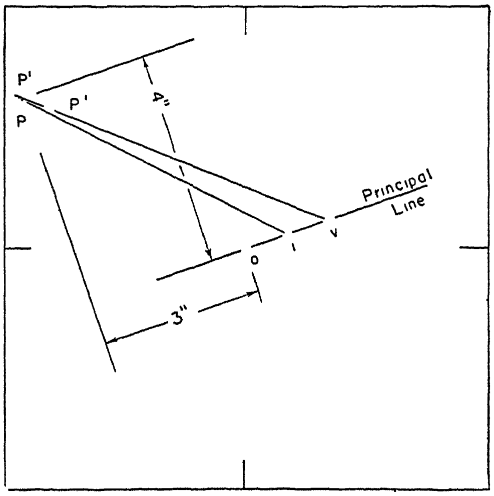


Fig. 5

The photograph coordinates, x, y, of the images may have been measured with the principal point as origin and with the axes defined by the fiducial marks It is assumed that the tilt t, the swing s, the flying height H, the camera focal length f, and the elevations of the objects, h_A, h_B, are known It is desirable that the image coordinates, x', y', be expressed using the nadir point as origin and the direction vo as the positive y'-axis. The values, x', y', may be measured directly or computed from the following equations for rotation and translation.

$$x' = -x \cos + y \sin s$$
$$y' = -x \sin s - y \cos s + f \tan t \qquad \text{Equations 30}$$

where the definition of swing s is the angle at the principal point of the photograph measured clockwise from the positive y-axis to the nadir point.

In Figure 6 the coordinates, x', y', of the point p are wp and vw, respectively. The line kw is constructed horizontal. Since wp is also truly horizontal, the triangle kwp may be considered as the plane of a truly vertical photograph for the image p, whose focal length is Lk, and where the coordinates of the point p are wp and kw. But it is evident from the geometry of the figure that

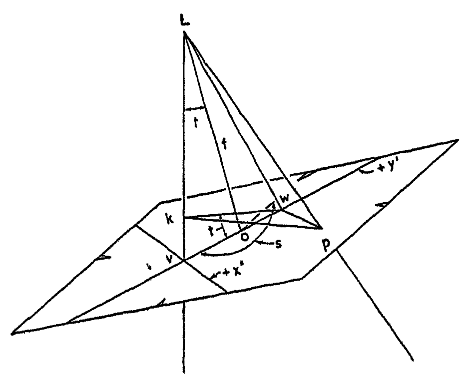


Fig. 6

$$Lk = (f \sec t - y' \sin t)$$
$$wp = x'$$
$$kw = y' \cos t.$$

From these values derived separately for two images, we may proceed in exactly the same manner as in Section 2 to find a corresponding set of ground coordinates for each point and thence the ground distance between the points

An example is shown where the image coordinates of the two images are given with respect to the principal point and the fiducial marks are known as well as the other necessary data It is required to find the exact ground distance between the two objects.

Given

	x	y	
a	− 79 83 mm	− 89 38 mm	$t = 2°33\ 3'$
b	− 20 33	+ 73.51	$s = 250°10\ 9'$
			$H = 13{,}376$ feet
			$f = 210\ 06$ mm
			$h_A = 692$ feet
			$h_B = 521$ feet

Solution

Tabular values needed

$\tan t = .04463$	$ov = 9.38$ mm	$\sin s = -.94080$
$\sin t = .04458$	$f \sec t = 210.27$ mm	$\cos s = -.33899$
$\cos t = 99901$		

Rotation and translation

	$-x \cos s$	$+y \sin s$	$-x \sin s$	$-y \cos s$	x'	y'
a	−27 06	+84 09	−75 10	−30 30	+57 03	−96 02
b	− 6 89	−69 16	−19 13	+24 92	−76 05	+15 17

	$y' \cos t$	$y' \sin t$	$f \sec t - y' \sin t$
a	−95 92	−4 28	214.55
b	+15 15	+0 68	209 59

	$H-h$	$(H-h)/(f \sec t - y' \sin t)$	X'	Y'	$\Delta X'$	$\Delta Y'$	D
A	12,684	59 119	+3372	−5671	+8036	−6600	10,399
B	12,855	61 334	−4664	+ 929			

Hence the exact ground distance between the objects A and B is 10,399 feet.

The method of finding the exact flying height when a distance is known was demonstrated in Section 4 for an untilted photograph It is apparent that the same system is employed for a tilted photograph, incorporating the principles demonstrated above in this section. The only change to be suggested is that in obtaining a trial flying height H' from Equation 7, the work may be shortened somewhat by using the average value of $(f \sec t - y' \sin t)$ in place of the value f.

PROBLEMS

1. Using a camera with a 6 54 inch focal length, at what height should it be flown over flat terrain in order to obtain photographs with a scale of 1:15,840? (8633 feet)

2. On a photograph of flat terrain the distance between two images is scaled at 3 46 inches. On a map whose scale is 1:31,680 the distance between the same two points is 5 72 inches Compute the scale of the photograph. (1:52,373)

3. Given the following conditions. the scale of a vertical aerial photograph is 1:15,000; the focal length of the lens used is 8 inches; the image of the top of a hill appears in the photograph 3 42 inches from the principal point; the height of the hill is 400 feet above the ground elevation at the center of the photograph. Compute the relief displacement of the hilltop on the photograph. (0 274 in.)

4. Given the following conditions· the scale of a vertical aerial photograph is 1:12,000; the focal length of the lens is 8 inches; on the photograph the image of a flagpole base is 3.92 inches and the image of the top 3 96 inches measured radially from the principal point. Determine the height of the flagpole (80 8 feet)

5. Find the elevation of a point for which the difference in parallax measures 1.37 mm. when H is 7942 feet, B is 2576 feet, f is 209 5 mm , and the lower point is on ocean shore line (157 feet)

6. A parallax difference of 0 32 mm. is measured between the image of the top of a water tank and an image near the base of the tank. The nominal flying height is 30,000 feet and the distance between the principal points and transferred principal points averages 79 2 mm. What is the approximate height of the tank? (121 feet)

7 Given: an image lies on the upper side of a tilted photograph a distance of 4.23 inches from the axis of tilt; the angle at the isocenter between the image and the principal line is 45°; the focal length of the air camera is 6 27 inches; and the tilt of the photograph is 2° 18′. (a) Compute the displacement of the image due to tilt (b) What would the displacement be on the lower side? (a) 0 1665 in inward (b) 0 1577 in outward

8. There is a tilt of 2° 52′ present in a multiple lens photograph whose focal length is 209.5 mm How much and in what direction has an image been displaced radially because of tilt, which lies a distance of 379 3 mm. from the axis of tilt on the upper side and when the angle at the isocenter between the principal line and the image is 35° 29′? (46 44 mm. inward)

9. Find the exact ground distance between two points whose images appear on an untilted photograph taken from an altitude of 13,781 feet, given the following data:

	x	y		h	
g	+243.8 mm.	−415.6 mm.	G	10 feet	$f=209$ 5 mm.
h	+203.0	+380 5	H	34	

($GH=52{,}357$ feet)

10. Solve the above problem for exact flying height if it is given that GH is 52,356 feet. ($H=13{,}781$ feet)

11. Find the exact ground distance between the two ground points whose images appear on a tilted photograph taken from an altitude of 13,980 feet from the given data:

	x	y		h	
					$t = 1° \ 06 \ 9'$
a	+326 8	+140.1 mm	A	21 feet	$s = 19° \ 02'$
f	−357 8	+347.9	F	5	$f = 209.5$ mm.

($AF = 46{,}747$ feet)

12 Solve problem 11 for exact flying height when it is given that the ground distance between the points is 46,749 feet and the flying height is unknown.

($H = 13{,}980$ feet)

AN IMPROVED METHOD OF DETERMINING TILT FROM SCALE CHECK LINES

Jack Rihn

There was a little girl
Who had a little curl
  Right in the middle of her forehead
When she was good
She was very very good
  But when she was bad, she was horrid

TILT in vertical aerial photographs is like the little girl, when it is bad, it is horrid. More and more, photogrammetrists are demanding a clear, simple but accurate method of calculating tilt The need for restitution of tilted pictures greatly increases this demand. It is partly answered by the system originated by Mr. R O Anderson. However, under some conditions the results obtained by this system are too approximate for most applications An improvement has been made on the original that reduces some of the disturbing factors, with attendant improved accuracy and speedier results

The description of the improved method that follows has been divided into three parts·

Part I An outline of the steps of the procedure and an example, for those interested in results only.
II. Mathematical proof of the improved method.
III. Comparison of Anderson and Rihn methods under unfavorably conditioned control.

Part I. Steps in the Procedure

1. Select three control points whose elevations are known. Although desirable, the position of the points on the photograph need not be symmetrical about the center.
2 Set the datum plane at the elevation of the lowest control point
3 Determine (H) the height of the air-plane above the datum plane from·

$$H = \frac{f}{S} - e$$

where (f) is the focal distance, (S) a fraction, is the map scale or intended scale of photograph, and (e) is the height of the lowest control point above the map datum elevation.

4 Find (r) the relief displacement of the two control points whose elevations are above the datum plane from:

$$r = \frac{Rh}{H}$$

where (R) is the radial distance from the center of the photograph to the image of the control point and (h) is the elevation of the control point above the datum plane.

5. Measure the two displacement distances radially toward the center of the photograph. The sides of the triangle formed by these two new points and the control point having the lowest elevation are termed "datum lines."

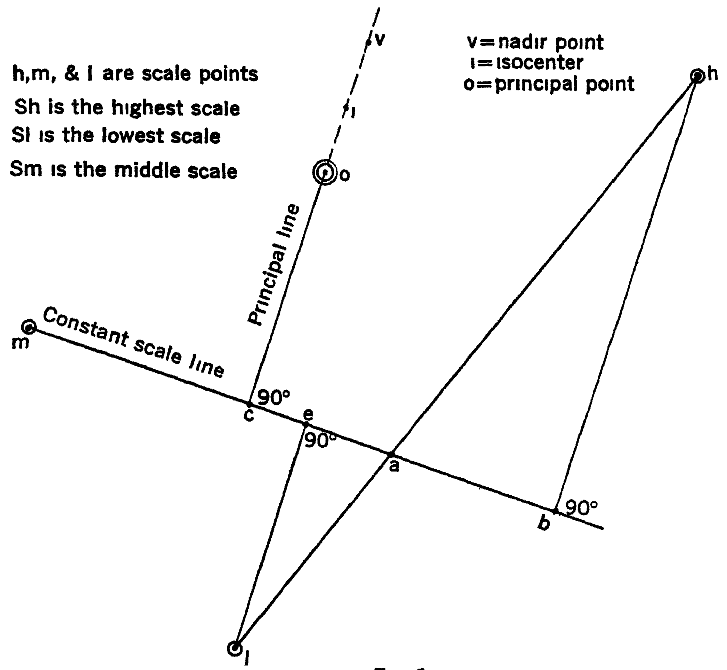


Fig. 1

CONSTRUCTION LINES TO FIND TILT AND PRINCIPAL LINE

6. Measure and record the length (P) of each datum line.
7 Determine and record the ground length (G) between each pair of control points.
8. Calculate and record the scale ratio $S=P/G$ for each of the datum lines. (P) and (G) need not be in the same units. The decimal point of the scales may be moved to any position convenient for arithmetic solution provided the movement is consistent for all scales.
9. Locate the scale point for each datum line as follows:
   From the center of the photograph drop a perpendicular to the datum line. Measure the distance from the foot of the perpendicular to the nearest end of the datum line Measure off this same distance on the datum line from the other end. This will be the scale point. See Figure 2. If the perpendicular falls on an extension of the datum line, the scale point will also be on an extension at the other end. In both cases the mid-point of the datum line will be midway between the foot of the perpendicular and the scale point.
10. In figure 1; h, m, and l are three scale points. Find point "a" on line hl from.

$$ha = hl\frac{S_h - S_m}{S_h - S_l}, \qquad \text{or} \qquad la = hl\frac{S_m - S_l}{S_h - S_l}.$$

11. Draw line ma and drop perpendiculars to the line ma from the center of the photograph "o" and from either h or l whichever would give the longest perpendicular.
12. Compute the rate of change of scale (dS) from:

$$dS = \frac{S_h - S_m}{hb}, \qquad \text{or} \qquad dS = \frac{S_m - S_l}{le} \cdot$$

13. Determine the scale at the center (S_o) from

$S_o = S_m + (oc)(dS)$ when "o" is the on the same side of line ma as "h"

$S_o = S_m - (oc)(dS)$ when "o" is on the same side of line ma as "l"

14. Solve for the angle of tilt (t) in the equation:

$$\sin t = \frac{fdS}{S_o} \cdot$$

15. The isocenter "i" and the nadir point "v" will be on the principal line oc The distance to i and v will be measured from o toward the area of high scales Solve:

$$oi = f \tan \frac{t}{2} \qquad (\tfrac{1}{2} f \sin t \text{ may be used})$$

$$ov = f \tan t \qquad (2oi \text{ may be used}).$$

16. The result may be improved by a second solution observing the following corrections:
    a. Use (S_i) instead of (S_o) in step 14

    $$S_i = S_o + (oi)(dS)$$

    b. Obtain a better value for H from:

    $$H = \frac{f}{S_i} \text{ in step 3.}$$

    c Use the nadir point (v) instead of the principal point (o) to measure (R) in step 4.
    d. Use (v) instead of (o) as the origin in step 5.
    e. Use the isocenter (i) instead of (o) as the origin of the dropped perpendiculars in step 9. This is the most important correction.
    f. In step 4, a better value for (r) is obtained from

    $$r = \frac{hR(f \pm x \sin t)}{Hf} \qquad \begin{matrix}(+ \text{ on nadir point side}) \\ (- \text{ on principal point side})\end{matrix}$$

    where (x) is perpendicular distance from image point to axis of tilt. Ignore this correction if h is less than 150 feet.

When applying this method, it would be wise to select control lines that are at least four inches in length, preferably six inches or more. Although a good solution can be obtained even though the control points or lines are very unsymmetrical in relation to the center of the photograph, a better solution might be obtainable by using points that are more symmetrical.

It is possible and sufficient to make measurements to the nearest 1/200 of an inch, i.e., 0.005″ or approximately 0 1 millimeter. Corresponding ground distances should be to the nearest foot. Under some conditions such as control lines less than four inches, a tilt angle of less than one degree, or a focal length greater than twelve inches, more precise measurements are necessary. With good

instruments and some care, distances can be found to the nearest 1/500 of an inch, i.e , 0 002″ or approximately 0.05 millimeter. Corresponding ground distances should be to the nearest half foot Scale ratios should have one significant figure more than the measurement with the most significant figures. The allowable error in the elevation of control points will vary depending on the circumstances. For instance a ten foot error in the elevation will affect the relief displacement distance less than 0.005″ if the scale is 1:20,000 and if the image point is within 6″ on the center, but on a multiple lens photograph with the image point within 15″ of the center and at a scale of 1:10,000 a three foot error in elevation will affect the relief displacement distance the same amount.

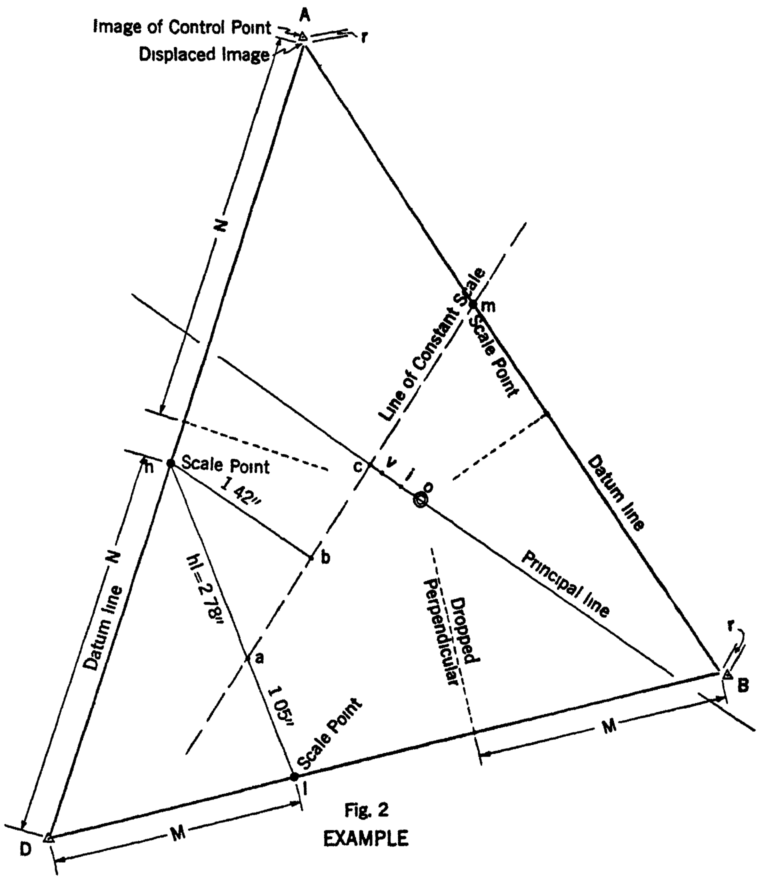


Fig. 2
EXAMPLE

The chief source of error that a second determination will eliminate is the error in using the principal point and not the isocenter as the origin of the dropped perpendiculars This error in percentages is approximately equal to $100(oi/hb)$ (see Figure 1) This means that the larger the tilt angle is, the more necessary is a second determination.

Part II. Mathematical Proof of the Improved Method

Ordinarily when the term "scale" is used in describing a map it refers to the ratio of the map distance to the actual ground distance and is usually indicated by a small fraction such as 1/62,500 or 1/10,000. When a scale is said to be

EXAMPLE (See figure 2)

$f=8$ 27 inches Approximate $H=13,780$ feet

Point	Elev	*h*	*R* (in)	*r*
A	692	171	3 90	0 05
B	809	288	2 91	0 06
D	521	0	—	0 00

Line	*G* (ft)	*P* (inches)	*S*	
AB	9,574	6 21	648 65	*Sm*
BD	8,894	5 74	645 42	*Sl*
DA	10,398	6 80	653 98	*Sh*

hl measured 2 78″, therefore $al=2$ 78×3 23/8 56 or 1 05″
hn measured 1 42″
$dS=(653$ 98−648 65)/1 42 or 3 75 change in scale per inch
oc measured 0 52″
$S_0=648.65-(0$ 52×3 75) or 646 70
sin $t=8$ 27×3 75/646 32 or 04796
tilt angle $(t)=2°45'$
$ov=8$ 27× 04798 or 0 397″
$oi=0$ 20″

larger than another, it means that the fraction representing one scale is larger than the other. A scale of 1:5000 is larger than a scale of 1:40,000

When used in connection with an aerial photograph, the term "scale" becomes very complex because of two disturbing factors—relief and tilt. If either relief or tilt is present, the scale will not be uniform over the entire picture. But if the effect of tilt is zero or negligible, and if the ground points are all on a common datum plane, then the scale will be constant over the entire photograph. The scale can be obtained by either of two formulas·

$$S = \frac{P}{G}$$

$$S = \frac{f}{H}$$

P = distance between two image points.
G = horizontal distance between corresponding ground points.
f = focal distance of lens of camera.
H = height of lens above datum plane.

Whenever tilt is appreciable these relationships will no longer hold. The effect of tilt can more easily be understood by first considering its effect when there is no relief present Even when the ground points are all at the same datum elevation, the scale of lines on the photographs will not be constant or uniform. It can be easily shown that the scale of these lines would vary from one area of the photograph to another.

In Figure 3, assume that area *abcd* is a rectangular portion of a tilted picture whose axis of tilt is perpendicular to two sides of the rectangle. Also assume that all points on the ground are at the same datum elevation.

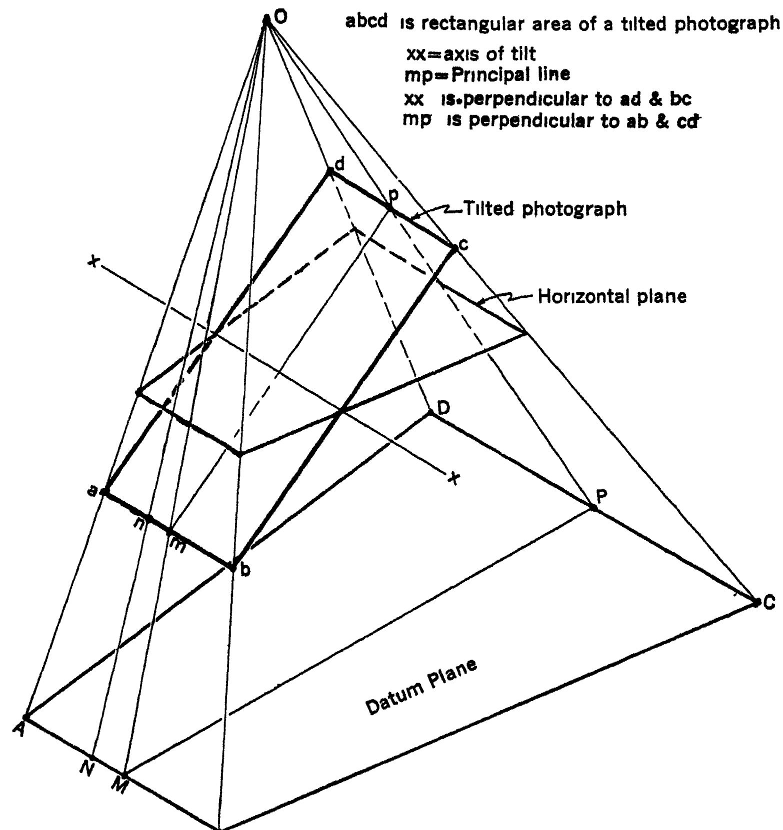


Fig. 3
LINES OF CONSTANT SCALE

Any line in the photograph that is parallel to the axis of tilt (XX) such as (ab) or (cd) will also be parallel to the ground or datum plane. Then the scale of all segments of one of these lines will be constant. For example:

$$\frac{an}{AN} = \frac{nm}{NM} = \frac{mb}{MB} = \frac{ab}{AB} = S_{ab} \text{ (the scale along line } ab).$$

From similarity of triangles

$$\frac{mb}{MB} = \frac{Om}{OM} = S_{ab}$$

the scale along line cd will also be constant but will be $S_{cd} = (Op/OP)$.

Figure 4 is the result of passing a plane through the pyramid at OMP.

f = focal length
OL = the plumb line
i = the isocenter
m = the foot of perpendicular from isocenter to line (ab)
x = the normal distance from the isocenter to line (ab)

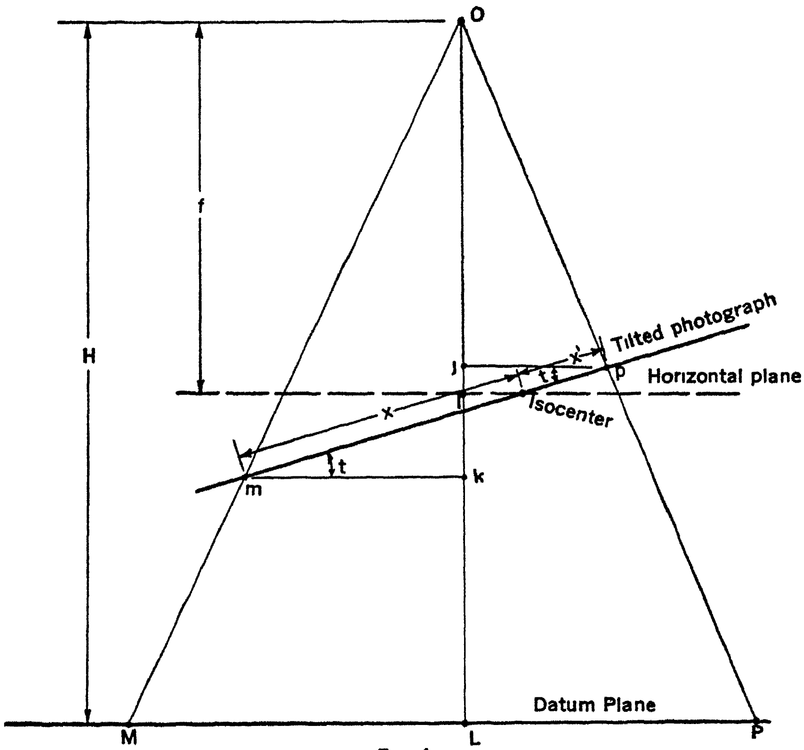


Fig 4
RELATIONSHIPS OF SCALE TO POSITION FROM ISOCENTER

Triangle Omk is similar to triangle OML

$$\therefore \quad \frac{Om}{OM} = \frac{Ok}{OL}.$$

Since

$$S_{ab} = \frac{om}{OM}, \qquad \text{then} \qquad S_{ab} = \frac{Ok}{OL}$$

$$ok = ol + lk = f + x \sin t$$

and

$$OL = H$$

therefore,

$$S_{ab} = \frac{f + x \sin t}{H}$$

similarly

$$S_{cd} = \frac{f - (ip) \sin t}{H}.$$

If the x' distances are considered negative on the raised or optical center side of the axis of tilt, then·

$$S_{cd} = \frac{f + x' \sin t}{H}.$$

At the axis of tilt $x=0$, so along the axis the scale is: $S_t=(f/H)$

It can be seen that all of the lines on the photo that are parallel to the axis of tilt have constant scales but that their scales differ from each other depending on their distance from the isocenter.

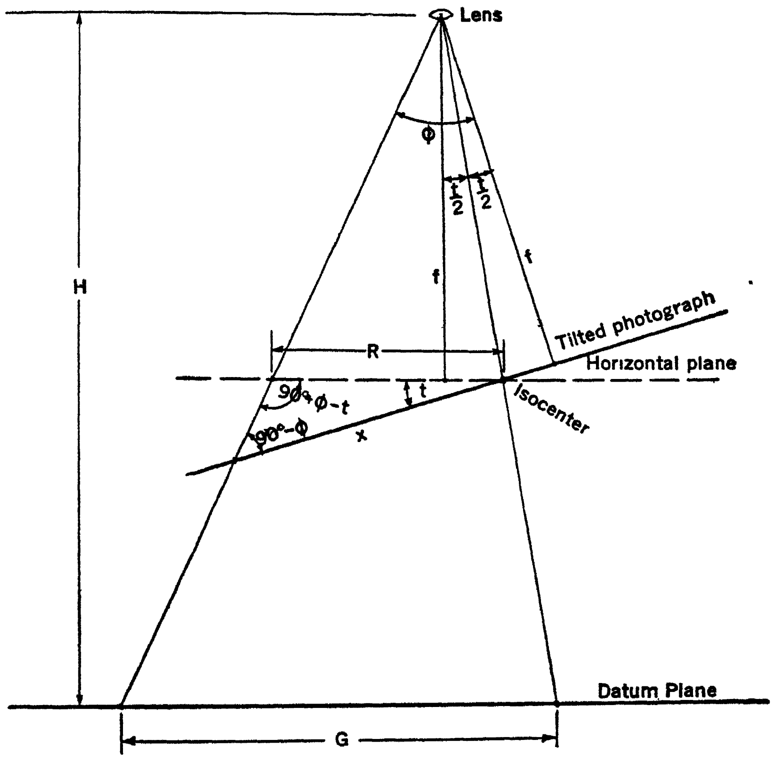


Fig 5
GROUND DISTANCE BETWEEN SCALE LINES

Other non-parallel lines will not have a constant scale throughout their length. However, any particular segment of a line considered in its entirety will have but one definite scale ratio; namely, its total photographic distance divided by its total corresponding ground distance. Whatever the scale may be, there will be a line parallel to the axis of tilt that will have exactly the same scale. This parallel line and the non-parallel line or its extension must intersect at some point in the plane of the photograph. This point is called the "Scale Point" of the non-parallel line. The dropped perpendicular method described in step 9 of the procedure will give a usable close approximation of the actual position of the scale point for any line on a photograph This has been checked by formulas giving the exact position of the scale point and are derived as follows:

First, consider a segment of the principal line, x in Figure 5. G is the corresponding ground distance and R is the rectified print distance. x would equal R if there were no tilt.

By the law of sines,

$$\frac{x}{\sin (90^\circ + \phi - t)} = \frac{R}{\sin (90^\circ - \phi)}$$

$$\frac{x}{\cos (\phi - t)} = \frac{R}{\cos \phi}$$

$$R = \frac{x \cos \phi}{\sin \phi \sin t + \cos \phi \cos t}$$

dividing numerator and denominator by $\cos \phi$

$$R = \frac{x}{\tan \phi \sin t + \cos t}$$

but

$$\tan \phi = \frac{x + f \tan \frac{t}{2}}{f}$$

$$\therefore \quad R = \frac{fx}{x \sin t + f\left(\tan \frac{t}{2} \sin t + \cos t\right)}$$

since

$$\tan \frac{t}{2} \sin t + \cos t = 1$$

then

$$R = \frac{fx}{f + x \sin t} \cdot$$

Now

$$G:R = H:f$$

or

$$G = \frac{RH}{f} = \frac{Hx}{f + x \sin t}$$

$$S_G = \frac{x}{G} = \frac{f + x \sin t}{H}$$

which incidentally is the same as the scale of the line parallel to the axis of tilt and an x distance from the isocenter. Therefore, the scale point of this particular line would be x distance from the isocenter which would be at the very end of the line.

Next consider any line on a photograph such as line (ab) in Figure 6 The distances of the coordinates x and y are positive and negative in the direction indicated. The lines y_1 and y_2 are not only parallel on the photograph, their traces on the datum plane are also parallel The distance between them is

$$\frac{Hx_2}{f + x_2 \sin t} - \frac{Hx_1}{f + x_1 \sin t} \cdot$$

The scale along the y_1 line is equal to:

$$\frac{f + x_1 \sin t}{H} \cdot$$

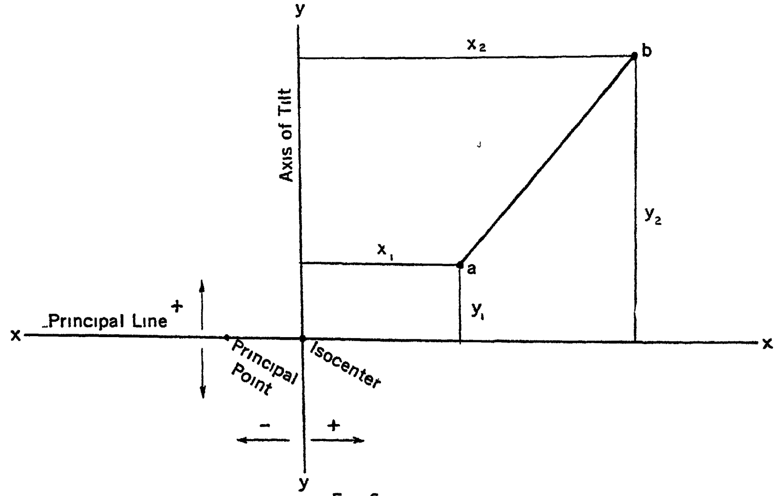


Fig 6
COORDINATE SYSTEM FOR ANALYZING SCALE POINTS

The distance on the datum plane representing y_1 is

$$Gy_1 = y_1\left(\frac{H}{f + x_1 \sin t}\right).$$

Similarly

$$Gy_2 = y_2\left(\frac{H}{f + x_2 \sin t}\right).$$

The ground distance of line ab is

$$G_{ab} = \sqrt{\left(\frac{Hx_2}{f + x_2 \sin t} - \frac{Hx_1}{f + x_1 \sin t}\right)^2 + \left(\frac{Hy_2}{f + x_2 \sin t} - \frac{Hy_1}{f + x_1 \sin t}\right)^2}.$$

The distance on the photograph of line ab is.

$$P_{ab} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The scale of line ab is

$$S_{ab} = \frac{P_{ab}}{G_{ab}} = \frac{(f + x_2 \sin t)(f + x_1 \sin t)\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{H\sqrt{(fx_2 - fx_1)^2 + (fy_2 - fy_1 + y_2x_1 \sin t - y_1x_2 \sin t)^2}}.$$

If the scale point for line ab is X distance from the axis of tilt, the scale at the scale point is

$$\frac{f + X \sin t}{H} = S_{ab}.$$

Substituting for S_{ab} and solving for X

$$X = \frac{(f + x_2 \sin t)(f + x_1 \sin t)\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{\sin t\sqrt{(fx_2 - fx_1)^2 + (fy_2 - fy_1 + y_2x_1 \sin t - y_1x_2 \sin t)^2}} - \frac{f}{\sin t}.$$

Substituting the actual values for any line and solving this equation will give

the normal distance from the axis of tilt to the true scale point. This equation for the exact location of the scale point was used to test the accuracy of the approximate location of the scale point by the dropped perpendicular method. Although there were differences, they were small enough to be disregarded For any segment of a radial line, the equation simplifies to

$$X = x_2 + x_1 + x_2 x_1 \frac{\sin t}{f}.$$

Knowing that scale points and the scale ratios at those points can be found, a formula for the angle of tilt can be easily derived. The general formula for the scale at any scale point is:

$$S = \frac{f + x \sin t}{H}.$$

Here x is the normal distance on the photograph from the point to the axis of tilt. The distance will be plus on the plumb point side of the axis and minus on the optical center side. The scale at two scale points "a" and "b" will be·

$$S_a = \frac{f + x_a \sin t}{H}$$

$$S_b = \frac{f + x_b \sin t}{H}$$

$$\therefore \quad S_a - S_b = \frac{f + x_a \sin t}{H} - \frac{f + x_b \sin t}{H}$$

$$H(S_a - S_b) = \sin t(x_a - x_b)$$

solving for $\sin t$

$$\sin t = H \frac{S_a - S_b}{x_a - x_b}.$$

Let

$$dS = \frac{S_a - S_b}{x_a - x_b}.$$

The quantity (dS) would be equal to the change in scales normal to the axis of tilt per unit of length. The value for $(x_a - x_b)$ would be the same whether referred to the axis of tilt or a line parallel to the axis of tilt.

Then

$$\sin t = HdS. \tag{c}$$

Since the scale at the isocenter is

$$S_i = \frac{f}{H}$$

then

$$H = \frac{f}{S_i}$$

and,

$$\sin t = \frac{fdS}{S_i}. \tag{b}$$

This is known as the general tilt equation. It should be noted that two scale points which represent the scale of two lines were used in the derivation. Equation (a) is not as accurate as equation (b) because an assumption must be made to find (H), the height of the air-plane above the datum plane.

Heretofore, the factor of relief has not been considered. The only pictures that have been considered are those of flat terrain where the ground points are all at the datum plane elevation. Tilt can be determined using control points with large differences in elevation. It is only necessary to introduce one step at the very beginning of the calculations. This step will remove the relief displacements of the image points. The resulting position will be the position that the image points would have had if they were all at a common elevation. They can then be used as previously indicated. The formula for image displacement because of relief is:

$$r = \frac{Rh}{H}.$$

R = distance from the nadir point on the picture to the image point
h = height of the ground point above the datum plane
H = height of the lens above the datum plane

This formula is exact only for untilted photographs or prints that have been correctly rectified. If the photograph is tilted, the relief displacement distance will also be affected The distance will be either shortened or lengthened depending on its location However, the total amount of change will be small enough to be negligible in all but the exceptional case. With extreme conditions a more accurate formula can be used·

$$r = \frac{hR(f + x \sin t)}{Hf}$$

where (x) is perpendicular distance to axis of tilt.

Extreme conditions could be said to exist if

$$hx \sin t > 40$$

where (h) is in feet and (x) is in inches. Under these conditions the correction would amount to more than 0 005″.

It will simplify matters if the datum plane is taken at the elevation of the lowest control point. The actual value of H is unknown and must be assumed. Use the formula

$$H = \frac{f}{S} - e$$

f = focal length
S = map scale or intended scale of photographs
e = height of the lowest control point above the map datum elevation

The error in this assumption will certainly not be greater than 2 per cent. If modern improved instruments are used in the airplane and if proper care is taken by the aviator, the error will be considerably smaller. When H is know for some other picture in the same flight of photographs, use this as the value for H.

Assuming that the datum plane has been set at the elevation of the lowest control point, all of the relief displacements "r" will be measured on the radials *toward* the optical center. Pencil lines can then be drawn on the print connecting

all of the corrected control points. These lines may be called "datum lines" because they represent the position the photograph lines would have if there were no relief present Of course, the "datum line" distance, *not* the actual photograph line distance, is used in subsequent calculations

Because of the graphical method used to locate the principal line, it is necessary to use three check lines The elevation of both terminals of all three lines must also be known in order to make relief corrections The method will be considerably simplified if the three check lines form a closed triangle Then there will only be three control points whose elevation must be known

The scale point system even with the improvements cannot be called an exact method of determining tilt. The weakness of the method derives from the use of dropped perpendiculars to locate the scale points. Scale points found in this manner are usually very close to their true position. Nevertheless, this difference in position introduces a small error that is never eliminated If there were a simple way of finding the true position of scale points, the determinations would then be more exact than the limits of accuracy in measurements would require.

The system may not be perfect but it has definite practical applications. It can be used for rectifying photographs to be used in mosaic maps or radial plots It can also be used to improve radial plots of either hand or slotted templates without rectifying the photographs The isocenter can be found and used as the origin of the radials provided the terrain is fairly level as along the Atlantic coast or Mississippi River The nadir point should be used as the origin in rugged country. Of course, if the photographs are rectified, the nadir point must be used regardless of the terrain

Part III. Comparison with Original Method

As has been stated in the beginning of this article, this method of calculating tilt is neither new nor original It is only an improvement of the praiseworthy work done by Mr. R. O. Anderson. Photogrammetrists familiar with his method will have noticed two major changes in the improved method. First—the use of (P/G) as the scale fraction at the scale points instead of (G/P) or (D/P) the

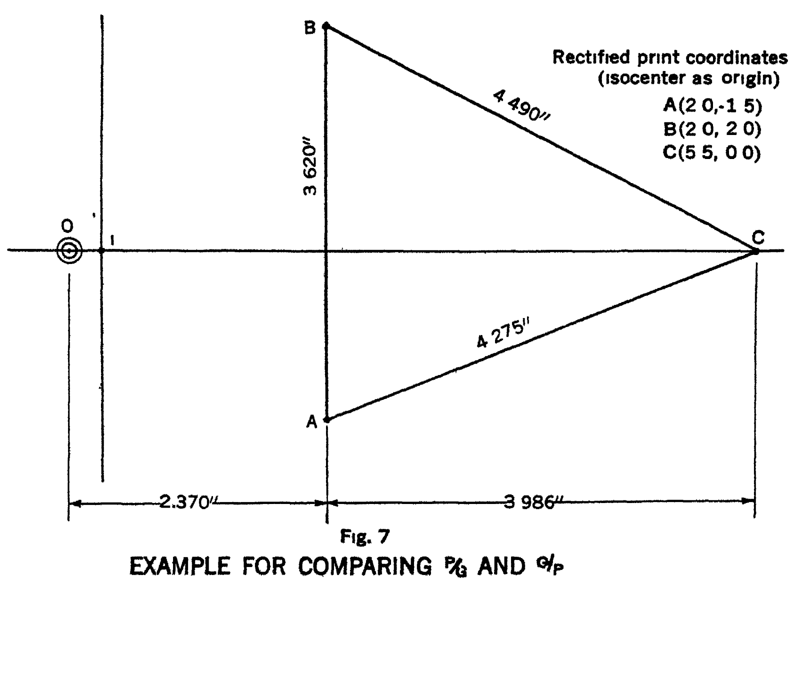


Fig. 7

EXAMPLE FOR COMPARING P/G AND G/P

symbol used in the original system Second—the use of relief displacements instead of equivalent elevations to remove the effect of relief in the calculations Under some conditions, not all, the original method becomes very approximate. If the two changes embodied in the improved method were used, the results would be noticeably improved.

The value of using (P/G) becomes most evident whenever the control points or lines are not fairly symmetrical about the center of the photograph Control points are not always found on photographs where they are most desirable. Therefore, asymmetry occurs fairly frequently. An extreme example of this is shown in Figure 7 where the three control points are all on one side of the photograph. For purposes of comparison between the two systems, all three points are at zero elevation, thus eliminating the effect of relief The pertinent data are as follows

$S=1$ 20,000 $H=10{,}000'$ $f=6''$ $t(\text{actual})=5°44$ 3′

	Improved			Original	
	P	G	P/G	G/P	R
AB	3 620	5833 3′	62 057	1611 4	96684
CA	4 275	6346 5′	67 360	1484 6	89076
BC	4 490	6718 6	66 829	1496 4	89734

$dS=9$ 6832 $\sin t=$ 09727 $\Delta R=$ 01382 $\sin t=$ 08298
$S_0=59$ 728 $t=5°34$ 9′ $R_0=$ 99972 $t=4°45$ 6′

The original system using G/P gives an answer for the angle of tilt of 4°45.6′, an error of 58.7′, while the improved method using P/G gives an answer of 5°34.9′ an error of only 9 4′. A second determination using the calculated position of the isocenter as the origin of the dropped perpendiculars improves both answers The error in the original system result is reduced to 46 9′ and in the improved method to 7 9′.

There are other conditions and positions of the control points that prevent the equivalent elevation correction from giving reasonably close results, whereas

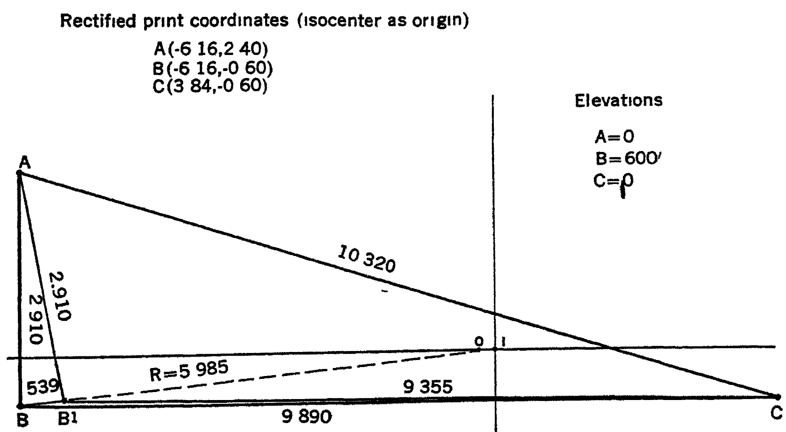


Fig. 8

EXAMPLE FOR COMPARING RELIEF CORRECTIONS

the use of relief displacements would give fairly close determinations. For instance, compare the results of the two methods of determining the tilt of the photograph shown in Figure 8. Point B is 600 feet higher than the other two points An unusual condition, but used to illustrate the advantage of the improved method The pertinent data are as follows·

$S=1$ 10,000=833 33′/″ $H=6{,}666$ 7′ $f=8''$
t(actual) $=2°17.5'$ relief displacement of $B=$ 539″

	P	G	G/P	X	X/P
AB	2 910	2,490 4	855 81	0 580	.199
BC	9 890	7,859 9	794 73	5.955	602
CA	10 320	8,690 3	842 08	—	—

	h_e	h_e/f	S_d	R_d
AB	119 6	14 95	870.74	1.04489
BC	361 3	45 16	839.90	1.00788
CA	0	—	842 08	1.01050

$\Delta R=$ 010115 $R_0=$ 99199
sin $t=.08157$ $t=4°40.3'$ error $=2°22.8'$

The original method recommends the application of an additional correction for problems of this nature. Therefore, avoiding the short cut and applying the most accurate formula for the "equivalent elevation convergence correction," the additional data obtained are as follows·

	N	N/P	h_c	h_{ec}	h_{ec}/f	S_d	R_d
AB	5 957	2 0470	115 06	4 5	.56	856 35	1.02762
BC	0 600	0.6066	10.49	350 8	43.83	838 57	1 00629
CA	—	—	0	0	0	842.08	1.01050

$\Delta R=$ 004116 $R_0=1$ 00059
sin $t=$ 03291 $t=1°53$ 2′ error $=24$ 3′
Swing angle error $=14°30'$
Isocenter position error $=.060''$
Nadir point position error $=.121''$

Improved method

	P	G	P/G
AB	2 910	2,490 4	116 849
BC	9.355	7,859.9	119 022
CA	10.320	8,690.3	118 753

$dS=.51225$ $S_0=119.802$
sin $t=.03421$ $t=1°57$ 7′ error $=19$ 8′
Swing angle error $=5°30'$
Isocenter position error $=.025''$
Nadir point position error $=.050''$

The position errors of the isocenter and nadir point using the original method are 140 per cent *greater* than those of the improved method. It should also be noted that the improved method is much simpler. In the above illustration there are four less measurements and eighteen less arithmetical calculations If all three elevations had been different, there would have been still more measurements and calculations It is also much simpler as well as more accurate to determine the scale of untilted photographs with relief displacements rather than equivalent elevations.

PRACTICAL TILT CORRECTION FOR SINGLE LENS AERIAL PHOTOGRAPHS

C P Van Camp

THE average relative-tilt method is one of the special methods developed by the U S. Geological Survey for increasing the accuracy of map information compiled from aerial photographs It is a practical method of reducing the errors caused by tilt in vertical photographs without using such costly instruments as the aerocartograph or the multiplex. It has been used at the Sacramento office of the Geological Survey since 1938 as one step in the mapping of more than 80 quadrangles covering approximately 16,000 square miles

These quadrangles were covered by photographs which are typical of the vertical photographs most generally available They were made by many different contractors and for many different agencies. A variety of cameras were used, and the photographs were printed on several kinds of photographic paper. Most of the photographs were single lens contact prints, but some were composite photographs and others were enlargements Practically all of the photographs, however, were taken in flight lines that were approximately straight and at constant altitude. Likewise the photographs were all tilted from the vertical. Figure 1 shows the variations in flight line and tilt for a flight near the Oregon coast that is typical of the photographs for all of the quadrangles The variations are similar to those found by Jones and Griffiths[1] as a result of tests made in England between 1920 and 1923.

An attempt was made to use the photographs without spending time to determine how much each was tilted The common practice of using the principal points as centers for radial triangulation was tried and found to be unsatisfactory for areas of high relief or for areas where ground control could not be easily extended. Nearly all of the quadrangles included mountainous areas with an average relief of about 2000 feet, and some quadrangles covered areas with 4000 to 5000 feet of relief. Extensive ground control could not be easily obtained because most of the quadrangles covered undeveloped areas, either remote or heavily wooded Under such conditions some method was needed to compensate for the effects of tilt

A study of the methods of determining tilt revealed that the relative tilt between two overlapping photographs can be determined without the use of ground control From successive determinations of relative tilt an average tilt can be determined and used as an approximately untilted position for all of the photographs in a flight McAdam[2] has described a practical procedure for five lens photographs which was developed by the U. S. Geological Survey in cooperation with the Tennessee Valley Authority For single lens photographs G. S Druhot of the Geological Survey developed a graphical method for determining the component of relative tilt perpendicular to the line of flight. These methods led to the development of a procedure for determining both components of relative tilt of single lens photographs.

Determination of Relative Tilt

Some of the effects of relative tilt are revealed by the photographed positions of a series of lines parallel with the air base The relative position of such lines

[1] Jones, B. M , and Griffiths, J. C , Aerial Surveying by Rapid Methods, Chapter IV, Cambridge, 1925

[2] McAdam, C. B., Practical Tilt Correction of Five Lens Aerial Photographs: The Military Engineer, vol. 28, pp. 47–54, January–February, 1936.

on the overlapping photographs will depend upon (1) the tilt of each photograph with respect to the air base and (2) the tilt of the photographs with respect to each other In Figures 2, 3, and 4 the right photograph is superimposed on the left to show the way the lines are affected by tilt The effect is most

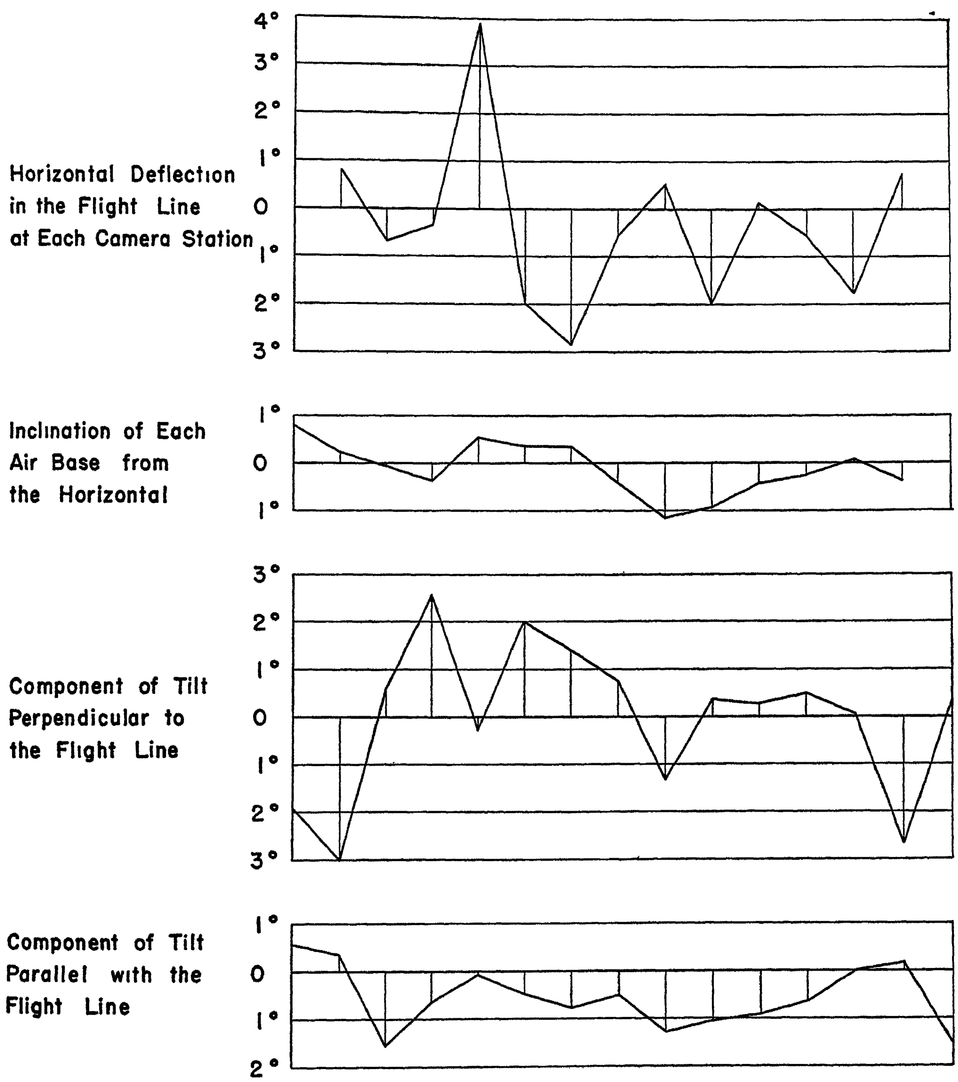


FIG 1

noticeable near the four corners of the area of overlap where the displacements or y-parallaxes are marked Δy_1, Δy_2, Δy_3, and Δy_4. In Figure 3, Δy_1 and Δy_2 are nearly equal and in the same direction so that $(\Delta y_1 - \Delta y_2)$ is practically zero In Figure 4, Δy_1 and Δy_2 are nearly equal but opposite so that $(\Delta y_1 + \Delta y_2)$ is practically zero Consequently the quantity $(\Delta y_1 - \Delta y_2)$ can be used as a measure of tilt in one direction and quantity $(\Delta y_1 + \Delta y_2)$ can be used as a measure of tilt

in the other direction. The quantities $(\Delta y_3 - \Delta y_4)$ and $(\Delta y_3 + \Delta y_4)$ can be used in a similar manner.

The relative tilt about the air base can be determined from either of two equations.

$$\Delta y_n' = \frac{f^2}{2a^2}(\Delta y_1 + \Delta y_2) \tag{1}$$

$$\Delta y_n' = \frac{f^2}{2a^2}(\Delta y_3 + \Delta y_4) \tag{2}$$

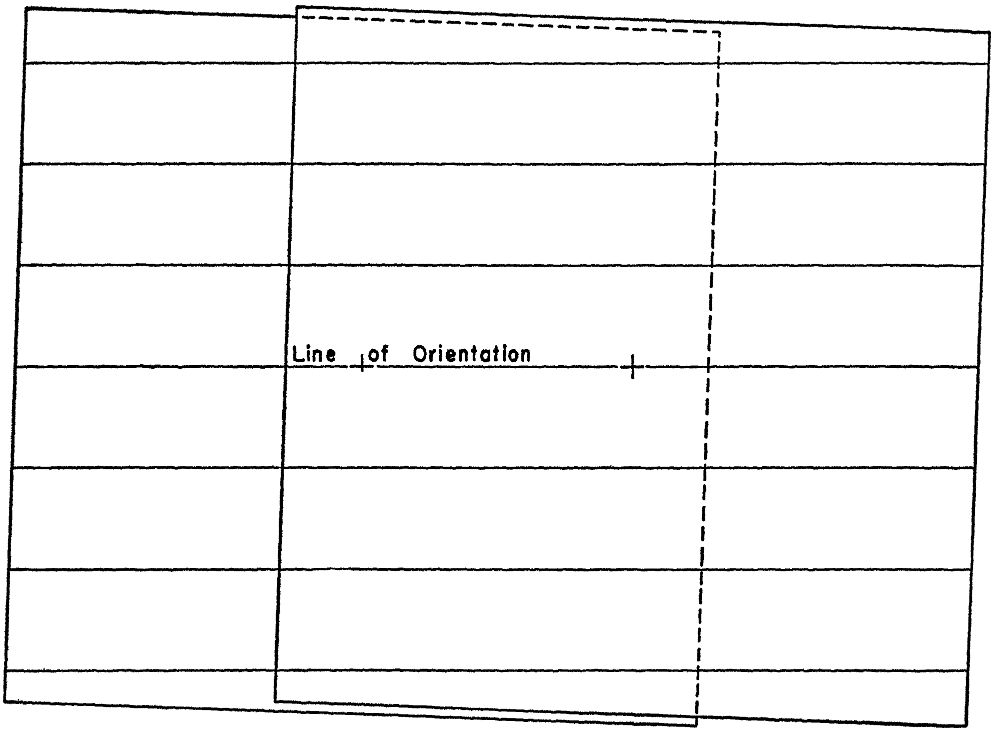


FIG. 2. Neither photograph tilted with respect to the air base or each other.

The expression $\Delta y_n'$ is the y-coordinate of the nadir point on the right photograph when the left photograph is not tilted. Ordinarily the left photograph is tilted, and $\Delta y_n'$ can be regarded as the difference which must be added to y_n on the left photograph to obtain y_n' on the right photograph.

If the air base is level the x-coordinate of the nadir point on the right photograph can be determined from the equation:

$$\Delta x_n' = \frac{f^2}{2ab}(\Delta y_1 - \Delta y_2). \tag{3}$$

The x-coordinate of the nadir point on the left photograph can be determined from the equation:

$$\Delta x_n = \frac{f^2}{2ab}(\Delta y_3 - \Delta y_4). \tag{4}$$

Whether the air base is level or not, the difference $(\Delta x_n' - \Delta x_n)$ can be regarded as the quantity which must be added to x_n on the left photograph to obtain x_n' on the right photograph.

In these equations, f is the principal distance of the photographs and can be considered equal to the calibrated focal length of the camera corrected for any change in size of the photographs caused by film or paper shrinkage or photographic enlargement As shown in the figures, a is the distance from the line of orientation to the place where the y-parallaxes are measured. The distance b is measured parallel with the line of orientation and is the x-coordinate of the point where the y-parallaxes are measured disregarding signs. It is measured on the right photograph for Δy_1 and Δy_2 and is measured on the left photograph for Δy_3 and Δy_4.

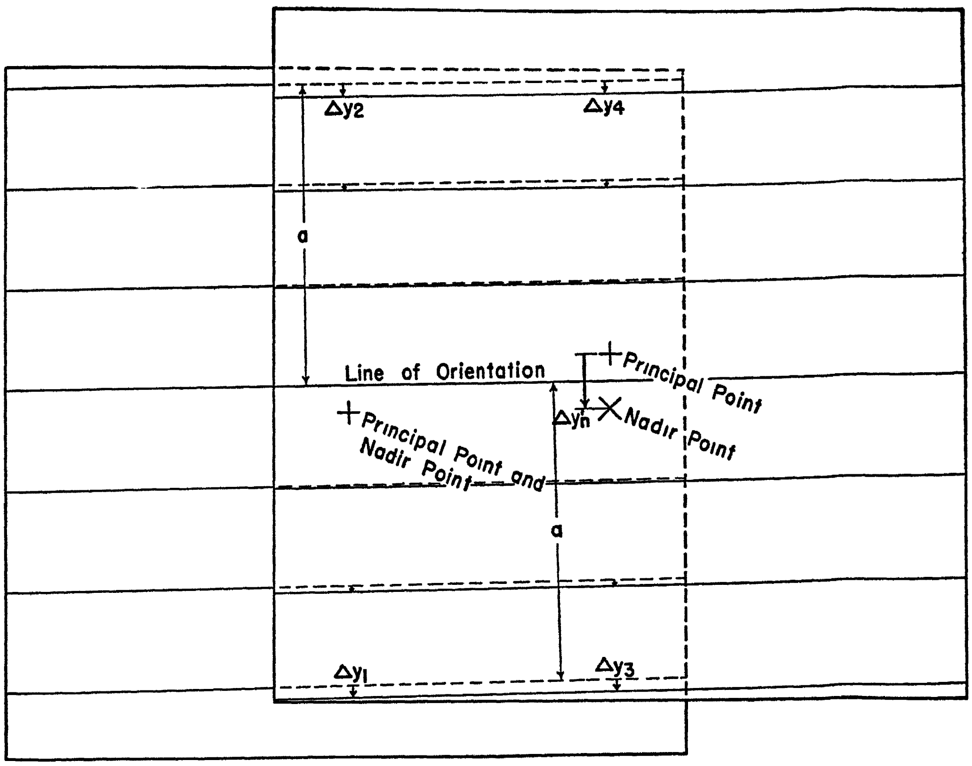


FIG. 3. Right photograph tilted with respect to the left photograph but not with respect to the air base.

Although these equations are not theoretically exact, they are both convenient and accurate for practical use. The exact equations, which can be derived from the geometry of tilted perspectives, contain several terms which are insignificant as long as tilt is limited to a few degrees. The insignificant terms have been eliminated by assuming that the tangents of small angles are proportional to the angles themselves and that the cosines of small angles are unity. The errors caused by such approximations are less than one minute of angle as long as the tilts are less than approximately five degrees.

The accuracy with which tilt can be determined is usually limited by the

accuracy with which the y-parallaxes Δy_1, Δy_2, Δy_3, and Δy_4 can be measured An error of 0.1 millimeter in each of the measurements will, if they are not compensating, cause an error in the angle of tilt amounting to ten or twelve minutes for $7'' \times 9''$ normal-angle photographs or an error of five or six minutes for $9'' \times 9''$ wide-angle photographs For practical work, the problem of determining relative tilt is mostly a problem of accurate measurement

Before the values of y-parallax can be measured accurately, the line of orientation must be determined A line passing through the principal points of both photographs can be used as an approximate line of orientation The quanti-

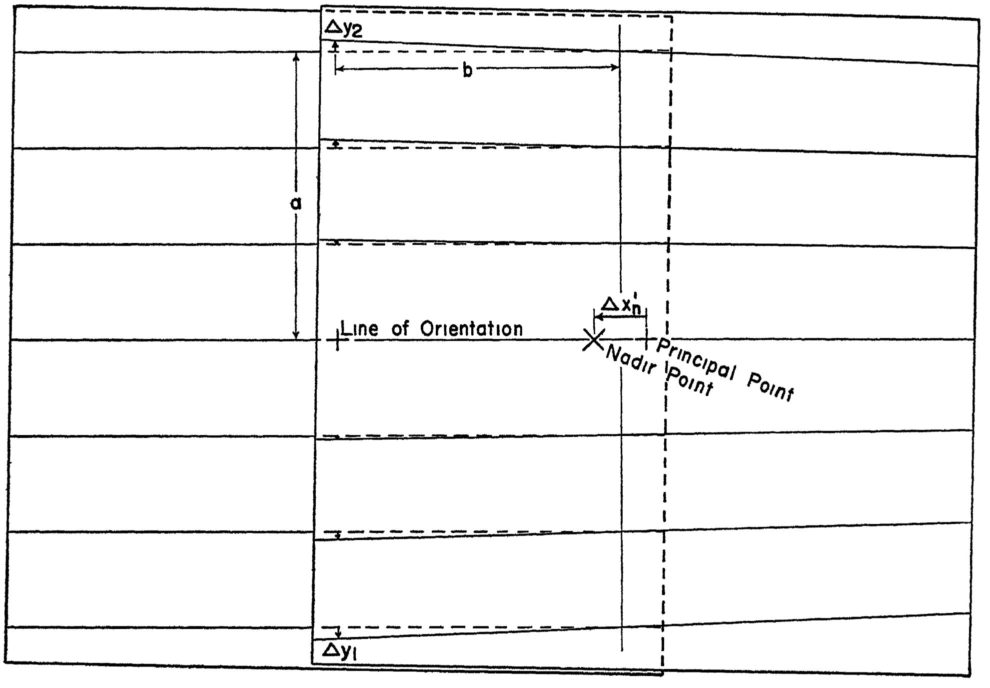


FIG. 4 Right photograph tilted with respect to the air base

ties Δy_1, Δy_2, and a can be measured on a perpendicular to such a line, and $\Delta y_n'$ can be determined approximately from equation (1). But if $\Delta y_n'$ is more than a millimeter or two, the line of orientation should be changed to pass halfway between the principal point and the nadir point on the right photograph as shown in Figure 3. This is especially true when the photographs cover areas of high relief because the displacements Δy_1 and Δy_2 will include a component of the parallax that is caused by difference in elevation unless the line of orientation is correct. If the flight line is fairly straight, an approximate line of orientation that is extended from the preceding pair of photographs will usually be better than a line through the principal points. It can be used until a trial value of $\Delta y_n'$ is computed. If such a procedure is followed, the first determination of $\Delta y_n'$ will usually allow the line of orientation to be plotted accurately enough for final measurements.

After the line of orientation is determined, the other lines shown in Figures 2, 3, and 4 can be drawn on the photographs, but they are not ordinarily neces-

sary Measurements can be made directly to image points which are already on the photographs Thus, for example, Δy_1 can be regarded as the difference between the y-coordinate of point 1′ on the right photograph and the y-coordinate of the corresponding point 1 on the left photograph

If measurements are to be made with an ordinary scale, the line of orientation must be carefully drawn on both photographs Special care should be taken to make it pass through the same image points on both photographs because an error in transferring the line will cause a magnified error in $\Delta y_n'$. A second line should be drawn approximately perpendicular to the line of orientation and through the principal point of each photograph. A soft chisel-pointed pencil may be used for light photographs, and if the photographs are dark the lines may be drawn with white or vermilion water color A well defined image point should be selected near each perpendicular and equidistant on each side of the line of orientation The distance a should be large so that the y-parallax can be measured accurately, but it should not be so large that the image points will be so close to the edge of the photograph that they are indistinct. A distance of 80 millimeters will usually be satisfactory for 9″×9″ wide-angle photographs, and 100 millimeters can often be used for 7″×9″ or 9″×9″ normal-angle photographs. The location is ordinarily less critical than the measurement, so a well defined point should be used even if it is several millimeters from the ideal location. The four points can be marked with vermilion on both photographs, the transfer of points from one photograph to the other being done with the aid of a magnifying stereoscope if such is available The y-coordinate of each point is then accurately scaled on both photographs so that the differences Δy_1, Δy_2, Δy_3, and Δy_4 can be obtained In equations (1) and (3) the average y-coordinate for points 1 and 2, disregarding sign, can be used for a, and the average x-coordinate measured from the perpendicular on the right photograph, disregarding sign, can be used for b. In equations (2) and (4) similar measurements can be made to points 3 and 4 Accurate measurements can be made by scale if sufficient care is taken.

If a stereocomparator is available, it can be used for rapid measurements which are usually more accurate than scaled distances, especially when the photographs cover heavily wooded areas. No lines nor image points need be marked on the photographs. Direct readings of y-parallax can be obtained so easily that each can be checked on several different images. The U. S. Geological Survey has used two different stereocomparators, the Stereocomparagraph manufactured by the Fairchild Aviation Corporation and the Abrams Contour Finder manufactured by the Abrams Instrument Company.

The Stereocomparagraph was not designed for measuring y-parallax, but the right hand mark may be moved in the y-direction by means of a small knurled nut The periphery of this nut can be divided into ten equal parts and marked with small numbers stamped by dies. Each rotation of the nut corresponds to a y-parallax of 1/32 inch, so each division would correspond to a y-parallax of 1/320 inch. The number of complete rotations can be determined by measuring, in thirty-seconds of an inch, the distance between the lens holder and the hinged frame with a small paper scale taped to the lens holder. The equations can then include a conversion factor to convert y-parallax to millimeters. Another device for measuring y-parallax with the Stereocomparagraph is a small millimeter scale glued by one end to the hinged frame and a vernier glued to the lens holder. The scale and the vernier can be scratched on pieces of celluloid cut from a triangle. If the triangle is held under the Stereocomparagraph, the edge of the lens holder can be used as a guide for the scratches while

the micrometer is used for spacing them. The scratches will show up clearly if the back surface of the celluloid is covered by black celluloid ink

The Abrams Contour Finder is equipped with a direct reading dial gauge for measuring y-parallax, but the range of one millimeter was found to be too small for approximately 75 per cent of the photographs used. The range was extended by adding to one of the floating dots a row of dots spaced one millimeter apart in the y-direction. One of these extra dots could then be fused stereoscopically with the other floating dot by a movement of less than one millimeter. The 7.5 millimeter range of the parallel measuring gauge was also found to be too small for photographs of mountainous areas, but the range was extended by inserting spacers between the gauge and the right hand floating dot.

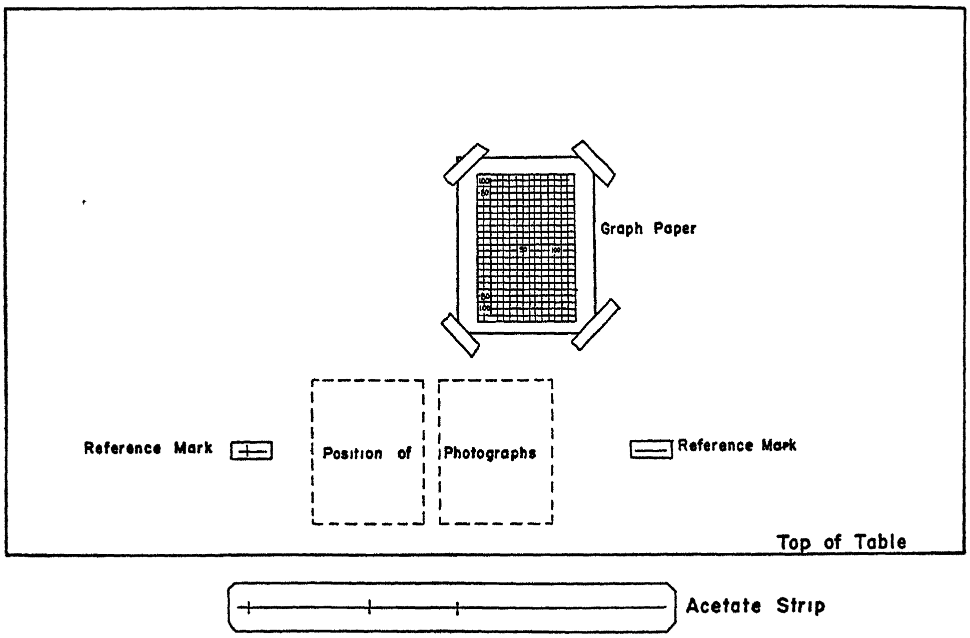


FIG. 5. Arrangement of the graph paper and the reference marks used with the Stereocomparagraph and the Abrams Contour Finder.

When a stereocomparator such as the Stereocomparagraph or the Abrams Contour Finder is used, the distances a and b can be easily measured by a sheet of millimeter graph paper located under the drawing attachment. The following steps are recommended for orienting the instrument, photographs, and graph paper.

1. Two reference marks are placed on the table as shown in Figure 5. They may be placed on pieces of scotch tape about two feet apart and six or seven inches from the front edge of the table.

2. A straight orienting line is scratched on a strip of cellulose acetate three or four inches wide and long enough to extend from one reference mark to the other. Short perpendiculars are added to indicate the positions of the left reference mark and the center of the left photograph. The range of the stereocomparator can also be marked off from the center of the left photograph as an aid in spacing the photographs under the instrument.

U S GEOLOGICAL SURVEY

RELATIVE TILT

Flight 1616

$\Delta y'_n = \frac{f^2}{2a^2}(\Delta y_1 + \Delta y_2)$ | $f = 132$ mm | $\Delta x'_n = \frac{f^2}{2ab}(\Delta y_1 - \Delta y_2)$ | Computer J A

$\Delta y'_n = \frac{f^2}{2a^2}(\Delta y_3 + \Delta y_4)$ | $\frac{f^2}{2} = 8710$ | $\Delta x_n = \frac{f^2}{2ab}(\Delta y_3 - \Delta y_4)$ | Date 12-1-42

Trial +27	$\Delta y_1+\Delta y_2$ +20	Δy_1 +15	Δy_2 +05					
$\Delta y'_n$ +26	$\Delta y_1+\Delta y_2$ +19	Δy_1 +14	Δy_2 +05	$\Delta y_1-\Delta y_2$ +09	$\Delta x'_n$ +12	a 81 78	b 78 83	Left 87
$\Delta y'_n$ +27	$\Delta y_3+\Delta y_4$ +20	Δy_3 +11	Δy_4 +09	$\Delta y_3-\Delta y_4$ +02	Δx_n +03	a 80 80	b 80 83	Right 88
+26	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = +09		sin δ = 0		
Trial −90	$\Delta y_1+\Delta y_2$ −66	Δy_1 −18	Δy_2 −48					
$\Delta y'_n$ −83	$\Delta y_1+\Delta y_2$ −61	Δy_1 −13	Δy_2 −48	$\Delta y_1-\Delta y_2$ +35	$\Delta x'_n$ +43	a 79 80	b 88 89	Left 88
$\Delta y'_n$ −84	$\Delta y_3+\Delta y_4$ −62	Δy_3 −33	Δy_4 −29	$\Delta y_3-\Delta y_4$ −04	Δx_n −05	a 80 80	b 88 95	Right 89
−8.3	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = +48		sin δ = +.01		
Trial −46	$\Delta y_1+\Delta y_2$ −34	Δy_1 −11	Δy_2 −23					
$\Delta y'_n$ −46	$\Delta y_1+\Delta y_2$ −34	Δy_1 −11	Δy_2 −23	$\Delta y_1-\Delta y_2$ +12	$\Delta x'_n$ +15	a 79 81	b 89 89	Left 89
$\Delta y'_n$ −46	$\Delta y_3+\Delta y_4$ −34	Δy_3 −04	Δy_4 −30	$\Delta y_3-\Delta y_4$ +26	Δx_n +32	a 82 79	b 87 89	Right 90
−46	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = −17		sin δ = −01		
Trial +62	$\Delta y_1+\Delta y_2$ +46	Δy_1 +20	Δy_2 +26					
$\Delta y'_n$ +67	$\Delta y_1+\Delta y_2$ +49	Δy_1 +22	Δy_2 +27	$\Delta y_1-\Delta y_2$ −05	$\Delta x'_n$ −06	a 80 80	b 85 85	Left 90
$\Delta y'_n$ +64	$\Delta y_3+\Delta y_4$ +47	Δy_3 +25	Δy_4 +22	$\Delta y_3-\Delta y_4$ +03	Δx_n +04	a 81 80	b 81 85	Right 91
+66	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = −10		sin δ = −.01		
Trial −46	$\Delta y_1+\Delta y_2$ −34	Δy_1 −09	Δy_2 −25					
$\Delta y'_n$ −52	$\Delta y_1+\Delta y_2$ −38	Δy_1 −10	Δy_2 −28	$\Delta y_1-\Delta y_2$ +18	$\Delta x'_n$ +24	a 79 79	b 84 81	Left 91
$\Delta y'_n$ −53	$\Delta y_3+\Delta y_4$ −39	Δy_3 −15	Δy_4 −24	$\Delta y_3-\Delta y_4$ +09	Δx_n +12	a 80 80	b 83 83	Right 92
−53	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = +12		sin δ = +.07		
Trial +14	$\Delta y_1+\Delta y_2$ +10	Δy_1 +14	Δy_2 −04					
$\Delta y'_n$ +14	$\Delta y_1+\Delta y_2$ +10	Δy_1 +14	Δy_2 −04	$\Delta y_1-\Delta y_2$ +18	$\Delta x'_n$ +2.5	a 80 81	b 81 79	Left 92
$\Delta y'_n$ +15	$\Delta y_3+\Delta y_4$ +11	Δy_3 +12	Δy_4 −01	$\Delta y_3-\Delta y_4$ +13	Δx_n +17	a 79 81	b 84 81	Right 93
+14	= Average $\Delta y'_n$			$(\Delta x'_n - \Delta x_n)$ = +08		sin δ = −04		

FIG. 6

3. The acetate strip is then placed over the reference marks on the table and held in position by weights or scotch tape near each end

4. The drafting machine connected to the stereocomparator is then clamped so that the floating dots of the instrument can be set on the orienting line when they are set for zero y-parallax.

5. Axes are drawn on the graph paper so that they intersect near the center on the left side, and additional lines for measuring a are drawn. The x-coordinates are numbered every ten millimeters so that values of b can be easily determined

6. The left floating dot of the stereocomparator is then placed at the mark on the acetate strip for the center of the left photograph, and the graph paper is placed so that the origin is under the pencil of the drafting attachment. The graph paper is adjusted until the x-axis is parallel with the orienting line on the acetate strip and is then taped to the table

7 The stereocomparator is then set aside while the photographs are oriented under the acetate strip and taped to the table. The acetate strip is removed until it is needed for orienting the next pair of photographs

After the photographs are oriented with respect to the instrument and graph paper, the measurements of y-parallax, a, and b are made in each of the four corners of the area of overlap, a total of twelve measurements which can be recorded on a form as shown in Figure 6. The four measurements of a and the measurement of b for points 3 and 4 can be read directly on the graph The distance b for points 1 and 2 can be read on the graph if the instrument is moved so the right floating dot is on the principal point of the right photograph while the dots are spaced for stereoscopic fusion on each point The distance a will be the same for both points of a pair except for minor variations made to use a distinct image on the photographs. The distance b will vary according to the elevation of the ground but the average distance for each pair will ordinarily give good results In fact, the variations in a and b will be so small that the averages can be determined at a glance. The calculations of $\Delta y_n'$, Δx_n, and $\Delta x_n'$ can be made readily on a slide rule

Several precautions should be kept in mind while measurements are being made with a stereocomparator. The measurement of y-parallax will be in error unless the instrument is kept parallel with its initial orientation, so the instrument should be moved without twisting. The y-parallax of a point near the center of each photograph should, as a check, read zero both before and after measurements are made on the perpendiculars. The measurement of y-parallax is also difficult for some operators who rely on stereoscopic fusion. The difficulty may be overcome by using image points with distinct edges parallel with the line of orientation and by blinking from one eye to the other while observing the dots.

Determination of Average Tilt

The average tilt for all the photographs in a flight can be determined after the relative tilt has been determined for each pair The average may be computed by a simple tabulation as shown for $\Sigma\Delta y_n'$ and y_n' in the following table, or it may be determined graphically by plotting the quantities on an enlarged scale as shown in Figure 7. The graphical solution is recommended because it can be more easily analyzed and checked for mistakes

The average will depend somewhat upon the number of photographs in the flight. In Figure 7 the exact averages of $\Sigma\Delta y_n'$ are shown for five, eight, and fifteen photographs. Theoretically the exact average for an infinite number of

CALCULATION OF AVERAGE y_n' FROM DETERMINATIONS OF $\Delta y_n'$

Photo No	$\Delta y_n'$	$\Sigma\Delta y_n'$	Average y_n'
87		0	+4 4
88	+2 6	+ 2 6	+7 0
89	−8 3	− 5 7	−1 3
90	−4 6	−10 3	−5 9
91	+6 6	− 3 7	+0 7
92	−5 3	− 9 0	−4 6
93	+1 4	− 7 6	−3 2
94	+1 4	− 6 2	−1 8
95	+4 8	− 1 4	+3 0
96	−3 9	− 5 3	−0 9
97	+0 1	− 5 2	−0 8
98	−0 5	− 5 7	−1 3
99	+1 0	− 4 7	−0 3
100	+6 3	+ 1 6	+6 0
101	−7 0	− 5 4	−1 0
		15)−66 0	0 0
		− 4 4	

photographs would give an untilted position if the tilts were entirely accidental. In practice the flight cannot include an infinite number of photographs and the tilts almost always contain small systematic errors For such conditions an exact average is no better than an approximate average, so the average may be selected on the graph by eye without making a computation.

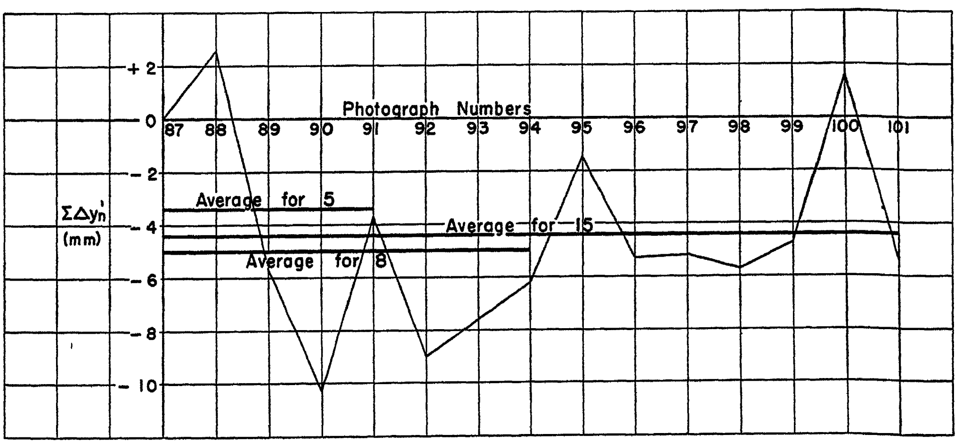


FIG. 7

All photographs in a flight should be referred to the same average, so the line which represents the average should not be inclined or broken without reason. However, it should also agree closely with the average for a section of the flight near either end. Ordinarily the average for $\Sigma\Delta y_n'$ will be approximately constant from one end of the flight to the other. If the average for a section

near one end is appreciably different from the average near the other, a search should be made for some mistake.

The values of $\Sigma(\Delta x_n' - \Delta x_n)$ are generally larger near one end of the flight than they are near the other, so the line which represents the average must be inclined. The inclination means a change in the average from one photograph to the next to correct for lens distortion. Lens distortion has an effect on y-parallax that is similar to the effect caused by tilt with respect to the air base, so each determination of $(\Delta x_n' - \Delta x_n)$ contains a small accumulative error. The error can be determined from camera calibration data and a correction can be applied to each value of $(\Delta x_n' - \Delta x_n)$. The correction, however, is practically constant if the distances a and b are uniform, so it can be applied graphically to each value of $(\Delta x_n' - \Delta x_n)$ by drawing the average on a slope to conform with the values of $\Sigma(\Delta x_n' - \Delta x_n)$ near both ends of the flight, as shown in Figure 8. All photographs made with the same camera will have the same distortion, so the corrected average will have the same slope for all flights

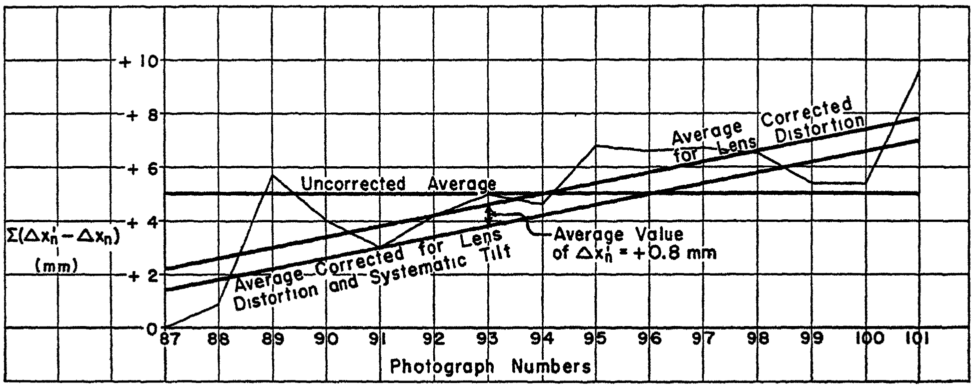


FIG. 8

Systematic tilt, such as that caused by incorrect adjustment of the level on the camera, will affect the averages of both components of tilt. The determination of $\Delta y_n'$ for each pair of photographs does not shed light on systematic errors which affect y_n', but systematic errors which affect x_n' can be estimated from the values of Δx_n or $\Delta x_n'$, which will often be predominantly negative or positive. The average of Δx_n or $\Delta x_n'$ for all the photographs in a flight is an approximate measure of the tilt between the average of $\Sigma(\Delta x_n' - \Delta x_n)$ and the average inclination of the line of flight which is usually almost zero. Unless there is reason to believe that the pilot did not attempt to maintain a constant flight altitude, the average of $\Delta x_n'$ should be added algebraically to all values of x_n'. This can be done graphically by shifting the average as shown in Figure 8. Whether the average of $\Delta x_n'$ is positive or negative will depend upon the direction of flight with respect to the direction in which tilts are computed, but the magnitude of the shift will be nearly constant for all flights in one project. Thus one component of systematic tilt can be determined approximately without use of ground control.

No additional refinements are ordinarily needed if the tilts are small and the flight line is fairly straight. A pilot can ordinarily maintain his course so that the deflection in the line of flight at each camera station is only a few degrees.

But whenever the tilts are large the deflections have a measurable effect even if they are small. In Figure 9 the deflection at photograph II is shown by the angle δ. The coordinates x_n' and y_n' measured with respect to the line of orientation from I to II become the coordinates x_n and y_n when measured with respect to the line of orientation from II to III. The relative tilt between II and III

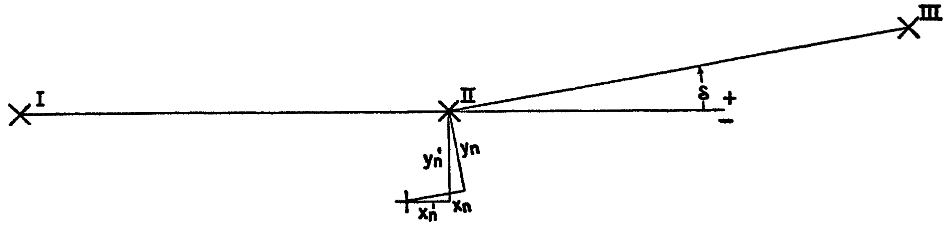


FIG. 9

should be applied to x_n and y_n, which may be computed from x_n' and y_n' by means of the common equations for rotation of axes:

$$x_n = x_n' \cos\delta + y_n' \sin\delta$$

$$y_n = y_n' \cos\delta - x_n' \sin\delta.$$

The value of cos δ can ordinarily be regarded as unity The value of sin δ can be determined with sufficient accuracy if the measured distance between the two lines of orientation on one photograph is divided by the measured distance to their intersection. If the value of sin δ is recorded on the form shown in Figure 6 it can, for sake of uniformity, be associated with the left photograph of each pair. Then when the components of average tilt are determined the value of sin δ can be plotted along with values of $\Sigma\Delta y_n'$ and $\Sigma(\Delta x_n' - \Delta x_n)$. After the averages are determined the values of $\Sigma\Delta y_n'$ and $\Sigma(\Delta x_n' - \Delta x_n)$ can be corrected for deflections in the line of flight.

The nadir can be plotted on each photograph by measuring from the principal point with a scale the coordinates x_n' and y_n' which are obtained by measuring the distance on the enlarged scale from the average to each value of $\Sigma\Delta y_n'$ and $\Sigma(\Delta x_n' - \Delta x_n)$.

Application

The photogrammetric control which is needed for preparing a planimetric map from the photographs can be easily obtained from the photographs themselves by nadir point triangulation. When the radials on each templet are carefully drawn from the nadir point, the templets can be assembled just as accurately as templets made from untilted photographs. The accuracy is usually limited by the precision of drafting Experience has shown that the templets for the average flight of 9″×9″ photographs can be assembled with an error of about 1 in 1000. That means the error in any ten-inch length need not be more than 0.01 inch. Such accuracy is high enough to eliminate much of the ground control that is needed when principal point triangulation is used in areas of high relief.

Elevations can be computed from parallax measurements which are corrected for tilt, but when the tilt of each photograph is determined from the average the elevations will be measured from a datum which is usually inclined a small amount from the horizontal The amount of inclination can be deter-

mined if several elevations computed from parallax measurements can be compared with the corresponding ground elevations. The work of computing accurate elevations is tedious because parallax measurements must be corrected for tilt, lens distortion, and variations in flight altitude, but the work is almost as simple when the tilt is determined from an average as when it is determined directly from ground control.

The cost of determining average tilt can be estimated from the time required. An experienced operator can orient a pair of photographs, measure the values of y-parallax, and compute the components of relative tilt in about twenty minutes, but to make the complete analysis for each flight he will ordinarily require an average of one hour for each pair of photographs. Part of the time required to determine tilt is balanced by the time saved in assembling templets afterward. However, most of the cost of determining tilt must be balanced against the cost of field control. The extra cost of determining tilt is certainly justified whenever it means a reduction in the total cost of mapping

CHAPTER VII

STEREOSCOPY

ELEMENTS OF STEREOSCOPY

Albert L. Nowicki

CLOSELY allied to the field of photogrammetry, which may be aptly defined as the science of the measurement of photographs, is the field of stereoscopy—the viewing of objects in three dimensions. Its application to photogrammetry is the observation of photographs with optical instruments for the purpose of measuring relative heights of objects thus shown, and also to define the shape and positions of such objects.

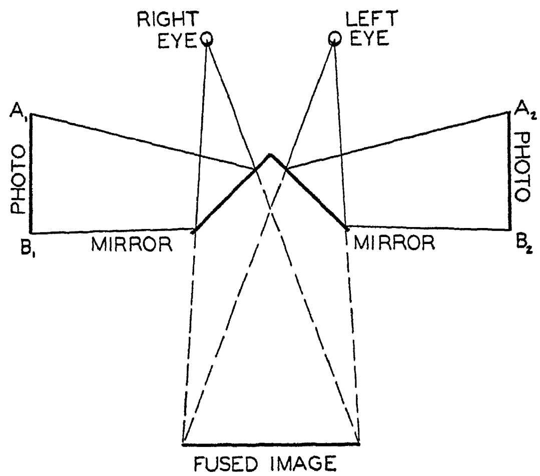


FIG. 1. Wheatstone Mirror Stereoscope.

Stereoscopic instruments may be of the mirror (reflecting) type, the prism type, or the lens (refracting) type, or a combination of all. The first recorded optical instrument incorporating the principles of stereoscopy was developed by Robert Wheatstone in 1838 This instrument consisted of two mirrors which reflected the images from a pair of stereoscopic pictures directly to the eyes. Figure 1 shows the Wheatstone mirror stereoscope, while Figure 2 shows the Helmholz (1857) four-mirror stereoscope, which is similar to those in use at present (see Figure 3). A few years after Wheatstone developed the reflecting stereoscope,

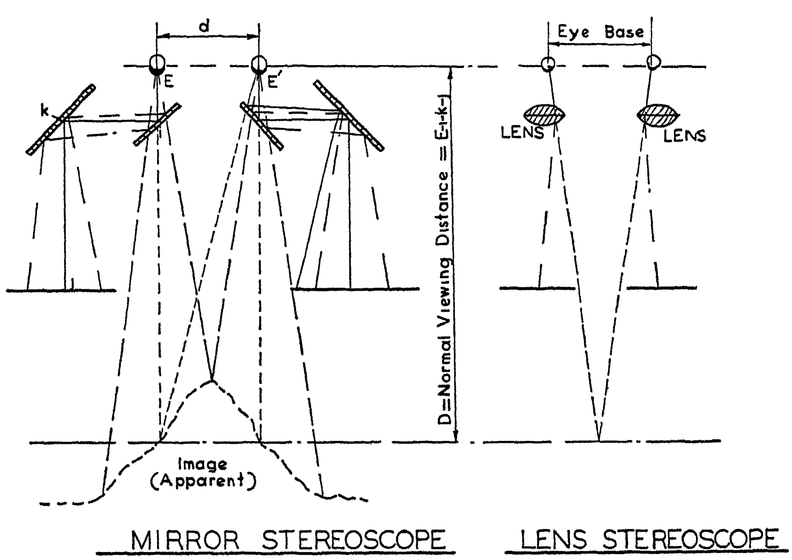


FIG. 2. Helmholz Mirror Stereoscope and Lens Stereoscope.

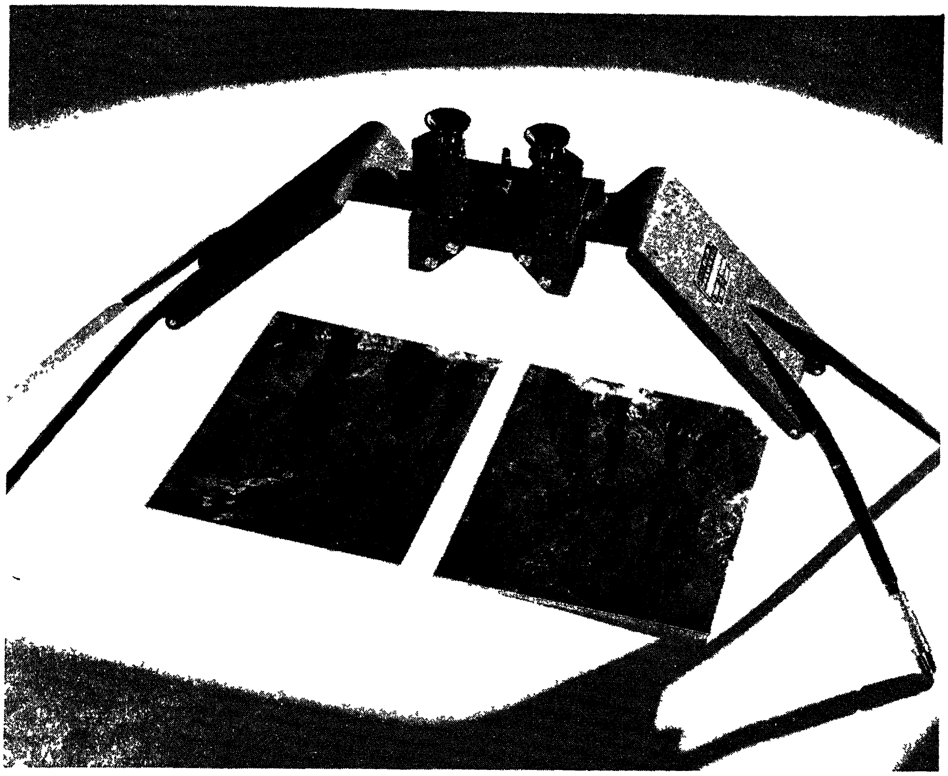


FIG. 3. Fairchild F-71 Stereoscope with Binoculars.

Sir David Brewster (1849) developed a lens stereoscope which consisted of two convex lenses separated about $\frac{3}{8}$ inch farther apart than the interpupillary distance of the observer's eyes. The characteristics of the lens stereoscope are also shown in Figure 2. Also see Figure 4.

FIG. 4. Abrams Folding Type Lens Stereoscope.

PRINCIPLES OF VISION

The principles and mechanics of stereoscopic perception are relatively simple and should be studied by every one who is to work in the field of photogrammetry. Some of the principles can readily be indicated by diagrams and simple formulae, but certain phases of these

phenomena must be considered from a physiological standpoint inasmuch as the workings of the human eye and mind also enter into the process.

The faculty of vision is so natural and customary that we seldom pause to appraise it or are in the least bit conscious of the intricate processes involved. In the process of vision, either monocular or binocular, three important elements appear to be linked together, namely, the eyeball, the optic nerve, and the visual centers of the brain The eyeball is globular in form and contains the dioptric apparatus and nervous mechanism which is sensitive to stimulation by luminous radiation (light) from without. The visual impulses thus produced by the impact of light are then transmitted to the brain where the sensation of vision comes to consciousness. Figure 5 shows a perspective view of the globular eye, and Figure 6 shows its internal arrangement

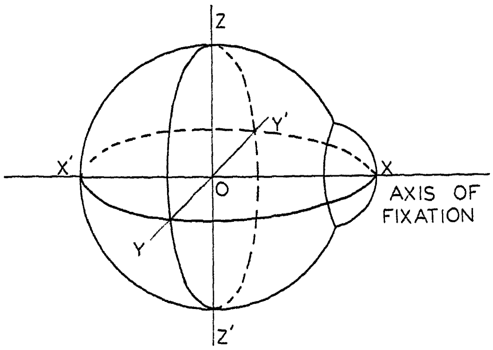


FIG. 5 Perspective View of the Globular Eye.

The retina, which constitutes the beginning element of visual perception, is perhaps the most important of all the eye components. A transverse section of the retina would show that it is made up of about seven million "cones" and about one hundred twenty-five million "rods" in addition to miscellaneous nerve fibres and cells As light falls upon the rods, a chemical change occurs within them which, in turn, stimulates the optic nerve and sends a message to the brain Each rod may be likened to the sensitive coating on a photographic film with the primary difference being that it has the power to regenerate itself. This latter process is continuously being carried out both in daylight and in darkness at a rate of about five hundred times per minute.

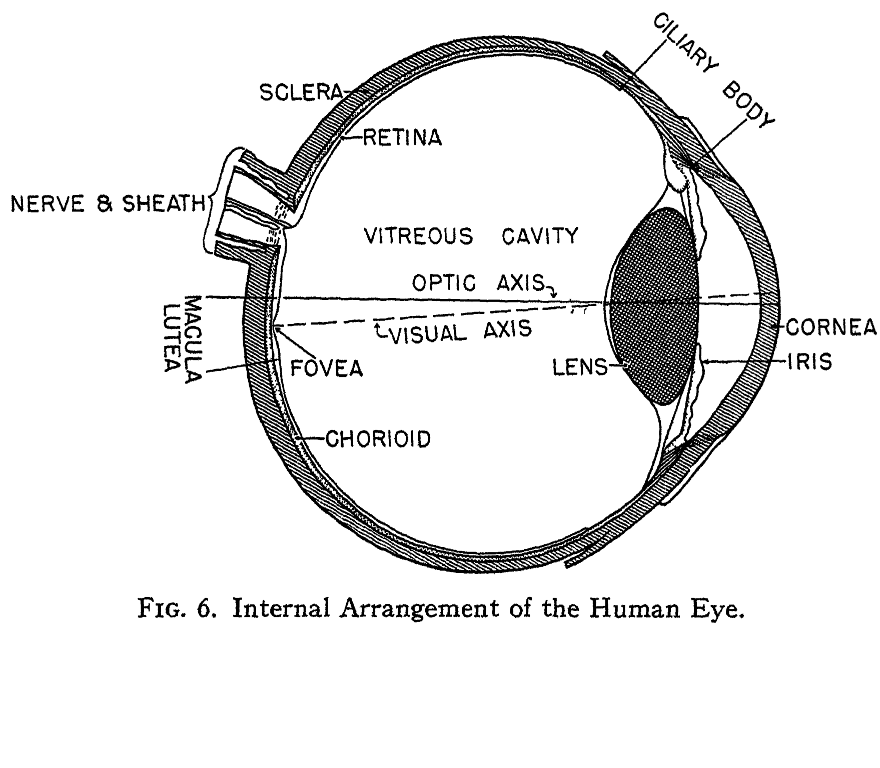


FIG. 6. Internal Arrangement of the Human Eye.

Whenever the eye fixes its attention on an object, the image is sharply focused on a small area of the retina called the macula lutea A high concentration of cones is located at this spot and tends to enhance the perception of detail and acute vision As a general rule, it may be stated that the cones make possible the abilty to see objects sharply over a small central field of view, while the rods dominate the viewing of movements and orientation of gross objects in the remainder of the outer portion of the field of view Nerve fibres leading from the retina to the brain carry the numerous stimulations that are thus set up and "develop" them by a mental process into a composite picture

Normally, the mobile human eye is capable of covering a horizontal field of view of about 45 degrees inward and 135 degrees outward and a vertical range of approximately 50 degrees upward and 70 degrees downward. In those cases where the eye is kept perfectly motionless (referred to as "instantaneous fixation"), the horizontal range is limited to about 160 degrees (45° plus 115°) Figure 7 shows the range of monocular and binocular vision

Although the single human eye (monocular vision) affords a wide range of view in a horizontal and vertical direction, it is very limited in its ability to convey accurate conceptions of depth Relative directions of objects fixed in space can readily be determined, but the process of being able to determine accurately (except by inference or association with other objects) whether one object is nearer or farther from another is impossible. A perspective view is all that can be ordinarily obtained.

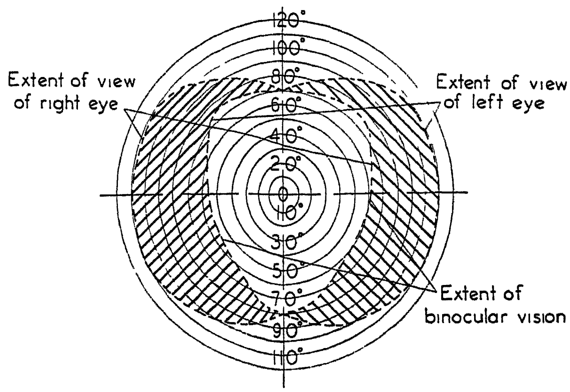


FIG. 7 Range of Monocular and Binocular Vision

BINOCULAR VISION

Fortunately, man is blessed with two eyes instead of one; thereby his faculty of vision is greatly enlarged and reinforced. Each eye is capable of executing its own movements, but constant training and use in the interest of distinct binocular vision has linked the units together to function as a "double eye." Reactions and movements are invariably made in unison. It will be found that the eyes will work together either as parallel lines of fixation (i e., in viewing a star which may be considered an infinite distance away), or as a duplex organ of sight in converging or diverging operations. In the process of convergence, the two eyes tend to work in unison whenever a change is made in the position of fixation. Such unified change in the position of fixation may occur outward or inward along the same line of vision or as a unified movement to another line of vision.

An optical characteristic which is often encountered in connection with binocular vision is that of a "double image." For example, when the eyes of the observer are focused for a certain distance, any object lying nearer or farther away will be seen as a double image. The following simple experiment may be used to substantiate this phenomenon: If an object on the opposite side of the

room, such as a small picture hanging on the wall, is observed momentarily with both eyes, and then the right eye is closed, the object apparently will shift its position to the right with respect to the wall. On the other hand, if the right eye is quickly opened and the left eye simultaneously closed, the object will appear to shift its position to the left. Or, if the same object is observed with both eyes, while holding one finger up about ten inches in front of the eyes in line with the object, it will be noted that the finger will appear to be doubled, i.e., two images will appear. Conversely, if the eyes are concentrated on the finger, the object on the wall will appear to be doubled.

In normal binocular vision, double images will not ordinarily be noticeable, for, as a rule, they are seen only when the viewer's attention is drawn to them by concentration. On the other hand, persons with defective vision, such as squint, heterotropia, or cross-eye, may see all objects doubly, although one of the images may be suppressed in consciousness Detailed studies of the human eyes have shown that there are corresponding places on the retinas of the two eyes which receive identical impressions and, conversely, that if the retinal images of one and the same object do not correspond, a double image will be seen

Additional factors which affect vision are intensity of light, differences in brightness between adjacent areas of an object, distance of an object away from the observer, and sharpness of boundary between adjacent areas. For instance, a brightly colored dot can easily be seen on black background but can hardly be seen on a background having a color which contrasts slightly from the color of the dot. An elementary experiment which will bring out clearly the basic factors involved in seeing is illustrated. Upon a sheet of white paper place a dark colored, irregular-shaped spot of a size just large enough to be visible at a convenient distance, i.e., fifteen feet. The sheet of paper should then be held so that the spot is viewed alongside of a relatively faint star in the sky (considered to be at an infinite distance from the observer). Under these conditions, both the star and the spot will appear to be the same size. However, as the observer approaches the sheet of paper, the spot will appear to get larger and more easily identifiable (the angle subtended at eye becomes greater), whereas the star will appear to remain constant in size (since it actually is an infinite distance away).

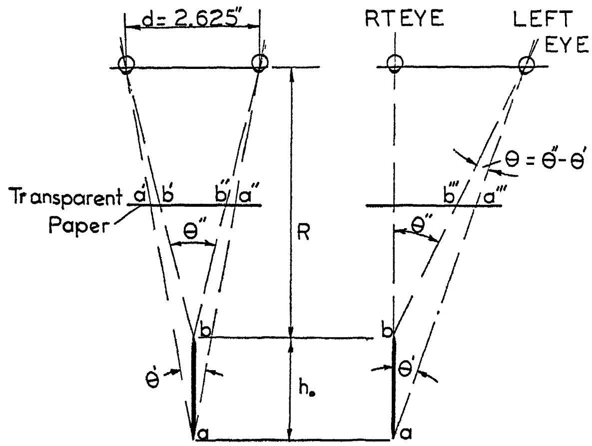


FIG. 8. Minimum Angle of Depth-Perception.

Definite conclusions may be gathered from the above experiment, namely, that vision depends upon the ratio of actual size of viewing distance (i e , angular size rather than actual size of the object), and that all objects appear about equal in size at the limit of visibility

RADIUS OF STEREOSCOPIC PERCEPTION

In monocular vision, as previously indicated, all that can be determined about the position of an object is its relative direction in the field of view. Binocular vision,

on the other hand, affords some estimate of distance and depth perception provided the image is not too far away as compared with the interpupillary distance (average value is about 2.625 inches) of the observer; see Figure 8. Even with normal binocular vision, it has been found that it is impossible to distinguish between objects if the difference of the angle of convergence ($\theta = \theta'' - \theta'$) is less than about 20 seconds of arc (0 000965 radians). Carl Pulfrich (1901) found, however, that for some extraordinary individuals this minimum angle of depth-perception was as low as 10 seconds.

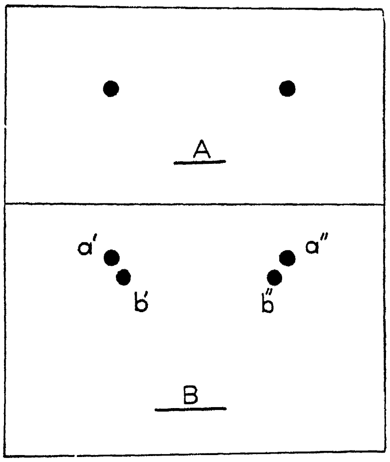


FIG. 9. Simple Stereogram.

Thus, for a range of interpupillary distance (d) of 1.97 inches to 2.85 inches (the general range for different persons) and angle (θ) of 20 seconds, the distance from an observer (R) that an object would still appear to have depth would be from 1700 feet to 2450 feet, respectively. Beyond that distance the naked eyes, alone, cannot discriminate differences of distance or depths of objects, since beyond that point all objects appear to be projected on the infinite background of space. The distance (R) has often been referred to as the "radius of stereoscopic perception."

When the distance (R) is relatively large the following relation can be used for approximate results:

$$R = \frac{d}{\theta} = \frac{d}{0.000097} = 10{,}315(d)$$

where both (R) and (d) are in the same linear units and (θ) is expressed in radians. This equation was based upon the relation that the arc distance (d) is equal to the radius (R) times the subtended angle (θ).

In aerial mapping operations involving the use of stereoscopic instruments, it will be found that visual acuity is dependent not only upon the inherent limitations of the instruments used and upon the physical nature of light but also upon the physiological state of the individual. Such factors as stimulants, fatigue, mental depression, distracting noises, unsatisfactory illumination, uncomfortable viewing position, and the improper humidity and ventilation of workshop —all tend to interfere seriously with results. Because of the number of complicated biological factors thus involved, it is very difficult to establish or confirm limits of visual acuity.

The manifestations of stereoscopic vision can best be studied and illustrated by means of pictorial views of the same object as seen from different angles by each eye separately Figure 9-A represents a pair of dots (images), which, when observed together through the process of parallel fixation, constitute a simple stereogram (spatial model).

Figure 9-B shows a simple stereogram consisting of two parallel rows of two dots each, with the lower set of dots spaced a little closer together than the upper set. By staring at the two left-hand dots with the left eye and the two right-hand dots with the right eye, it will be found that the dots (a') and (a'')

will fuse and the dots (b') and (b'') will also appear to fuse, but above the other fused pair. Figure 8 explains how this phenomenon of the "floating dot" is possible. Such a result is obtained because the angle of convergence of the inner (upper) row of dots is greater than the outer (lower) row. Hence, it is seen that such distances as ($a'b'$) and also ($a''b''$) may be used as a direct measurement of the relative heights of the objects they represent

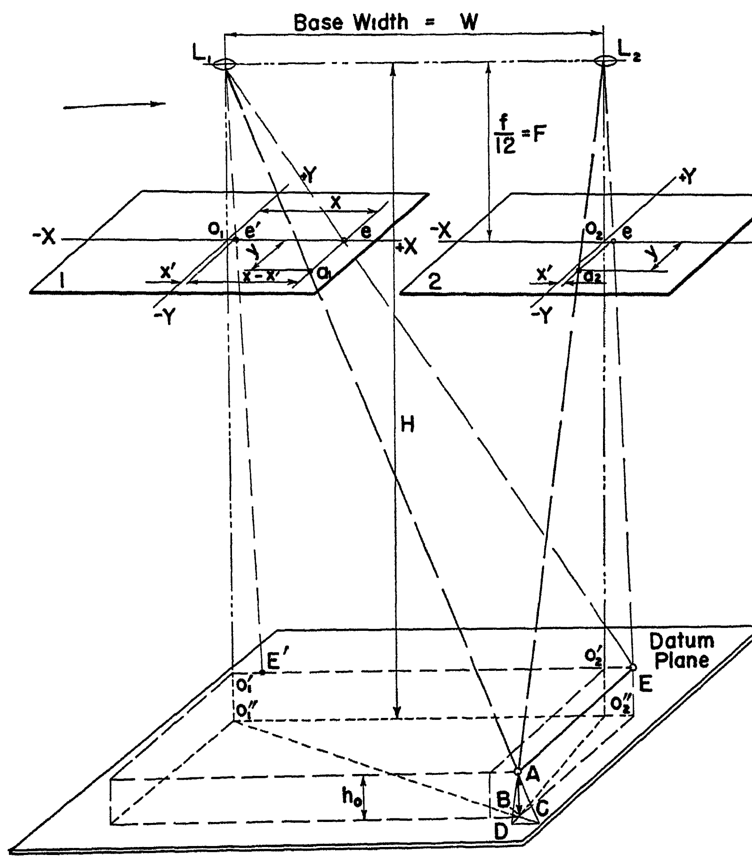


FIG. 10. Perspective View of Object being Photographed from Two Successive Camera Stations.

ABSOLUTE PARALLAX

Further reference to Figure 8 will show that the difference of the angles of convergence (i e , the angles of parallax θ'' and θ') may be used as a direct measurement of the height of the object (h_0). However, since it is quite difficult to measure the angles of parallax in the case of an actual pair of stereo photographs, it is much simpler to resort to linear measurements on the photos, such as the distances ($a'b'$) and ($b''a''$). Certain geometric relations involving such distances on the photo and the corresponding height of the object can be readily set up.

Figure 10 illustrates the perspective view of the two successive camera station positions (L_1 and L_2). Photographs 1 and 2 show the object point (A) as

falling at the points (a_1) and (a_2) respectively. Point (a_1) has the coordinate value of (x, y) and the point (a_2) the coordinate value of (x', y). It is to be noted that in the case of two photos in which the scale is the same (i.e., when photographed at the same height above the datum, with the same focal length camera, and in which no tilt exists) the successive position points, (a_1) and (a_2), will both lie at the same distance (y) from the line connecting the two centers of the photographs. This latter line is the flight line represented by the points (O_1) and (O_2). This feature is more clearly shown on Figure 11. In this case the object (AB) of Figure 10 is shown on photo 1 as the displacement (bc) and on photo 2 as (bd). These two distances are referred to as "relief displacement" distances.

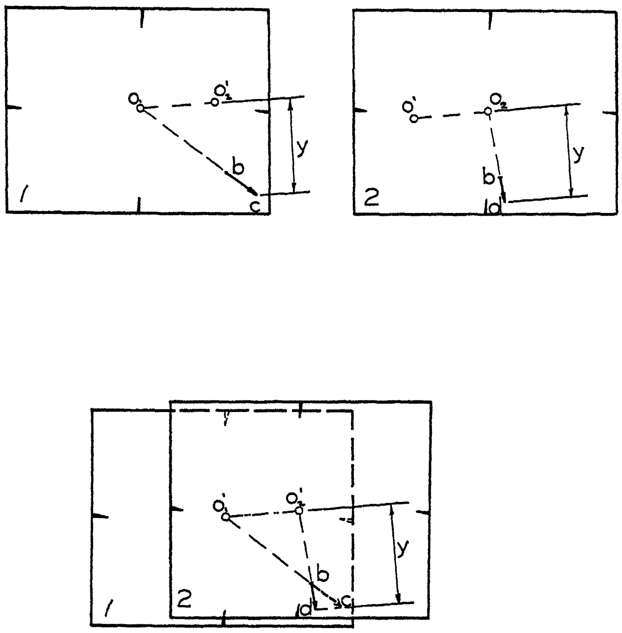


FIG. 11. Method of Obtaining "Differential Parallax Displacement."

When the two photographs are superimposed one upon the other so that the two flight lines and the two photo centers coincide, it will be found that the line connecting (d) and (c) will be parallel to the flight line (O_1'' and O_2''). By geometric relations it will be found that the line (dc) can also be used as a direct measurement of the height of the object (AB).

If, in Figure 10, the line (L_1E') is geometrically constructed parallel to (L_2E), the distance (O_1e') will then be equal to (O_2e). For convenience, both these latter distances are labelled as (x'). The distance ($x-x'$) is known as the "absolute parallax" of the point (A). The value of (x) and (x') must be added algebraically, however, if proper relations are to hold. The value of (x) and (x') are positive if they lie on the forward (as measured in the direction of flight) side of the photo and negative if they lie to the rear of the photo center. In the illustration, the values of (x) and (x') are both positive, as is the value of (y).

Parallax Equations

Figure 12 shows the principle of the many commercial stereoscopic devices which are available for measuring differences of relief. Instruments of this type are the Abram's Contour Finder, Figure 13, and the Fairchild Stereo-comparagraph, Figure 14 Figures 10, 11, and 12 will be used to derive the "parallax equations" which are the basis of all contour plotting machines and machines for determining the relative heights of objects

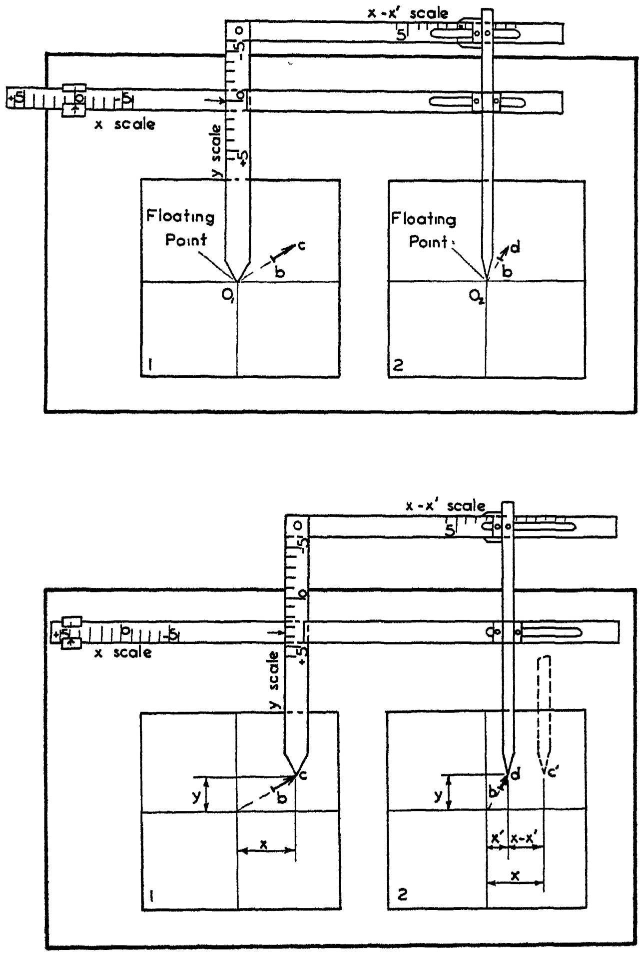


Fig. 12. Principles of Stereocomparator and Contour Finding Machines.

From the similarity of the various triangles in Figure 10, it is seen that:

$$\frac{EO_1'}{eO_1} = \frac{EE'}{ee'}\cdot \quad \text{Let:} \quad X = EO_1'$$

$$x = eO_1$$

$$x - x' = ee'$$

$$EE' = L_1L_2 = \text{Air Base width} = W.$$

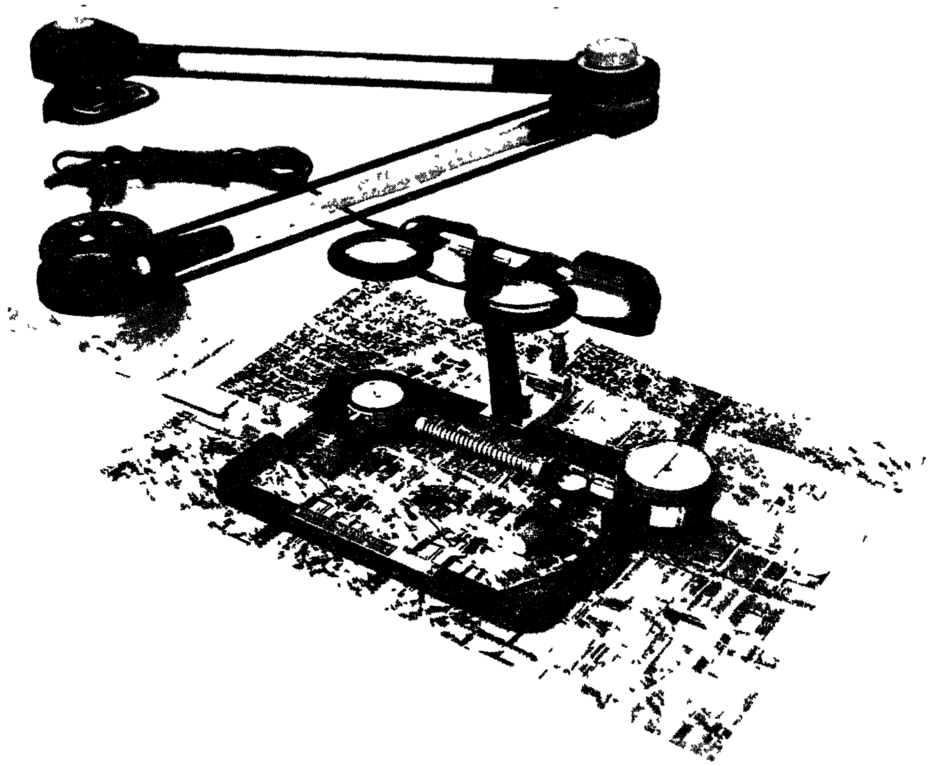


FIG 13. Abrams Contour-Finder ready for operation.

Then:

$$\frac{X}{x} = \frac{L_1L_2}{x - x'} = \frac{W}{x - x'}$$

Likewise·

$$\frac{AE}{a_1e} = \frac{EE'}{ee'} \cdot \quad \text{Let: } Y = AE.$$

Then:

$$Y = \frac{y\ W}{x - x'} \quad \text{where } y = a_1e = a_2e.$$

Also:

$$\frac{L_1O_1'}{f} = \frac{EE'}{ee'}, \quad \text{Let: } Z = L_1O_1' = H - h_0.$$

Then:

$$Z = H - h_0 = \frac{f\ W}{x - x'} \cdot$$

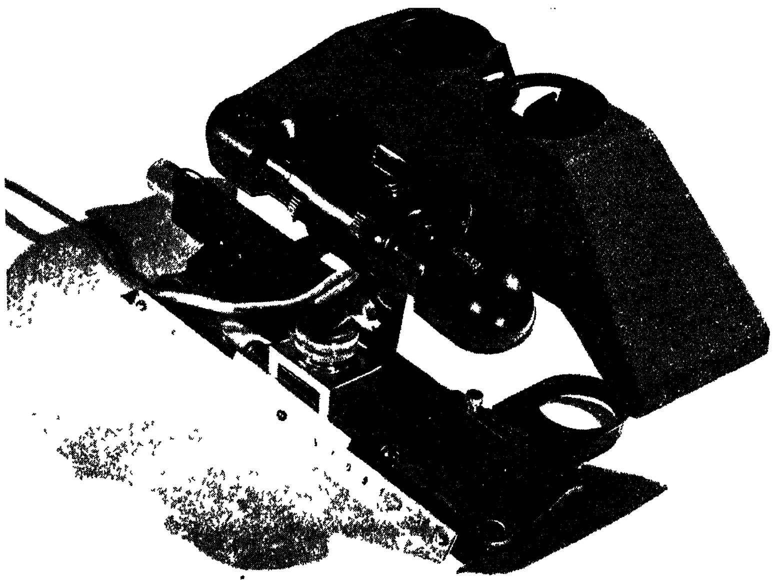


FIG. 14. Fairchild Stereo-Comparagraph

Or:

$$x - x' = \frac{f\ W}{H - h_0} = \text{“parallax distance.”}$$

And:

$$h_0 = H - \frac{f\ W}{x - x'}.$$

DIFFERENTIAL PARALLAX DISPLACEMENT

Reference to Figures 10 and 11 will show that the distance (dc) is equal to the value of $(x-x')$ minus the distance (O_1O_2'). To provide for a simpler expression let $(D_p)=(x-x')-(O_1O_2')$, which may be defined as the "differential parallax displacement" of the point (A).
Then:

$$\frac{O_1O_2'}{W} = \frac{F}{H} = \frac{\frac{f}{12}}{H}$$

Therefore:

$$O_1O_2' = \frac{F\ W}{H}. \quad \text{And: } x - x' = \frac{F\cdot W}{H - h_0}.$$

Hence:

$$D_p = \frac{F\ W}{H - h_0} - \frac{F\cdot W}{H} = \frac{F\ W\cdot h_0}{H(H - h_0)}.$$

Or:

$$h_0 = \frac{D_p\ H^2}{D_p\ H + F\ W}$$

in which

D_p = differential parallax displacement for point (A), in feet
h_0 = elevation of point (A) above the datum plane, in feet.
W = distance between successive exposure stations, in feet.
H = elevation of camera lens above datum, in feet.
f = focal length of camera, in inches.
F = focal length of camera, in feet

Since:

$$\frac{\frac{f}{12}}{H} = \text{R.F scale} = S, \quad \text{Or: } \frac{f}{H} = 12\ S.$$

Then:

$$D_p = \frac{S\ h_0\ W}{(H - h_0)}, \qquad \text{Or. } h_0 = \frac{H\ D_p}{(D_p + S\ W)}.$$

It is evident from the above equations that all points which have the same differential parallax displacement, regardless of their positions on the photographs, will have the same elevation above datum. Or, conversely, all points at the same height above a given datum plane will have the same parallactic displacement

Stereocomparator and Contour Machines

The commercial comparator machines are fairly precise instruments in which the (x), (y), and ($x-x'$) distances are measured by means of micrometer screw adjustments which may be read to the nearest 0.01 mm. Each photo of a stereo pair is attached to a drafting table with the center of each in direct alignment and in a vertical plane containing the line joining the two pointers, as shown in Figure 12. When the pointers are placed in this position the (x), (y), and ($x-x'$) scales should all read zero

Instead of a pointer, the actual contour machines have two "floating dots," or "grids," placed on transparent glass plates, located in the plane of the photographs. The dots are called floating dots because when set apart at the proper distance they will fuse into a single dot which will appear to be "floating" beneath the eyepieces of the stereoscope. If the dots are moved further apart, by means of the ($x-x'$) vernier attachment, the fused dot will appear to be lowered vertically through space away from the observer, and if the dots are brought together, the floating dot will appear to rise, relative to the landscape. Hence, by varying the spacing between the dots, the fused image can be made to approach, recede from, or actually touch a given portion of the ground.

A "Universal Drafting Attachment" is connected to the stereocomparator machine to insure that any movement of the entire unit (i.e., the stereoscope and the floating dots) will occur only in a direction parallel to the X-axis (flight line). In moving the unit from the original zero position of the dots to another position for which the new (x), (y), and ($x-x'$) values are to be found, the following

procedure is carried out: The left-hand floating dot is moved to the new object (point (c) in Figure 12) whose coordinates are shown as (x, y). The right-hand dot will then be at the position marked (c'). At this point the two dots will appear separately to the observer. The right dot is moved parallel to the X-axis by means of the $(x-x')$ adjusting screw to the position of (d) at which point it will be fused with the left-hand dot. In this position the fused dot will appear to be just touching the object whose elevation and coordinates are being determined. The scale reading on the $(x-x')$ scale may then be substituted in the formula:

$$h_0 = H - \frac{f\,W}{x - x'}\,.$$

If it is desired that contours be plotted with the stereocomparator machines, the $(x-x')$ scale reading can be set for the proper contour level and retained at that value as the floating mark is moved from point to point. Under these conditions the $(x-x')$ scale value will remain unchanged but the (x) and (y) coordinate values will be continuously changing as the instrument is moved about. Contours may then be traced by moving the unit so that the floating dot will always be kept in apparent contact with the ground as it thus moves in a horizontal plane. The path of the floating dot is traced upon a map sheet by means of a pencil that is located a suitable distance from the stereoscope and which is held in place by an extension arm leading from the stereoscope. If a new contour line is to be plotted, then the $(x-x')$ scale value must be calculated and properly set in place.

In actual contour mapping operations resort is made to "parallax distance" charts which are computed by measuring the parallax of three points which appear on the overlapping photographs and whose elevations are known. If the elevations above a given datum plane are plotted as ordinate values and the parallax distances as abscissa values, a curve will be obtained which deviates slightly from a straight line. It will be convex toward the ordinate axis. However, where only slight variations in elevation occur on a pair of photographs, this graph can be assumed as a straight line.

Usually the horizontal plane at the elevation of the lowest ground control point is chosen as the datum plane and the graph plotted from that point upwards. With such a graph available for a pair of photographs the parallax distance reading can be found for any desired level, and, if such levels are taken so as to coincide with contours, a satisfactory topographic map can be compiled.

In order to determine the value of the air base width (W) in the formula:

$$(x - x') = \frac{f\,W}{H - h_0}$$

the $(x-x')$ values of the three ground control points can be found by means of the stereocomparator machine and then substituted separately in the above equation. By solving any two of the three equations simultaneously the value of (W) can be obtained. This constant value can then be used for finding the elevation (h_0) of any other point falling on the overlapping portions of the two photographs.

Limitations of Stereo Perception

Simple and obvious as the above geometric relations appear, it cannot be inferred that stereoscopic perception can always be actually verified by one's own eyes. It has been shown that there is a definite limit (20 seconds of arc) to the

difference between the two converging angles of fixation upon an object. Such limit would be equivalent to a minimum measurement of differences of elevation of two objects of approximately 0 004 inches (i.e., without magnification). At a scale of 1/20,000, differences of elevation of seven feet or more could be determined by the above process, but a difference of less than seven feet could be determined only by means of some type of magnifying apparatus

From the formula for determining the radius of stereoscopic perception, it was found that a value of about 2200 feet would result if an average interpupillary distance of 2.55 inches was chosen. It can further be shown that differences in distances can be determined stereoscopically up to the square of the distance to the object divided by the above figure of 2200 feet Thus, if an object is 1000 feet away from the observer, other objects up to $(1000)^2/2200 = 455$ feet beyond the first object cannot be perceived stereoscopically because up to that point, the difference in the angles of convergence is 20 seconds or less.

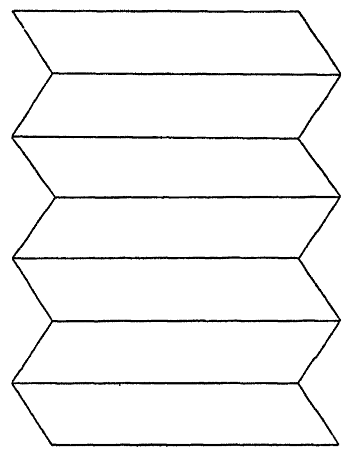


FIG 15 Optical Illusion of Depth.

Oftentimes, certain geometric figures present themselves as illusions which are purely mental That such illusions have nothing to do with binocular vision may be proved by the fact that they are more obvious when regarded only with one eye. Monocular conception of depth appears, therefore, to be more of a mental than optical process as differentiated from binocular vision which is strictly optical. Figure 15 illustrates such an illusion

Artificial enhancement of the power of stereoscopic vision can be obtained by increasing the virtual base-line (i.e , interpupillary distance) or by the introduction of a magnifying optical instrument (which directly tends to lower the effective value of the angle (θ)). The "stereo-power" of a binocular instrument is found by multipying the magnifying power of the lens, designated by the letter "M," by the ratio of increased optical base to interpupillary distance. The latter ratio can be designated by the letter "c." Thus, if a binocular instrument has $M=3$ and $c=2$, its stereo-power value would be 3×2, or, 6. An illustration of the above type of instrument would be the prism binocular or the prism

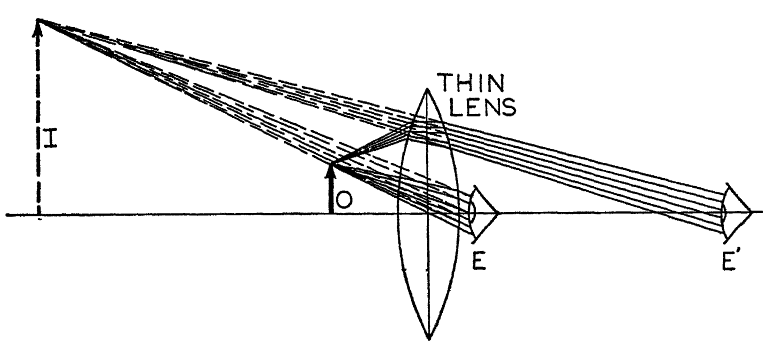


FIG. 16. Relationship Between Size of Object and "virtual" Image.

stereoscope. The above instrument would result in an increase in the power of depth perception of an object six times the distance at which stereoscopic vision could be obtained without the use of the instrument

Methods for the determination of lens magnification are discussed in Chapter II. Judging from the practical requirements of a lens stereoscope, it would appear that the lens focal length be so selected as to give an object distance (from photograph to lens) of at least three inches This would allow sufficient clearance for the tracing of topographic details or for the pricking of points on the photographs. In using a magnifying lens of a stereoscope, the eye should be kept as close to the lens as possible (i.e , position (E) in Figure 16 rather than at (E') and the lens in turn should be brought up to the object (photograph) until

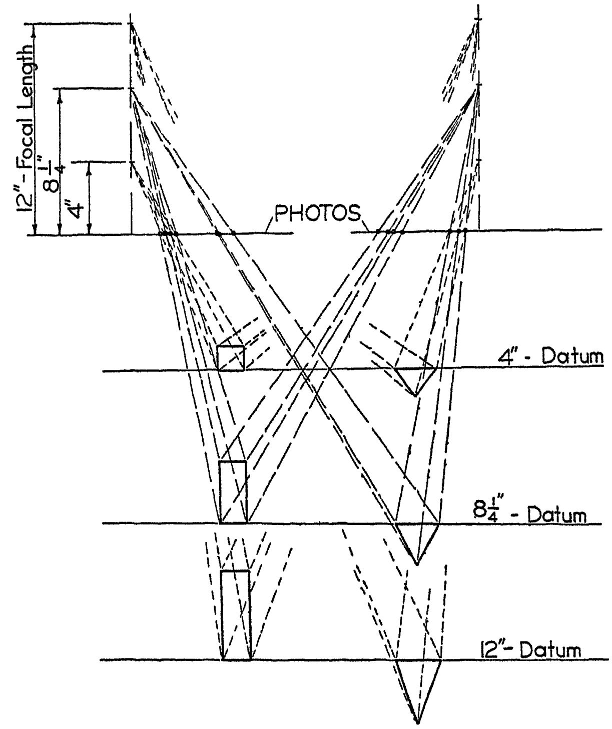


FIG. 17. Correct and Incorrect Stereoscopic Systems for Observation of Spatial Models

the latter is seen as distinctly as possible. This results in a condition in which the rays from all parts of the photo object come to the eye through the central part of the lens, thus reducing the possibilities of spherical and chromatic aberration. On the other hand, if the eye is placed at position (E') the rays from the object (O) would be refracted in the outer portion of the lens, resulting in some distortion.

PROPER STEREOGRAPHIC OBSERVATION

Figure 17 shows the result of observing objects through different stereoscopic systems in which the angles of convergence of emerging bundles of rays are not

the same as the angles of convergence in the original taking cameras. In the diagram, it is assumed that the $8\frac{1}{4}''$-focal length is the effective focal length which results in the proper reproduction of the model whereas the $4''$- and the $12''$-focal lengths are not.

Only when the observations are made with the proper stereoscopic system will the spatial model have the same vertical and horizontal scale and, hence, be a true reproduction of the original model.

Anaglyphs

Another singular effect which may be produced with stereoscopic pairs of photos is that of the "anaglyph." In this case, two separate pictures of a stereo pair are printed in complementary colors and then superposed upon a single sheet of paper. The image that is to be observed by the right eye is printed in blue-green and that for the left eye in red. Binocular fixation of corresponding points is then obtained by observing the dichromatic over-print with a pair of goggles with a blue-green glass filter or celluloid film, in front of the left eye and a red one in front of the right eye Thus, the blue-green image will be seen by the right eye alone and the red one by the left eye alone. The resulting effect will be a spatial model, in black and white, that had been formed mentally by observation of the two different optical impressions. In general, it may be stated that the ordinary anaglyph serves no practical purpose except as an instructional aid in attaining stereoscopic fusion.

Anaglyphs are printed by the half-tone process in order to insure satisfactory results. The half-tone process consists of photographing each print through two glass plates upon which have been engraved at least 120 fine lines per inch. The plates are placed so that the lines are at right angles to each other, resulting in a checkered pattern of fine dots. In order to prevent the dots of the blue-green print falling on those of the red print, the half-tone plate of one is rotated about 30 degrees with that of the other. If the images in the foreground of the two half-tones are matched one on top of the other, the resulting spatial model appears to be behind the plane of the paper. This affords a clearer mental impression of the anaglyph than if the objects in the background were thus matched.

Similar depth-impression effects may be obtained by projecting upon a single screen two stereoscopic pictures that have been illuminated by light of two different colours. Spectators are then able to observe the effect of relief by the use of suitable goggles as indicated above. It is this same principle which is followed in the observance of spatial-models in connection with the Multiplex Plotting Machine (see Chapter XI).

White light, which has been polarized in two planes at right angles to each other, may also be used to illuminate two stereo pictures instead of the two colours (i.e , red and blue). It is necessary, however, that "polaroid" spectacles be used in the observance of the model which is formed by the illumination of each picture with a different beam of polarized light. In this method of relief visualization, the right eye will receive one kind of polarized light while the left eye will receive the opposite kind (also see vectographs, page 327).

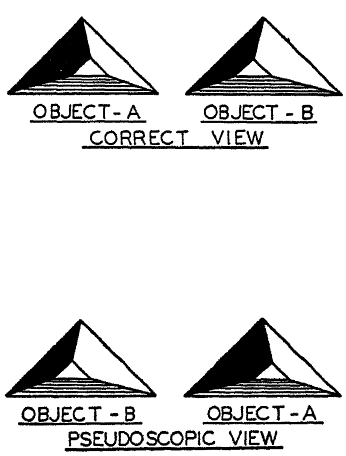


FIG. 18. Stereograms Showing Correct and "Pseudoscopic" Views.

Pseudoscopic Views

In observing an anaglyph or a stereogram, care must be taken to assure that a reversal of relief is not obtained. Such an effect is known as a "pseudoscopic illusion." A reversal of relief is obtained if the photo originally intended to be observed by the left eye is placed at the right-hand side of the stereoscope (or observed through the red-colored glass in the case of the anaglyph), and if the photo designated to be seen by the right eye is placed at the left-hand of the stereoscope (or observed through the blue-colored glass). The same result can be obtained with the anaglyph by merely rotating it through 180° (i.e., the top of the picture is placed at the bottom) and observing it with the spectacles in their original position. Figure 18 shows a simple geometric stereogram in correct (orthoscopic) position and also in reversed (pseudoscopic) position.

Pseudoscopic illusions can be obtained much more easily through the use of simple geometric figures than with complicated objects In the first case, the converse figure is as easy to visualize as the original one because it is one of frequent occurrence. On the other hand, objects such as buildings, hills, and valleys, are difficult to observe conversely because no previous observation in nature has been encountered.

Sometimes the term *pseudoscopic* is also used to describe the apparent reversal in relief seen on viewing a single photograph in different orientations. The correct relief impression usually being obtained when the shadows fall toward observer For instance, the valleys shown in Fig. 19 appear as ridges when the photograph is viewed upside-down.

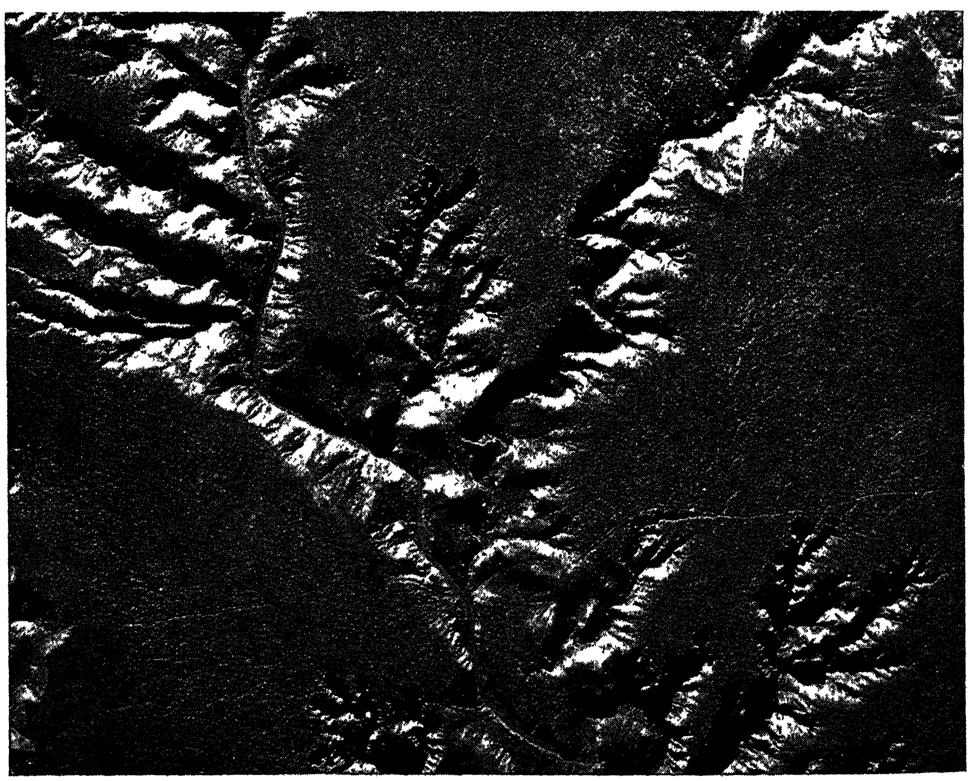


FIG. 19. Effect of shadows on relief perception.

ORIENT YOUR STEREOSCOPE CORRECTLY

Revere G. Sanders

THE use of aerial photography since early 1941 has grown to such tremendous proportions that only the roughest estimate of the rate of growth can be made. Along with the increase in the use of aerial photography has come a proportionate increase in the use of stereoscopes for proper evaluation of the photographs made by the rapidly mounting quantities of aerial cameras in use New people are being called upon constantly to use these stereoscopes for the study of aerial photographs for the many varied purposes to which they are put in connection with war time photography.

Instruction and practice is recognized as being necessary in order to get the most out of aerial photographs. Both the Army and the Navy are conducting schools by which they are training men heretofore unfamiliar with aerial photographs, in the intricacies of photo-interpretation, camouflage detection, and the many types of war time mapping. These Army and Navy schools are being augmented by the existing colleges and universities which are teaching courses under the defense training program in photogrammetry, camouflage and map making.

The principal instrument used in all of these training courses, and in the actual work which is done at the conclusion of the training courses, is the simple stereoscope. The stereoscope is the most important single instrument to anyone involved in the use and evaluation of photographs, and its importance is second only to the aerial photographs themselves. Unfortunately too few people really understand and appreciate the correct way to set up aerial photographs for stereoscopic study. Consequently few people get the full value of the three dimensional view which can be seen through a properly used stereoscope. In addition, many people undergo undue eyestrain because they do not know that it is possible to set up the photographs and the stereoscope in a way that will eliminate eyestrain even when used constantly over long periods of time.

Many people who use stereoscopes have learned through trial and error After sufficient practice, such people are able to see three dimensionally to a certain degree and are satisfied. However, it is only when they look through a properly adjusted stereoscope at properly oriented photographs that they realize what they have been missing through the use of "rule of thumb" methods So many users of stereoscopes know of no other system than to juggle a pair of photographs beneath a stereoscope until finally a three dimensional view of sorts is obtained It is no wonder that a beginner looking through a stereoscope at a pair of photographs so oriented finds it very difficult or impossible to get the stereoscopic view.

The correct method of setting up aerial photographs for stereoscopic study takes a small amount of time and trouble. Naturally there are many conditions under which it is unnecessary to take the time and trouble necessary to set the photographs up the correct way Approximate methods can be used for brief or casual stereoscopic study of only a few minutes' duration. However, even to set up the stereoscopes for a casual study requires a thorough knowledge and past practice of the correct method. A thorough grounding in the correct practices enables a person to approximate very closely to the correct method without actually going through the operations necessary.

The first step to be learned is how to avoid what is known as the "pseudoscopic" effect. The pseudoscopic effect is the exact opposite of the stereoscopic effect. Instead of houses, trees and hills extending upward in a natural fashion

as they do in a stereoscopic image, the houses, trees and hills appear to project vertically downward into the ground. Thus valleys become hills and vice versa. Sometimes the pseudoscopic effect is desired to aid interpretation, but that is the exception to the rule. Consequently, it is usually desired to know how to avoid the pseudoscopic effect rather than how to get it

Inspect one photograph of the stereoscopic pair of photographs to be viewed and see if shadows in the photograph can be seen Trees, houses or other objects will cast shadows and these shadows will be visible to a certain extent. If high hills or mountains are in the photographs, the shadows will be apparent on the slopes of the hills away from the sun. Lay one photograph of the pair on the

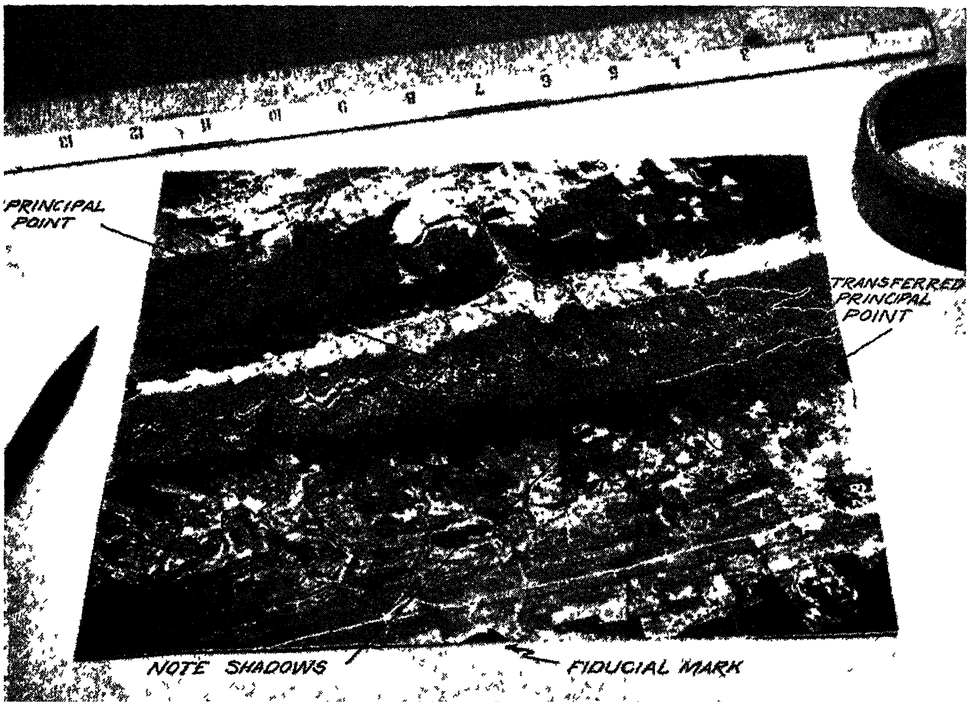
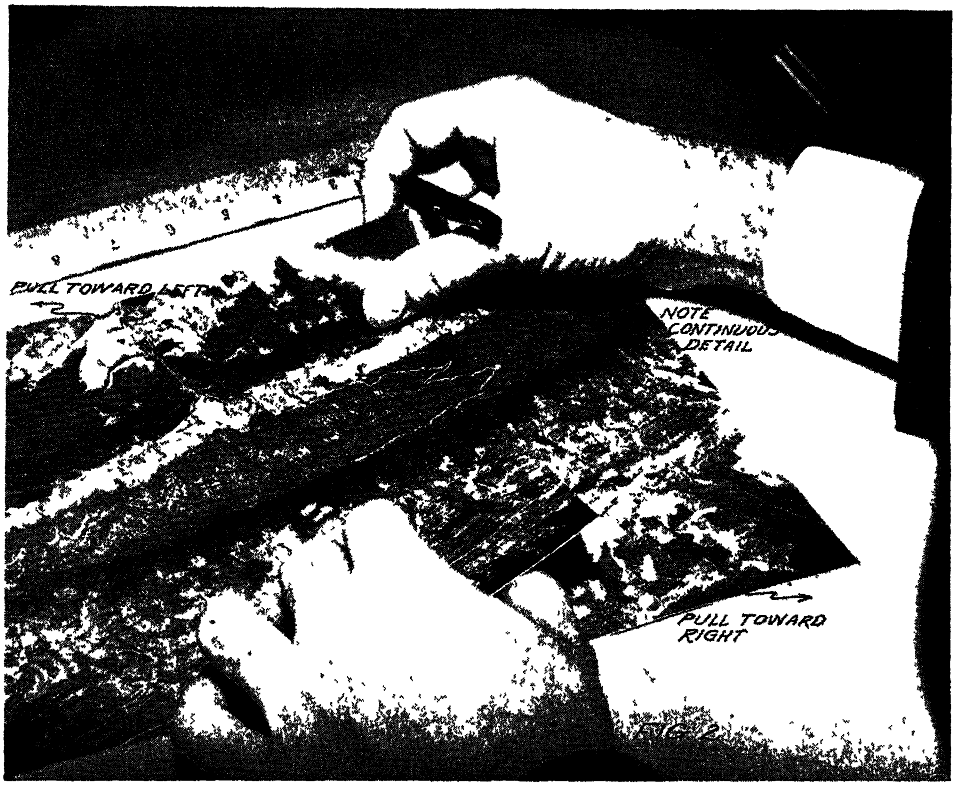


table so that the shadows appear to be cast by a light somewhere out beyond the upper left-hand corner of the photograph. This orientation is shown in Figure 1.

Each photograph of a properly taken stereoscopic pair of photographs contains about 60% of the details shown in the other. This is shown in Figure 2. Using the second photograph of the pair, lay it over the first picture which is lying on the table, so that the detail is continuous as shown in Figure 2. Then pull the two photographs apart so that the overlapped portions are adjacent to each other (Fig. 3). If this is done, and if the operation to be explained later is followed, a pseudoscopic image will not be obtained. The photographs can now be considered as the left-hand photograph and the right-hand photograph respectively.

Each photograph contains what is known as fiducial marks. Figure 1 illustrates one form of fiducial marks commonly used. However different types of cameras have different types of fiducial marks but they are all readily apparent either being in the center of the sides or at the corners of the photographs. Their purpose is to enable a person to locate the center of the photograph. The center of the photograph is commonly referred to as the principal point (Fig. 1). The principal point is simply determined by drawing lines between opposite fiducial marks and the intersection of the lines is the principal point of the photograph Actually, a line is not drawn across the entire photograph but only a short section of the line near the center as shown in Figure 1 Sometimes when the princi-



pal point is found it is marked merely by a pin prick with a small circle around it and other times a cross is used. It was explained above that each photograph of a stereoscopic pair includes about 60% of the photographic detail in the other. Consequently the principal point of one photograph will likewise appear in the other. This can be understood by reference to Figure 2. For instance, when the principal point of the left-hand photograph is located, it will probably be found that it lies on top of some recognizable spot of detail. If one looks closely in the other photograph, this same spot of detail will be found and a pin prick can be made or a cross made to indicate the point to which the principal point of the left-hand photograph has been transferred. Likewise the principal point of the right-hand photograph will be found to lie on top of a recognizable bit of detail which can be likewise found in the left-hand photograph. As a result of the deter-

mination of the principal points and the transference of the principal points, four definite points exist in the stereoscopic pair of photographs (Figs 3 and 4)

Fasten the left-hand photograph to drawing board or table by means of a fine needle or pin, pushed through the principal point (Fig. 3). For convenience, it will be found that the bottom edge of the photograph should be approximately five inches in from the edge of the drawing board or drawing table, when the photograph is thus fastened down. The next step is to fasten the right-hand photograph in position The distance of the right-hand photograph from the left-hand photograph is important Most reputable manufacturers of stereoscopes provide this information in their descriptive handbooks The separation is meas-

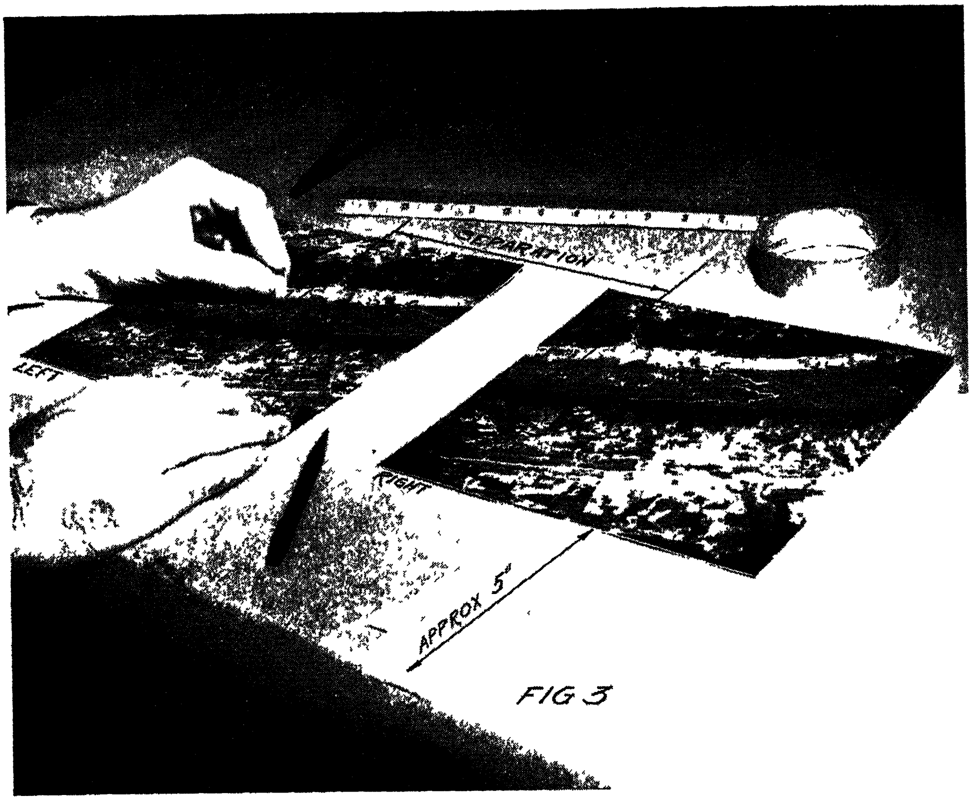


FIG 3

ured from a point in one photograph to the same point where it occurs in the other photograph. Usually the measurement is made from the principal point of the left-hand photograph to the transferred position of that point in the right-hand photograph (Figs 3 and 4). This dimension for the Fairchild Model F-71 (Air Forces Type A-6) Stereoscope is $9\frac{3}{4}$ inches, and for the Fairchild Stereo-comparagraph is $6\frac{1}{4}$ inches.

Lay the right-hand photograph on the drawing board opposite the left-hand photograph by the correct separation (Fig. 4). Push a thin needle through the principal point (center) of the right-hand photograph into the drawing board or table. Thus this photograph, like the left-hand photograph can be rotated about its center point. Lay a straight-edge across both photographs so that it is held against the two needles (Fig 4). Rotate the left-hand photograph under-

neath the straight-edge in this position until the transferred principal point in the left photograph is directly beneath the edge of the straight-edge By means of Scotch tape or thumb tacks, fasten the left-hand photograph permanently in this position Then rotate the right-hand photograph underneath the straight-edge in a similar fashion until the transferred principal point of that photograph is directly under the edge of the straight-edge. Then fasten the right-hand photograph securely to the drawing board and the orientation of the photograph has been accomplished. The needles can now be withdrawn.

With both the left and the right photographs now secured to the table in proper orientation, the stereoscope should be set up over the photographs (Fig

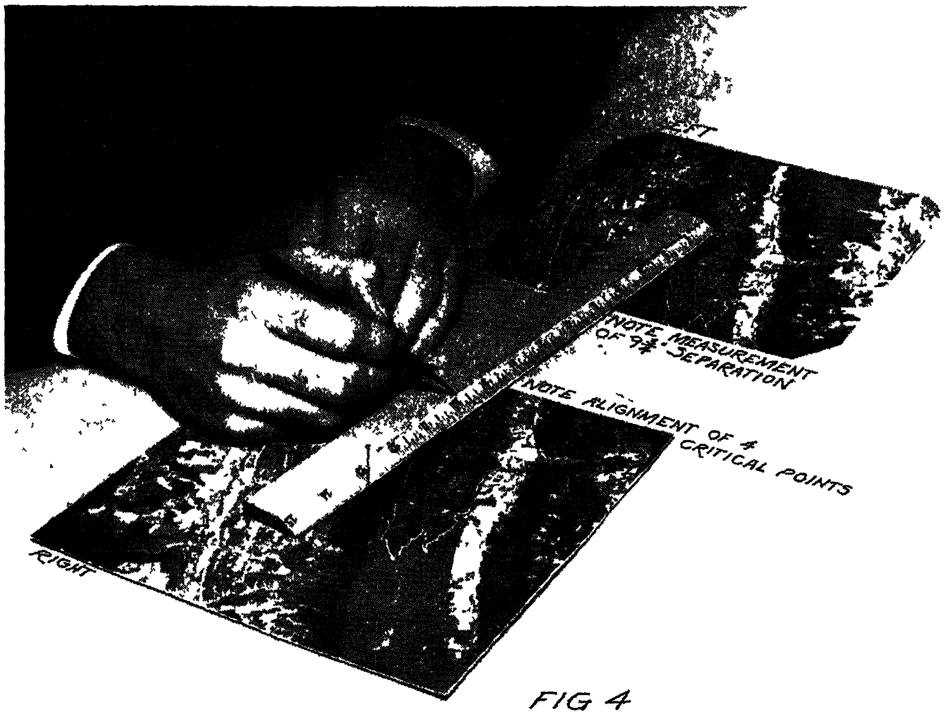


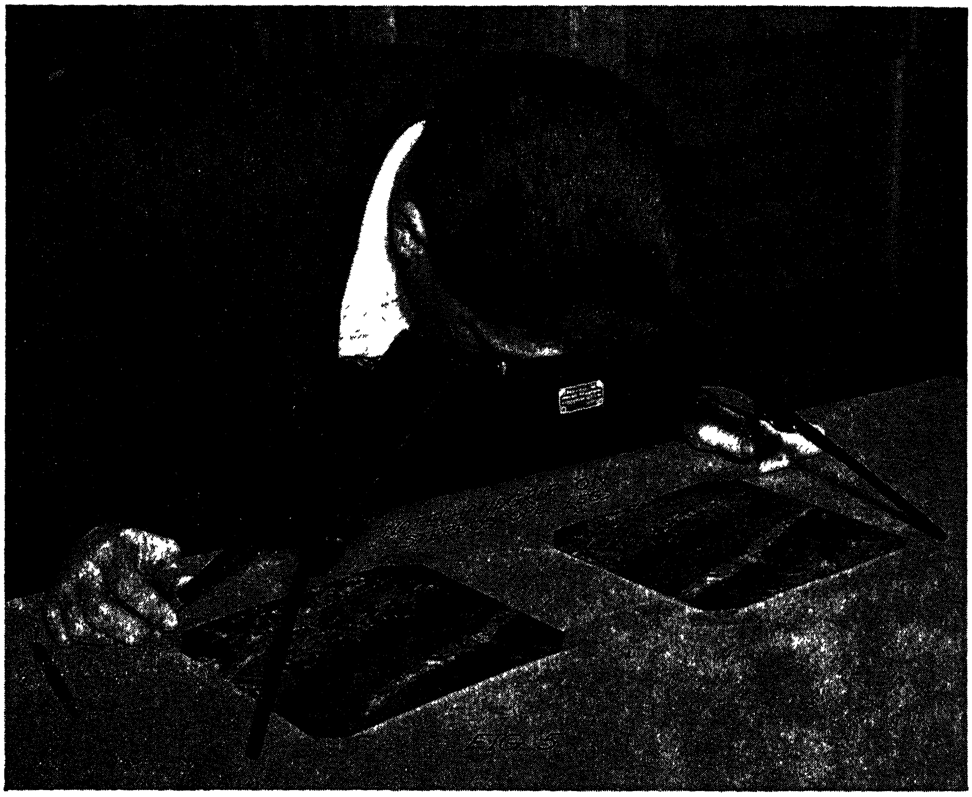
FIG 4

5). In setting up the stereoscope, a line through the centers of the eye lenses or viewing points should be parallel to the imaginary line through the four points on the photographs. A good test is to lay a straight-edge along the four points and then look through the stereoscope. If the straight-edge appears to be non-continuous, it is only necessary to rotate the stereoscope clockwise or counter-clockwise until the straight-edge appears to be continuous across the entire field of view When all of this is done, the three dimensional view will literally pop up at the observer. Furthermore, the observer can step back with confidence and call up a novice without making excuses about slipshod orientation.

To describe a simple operation such as the foregoing frequently requires, as in this case, a long space of time. However, if the Stereoscope, the stereoscopic pair of photographs, a ruler, thumb tacks or Scotch tape, and pins or needles are

available to start with, this whole procedure need not take longer than 3 or 4 minutes. This method should always be used if the photographs are to be studied for long periods of time The method should be used if the photographs are to be studied for the purpose of interpreting or detecting information of great importance. This method must be used if there is any intention of making parallax measurements for the determination of elevation.

A student or instructor who is thoroughly familiar with the above method can readily line up a pair of photographs without resort to the procedures outlined previously and can get the photographs reasonably close to the desired orientation Such approximation serves for the quick inspection of a pair of



photographs for the information of the observer himself. However no matter how experienced the observer is, if he must study the photographs for a long period of time, the approximate method is not sufficient. In conclusion the essential steps to be followed are here given.

a) Select the left and right photographs to avoid the pseudoscopic effect.
b) Determine the principal point of each photograph and transfer each to the other photograph.
c) Affix the photographs to the table along the base line with the proper separation.
d) Set up the stereoscope with the eye base of the instrument parallel to the stereoscopic base of the photographs.
e) Look through the stereoscope and view the three dimensional model.

THREE DIMENSIONAL VECTOGRAPHY

*Commander R. S. Quackenbush, Jr.**

VECTOGRAPHY is a new medium for the presentation of stereoscopic views. It provides an easy way for nonexperts to see—and use—the three-dimensional visual model of the land previously restricted to the expert equipped with a stereoscope. Easy and quick to reproduce in quantity, requiring only a pair of spectacles for viewing, capable of presenting very large areas of terrain in a single view, and making no demands on the skill of the observer, the vectograph presents a number of new possibilities in the field of military and civil photography.

In military operations, the vectographs have accomplished what no other form of presentation has ever been able to do, as it has given the field commanders a photographically precise scale model of the terrain, a model which needs no interpretation or special knowledge

Tactically, the existence or nonexistence of a low hill or a shallow ravine can be the key to the success or failure of an operation The troop commander in the field has to know the lay of the land. In the past he has been able to extract information on land formation from maps and information on vegetation cover from photomaps. For the first time it is now possible for him to get both types of information in the same picture The vectograph shows him the whole situation at a glance With no special knowledge or skill in interpretation, he can tell what he is running into on a strange beach, how his men can use natural land and vegetation cover, and how to lay out his communication lines.

Experience in the war theatres has indicated the value of the vectograph in combining the precise detail of the aerial photograph with the relative-elevation information conveyed by the contoured map; a value particularly important in areas where maps are likely to be crude.

Single Prints for Direct Viewing or Projection

To the casual glance, vectographs look very much like glossy paper prints. They can be held in the hand, mounted in books or reports, assembled on a plotting table and otherwise handled much as paper prints are handled.

The viewing device is a pair of polarizing picture viewers, simple spectacles fitted with polarizing disks. When viewed through the spectacles, the vectograph appears as a three-dimensional model of the terrain

Vectographs may be prepared in several different forms, corresponding to the forms of the photograph: reflection prints, resembling photographic paper prints; transparencies, for direct viewing against a light box; lantern slides for projection on a screen for group viewing.

Three-dimensional vectographic slides may be projected in any lantern slide projector. The vectograph is its own polarizer, so no polarizing filters are involved. Both images occupy the same film area, so there is no need for beam-splitting attachments. The relationship of one image to the other is determined when the vectograph is made, so that beam-adjusting arrangements are neither desirable nor possible. The vectographic slide or film simply takes the place of the conventional slide.

The only requirements are that the screen MUST have a metallic surface in

* The opinions or assertions contained herewith are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department or the Naval Service at large.

order to preserve the polarization of the light that forms the images and each person in the audience must have his polarizing viewers, either the simple card type or the spectacle variety

The stereoscopic model presented by the vectograph is essentially the same in appearance as that presented by paper prints viewed in the stereoscope, except that in vectographs made from current materials the image is sepia in color rather than black The process is capable of continuous tone reproduction with definition of about 50 lines per mm

FIG. 1. Viewing a vectograph.

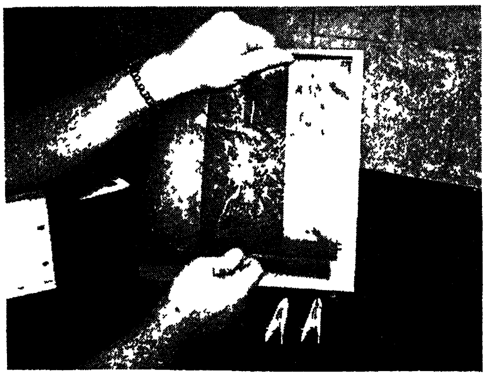


FIG. 2. One step in processing a vectograph.

FIG. 3. Printing a vectograph.

Vectographs may be made to cover any area of terrain the situation demands Using an extremely rapid, simple technique developed through the joint efforts of Lt. Hubert Dogan (USMC), Commander Roswell Bolstad (USN), Clarence Romrell (Polaroid War School) and others, service photographers are producing vectograph mosaics as large as seven by eight feet, presenting complete steroscopic models of many hundreds of square miles of terrain at original photographic scale. These large mosaics may be viewed as a whole, by a number of observers simultaneously; a feature of considerable interest in military staff work A single set of paper prints of the terrain is laid down, first with left-hand halves of the prints exposed, to form a "right-eye" view of the terrain, then by re-lapping, with right-hand halves exposed to form a left-eye view. The two views, recorded on copy-negatives, are then treated as if they were a simple stereo pair.

PROCESSING

Almost all stereograms suitable for viewing in the stereoscope may be reproduced in vectographic form Vectographs are made from negatives made according to standard mapping procedure, with 60% forward overlap and 30% sidelap. They may also be made, from aerial obliques or from stereograms of nonaerial subjects obtained with stereoscopic cameras or with a single camera moved between exposures.

A simple imbibition transfer process is being used at present in reproducing the views on the vectograph film Each of the two stereoscopic negatives is printed photographically on wash-off relief film on which the picture images appear in relief in the gelatin emulsion, the degree of relief varying with the density of the photographic image These relief films are placed in approximate register, emulsion sides together, soaked briefly in printing solution and then, with a sheet of vectograph film inserted between them, passed through a common clothes wringer. After the vectograph film has had an opportunity to imbibe the printing solution from the relief film, the relief films are stripped off and the vectograph is given a brief bath in a fixing solution. At this point, it is ready for use as a transparency or as a lantern slide

For finishing as a reflection type print, it is painted with clear lacquer on the front and with aluminum lacquer on the back Army and Navy photographers, after two practice runs through the process, are usually able to turn out a print, dried and finished, in about 35 minutes. Succeeding copies take about 1minute each

The process is the invention of Edwin H. Land, President and Director of Research of Polaroid Corporation, working in collaboration with Joseph Mahler. Announced as a laboratory achievement in 1939, it was quickly developed into practical form for war use with the encouragement of the Navy, Marine Corps and the Army Air Forces.

Military Applications

Army, Navy and Marine Corps photographic units are producing vectographs in the combat theatres for a number of uses. Lieutenant Colonel M. E. Parks describes the military uses of the vectographs as follows

> This new method is now an established service in the AAF, the Navy, Marine Corps, and the RAF. Air and ground force groups in the South Pacific, the Aleutians, Africa and Britain are having them made up by photo units already trained and equipped to do the job.
>
> Although new uses for vectographs will arise from time to time, their principal military value may be summed up as follows:
>
> Briefing combat teams, such as bomber crews, assault parties, landmining groups, engineers, air support groups, ship-to-shore artillery units and parachute troops.
>
> Staff work—for tactical planning over unmapped or sketchily-mapped territory
>
> Intelligence reports—for conveying intelligence information to field officers who need not be equipped with stereoscopes in order to be thoroughly familiar with the location and appearance of enemy supply depots, dumps, road crossings, bridges, communication bottlenecks, fortifications, the effects of bombing and shellfire, disposition of enemy artillery and desirable target objectives.
>
> Training large classes of student mechanics, gunners, navigators, pilots and other personnel who must assimilate a lot of information rapidly on subjects and devices that are difficult to understand from a flat picture

For photographic interpretation, it appears that the stereoscope, in the hands of the skilled intelligence officer, is likely to hold its well-established place. The job of the vectograph is that of extending to all the nonexperts, the advantages of stereoscopic viewing formerly available to a few experts.

After painstaking examination by intelligence officers, for example, the aerial reconnaissance photos covering the entire area of an enemy base have been reproduced in vectograph form, turned over to the naval, ground and air commanders involved in the operation for their tactical planning, and then passed along to lower unit commanders so that all of those taking part in the operation

could have a clear grasp of the ground to be covered A platoon sergeant with no skill at map reading or photo interpretation gets the whole story at a glance

Civilian Applications

Only recently have vectograph materials been made available for civilian applications, so that there have been few opportunities for actual trial on actual operations It has been suggested, however, that a number of uses now served by the photomap may be served more effectively by the vectograph

Dr. Ian Campbell, of California Institute of Technology, has made use of vectograph mosaics as map supplements in field work for the USGS.

Mr. Louis A Woodward has proposed the use of vectographs in soil conservation field work.

Many other possible applications suggest themselves. in the preliminary studies for siting of highways, power lines, flood-control and regional planning projects; recording of progress on construction projects, lumbering and reforestation operations, wherever, in fact, the lay-of-the-land is an important part of the message that the aerial photograph is expected to convey.

References

1 "Vectographs Images in Terms of Vectorial Inequality and their Application to Three-Dimensional Representation" E. H Land, Journal Optical Society of America, September 1940
2 Lt Col. M. E Parks, "Depth Perception Simplified," Air Force Magazine, August, 1943.
3 Tech. Order 10-25-23, U.S A.A F
4 "How To Make Polaroid Vectographs," processing manual published by Polaroid Corporation, Cambridge, Massachusetts.
5. "Stereoscopic Photography," Richard T. Kriebel, "Complete Photographer," Issue 51—1943
6. "Vectograph Process," Richard T. Kriebel, "Complete Photographer," Issue 53—1943
7. Rule, John T, Polaroid and its Application to Aerial Photography and Photogrammetry," Photogrammetric Engineering, Vol VII, No. 1, pages 31–39 (1941).

CHAPTER VIII

INTERPRETATION

PHOTO-INTERPRETATION

William C. Putnam

Introduction

THE purpose of this chapter is to describe briefly the criteria used in the interpretation of aerial photographs. An understanding of the nature of the terrain is a fundamental pre-requisite in the construction of maps by photogrammetry and in the interpretation of aerial photographs for military or civilian use.

Vertical photographs at first may appear almost meaningless to the inexperienced but with practice they will reveal a wealth of detail unobtainable on any single map. The principal reason for the apparent difficulty in understanding them is the unfamiliar appearance of the landscape they portray. Most commonly, objects are viewed horizontally and the eyes are likely to "select" individual objects in a landscape rather than attempt to encompass the entire area at once. The side of a building has a more familiar appearance than its roof, and mountain ranges are more commonly viewed from the plain at their base than from the air above However, artificial features, such as buildings, roads, and railroads are not difficult to distinguish, as they resemble their conventional map representations. Mountains, valleys, streams, and coastlines seldom are shown in anything like their true complexity on maps and when seen vertically on a photograph they appear strange and unfamiliar. Oblique photographs are less difficult to comprehend, as they resemble a landscape viewed from a high mountain

The range of tones from light gray to black in aerial photographs depends on the amount of light reflected from the earth. Where much light is reflected the photograph is light-colored; where light is scattered or absorbed, the photograph appears dark In vertical photographs, water generally appears black, but if the sun's rays are reflected to the camera, the color may range from light gray to white.

Shadows are extremely important in interpretation. They reveal the outlines of buildings or silhouettes of such objects as ships, and may indicate the presence of tall structures, as derricks, smokestacks, and water tanks that are almost invisible when seen vertically. Without shadows to throw hills and valleys into relief, the terrain appears flat and almost featureless However, the pattern of light and shadow on an aerial photograph may at first lead to confusion in distinguishing ridges from valleys. The natural relief may appear to be reversed, the streams seeming to flow along ridge crests, and the ridges appearing to be valleys. This illusion may disappear if the photograph is held so that the shaded side of the ridges is nearest the observer and the light source away from him. Photographs taken in the northern hemisphere should be oriented so that the southern edge of the picture is at the top.

The characteristics of various terrain and cultural features are briefly listed in the following outline and some are illustrated by aerial photographs. Descriptions in the text apply chiefly to vertical photographs.

Identification of Man-made Features

Under this heading are considered some of the works of man, such as railroads, roads, bridges, canals, and buildings. These cultural features are commonly the first to be recognized on an aerial photograph and are the easiest to identify.

Railroads: These generally appear as dark, narrow bands. Curves are usually

of large radius and are separated by straight stretches of track, or tangents. Cuts and fills are common, and tunnels are numerous in mountainous terrain. Gradients are less than on highways, and in hilly country the ground on either side of the track may be smoke-darkened Accessory features, as stations, sidings, water tanks, loading platforms, over- or under-passes, classification yards, and the presence of trains on the track all aid in identification. Railroad bridges are generally narrower than highway bridges

Roads: Secondary roads are more readily distinguished from railroads than highways. Secondary roads are lighter-colored and in hilly terrain make a less regular pattern than railroads On nearly level ground secondary roads may make a more regular pattern than railroads, as the latters' alignment is controlled by topography Highways as a rule are wider than railroads. They have sharper curves, and secondary roads join them instead of crossing by over- or underpasses. Paved highways in hilly or rolling country are darkened where automobiles have discharged oil when accelerating on a grade In hilly country, secondary roads are more likely to follow contours than railroads or highways, and to utilize hairpin turns in climbing steep slopes. European roads are likely to be narrower and more winding than American ones, and have fewer road cuts In mountainous terrain many are supported on hill slopes by masonry retaining walls.

Bridges: These are readily recognized by the fact that they span water bodies. The type of bridge is most readily determined by its shadow. Among the more common forms which may be identified by their shadows are suspension, truss, cantilever, stringer, arch, and lift bridges of various sorts Pontoon bridges are identifiable by their floats.

Canals: If water-filled, these features may photograph darker than roads Under the stereoscope they are commonly recognized as depressions bordered by levees In level country they usually have a rectilinear pattern; in hilly areas they nearly parallel the contours. Drainage canals on the flood plain of a large river are generally aligned at right angles to the river bank. Roads and railroads cross canals by small bridges which may be identifiable

Urban Features: Residences, office buildings, factories, schools, churches, airports, docks, refineries, parks, and racecourses are not difficult to recognize, most of these features have distinctive plans or silhouettes. Many European cities are more congested than American ones, and may lack the familiar rectangular street pattern. Some show traces of ancient walls, and in the vicinity of some of the larger ones are 17 or 18th Century forts built on the star-redoubt pattern of salients and reentrants. The older area within the walls, is likely to have narrow, crooked streets; newer districts outside may have rectangular blocks and wide boulevards Many Asiatic towns and villages are even denser agglomerations of houses bordering narrow, winding streets

Agricultural Patterns· Farm boundaries in the western United States and Canada conform to the rectangular coordinates of section lines. In eastern United States and Europe their pattern is controlled to a large extent by the terrain European farms are particularly distinctive, as they are commonly divided into long narrow strips. These will be various shades of gray or black, depending on the crops and condition of the ground. Many European peasants live in small, densely clustered villages and walk to the fields to work. As a result, there are few isolated farmhouses or outlying structures resembling American barns or silos. Hillsides are commonly terraced with stone retaining walls. Terraces are also extensively utilized in rice-growing areas of the Orient. Rice paddies are surrounded by low, earthen dikes, and are generally laid out on a rectilinear plan

The paddies make an intricate pattern of various shades of gray, no two are an exactly identical shade

Grass and cereal crops photograph light-gray, vineyards are represented by rows of small dots, orchards appear as rectangular patterns of larger dots Hay stacks or corn shocks appear as light-colored dots Deciduous fruit trees, as apples and pears, show marked seasonal changes; evergreen trees, as oranges and olives, remain about the same throughout the year. European orchards, particularly in hilly parts of Italy and the Balkans, are less likely to have as regular a pattern as American orchards

FIG. 1. Delta and flood plain crossed by meandering stream. Note several cut-off meanders (ox-bows) and old-world agricultural pattern of irregularly-shaped fields bordered by hedgerows. USAAF.

VEGETATION

Next to man-made features, the vegetational pattern is perhaps the most readily identifiable on an aerial photograph. The distribution and character of plants on the earth's surface is controlled by many factors; among them are temperature, insolation, precipitation, altitude, and type of ground. In the interpretation of aerial photographs it is desirable to know the characteristics of regional assemblages of plants, as well as their relation to their environment. The

regional classification used in this chapter is generalized, and it should be recognized that gradations among these assemblages are the rule.

1) *Tropical Vegetation:* There are two main types; the rain forest and the savanna. The rain forest is prominent in the Congo and Amazon Basins and in southeastern Asia, including the islands of the southwest Pacific, and in southeastern Asia, including the islands of the southwest Pacific. The forest is a dense stand of broad-leafed evergreen trees forming a nearly continuous canopy with very little undergrowth Individual trees may be more than 200 feet high and are recognized stereoscopically by their rounded, dome-like tops Undergrowth is extremely dense where the canopy is broken and sunlight reaches the forest floor. The unbroken forest appears dark in the photograph; few clearings interrupt its monotonous expanse. The rain forest may extend up the flanks of tropical mountains, and on steep slopes may be gashed by landslides or deep gullies. Temporary clearings in the forest are made by natives, and these may survive for a few years before they are overgrown (Figures 7 and 8).

Savanna-type vegetation grows in parts of the tropics that have pronounced wet and dry seasons. The savanna is largely grassland with scattered groves of trees and some open forest Savannas show marked seasonal changes, in contrast to the rain forest In the dry season many trees shed their leaves, while the grass turns light yellow-brown and will photograph light-gray. In the wet season plants regain their foliage very shortly after the rains commence and the color of the entire region changes from a dusty brown to green. Ground appears darker in photographs taken during the wet season.

2) *Desert Vegetation:* Plants are widely spaced and appear as small, isolated dark spots on the photograph (Figure 3) The greatest concentrations are where water is close to the surface, as at seepage springs near the base of an alluvial fan Trees and bushes are concentrated along desert water courses Desert mountains may have slightly more vegetative cover than the plain at their base. There is little seasonal change in most deserts Many of the larger plants, such as cactus, have no leaves, or, like the sage brush, may have small, leathery evergreen ones.

3) *Mediterranean Vegetation:* This type is characteristic of regions of moderately high temperature with winter rain and summer drought. This climate typifies southern Europe, Chile, California, and South Africa. Most native trees and shrubs are evergreen. The landscape is green in late winter and dusty brown by summer Characteristic trees are eucalyptus, pepper, live-oak, orange, and olive. Open plains are likely to be treeless, or to have trees concentrated along watercourses and in irrigated areas. Hills are generally mantled by waist- or head-high evergreen brush—the *chaparral* of the western United States or *maquis* of the Mediterranean region. Irrigation is necessary for certain types of agriculture; ditches, canals, and diversion works should be looked for in aerial photographs

4) *Temperate Climate Vegetation:* This type is characterized by summergreen, broad-leafed trees (elm, beech, maple, oak, birch, and chestnut) Pine and other conifers are abundant in some parts of the region, as for example the pine forest of southeastern United States. The range of characteristic plants is great, and includes such diverse assemblages as the hardwood forest of eastern United States and the treeless prairie of the Western Missouri Basin. The most distinctive characteristic is the pronounced seasonal change; trees shed their leaves in autumn and the ground is barren or snow-covered in winter.

5) *Arctic and Alpine Vegetation:* High mountains and far-northern areas may be largely devoid of vegetation, except for dwarfed trees, low shrubs, grass, or

moss (Figure 15) Typical trees at lower altitudes, and in the sub-arctic zone, are conifers, readily recognizable in photographs by their tapering, spire-like shadows. Low ground near rivers is the site of an extensive forest of spruce, alder, willow, birch, and balsam poplar In the extreme north these forests yield to the treeless expanse of tundra; a snow-covered or barren region in winter and a lichen-, grass-, and moss-covered plain in summer.

Terrain Features

Many terrain features are immediately apparent in aerial photographs and an understanding of their nature is of importance in the photogrammetric preparation of maps Some are so obvious that they are readily recognized from the air, and for this reason should be represented on aeronautical charts. Others possibly of equal importance, require some knowledge of geologic processes before their significance is appreciated It is the purpose of this section to outline the criteria by which significant elements of the terrain are recognized

The topography of any region is the result of the erosion, through a long period of time, of rocks exposed at the surface of the earth In order to understand the nature of the terrain, it is necessary to know what processes of erosion have been active, how far they have advanced, and what sort of rocks have been affected and their structure.

Rocks

All rocks exposed at the earth's surface may be placed in one of three major classes. These are 1) *igneous*, 2) *metamorphic*, and 3) *sedimentary* Igneous rocks are crystallized from molten material and may have solidified below the surface, in which case they are intrusive (*granite* is a common example), or on the surface, in which case they are called volcanic (*basalt* is an example) Metamorphic rocks are recrystallized from pre-existing rocks by heat, pressure, and chemical activity. These processes are active deep within the earth rather than on the surface. Metamorphic rocks may be massive or banded (*gneiss*), or they may be foliated (*schist*), or they may separate readily into thin plates (*slate*) Sedimentary rocks consist of mineral or rock fragments deposited in layers (*strata*), and cemented by lime, silica, or iron oxide to form such rocks as *limestone*, *shale*, *sandstone*, and *conglomerate* (listed in order of increasing size of their fragments)

Based on their appearance in aerial photographs, rocks may be placed in two general groups; massive and layered rocks The massive rocks include most intrusive and many metamorphic rocks. The layered rocks include sedimentary, most volcanic, and such metamorphic rocks as schist, and slate.

Massive rocks appear to be essentially homogeneous over broad areas, and may be largely devoid of patterns, other than lines of fracture (*joints*), or *faults*. Some intrusive igneous rocks may be penetrated by *dikes* or *veins* which show in photographs as light- or dark-colored lines cross-cutting the massive rock

Bedded rocks make a definite pattern of parallel bands on aerial photographs (Figure 3). The character of the pattern depends on the structure (described in next section), and is accentuated by differences in resistance of the rocks to erosion and by the extent to which the soil derived from them limits the growth of vegetation.

Characteristics of some important rocks:

1) *Granite*, a light-colored, massive igneous rock. Fresh outcrops usually photograph light-gray. In arid climates the rock makes bold, rounded outcrops generally seamed by many joints. In humid climates granite may either produce

rugged mountains, or may weather deeply and make smooth and unbroken slopes. Most outcrops are rounded, and large boulders are common on the surface.

2) *Volcanic Rocks*, generally appear dark in photographs *Basalt*, an outstanding example, photographs black (Figure 11). The ropy, or slaggy surface of

FIG. 2 Very deep stream canyon cut in horizontally-stratified sedimentary rocks Note how their relief is accentuated by snow and how their pattern resembles contours Canyon rim is at extreme upper right corner. USAAF

the flows is a striking feature of recent lava. Cinder cones, fissures, and individual flows of various ages may sometimes be recognized

3) *Limestone*, a sedimentary rock, may appear as massive as granite, and in photographs the two may be confused. With careful study some evidence of stratification can usually be seen When compared to other rocks, limestone is quite soluble and may weather to an intricate topography of crag-like pinnacles and closed depressions, or *sink holes* (Figure 12). Limestone may survive as rock at the surface of the ground, even where deep weathering is the rule, as in the

tropics It may be covered by thin, stony, bright red clay (*terra rossa*) that will photograph light to dark gray

4) *Sandstone and Shale* in photographs range from dark gray to almost white. As a rule, outcrops of fresh rock are lighter in color than the surrounding soil In aerial photographs the layered appearance of interbedded sandstone and shale

FIG. 3. Cuesta eroded from tilted sedimentary rocks. These strata are inclined towards the left. Note braided channel of dry stream crossing center of photograph. USAAF

may be accentuated by marked differences in ground color (Figure 3). For example, sandstone strata may photograph light-gray and be separated by layers of darker-gray shale. Vegetational differences are sometimes pronounced Sandstone usually is more permeable than shale, and as a result soil overlying sandstone in some areas may support a dense growth of shrubs and trees, whereas shale is barren, or grass-covered.

Structure

By structure is meant the arrangement of rock masses in the earth's crust. Rocks, subjected to stress, may be deformed into folds, or fractures (faults). The alternation of strata of various colors is likely to make structures in sedi-

mentary rocks more readily identifiable in aerial photographs than structures in massive rocks like granite (Figure 4).

Horizontal Strata: Flat-lying strata, incised by stream canyons, crop out on valley walls as parallel, horizontal bands that essentially resemble the pattern of contour lines. That is, they point upstream in a series of sharp V's, make

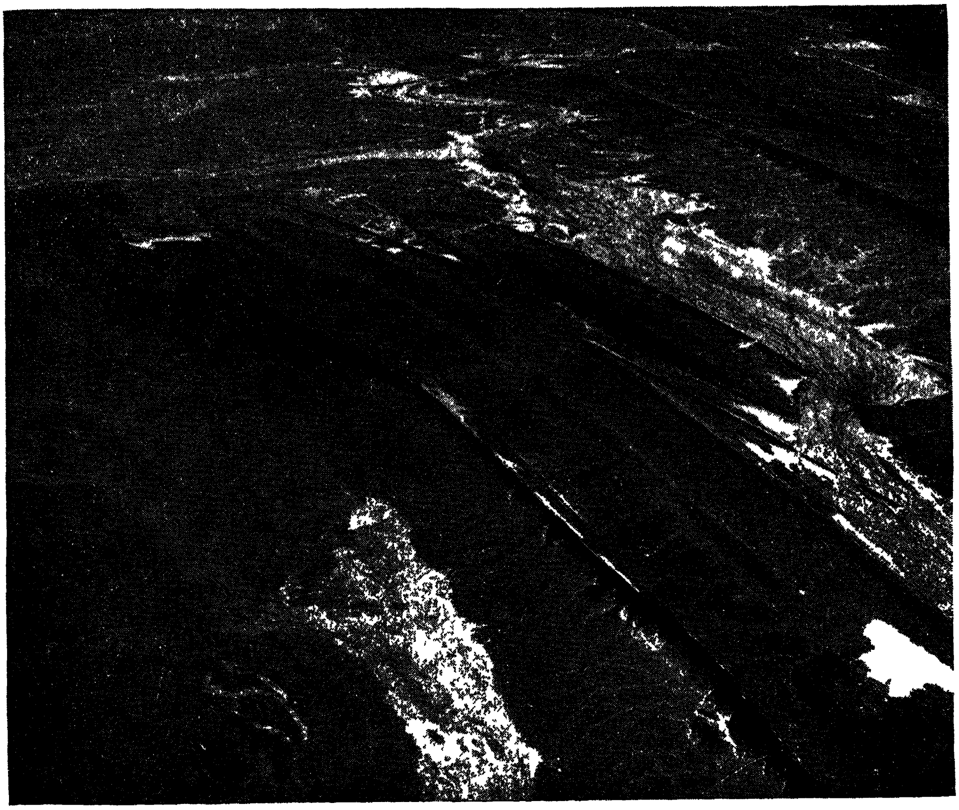


FIG. 4. Folded sedimentary rocks. Sequence of folds from right to left in central part of photograph is an anticline, a syncline, an anticline, and a syncline. Long, dark ridge in foreground is outer limb of nearest syncline. USAAF

convex curves around spurs, and form closed figures encircling isolated peaks and knobs (Figure 2). Extensive areas of flat-lying strata may make *plateaus*, as the Colorado Plateau at the Grand Canyon. Smaller or more isolated areas of flat-lying rocks cropping out in arid regions are *mesas*. The sharply defined rim of a plateau or mesa is an *escarpment*.

Tilted Strata: Sedimentary rock strata that are steeply tilted crop out as parallel bands. Where tilted rocks are cut through by a stream canyon, individual beds appear on the photograph to V up- or down-stream in the direction toward which the strata are inclined. Layers that are resistant to erosion may stand out as prominent ridges parallel to the trend of the strata (Figure 3). Such a ridge is known as a *hogback* if the two slopes are approximately equal; it is a *cuesta* if one slope is markedly steeper than the other.

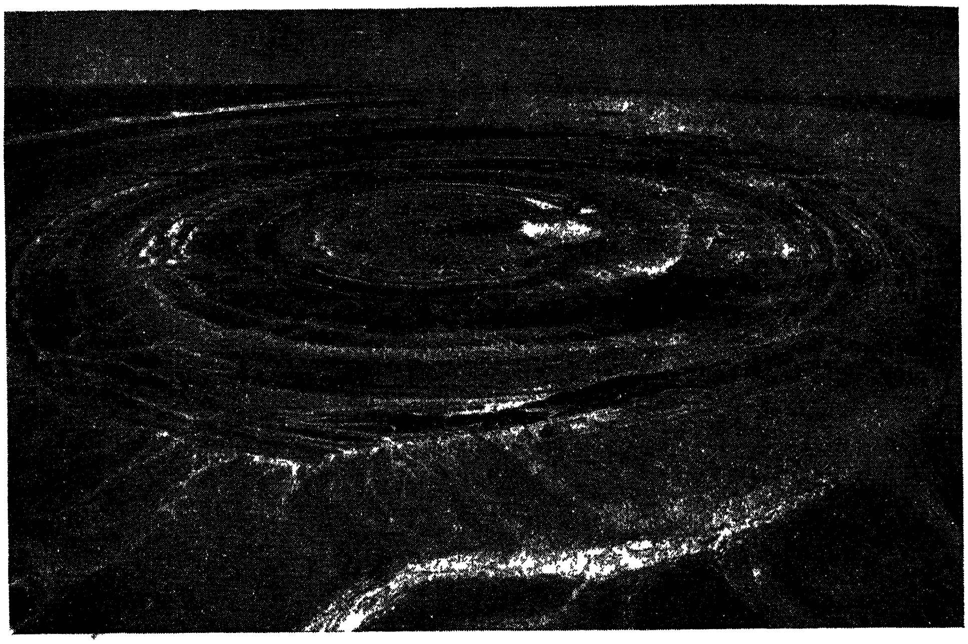


FIG. 5 A dome. Note how strata dip away from center of fold, top of dome has been eroded away. USAAF

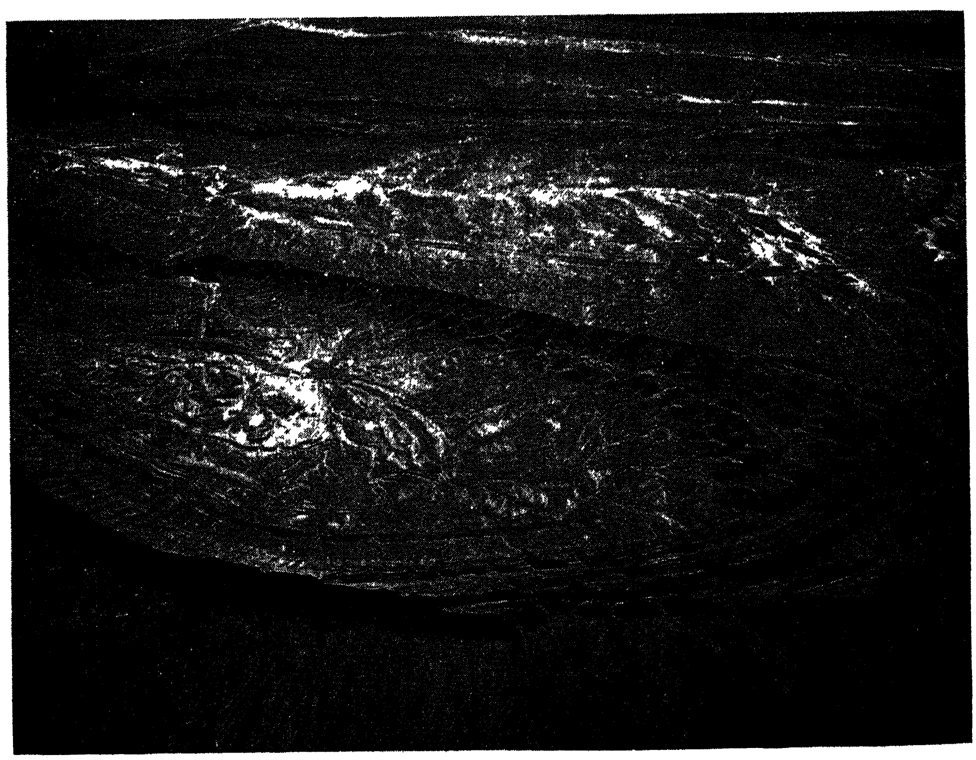


FIG. 6. A basin. In this fold all the strata dip towards center of the structure. USAAF

Folded Strata: Two common folds recognizable in some aerial photographs are *anticlines* and *synclines* An anticline, in cross-section, resembles an arch, and a syncline, a trough. Strata cropping out on the flanks of either of these folds may form parallel cuestas; in an anticline the more gentle slopes face away from

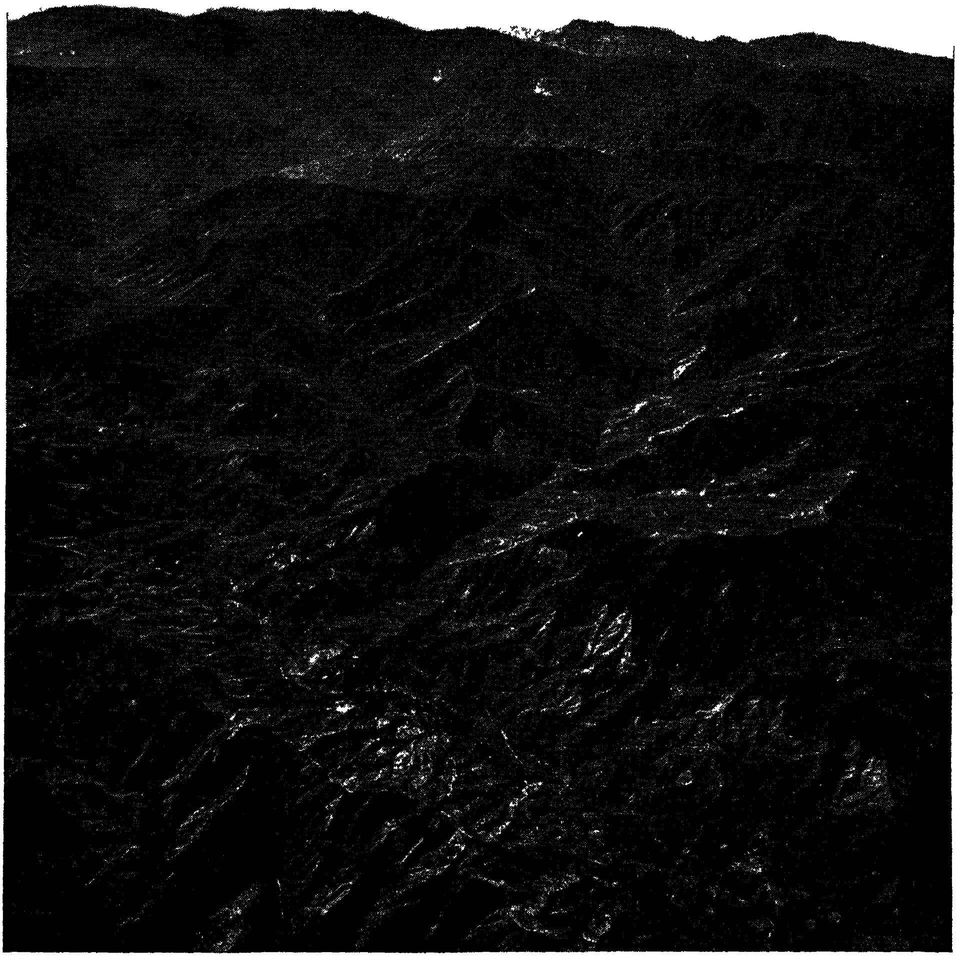


FIG. 7. Tropical mountains. Note branching network of main and tributary streams. So-called "tapestry" forest extends to summit of mountains. Patchy areas on steep slopes in foreground are small agricultural clearings made by natives. USAAF

the axis, and in a syncline the steeper slopes face away from it. Anticlines whose strata dip outward from a center are *domes* (Figure 5); a synclinal structure whose strata dip toward a center is a *basin* (Figure 6).

Faults: These are fractures along which rocks have been displaced. Some are of great size; the San Andreas fault in California is more than 600 miles long. Faults may appear as linear features on aerial photographs, and may be recognized by the fact that they interrupt the outcrop pattern of rocks, or bring un-

related rocks into contact with one another. Rocks in the vicinity of a fault may be shattered and distorted, and, as a consequence, may be readily eroded. Long, linear valleys may be developed by streams along the fault line, where rocks have been shattered or deformed.

Erosive Processes

Rocks cropping out on the surface of the earth are continuously subject to erosive processes. The character of the erosive processes depends on the type of rock and the climate. The purpose of this section is to indicate the results ac-

Fig 8. Stream pattern in tropical rain forest. Note branching tributaries and unbroken carpet of vegetation. USAAF

complished by various processes; the ones considered are stream erosion in humid regions and glacial erosion, and marine erosion, in arid regions.

Stream Erosion

Humid Regions: Most streams are perennial in humid regions. Rivers form elaborate, interrelated systems of tributary and main streams branching in a manner similar to the trunk and branches of a tree. Details of such a landscape are clearly revealed under the stereoscope, and constitute the type of terrain familiar to most Americans. The direction of stream flow usually may be determined under the stereoscope, or by the fact that on most streams the acute angle between trunk and tributary streams points downstream.

Streams crossing nearly level ground may follow curved or sinuous courses. The bends of such a stream are *meanders*, which are conspicuous features on

FIG 9 Ice-covered meandering stream. Note several cut-off meanders, or ox-bows Ground is under snow. USAAF

FIG. 10. Meandering stream. Note complex system of ox-bows and abandoned channels. What is the direction of stream flow? USAAF

some photographs (Figure 9) Cut-off meanders form crescent-shaped lakes (*ox-bows*), which in time are filled with sediment. Their pattern of crescentic loops may be identified by soil and vegetational differences for many years after the lake has disappeared Sand bars show up clearly as light gray areas in a river

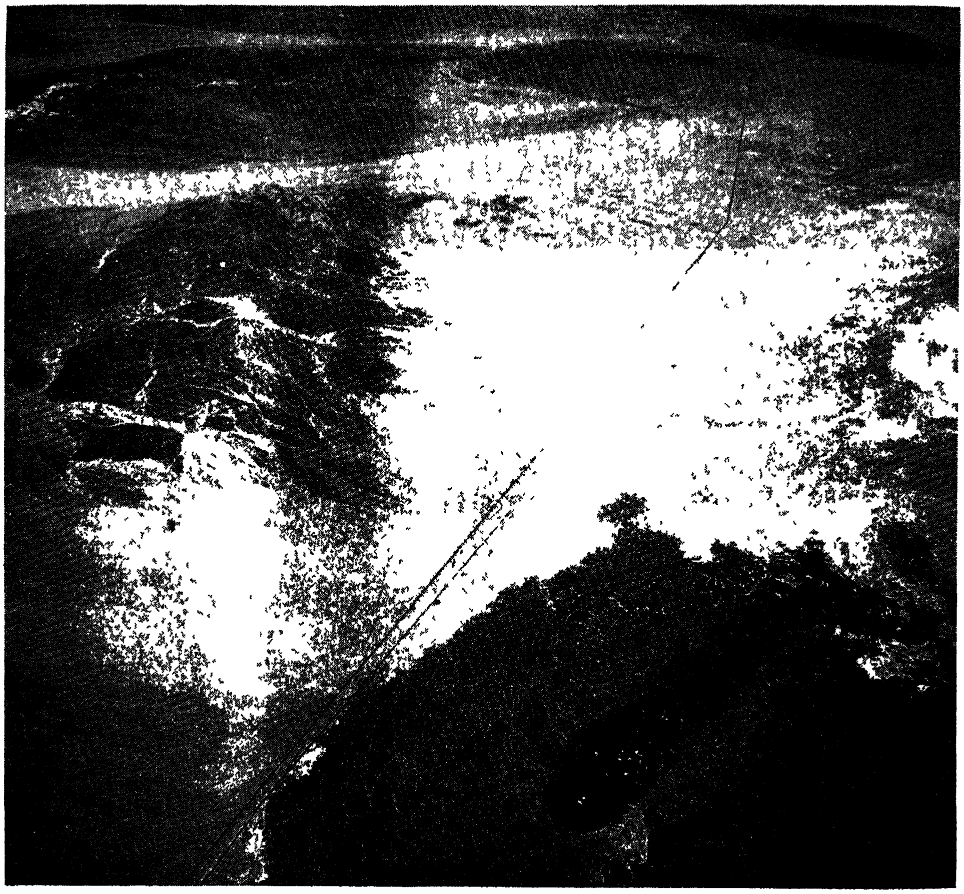


FIG. 11. Desert basin. Light-colored slope at base of mountains was formed by coalescing alluvial fans. Level, white area near right margin is saline playa. Rough ground near lower right corner is recent lava flow. Dark, circular area on lava flow is volcanic cone; dark streak leading away from it is wind-deposited volcanic ash on leeward side of volcano. USAAF

channel. They may be streamlined, the upstream end blunt, the downstream long, and tapering. Shallow water is generally indicated by a lighter shade and possibly by current ripples on the water surface. Waterfalls and cascades are revealed by broken water, and are readily apparent under the stereoscope.

Streams that are transporting large volumes of sediment may divide into many branching and rebranching channels. Such stream courses are called *braided*, or *anastomosing*. Individual channels are temporary features that may disappear in a few hours, and new ones develop with equal rapidity.

Most streams emptying into a standing body of water build deltas The delta is a nearly level, generally triangular, expanse of sediment which is crossed by numerous branching stream channels, or *distributaries*. Unlike tributaries, the acute angle at their intersection with the main channel points upstream. The delta surface is commonly interrupted by lakes and marshes, which are conspicuous features on aerial photographs Deltas of large rivers, such as the Mississippi, Ganges, Rhine, Nile, and Po, are flanked by broad, shallow lakes or lagoons The seaward margin of the delta may be bordered by long, narrow, sandy islands or bars. The upper part of the submarine portion of a delta may appear as a cloudy, light-colored area darkening seaward as the depth of water increases.

Arid Regions: Desert landscapes exhibit more variety than humid ones. Some deserts are mountainous; others are monotonous, desolate plains. It is characteristic of most arid regions that streams originating within a desert do not, as a rule, escape beyond its confines. The limited supply of water either evaporates or sinks underground, producing what is called an interior drainage In mountainous deserts, broad basins separating individual mountain ranges are floored with sand and gravel swept into them from adjacent ranges by torrential, short-lived streams. Stream deposits accumulate at canyon mouths as gravel-covered alluvial fans. These may coalesce to form a nearly continuous alluvial apron at the mountain base (Figure 11) Temporary lakes occupying lower parts of the basin floor are *playas*. Some are clay-floored and make ideal landing fields, but only when dry. Others are floored with salt, and in aerial photographs appear as a conspicuous expanse of white ground

Parts of many deserts, such as the Sahara, are broad dry plains of barren rock; other parts may be sand-covered. Sand is frequently drifted by the wind and accumulates in dunes. Some are crescent-shaped, with horns of the crescent pointing down-wind. Dunes of this type are *barchanes*. They are readily recognized from the air, and may be used as a rough guide to the direction of the prevailing ground wind. Other dunes may form long, parallel sand ridges, which are particularly conspicuous in the Sahara, and in parts of the Australian desert.

Stream erosion is very effective in many deserts, partly as a result of the lack of vegetation, and partly as a result of the torrential nature of desert cloudbursts. Deep, narrow, steep-sided canyons are rapidly excavated where rocks are poorly consolidated These dry channels are conspicuous features in many deserts

Limestone Regions: An unusual type of erosion may be found where extensive bodies of limestone crop out This rock is more soluble than most others, and as a consequence large subterranean caverns may be dissolved in it by ground water. Much of the surface water sinks underground through solution channels; vegetation is sparse, and most of the area is an expanse of barren, seamed, and fissured rock, even in regions of heavy rainfall. An outstanding example of such a district is the mountainous Adriatic coast of Yugoslavia, the Karst region. Many of the streams disappear underground, only to reappear several miles away, and others flow into large basins without surface outlets. Limestone may erode to form spire-like mountains, or narrow, sharp-crested ridges that rise precipitously from the surrounding plain. Examples of this spectacular terrain are in Indo-China, southern China, and Puerto Rico.

Sink holes are another characteristic feature of limestone regions (Figure 12). These are best identified under the stereoscope, and appear as nearly circular saucer-like depressions, many with steep walls. Some are water-filled; others may have level, clayey floors.

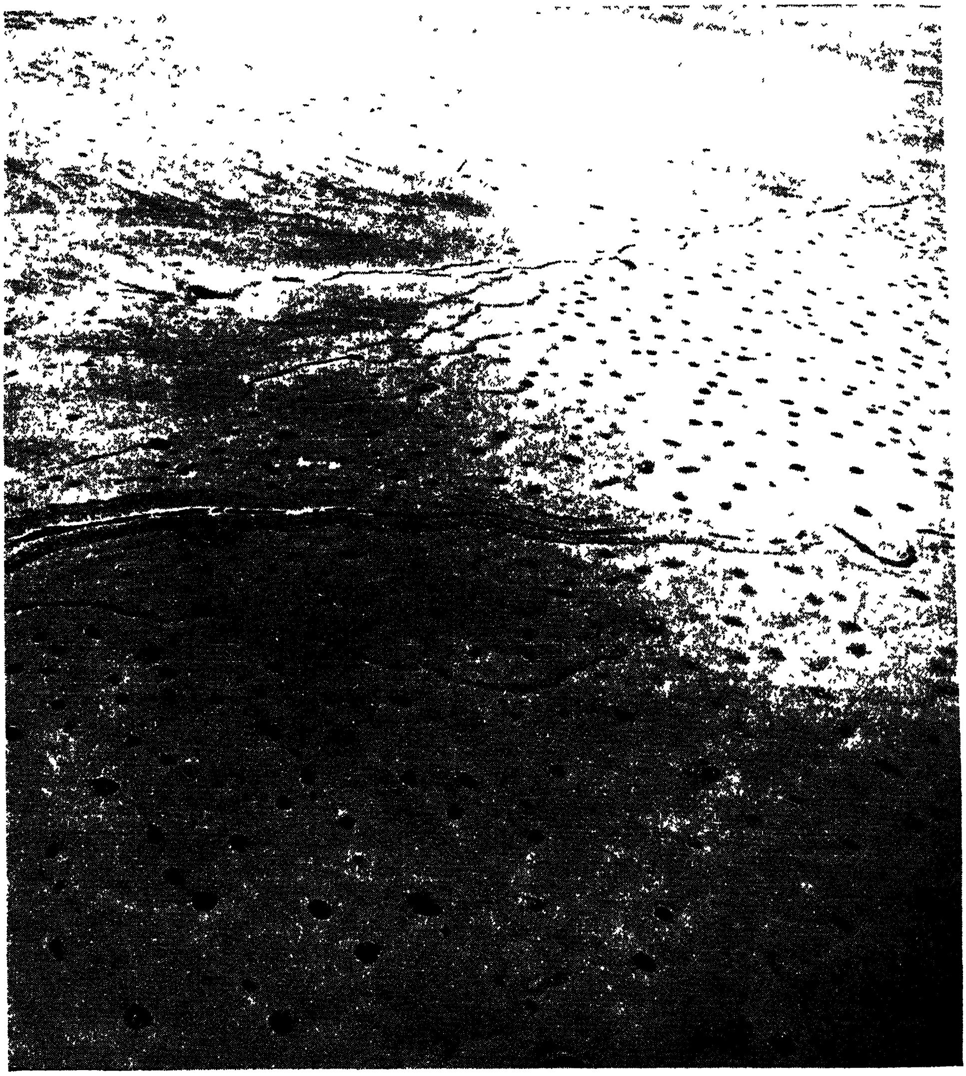


FIG. 12. Sink holes in limestone forming barren desert plain. Note how small stream channels lead to sink holes where they disappear. USAAF

Glacial Erosion

High mountains in all parts of the world, even in equatorial regions, are capped with permanent snow and ice fields. Large accumulations of ice that are moved downslope by gravity are glaciers. They are readily recognized from the air by the strong contrast of their light color with the somber background of bare rock above timber line or dark forest at lower altitudes. The upper end of a large glacier commonly merges with a snow field in a basin of accumulation; the lower is usually a blunt, snout-like or bulbous terminus far downslope in the bottom of a steep-walled valley. Some large glaciers, as in Alaska, may extend from the mountain onto the plain at their base. The surface of a glacier is likely to be seamed by large cracks, or *crevasses*. Dark bands may streak the surface of the

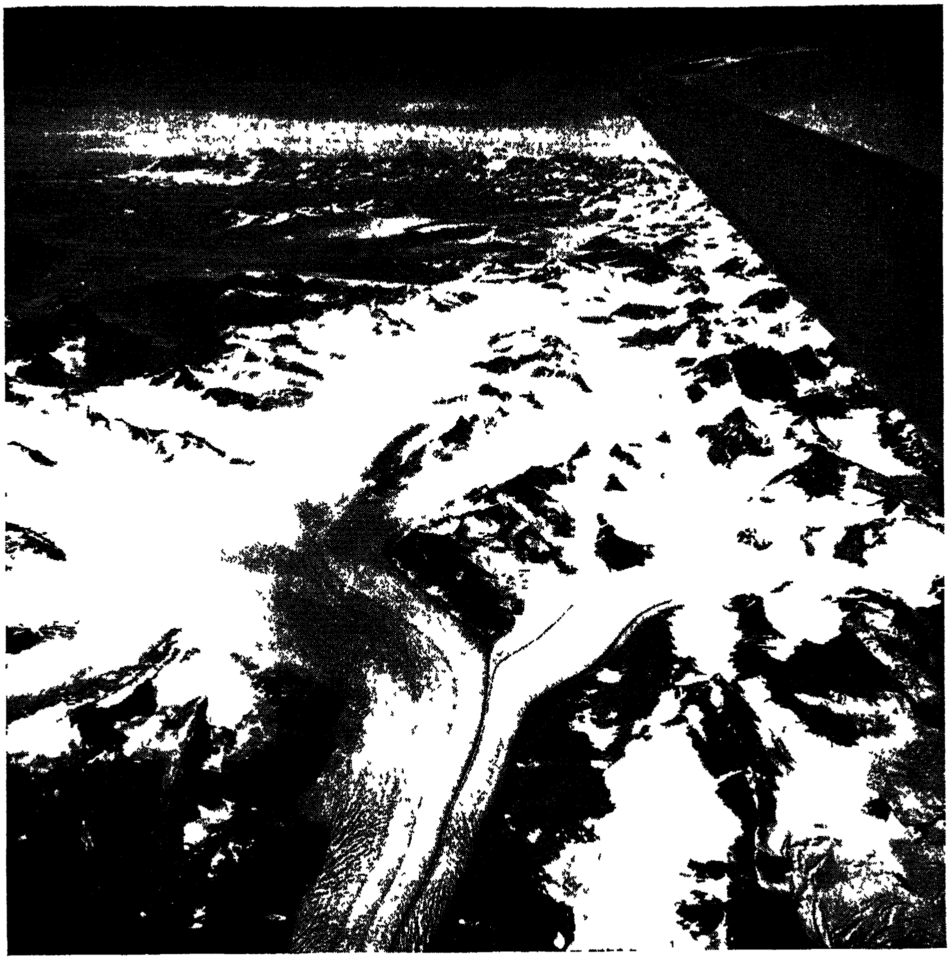


FIG 13

ice These are concentrations of rock fragments plucked by the ice from the valley walls, or from rock islands, *nunataks*, that rise above the glacier surface Debris transported by the glacier is deposited at its lower end in a crescentic embankment of rock fragments ranging in size from clay to boulders, tens of feet in diameter. This accumulation of glacial detritus is a *terminal moraine* *Marginal moraines* may flank the glacier on either side, and *medial moraines* may be found on the surface of the ice

A short while ago, as time is measured geologically, glaciers were larger than they are today Moraines beyond the limits of present-day glaciers are evidence of their former greater extent in the majority of the world's mountain ranges, such as the Sierra Nevada, Rockies, Andes, Atlas, Pyrennes, Alps, Caucasus, and Himalayas

Valleys excavated by former glaciers have steep, bare rock walls and comparatively flat floors in the lower part of the valley Some tributary streams do not enter the main glaciated valley at the same level as the floor, but reach it by a series of cascades or waterfalls (the Yosemite falls are an example). The floor

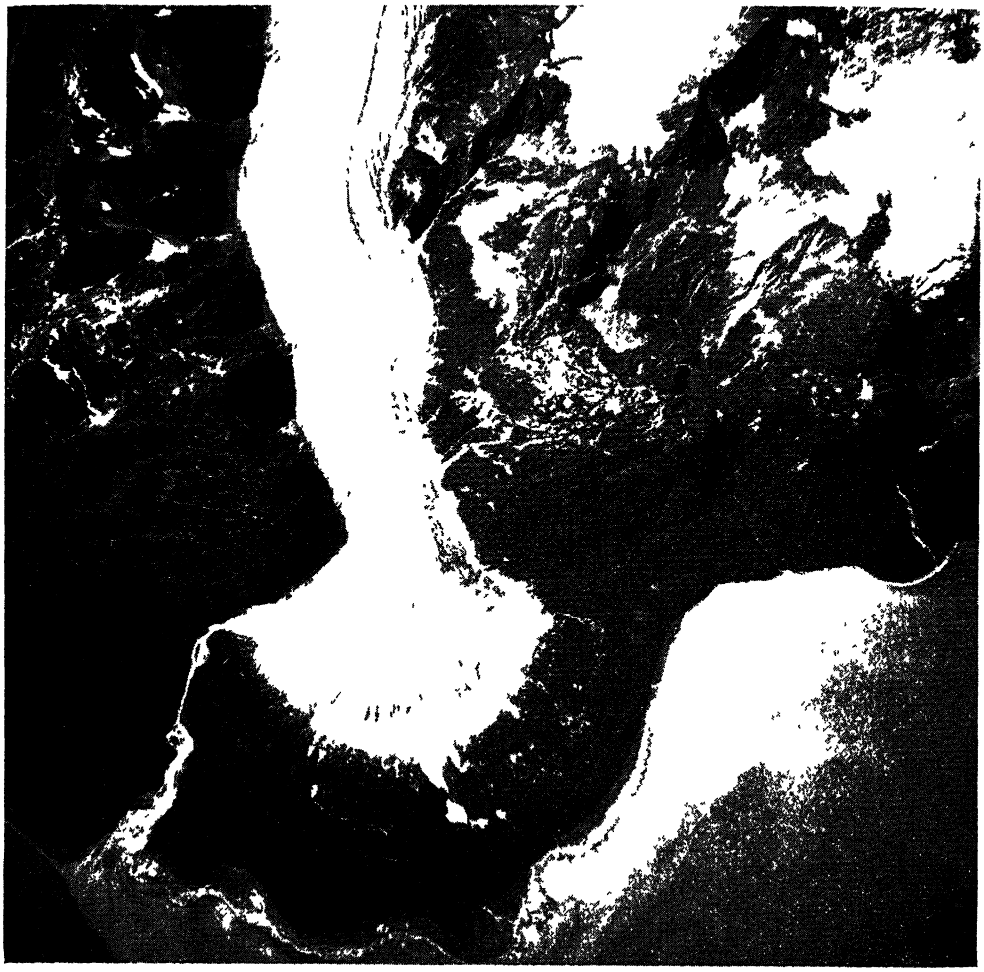


FIG. 13 AND 14. Vertical and left-wing oblique photographs of mountain glacier. Note catchment basin, nunataks, debris trains, crevasses, and tree-covered terminal morains. Small depressions on moraine are kettleholes. USAAF

of the upper part of a glacial trough may largely be bare rock, and may rise towards the valley head by a series of cyclopean steps The main stream descends this glacial stairway in a succession of waterfalls (examples of this type are Nevada and Vernal falls in Yosemite Valley). Customarily the upper end of a glacial trough is a theater-shaped valley head with precipitous bare rock walls. This crescentic, cliff-walled valley head is known as a *cirque.*

Small lakes (*tarns*) occupy ice-excavated basins in the rock floors of glaciated valleys. Larger lakes may be impounded by terminal moraines at the lower end of the valley; some, as lakes Como, Maggiore, Garda, Constance, and Lucerne at the margins of the Alps, are renowned for their scenery

In recent geologic time, most of Europe and North America, as far south as the Ohio River, was covered by immense ice sheets. These glaciers differed from the valley or mountain glaciers described in the preceding section in that they were extensive ice sheets that overrode the pre-existing hills and valleys. Soil

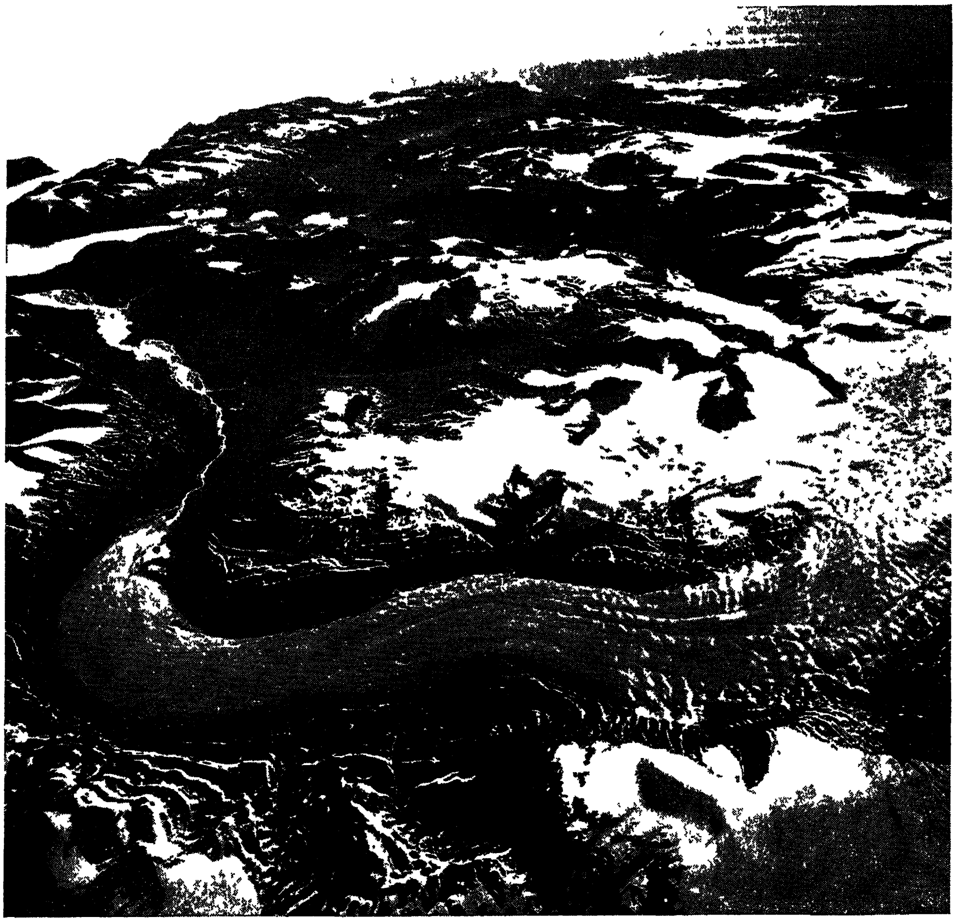


FIG. 15. Fjord coast Note braided stream beyond glacier terminus, steep-walled glacial valleys, and long inlets of the sea. USAAF

was stripped from broad areas by these *continental glaciers*, and bare rock was widely exposed. In northern and eastern Canada many basins excavated by the ice are now occupied by lakes (Figure 16).

Elsewhere, great thicknesses of glacial debris, in some places as much as 1,000 feet, were deposited. The southern margin of the ice was bordered by long terminal moraines. Complex systems of these morainal embankments cross northern Europe and the north-central United States and mark temporary halts of the ice Deposition occurred under the ice as well as along its margins Swarms of elliptical, rounded hills of glacial debris are common in formerly ice-covered regions such as central New York, Massachusetts, and Wisconsin. These *drumlins*, in general have their long axes oriented parallel to the direction of ice advance. *Eskers* are long sinuous ridges of glacial debris that appear to have been deposited by sub-glacial streams.

Blocks of ice transported by streams beyond the glacier front have been buried in outwashed detritus. When they have melted, the space they occupied

FIG. 16. Rock plain; soil has been stripped by glaciation Rocks are probably intrusive igneous and metamorphic. Note many rock-basin lakes USAAF

is marked by a pit, or *kettlehole* (Figure 14) Superficially these resemble sink holes, but their association with other glacial features helps to identify them.

SHORELINES

Several classifications of shorelines are recognized, but from the photogrammetric point of view four major types are significant. These are embayed coasts, plains coasts, fjord coasts, and coral reefs. Many unusual types might be listed, but are likely to be of academic interest only.

Embayed coasts are characterized by inland extensions of the sea in bays, estuaries, or sounds that occupy the drowned seaward ends of stream valleys Chesapeake Bay and New York Harbor are excellent examples The nature of such a coast largely depends on the nature of the adjacent land area. It may be steep and rugged, as the coast of Yugoslavia, where the sea reaches far into the interior in cliff-walled bays, or it may be a region of moderate to slight relief, like eastern Maryland along Chesapeake Bay.

Embayed coasts make a diversified pattern of cliffed headlands, coves and pocket beaches, islands, estuaries, and coastal marshes Most streams that empty into coastal bays end in deltas. With favorable lighting, shoal areas on the

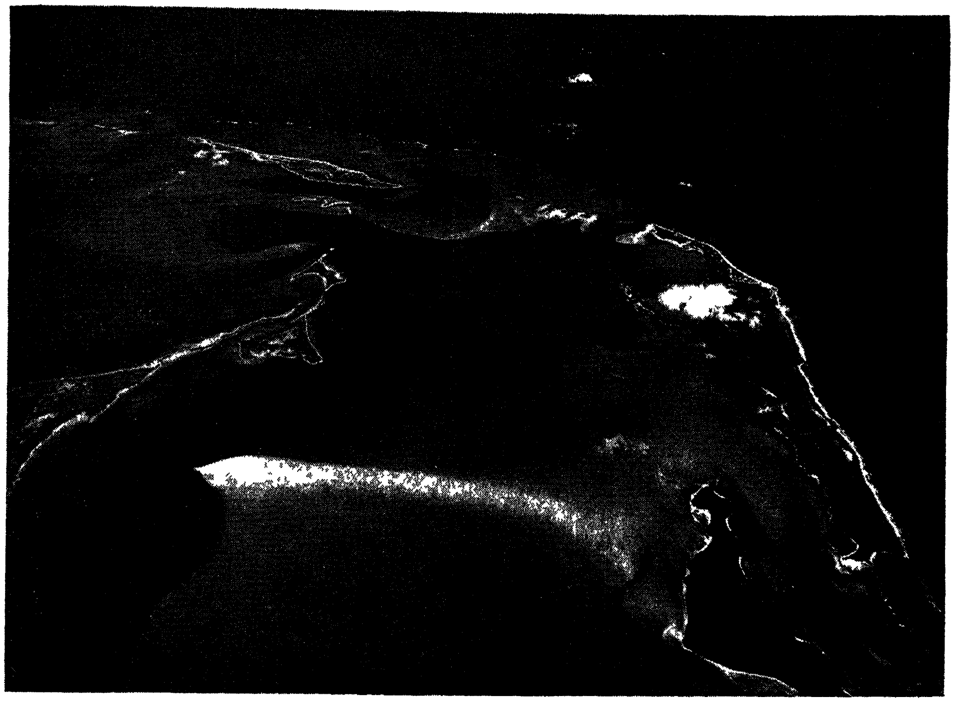


FIG 17. Coral reef. Dark areas outlined by white margins are vegetation-covered coral islands. Large area of ripples in foreground is sandbar on lagoon floor. USAAF

sea floor appear lighter-colored than deep water, and a qualitative estimate may be made of the depth of water, or possibly the location of the main channel may be determined.

Plains coasts are found where land bordering the sea is low and nearly level, and where shallow water extends a long distance offshore. *Offshore bars* are a characteristic feature of this type of shoreline. These are low, sandy islands paralleling the coast and separated from it by shallow lagoons. Some of these islands are many miles long and may be 10 miles or more offshore. They are interrupted at intervals by shallow passes, usually located opposite stream mouths. The mouths of these passes tend to shift their position in the direction of the coastal current After some time a new pass may break through opposite the stream mouth and the former channel fills with sand.

Sand bars and shoals in the shallow water of the lagoon may be clearly apparent in aerial photographs. In many cases a tidal delta is visible inside the mouth of a pass where sand has been swept into the lagoon by the flood tide. The landward margin of the lagoon may be fringed by long reaches of coastal marsh. The swamp vegetation photographs dark, and the marsh is seen to be crossed by many intricately meandering and branching tidal channels.

Fjord coasts are a special type of embayed coast whose occurrence is limited to recently glaciated coastal mountains. The sea extends far inland along glacial valleys laid bare through withdrawal of the ice. Fjords are characterized by deep water close to shore, steep coastal cliffs, waterfalls, and extensive areas of bare rock (Figure 15). Almost all fjords are continued inland for some distance

FIG. 18. Coral reef Ocean is to left; lagoon to right. Irregular patches in center of area are coral Dark area to left is vegetation-covered part of reef above sea level. USAAF

by the non-submerged part of the glacial trough. Some of the better known examples of fjord coasts are Norway, Scotland, Iceland, Greenland, Labrador, British Columbia, Alaska, southern Chile, and New Zealand.

Coral reefs are an unusual type of coast found where the sea is warm, the water shallow, and the proper food supply available for corals to flourish. Corals are animals that secrete a limy "shell." After the individual coral dies this remains as a foundation to which his successors attach themselves. In time, a hard stony submarine wall, or reef, of limestone is built to sea level. Corals prosper only where rather exacting requirements of temperature, salinity, turbulence of sea water, and sunlight are met They are largely restricted to tropical waters, but not every tropical coast has a reef bordering it.

There are three important types of reefs: *fringing reefs*, *barrier reefs*, and *atolls*. Fringing reefs are attached directly to the mainland. Barrier reefs occupy the same relative position as offshore bars, but consist of limestone rather than sand. Like offshore bars they are interrupted by passes, usually located opposite stream mouths, and are separated from the mainland by shallow lagoons The largest is the Great Barrier Reef, 1,200 miles long, off the northeast coast of Australia. Many islands in the southwest Pacific are encircled by reefs. The New

Caledonian reefs are particularly impressive as they flank the northeast and southwest coasts of the island for 400 miles

Atolls lack a large central island, and consist essentially of a reef encircling a lagoon A coral island built upon a reef is seldom over 20 feet high; that part above sea level is composed principally of coralline sand dunes Wake and Midway Islands are well known examples

Coral photographs in light colors, although vegetation growing on its surface appears dark The outer margin of the reef is commonly marked by a white line of breaking surf, and the reef itself stands in strong contrast to the surrounding ocean The shallow water of the lagoon appears only slightly less dark than the reef, and, in clear tropical water, submarine features such as sand bars and shoals appear almost as clearly as ones above sea level

The writer gratefully acknowledges the assistance and advice of the following persons in the preparation of this chapter Alice Allen, J E Mundine, and L L. Ray, all of the U S Geological Survey; J I Davidson of the Aeronautical Chart Service, USAAF, and Lt Col G. FitzGerald, USAAF For reasons of security it is not possible to give the locations of aerial photographs used as illustrations, all were taken by the U. S Army Air Force

CHAPTER IX

RADIAL PLOTTING METHODS

RADIAL PLOT

C. E. Kowalczyk, L. F. Fish and A. P. Dill

General Discussion

THE RADIAL plot is a graphic method of obtaining photogrammetric triangulation. As the basis for the use of slotted templets, hand templets, metal templets, and overlay photogrammetric compilation, it has its greatest application in the location of photogrammetric control points between widely spaced horizontal ground control points Its principles date back to 1893, when a U. S. patent was issued to C B Adams for use with photographs taken from balloons Its use was advocated by T Scheimflug in 1906, by S Finsterwalder in 1921 and probably given its greatest impetous by Maj. J W. Bagley in 1923

The radial plot is based upon the principle that true horizontal angles can be measured around the nadir point of a photograph regardless of scale change or relief displacements because such errors are radial from the nadir[1] point

The radial plot method presupposes the use of vertical photographs While relatively few so called verticals are truly vertical, for variations up to 3° the principle point[2] may be considered coincident with the nadir point. The isocenter[3] from which displacements due to tilt radiate, is coincident with nadir and principle points in a true vertical photograph

Factors governing the amount and direction of relief displacement of a point on the photograph (either away from or towards the nadir point) are: *focal length of lens, flying altitude*, and *distance* of ground point from the nadir point.

The variables to be considered in evaluating the positions of points on a

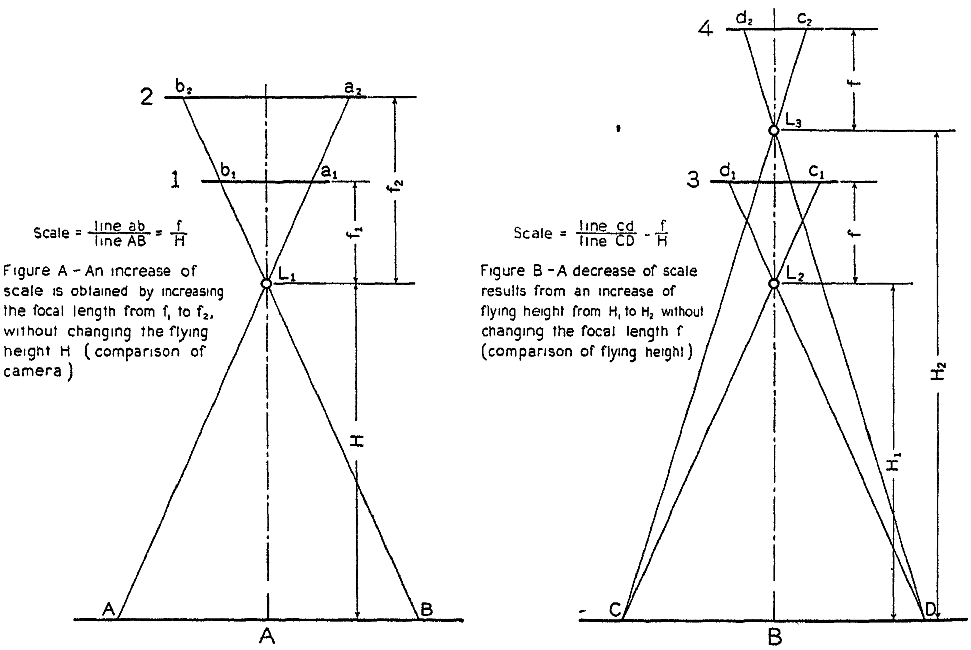


FIG. 1. Altitude and Focal Length Relationship.

[1,2,3] See definitions in Chap. XVII "Accepted Definitions and Nomenclature."

photograph in relation to the ground points they represent or simply to other points on the photograph are

1. *Scale.*
2 *Variation in heights of terrain.*
3. *Tilt of the camera from the vertical*
4 *Photographic errors, camera, film,* and *paper*

1 SCALE—Two factors determine the scale of an aerial photograph; namely, the altitude of the plane, and the focal length of the camera lens Figure 1 shows the effect of variations in focal length for a given flying altitude, and variations of flying altitude when a given focal length is used.

2 VARIATION OF HEIGHTS OF TERRAIN—A photograph shows the positions of objects projected through a single point, the lens. Since it is rare, if ever, that perfectly level terrain is encountered, it is evident from a study of Figure 2 that the effect of relief distortion is present in most aerial photographs.

In Figure 2.

V = Ground nadir point.

V' = Photo nadir point.

L = Lens.

f = Focal length of lens.

B is a ground point which is at a height h above the datum plane.

A is the point of intersection that point B would make with the datum plane if projected vertically

b is the position B would take on a photograph (i e. photo position of B)

a is the position A would take on the photograph (i e., map position of B)

H = the height of the lens above the datum plane

E = the point of intersection of a horizontal plane passing through B, with the plumb line VLV'.

h = amount of relief of the point B.

Z = relief distortion.

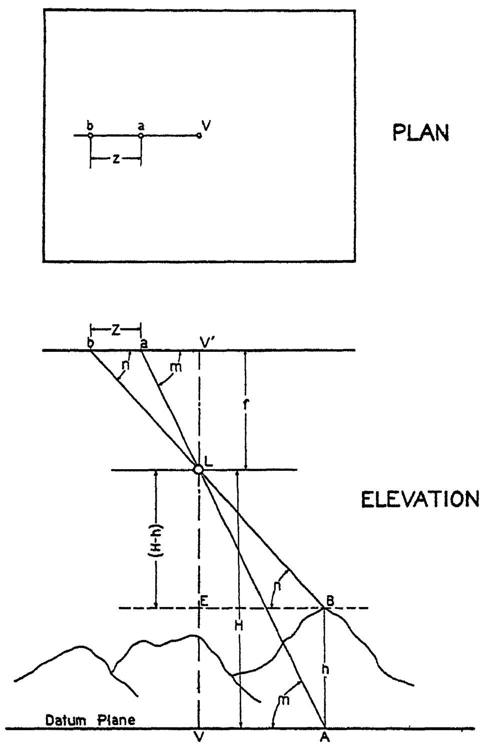


FIG 2 Displacement Due to Relief.

It is evident that the location of the nadir point V' is not affected by relief, and that in a truly vertical photograph the nadir point V' coincides with the principal point.

$$\tan n = \frac{f}{V'b} = \frac{H - h}{AV}$$

$$\tan m = \frac{f}{V'a} = \frac{H}{AV}$$

$$\frac{\tan n}{\tan m} = \frac{V'a}{V'b} = \frac{V'b - Z}{V'b} = \frac{H - h}{H}$$

$$1 - \frac{Z}{V'b} = 1 - \frac{h}{H}.$$

Therefore

$$Z = \frac{h}{H} V'b. \qquad \text{eq. (1)}$$

Example:

H = 10,000 feet, h = 2,000 feet, and $V'b$ = 6 inches.

Then

$$Z = \frac{2{,}000}{10{,}000} \times 6 = 1.2 \text{ inches.}$$

In addition to the range in relief h of the ground, the magnitude of displacement Z will also depend on the datum chosen for the plane of reference This is merely a conventional term defining the scale of the plot If, for instance, a plot is made at a scale represented by $f/(H\text{-}h)$ the corresponding plane of rerference will pass through B, Figure 2, which will accordingly be unaffected by height displacement.

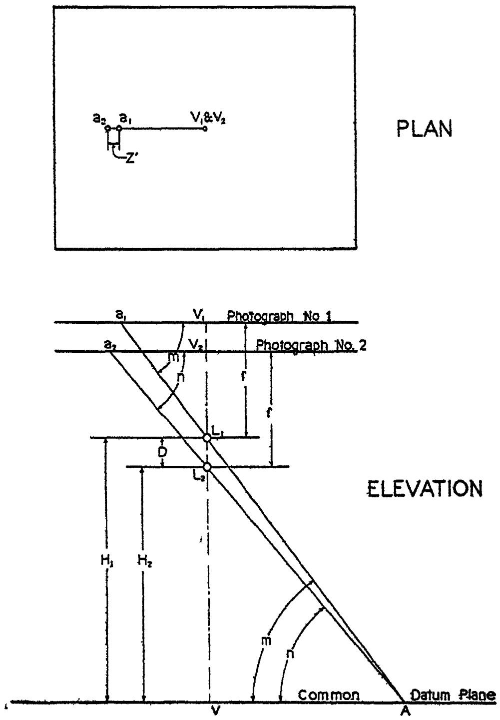


FIG. 3. Displacement Due to Variation in Flying Height.

In any photographic flight considerable variation is to be expected in the flying height, thus producing photographs of different scales. This variation of flying height means that the photographic scale will also vary. It is evident that by obtaining the average scale of the flight, and plotting the photographs to this scale, the airplane will then be considered as having flown at a varying height, H, above the resulting common plane of reference. The result will be a further set of height displacements introduced for different photographs

Suppose that the plot is being made at the average scale of the photographic flight, giving an equivalent common datum plane, Figure 3, which is H_1 below the common flying height. If all the photographs were flown at equal heights, and if there were no relief in the terrain, there would be no displacement of image points on the photographs. Since, as stated above, considerable variation in the flying height is to be expected, points in the same hori-

zontal plane will be displaced on the photograph in reference to the plotting scale

In Figure 3

A is a point on the common datum plane

V is the nadir point

H_1 is the common flying height above the common datum plane

V_1 is the photo nadir point of Number 1 photograph

a_1 is the image point A on Number 1 photograph This would correspond to the map position that A would take if plotted to a scale equal to the common datum plane

V_2 is the photo nadir point of Number 2 photograph

a_2 is the image point A on Number 2 photograph

H_2 is the height of Number 2 photograph above the common datum plane.

Z' is the displacement due to Number 2 photograph having been flown below the common flying height. In this case a_2 is displaced outward along a line V_1a_1. If number 2 photograph had been flown higher than the common height, then, a_2 would be displaced toward V_1 along a line V_1a_1; i e , a_2 will be displaced from a_1 a distance Z' depending on the algebraic difference between H_1, H_2.

$$\tan n = \frac{f}{V_2a_2} = \frac{H_2}{AV}$$

$$\tan m = \frac{f}{V_1a_1} = \frac{H_1}{AV}$$

$$\frac{\tan n}{\tan m} = \frac{V_1a_1}{V_2a_2} = \frac{V_2a_2 - Z'}{V_2a_2} = \frac{H_2}{H_1}$$

$$1 - \frac{Z'}{V_2a_2} = 1 - \frac{H_1 - H_2}{H_1}.$$

Let D = the algebraic difference between H, H_2

Then

$$Z' = \pm \frac{D}{H_1}(V_2a_2). \qquad \text{eq. (2)}$$

Example·

Let $H_1 = 10{,}000$ feet, $H_2 = 9{,}000$ feet. D would then equal 1,000 feet, $V_2a_2 = 6$ inches.

Then,

$$Z' = \frac{1{,}000}{10{,}000} \times 6 = .6 \text{ inches},$$

i e., a_2 would be displaced .6 inch outward from a_1 along V_1a_1).

In the reverse case:

Let $H_1 = 10{,}000$ feet, $H_2 = 11{,}000$ feet. D would then equal $-1{,}000$ feet, $V_2\ a_2 = 6$ inches.

Then,

$$Z' = \frac{-1{,}000}{10{,}000} \times 6 = -.6 \text{ inches},$$

(i.e., a_2 would be displaced 6 inch toward V_1 from a_1).

3. TILT OF THE CAMERA FROM THE VERTICAL—If, at the time of exposure, the camera axis is tilted through a small angle, the image points will not occupy, on the tilted photograph, the same position that they would have occupied had the camera axis been truly vertical If the amount and the direction of tilt is known, a corrected position to each image point can be applied

The following expressions have been given without complete mathematical proof (For further discussion see Chapter VI.)

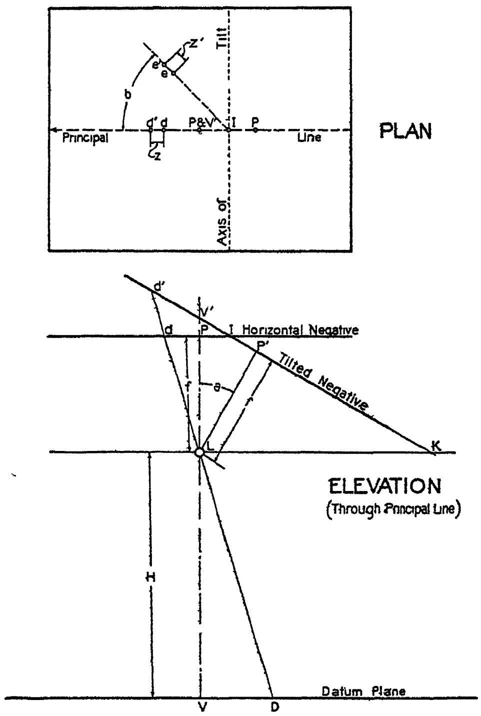


FIG. 4. Displacement Due to Tilt.

In Figure 4.

- V is the nadir point, which, in a truly vertical photograph coincides with the principal point P on the horizontal negative.
- V' is the point where the plumb line intersects the plane of the tilted negative and is referred to as the photograph nadir point
- D is an object on the ground plane.
- L is the lens.
- f is the focal length of the lens.
- a is the angle of tilt, positive in direction toward the nadir point from the principal point.
- P is the principal point of the horizontal negative.
- P' is the principal point of the tilted negative.
- d is the image point D on the horizontal negative.
- d' is the image point D on the principal line of the tilted negative.
- I is the isocenter, a point common to both horizontal and tilted negatives in a plane perpendicular to each through the optical center of the lens, and serves as an origin for measurements made in the planes of both negatives. It is also evident that the plane of the tilted negative intersects the plane of the horizontal negative on a line passing through the isocenter I and normal to the plane of the drawing. Therefore, there is no displacement of image points along this line of intersection. K is the intersection of the horizontal plane passing through L and the plane of the tilted negative.

$KP'IV'd'$ is the principal line of the tilted photograph

The distance

$$P'V' = f \tan a$$

The distance

$$P'I = f \tan \frac{a}{2}$$

Let Z equal the difference in distance between Id' on principal line of tilted

negative, and Id on horizontal negative (i.e , tilt distortion in distance Id'). Then, the tilt distortion Z, in the total distance Id' may be expressed by the ratio:

$$\frac{Z}{Id'} = \frac{Id' - Id}{Id'} = 1 - \frac{Id}{Id'}$$

and since $KI = KL = f$ cosec a
we have

$$\frac{Z}{Id'} = 1 - \frac{Id}{Id'} = 1 - \frac{KL}{Kd'}$$

$$= 1 - \frac{KL}{KI + Id'} = 1 - \frac{f \text{ cosec } a}{f \text{ cosec } a + Id'}$$

$$= 1 - \frac{1}{1 + \dfrac{Id'}{f} \sin a}$$

or

$$\frac{Z}{Id'} = \frac{Id'}{f}\, a \text{ (radians)},$$

approximately, when, as is always the case, a is small.
Therefore,

$$Z = (Id')^2 \frac{a}{f}\cdot \qquad \text{eq. (3)}$$

Example:
Let $Id' = 6$ inches, $a = 3$ degrees = 0524 radians, and $f = 8$ 25 inches
Then,

$$Z = 6 \times 6 \frac{0524}{8\ 25} = .23 \text{ inches (approximately).}$$

Equation (3) gives the amount of the distortion of the image poing d' which falls on the principal line, that is, on the line of the greatest displacement. If the image point does not fall on the principal line, the displacement due to tilt is expressed below

Let Z' = difference in distance between Ie' on tilted negative, and Ie on horizontal negative (i.e , tilt distortion in distance Ie'), and b = angle $V'Ie'$ = angle which the direction from I to the image point e' makes with the direction of the principal line, considered positive in direction of plumb point V:

$$\frac{Ie'}{Ie} = \frac{Id'}{Id}, \qquad \text{or} \qquad \frac{Ie'}{Id'} = \frac{Ie}{Id}\cdot$$

The tangent of the angle VIe' equals the tangent of angle formed by the corresponding lines in the horizontal photograph, i.e., *the two photographs are angularly equal at the isocenter I*

Since, $Ie' \cdot \cos b$ equals projection of Ie' on the principal line, by substituting

(Ie' cos b) for Id' (3) written in the form $Z=(Id')^2\ a/f$, we obtain Z' on the principal line or·

$$Z' = (Ie' \cos b)^2 \frac{a}{f}$$

Therefore on a line inclined by an angle b to the principal line, Z' on the inclined line will be expressed by.

$$\frac{(Ie' \cos b)^2 \dfrac{a}{f}}{\cos b}$$

or

$$Z' = (Ie')^2 \frac{a}{f} \cos b. \qquad \text{eq. (4)}$$

Example

Let $Ie'=6$ inches, $a=3$ degrees= 0524 *radians*, $b=45$ degrees, and $f=8$ 25 inches

Then,

$$Z' = 6 \times 6 \times \frac{.0524}{8\ 25} \times .70711 = .16 \text{ inches (approximately).}$$

4 PHOTOGRAPHIC ERRORS—These errors are due to aberrations in the lens system and the mean and irregular distortions of film and paper. These errors are usually too small to affect planimetric detail taken from a single photograph They are of considerable importance in contouring by stereoscopic measurements between vertical control and tend to accumulate rapidly in any process of extension of control, either vertical or horizontal. Nothing can be done towards correcting or eliminating these errors by any simple method of plotting.

To summarize, it is evident that a change in scale in the photograph as well as photographic errors are radial about the *principal point;* that displacements due to ground relief are radial about the *nadir point*, that displacements due to tilt are radial about the *isocenter*. Since the greatest source of error in the radial direction, comes when the photograph is tilted, a point halfway between the nadir point and the isocenter should be used whenever it can be located and is the best approximation that can be made for the origin of radials. Where the nadir point and the isocenter are unknown and cannot readily be determined, the next best approximation is to assume as correct, directions from the principal point, which will not materially affect the accuracy of graphic triangulation providing that the tilt is not excessive. It must be clearly understood that, pending development of means to eliminate tilt, this principal point is a compromise which at once limits the application of any method based on it.

It will be seen from the errors described above that distances cannot be measured on photographs with the view of applying them directly to the map. If any attempt is made to measure distances on the photographs as a means of plotting, the whole effect of the above displacements is likely to be introduced, instead of only a small part of them. The accuracy of any measurement must be examined on the lines of the above discussion. However, directions may be de-

termined with very little error when all precautions are taken and the best of camera equipment and photographic materials are used Directions about the principal point are correct for all normal conditions of tilt and relief. Therefore, to prepare an accurate map from aerial photographs, use must be made of *angular values* rather than scaled distances It will be appreciated from the above that aerial photographs, in themselves, are not surveying records of geodetic precision but rather a means of mapping with a resulting accuracy of fourth or possibly third order, depending, of course, on both the vertical and horizontal control obtained. Sixty percent overlap between successive photographs is essential. This will also facilitate the examination of each pair of overlapping photographs under the stereoscope for satisfactory study of details. For graphic mapping, however, the vertical photographs give the advantages of speed, economy, and a complete detailed and permanent record of topographic features not otherwise so readily available

Errors in the Radial Assumption

Application of corrections for tilt and relief displacements requires that the amount and direction of the tilt be known This necessitates the knowledge of the X, Y, and Z, coordinates of at least three ground control points within the photograph The computation in this determination is involved In general, it is assumed that radials from the principal point are angularly true The errors in this assumption are shown below

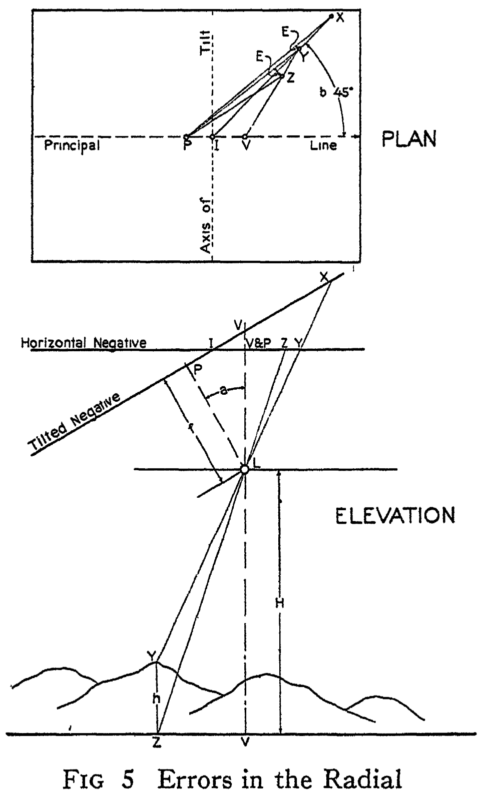


FIG 5 Errors in the Radial Assumption.

In Figure 5·

- P is the principal point.
- I is the isocenter.
- V is the nadir point.
- Z is the correct map position of a point located at a height h above the datum plane
- Y is the position Z would occupy, due to relief, on an untilted photograph.
- X is the position Z would occupy, due to relief and tilt, on a tilted photograph.
- f is the focal length of lens.
- a is the angle of tilt.
- b is the angle VIX (positive in direction of Nadir point).
- H is the height of lens above the datum plane.
- PIV is the principal line of the tilted photograph.
- XY is tilt distortion $=(Ie')^2\ a/f \cos b$ eq. (4).
- E is the error in assuming that the tilt is radial from the principal point $=XY \sin YXP$.

$$\text{Angle } XIP = 180° - b$$

$$IP = f \tan\frac{a}{2}$$

By the law of sines, in triangle YXP (see plan):

$$\sin YXP = \frac{IP \sin b}{XP} = \frac{f \tan\frac{a}{2} \sin b}{XP}$$

then,

$$E = XY \sin YXP = \frac{(XY)\, f \tan\frac{a}{2} \sin b}{XP}.$$

Substituting for XY from (4), writing IX for Ie',

$$E = \frac{(IX)^2 \frac{a}{f} \cos b \left(f \tan\frac{a}{2} \sin b\right)}{XP}.$$

When a is small, tan $a/2$ becomes $a/2$ radians, and $(IX)^2/XP = IX$ (approximately). Hence, the above expression reduces to:

$$E = \frac{IX}{4} a^2 \sin 2b. \qquad \text{eq. (5)}$$

Example:
Let $IX = 6$ inches, $b = 45$ degrees, and $a = 3$ degrees $= .0524$ radians.
Then,

$$E = \frac{6}{4} \times .0524^2 \times 1 = 00412 \text{ inches.}$$

This error is zero when X falls on the principal line or on the axis of tilt, and is at a maximum when X is at a maximum distance from I on the photograph, and the angle b is 45 degrees.

Next examine E', the error in assuming that the relief distortion YZ is radial from the principal point

$E' = (YZ) \sin ZYP$. Substituting YZ for Z in (1) and writing YV for $V'b$, we have:

$$E' = \left(\frac{h}{H} YV\right) \sin ZYP. \qquad \text{eq (6)}$$

Sin ZYP is zero when Y is on the principal line, and the maximum value of sin ZYP occurs for any particular value of YV when angle VPY becomes a right angle.

Then,

$$\sin ZYP = \frac{f \tan a}{YV}$$

and (6), giving the maximum extent of E', becomes·

$$\frac{h}{H} YV \frac{f \tan a}{YV}$$

Then,

$$E' = \frac{h}{H} f \tan a. \qquad \text{eq. (7)}$$

Example:
Let

$h = 2{,}000$ feet, $H = 10{,}000$ feet, $= 8.25$ inches,

and

$a = 3$ degrees.

Then,

$$E' = \frac{2{,}000}{10{,}000} \times 8\ 25 \times .0524 = .0865 \text{ inches (approximately).}$$

By combining the errors expressed in (5) and (7), the total amount (0906 inches) of error is reached when X is at a distance of 6 inches from I and at a height h of 2,000 feet, and at an angle of 45 degrees

It is seen by these errors that the tilt must be held below 3 degrees in photographs over rough areas, otherwise the nadir point must be located and used as the origin for radials, instead of the principal point.

INDEXING OF PHOTOGRAPHS

Upon receipt of photographs the first step is to prepare an index map of the photography. This is accomplished by indicating on the best available map the position of the photographs with respect to ground features. First, determine the scale of the photographs in relation to the scale of the map and construct a cellulose templet.

For example· Suppose the photographs are 7×9 at a scale of 1:20,000 and the map scale is 1:90,000, then:—

$$\frac{\text{Width of photograph in inches} \times \text{photograph scale}}{\text{Map scale}} = \text{width of templet in inches}$$

also

$$\frac{\text{Length of photograph in inches} \times \text{photograph scale}}{\text{Map scale}} = \text{length of templet in inches.}$$

In above example:

$$\frac{9 \times 20{,}000}{90{,}000} = 2 \text{ inches, and}$$

$$\frac{7 \times 20{,}000}{90{,}000} = 1.6 \text{ inches.}$$

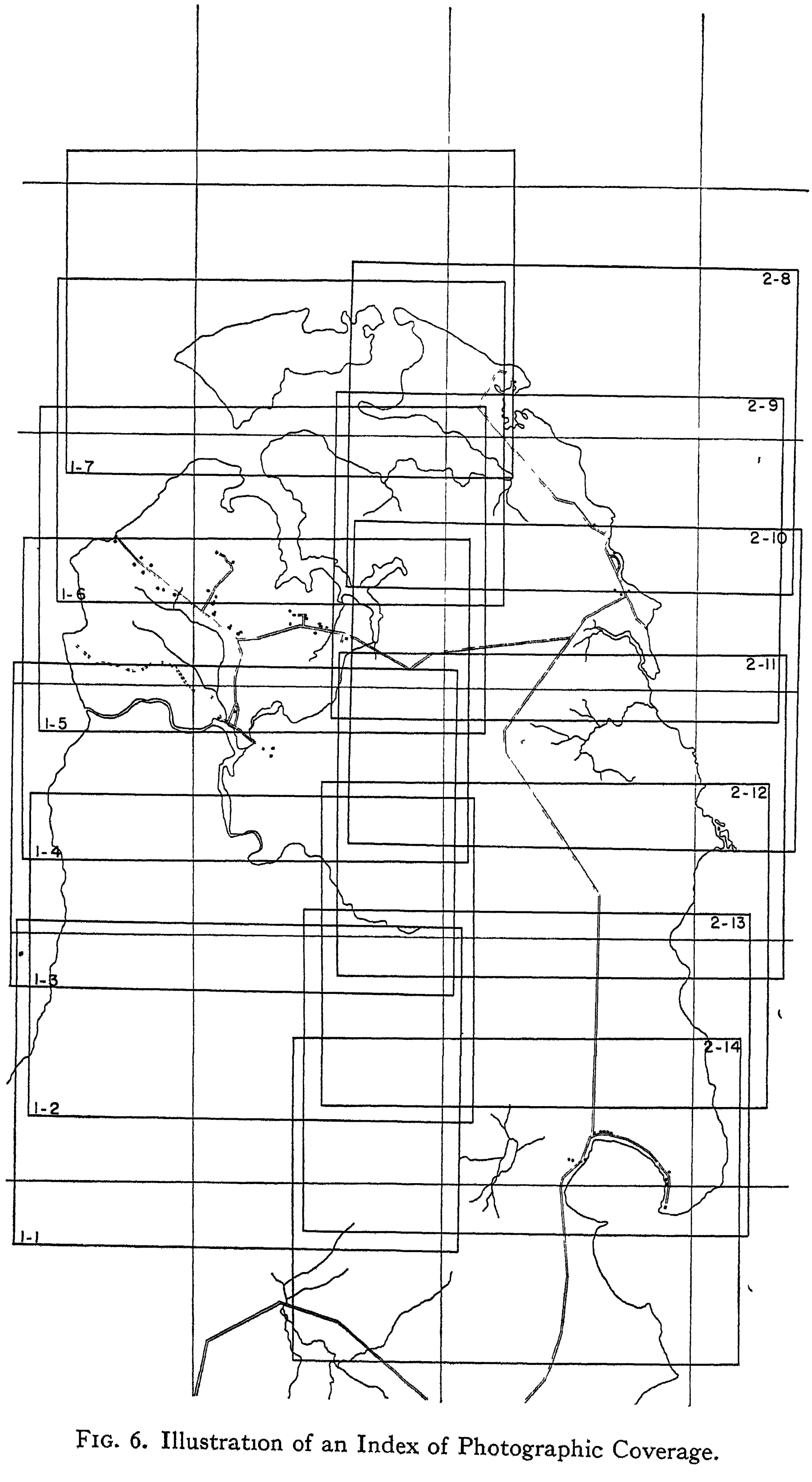


FIG. 6. Illustration of an Index of Photographic Coverage.

Therefore,

Templet would be 2×1 6 inches

The photographs are then laid out by strips, and the templet is placed on the map and shifted about until its outline covers the detail shown on the photograph The templet outline is then traced with pencil The process is repeated for any number of photographs in the strip, and then for photographs in adjacent flight strips. The exposure number of the photograph is entered in the outline. The resulting diagram is an index-map of the photography (See Figure 6)

Overlay Compilation Procedure

It has been shown that photographs in areas of moderate relief where the tilt has been kept to a minimum, displacement of images points due to tilt, relief of the ground, changes in flying altitude and photographic errors, may be considered to take place along a line joining the image point to the principal point of the photograph In other words, the photograph may be considered as *angularly-true* at its principal point

If vertical photographs are taken in a strip with proper longitudinal overlap (approximately 60 percent), a graphic triangulation net of the strip can be constructed on the above assumption

In the overlay compilation procedure, reference will be made to the following kinds of control:

a. *Ground control*—already established control of third or higher order.

b *Pass points*—photogrammetric control used in bridging between ground control

c. *Detail control*—control used to bring detail to proper scale and orientation.

The following will discuss in detail the progressive steps used in the overlay compilation procedure (It is recommended that the reader follow these steps graphically in Figures 7 through 12)

1. Identification of control points
2. Location and transfer of principal points
3. Selection of pass points.
4 Drawing radials.
5. Preliminary plot to determine average scale
6 Construction of projection and plotting ground control
7. Control plot.
8. Selection and establishment of detail control.
9 Delineation.

1. Identification of Control Points—The accuracy of the plot depends on the accuracy of the control and its proper identification on the photograph. Whenever possible these control points should be spotted on the photograph in the field. On the back of each control photograph should be a detailed description of the control.

When the photographs are received from the field, the identification of the control points should be checked to assure that the points are accurately pricked as described. When a point is repricked, the correct location should be plainly indicated to assure that it will be used in any subsequènt identification. All control should be transferred from the photograph on which it was identified in the field to all others on which it appears and marked with a quarter-inch green circle and named according to the field records. The accuracy of the entire plot is dependent upon the care exercised in the identification and marking of these points on every photograph upon which they appear

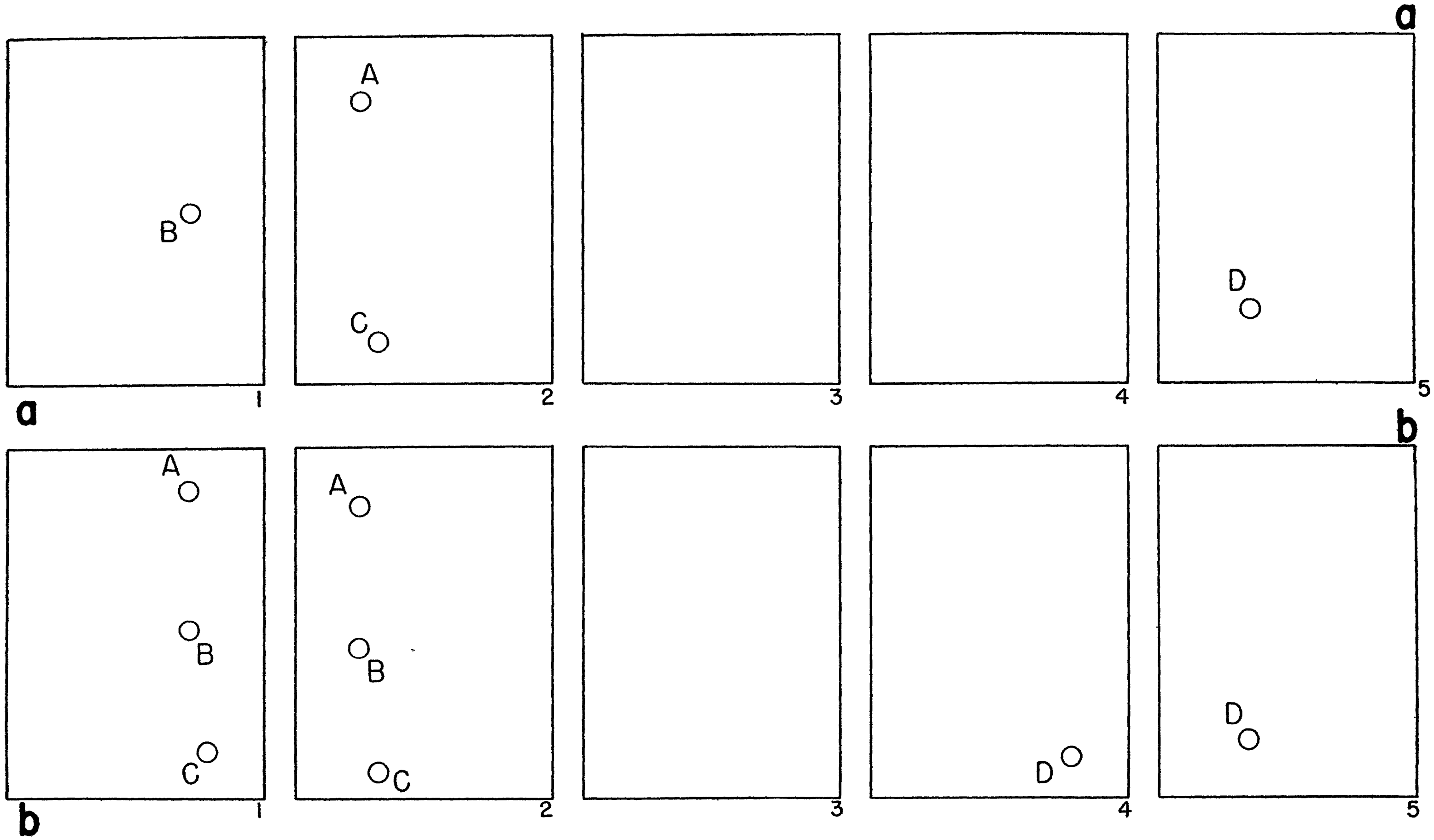


FIG. 7a. Five Photographs Selected, with control points (*A-B-C-D*) as located in the field
FIG. 7b. The control points are transferred to all photographs on which they fall.

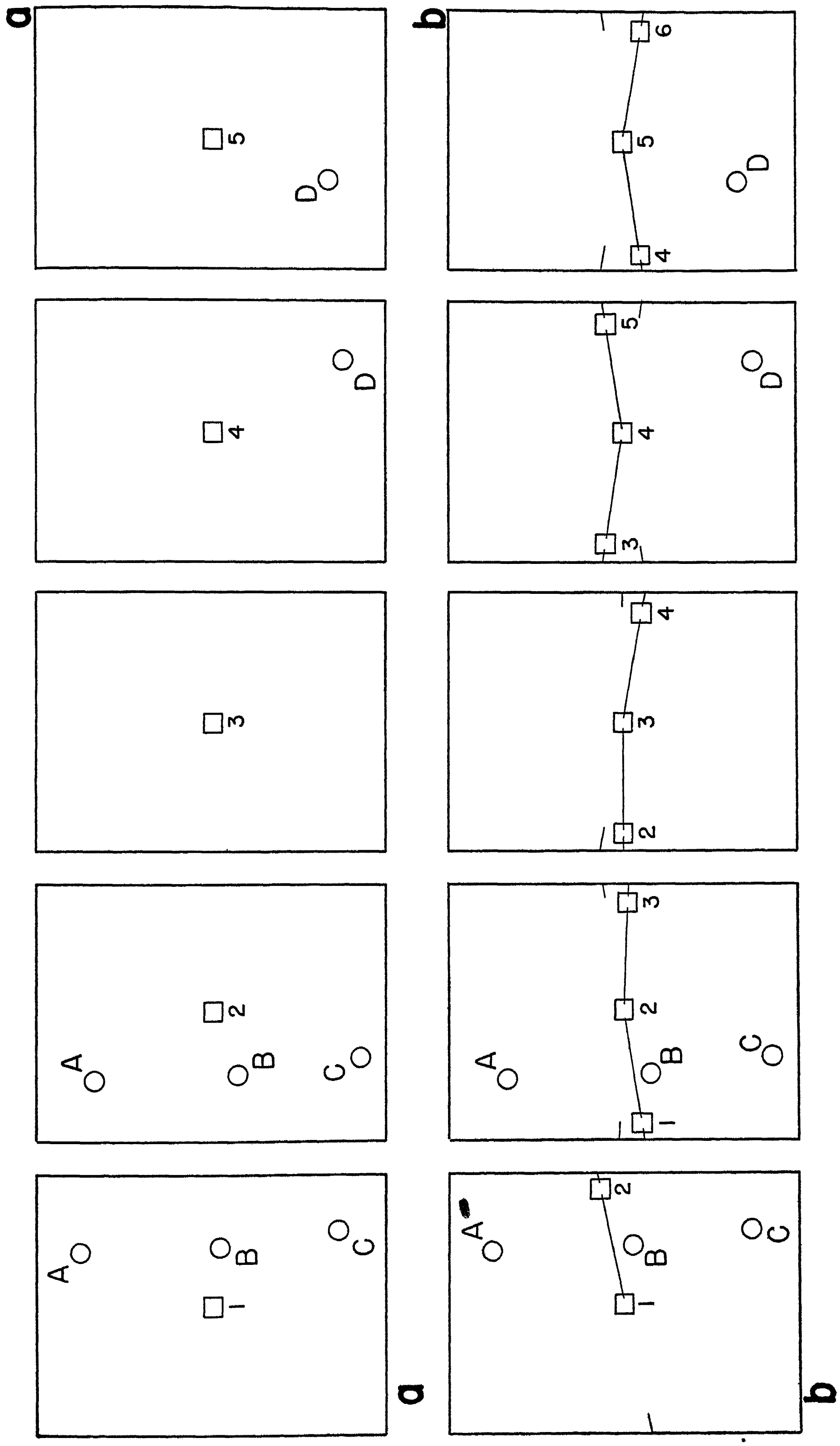


FIG. 8a. Location of the Principal Point on all photographs
FIG 8b. The Principal Point transferred to the succeeding and preceding prints.

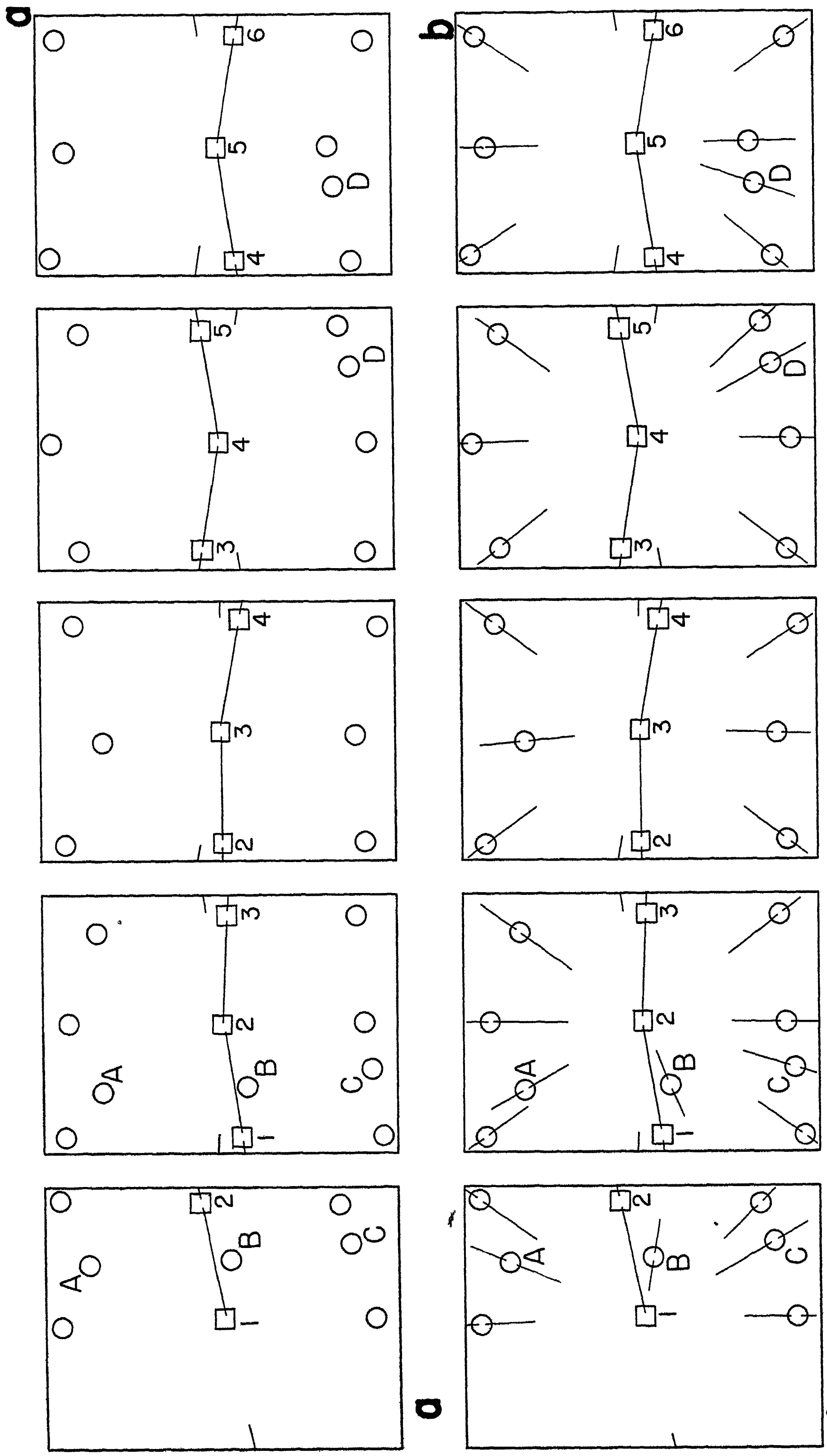


FIG 9a. Selection of Pass Points
FIG 9b. Radials drawn through all control points, from principal point.

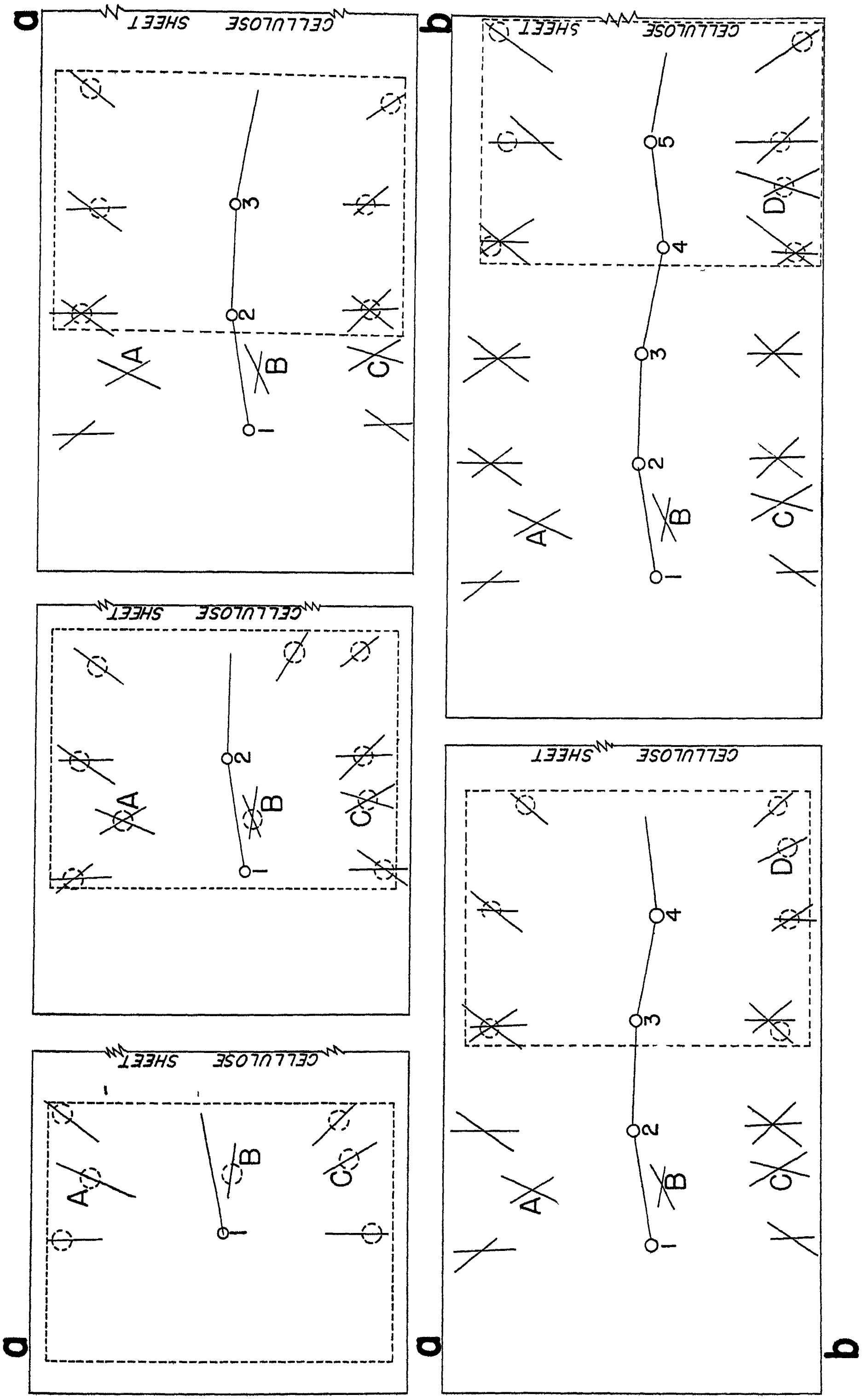


FIG 10a. Beginning the preliminary radial plot.
FIG. 10b. Preliminary radial plot extended.

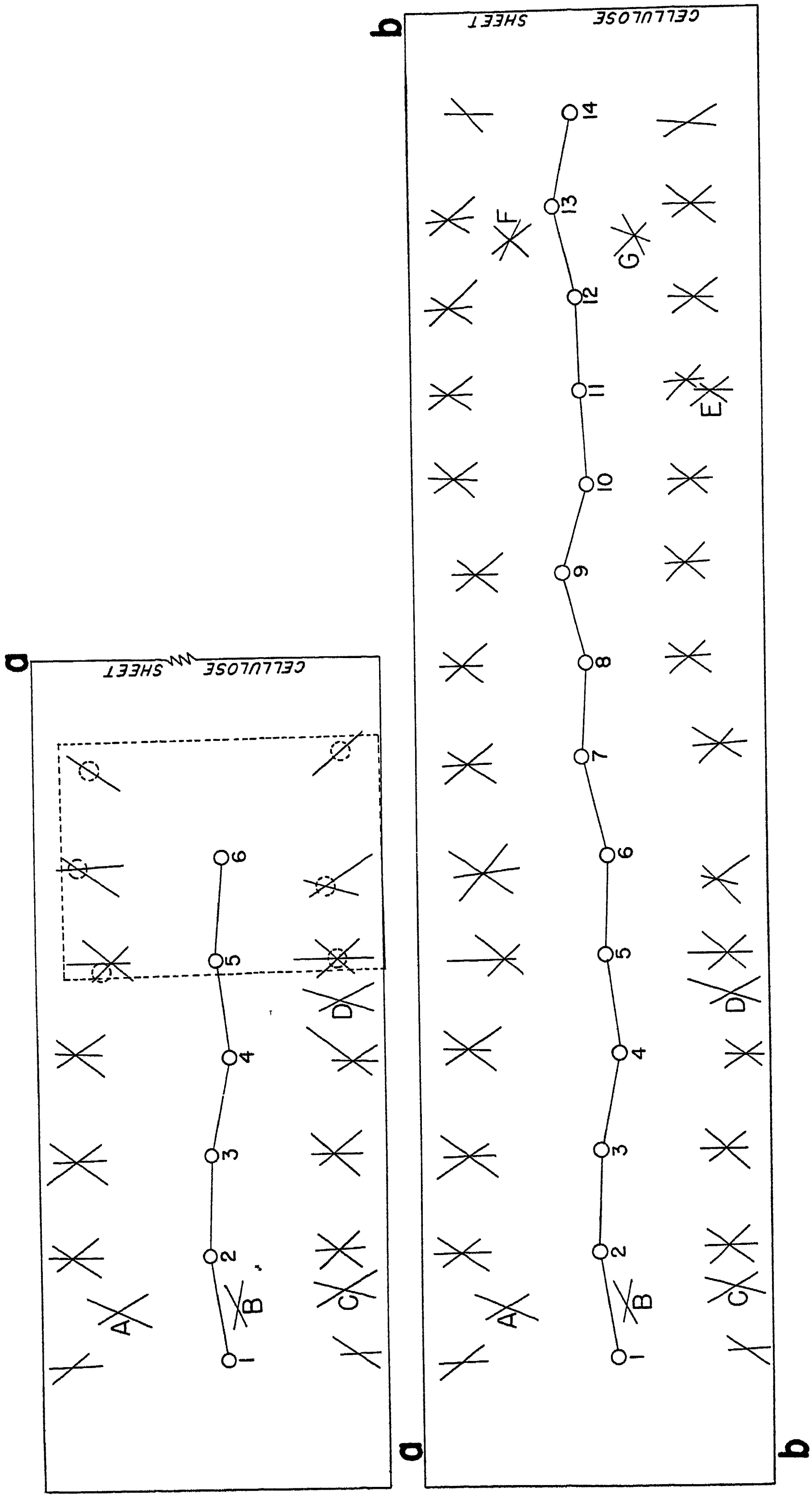


FIG 11a Preliminary radial plot extended
FIG 11b. Preliminary radial plot concluded.

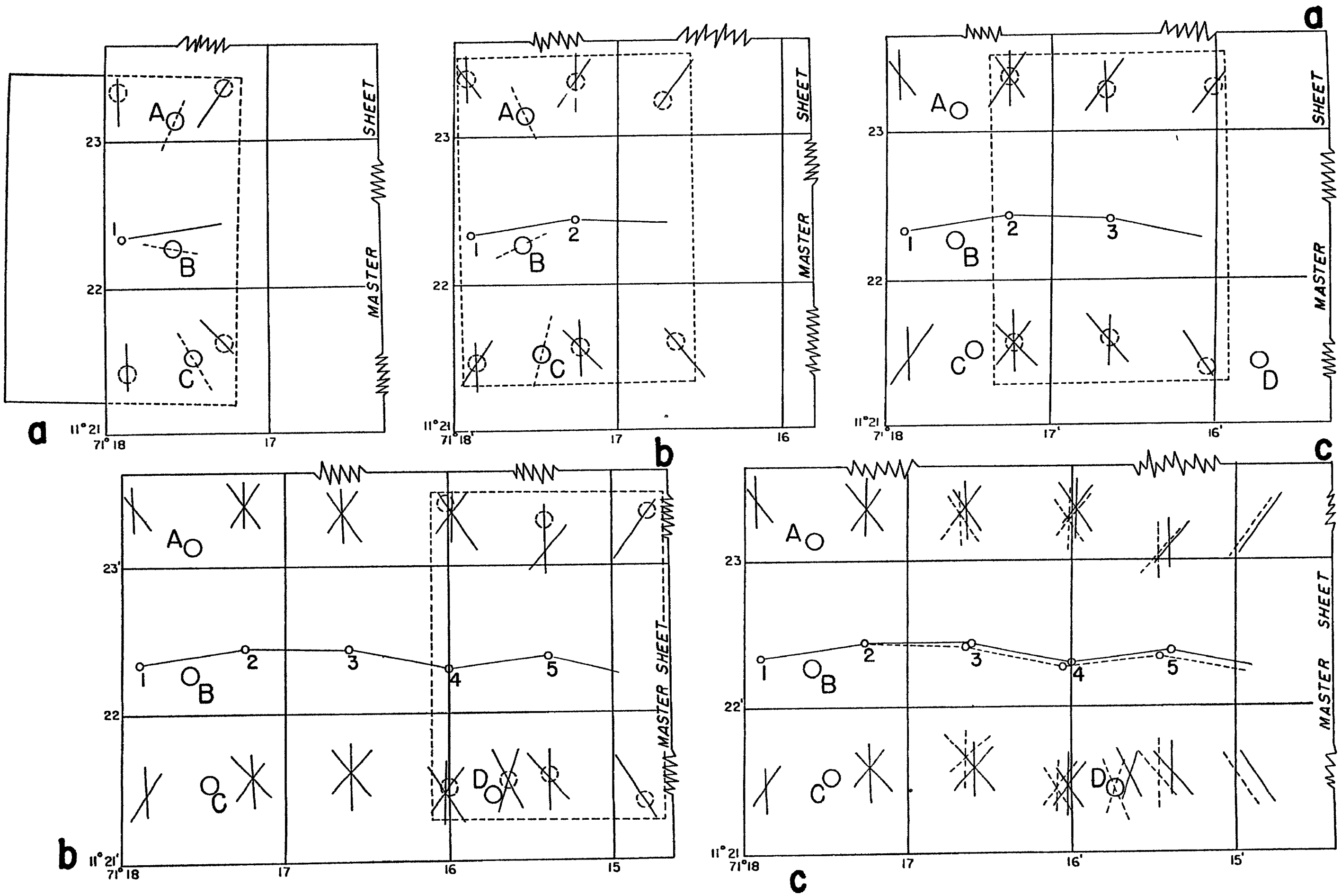


FIG 12a Starting the radial plot
FIG 12b Radial plot extended to the nearest ground control point.
FIG 12c Adjustment of the plot to the ground control Solid rays indicate error made in change of azimuth and scale. Dashed rays indicate adjustment to ground control points

2 LOCATION AND TRANSFER OF PRINCIPAL POINTS—The principal points are now located by drawing fine short intersecting lines joining opposite fiducial marks The intersection of these two lines should be pricked with a fine needle and encircled. This principal point is used as the origin of all radials on the photograph Its corresponding point on the succeeding photograph is then located with the aid of a stereoscope

The two prints should be brought into stereoscopic fusion and held in position by weights or tape Then with a fine needle transfer the position of the principal point of each photograph to its adjacent photograph

If this point falls in water, a definite position can not be located; therefore, if the point is not too far from the shoreline, it may be transferred with a pair of markers as shown in Figure 13 These markers are made by drawing fine perpendicular crosses on clear photographic film

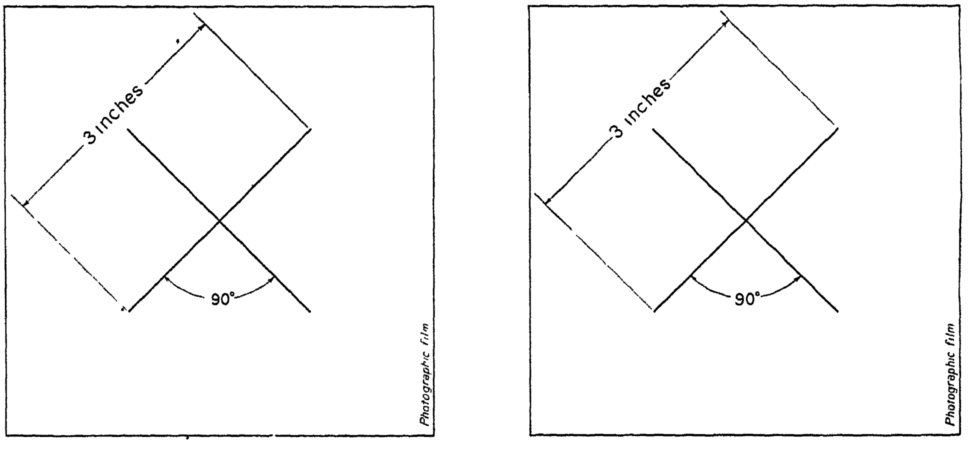


FIG. 13. Markers used to transfer points from one photograph to another.

One marker is placed over the principal point to be transferred and is guided into place with a fine needle passed through a small hole at the center of the cross. The lines of the cross should be at an angle of approximately 45° with the principal point base line The second marker is placed on the other print, over the apparent position of the first marker, both prints being observed stereoscopically. This will show a single cross apparently floating in space, and the second marker will indicate the transferred principal point when the center of the spacial cross appears to touch the ground. If the spacial cross appears above the ground, the second marker must be moved away from the first, and vice versa. If the spacial cross appears to be split, presenting the appearance of two single lines in space, the second cross must be moved up or down at right angles to the base line. When the correct fusion is observed, the new point is pricked through the center of the cross and marked. This method requires some practice but will give remarkable accuracy in minimum time.

The principal points transferred from adjacent prints are also marked with small red circles Lines are drawn connecting these points with the principal point of each print and are extended through it to each side of the print. This connecting line is called the *azimuth line*.

3. SELECTION OF PASS POINTS—Pass points are now chosen. These should

be readily identifiable points of detail such as houses, road intersections or any feature of sufficient clarity. In addition they should be selected so as to fall in the overlap zone common to three overlapping photographs They should be near a line which might be drawn at right angles to the azimuth line and at a distance approximately equal to the length of the air base This is the ideal selection to give maximum strength to the plot although it will be found in general practice that this condition can not always be met.

It is recommended that the selection of these points be made without the aid of a stereoscope although a stereoscope may be used to check the transfer of these points from one print to the next. These points should be transferred to the prints on which they are common They may be used in overlapping flights provided their usage does not materially retard the work. Care should be taken to ensure their proper location as the points are viewed from different perspectives in different flights.

4 Drawing Radials—When the prints have been properly marked with common points and all control stations located, the radials are drawn, in ink of suitable color, through the points which have been pricked and marked on the successive photographs. The lines are drawn to radiate from the principal point Where the terrain is flat, the radials need not be over one to one and one-half inches in length, but in mountainous area they will have to be somewhat longer. Great care should be taken to draw the radials accurately through the pricked points, and to keep the lines as fine as practicable.

5. Preliminary Plot to Determine Average Scale—One method of obtaining the approximate scale of a photographic flight is by the focal length and altitude relationship; for example

Focal length	8 25 inches
Altitude of flight.	13,750 feet, then—

$$\frac{\text{Focal length}}{\text{Altitude} \times (12)} \quad \text{or} \quad \frac{8.25}{13{,}750 \times (12)} = \frac{1}{20{,}000}.$$

Since the above formula can only give approximate scale, in that the absolute altitude of the plane is unknown, the average scale of the photographic flight should be determined by a preliminary radial plot.

The first photograph of the flight is placed under a transparent sheet and oriented so that the flight will not run off the sheet All lines are drawn on the transparent sheet coincident with the radials drawn on the photograph. The principal point and the principal point of the succeeding photograph are marked on the transparent sheet The next photograph is then oriented under the sheet, making the principal point of this and the preceding photograph coincide with the principal points previously marked on the sheet. When these points will not coincide, the difference is averaged and the azimuth retained Lines are then drawn coincident with the radials on the photograph, intersecting lines previously drawn and lines to new points The same procedure is followed throughout the strip with the remaining photographs. Intersections of the radials to the ground control stations establishes these stations to the average scale of the photographs. The distance between the intersections of the ground control stations as established by the radial plot is determined. The distance compared with the known ground distance will give the average scale. Thus,

$$\frac{\text{Actual ground distance}}{\text{Approximate photo distance}} = \text{factor}$$

which when multiplied by the scale used, will give the average scale of the radial plot

It is preferable, however, to have at least four or more of these ground control stations identified. A good average scale can then be obtained. If two or more flights are to be used, the average scale for each flight must be determined and a mean average of all the flights used This method will prove very satisfactory, provided there is not over 5% difference in the average scale of the flights Any difference greater than this will cause some error and will slow down the final and tedious work of taking off detail from the photographs In this event, it would be advisable to compute a separate projection for each flight.

6 CONSTRUCTION OF PROJECTION AND PLOTTING OF GROUND CONTROL—In the overlay method of compiling detail from aerial photographs it is necessary to plot the control on transparent material as the photographs must be adjusted beneath the plot for proper resection Several types of transparent material of relative stability under wide range of temperature and humidity are available In this discussion the transparent material referred to is cellulose acetate in that it is probably the most widely used at present

A projection of the desired limits is constructed at the average scale of the photographs (For a complete discussion on projections see Chapter I) All ground control to be used is plotted These control points should be pricked with a fine needle and properly marked and identified. Great care should be exercised in plotting the control to avoid errors later in the radial plot

The following factors govern the amount of ground control necessary to control a plot; namely, the photographic scale, the finished map scale, its intended use, the cost and the time involved, and the density of the culture. The ideal would be a density of control sufficient to resect each photograph on a minimum of three properly distributed ground control points This condition would simplify plotting from the photographs to a great extent but it is impractical and unnecessary to achieve the accuracy usually desired.

It is suggested that to achieve a good compilation at least three ground control points must be established in the overlap zone of two photographs in the same strip.

7. CONTROL PLOT—A photograph is selected, regardless of its position in the flight, having the most ground control located on it It is placed under the projection sheet and resected under the plotted control.

If there were no tilt in the photograph and no ground relief, and if the scale of the photograph exactly equalled that of the projection sheet, the points on the photograph would coincide with the true positions of those points as plotted. As this seldom occurs, it is necessary to make the radials from the center of the photograph to each control point coincide with the true direction to that point. If the photograph is shifted until each of the points on the projection sheet fall somewhere along the radials as drawn on the photograph through the points, this condition is satisfied and the photograph is properly oriented. *This constitutes resection.*

After the photograph is properly oriented, short radials are drawn on the sheet through the points previously marked on the photograph, and also along the azimuth line These lines are drawn to coincide with the lines on the photograph.

The first photograph is now replaced by the second, which is oriented along the azimuth line, and resected as before. The radials are drawn and centers of the photographs are properly marked

From this stage of the plotting and on throughout the entire plot, there are,

in addition to the ground control points, the intersections of the radials of the two previous photographs, which provide several other true locations to assist in resecting the next photograph.

Lack of coincidence of intersections throughout a photograph may be caused by carelessness in identification of control points, improper identification of points on adjacent prints, faulty drawing of radials, and errors due to tilt. Tests have shown that averaging the error at all points produces the best results. *It is improper to place more reliance on one point than on any other.* Each radial may be considered as a line of sight, taken through a transit from the principal point, and recorded as an angle from some other line, and as all such transit cuts would be treated with equal weight, so should all radials drawn from the principal point; which corresponds to the transit station, be treated. At some

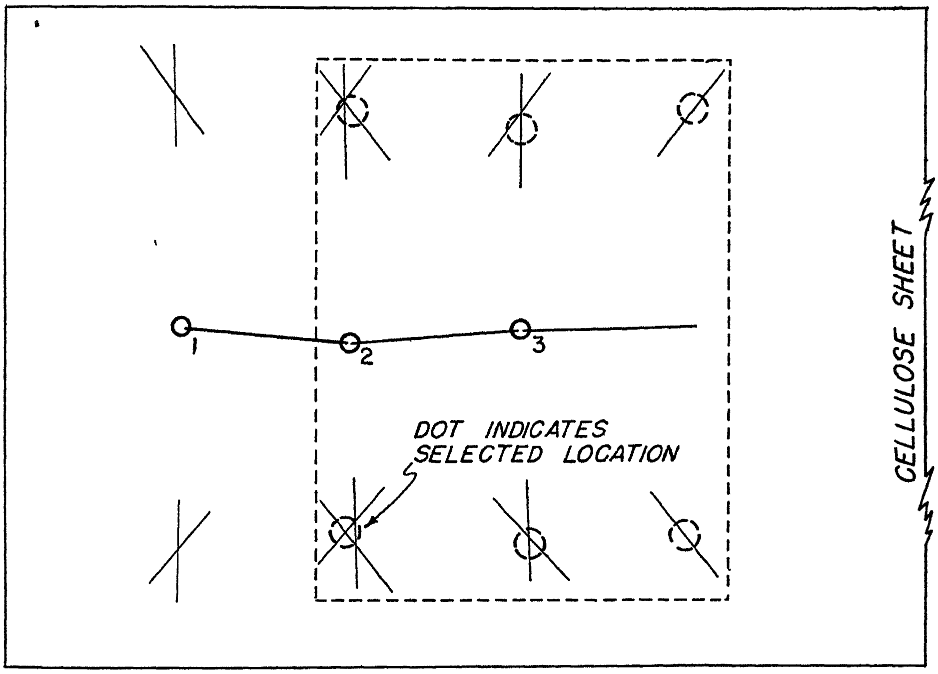


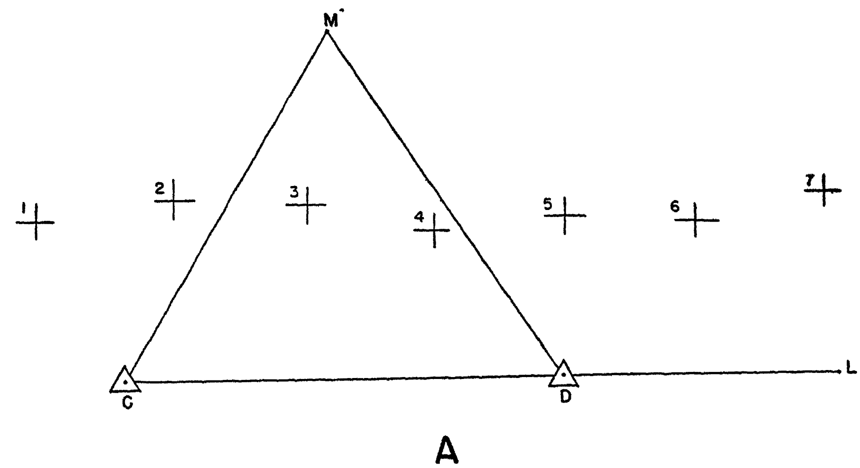
FIG. 14 Illustrating triangle of error

intersections, small triangles of error will occur. The usual causes for such triangles are errors in drafting, marking of points, or a combination of tilt and relief If after checking, the error persists, the center of the triangle may be used as the location of the point (See Figure 14)

By this method of using successive photographs, the extension of photographic control is carried forward. Eventually ground control points will be reached and will give a chance for checking the accumulated error of the plot. In most cases some error of closure will exist. This error should be adjusted before proceeding any further. It is adjusted by means of a straight line adjustment back through *all* of the pass points, that is, working back along the center traverse and adjusting the error in an amount proportional to the distance from the beginning of the strip. In no case should the plot terminate until some ground control is reached, so as to check the accuracy of the plot.

A more detailed method of making these adjustments is a graphic triangular or quadrilateral resection. It is as follows:

Figure 15 a shows principal points *1* to *6* of the strip on which ground control points *C* and *D* have been identified and plotted. The wing points are omitted in order to avoid confusion



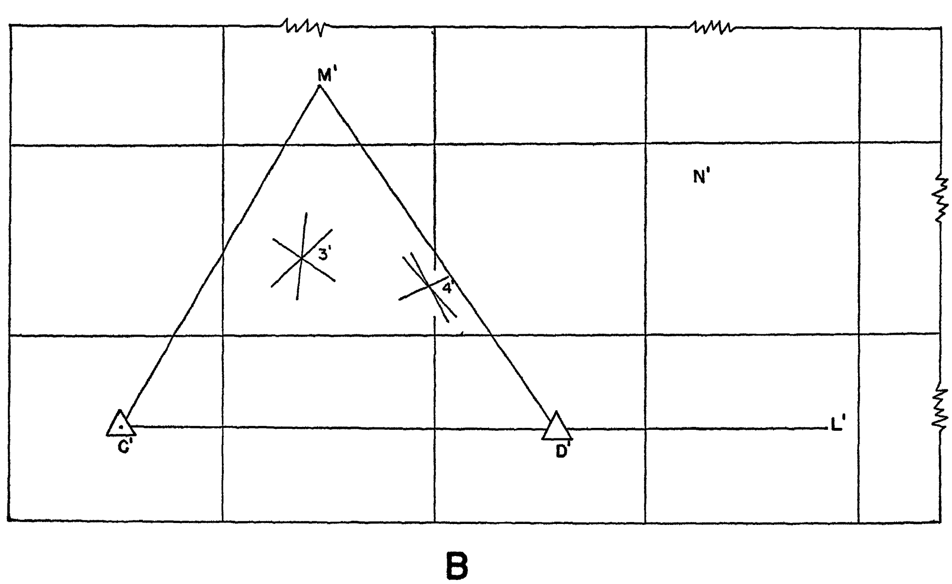


FIG. 15. Triangular closure adjustment.

In Figure 15*b*, *C′* and *D′* are the same two ground control points plotted on the projection sheet In order to reduce the length of *CD* to the length of *C′D′* and plot the traverse to the same scale, select a point *M* in the position shown, fulfilling the conditions described in the following paragraph.

Draw *CD*, *CM*, *MD*, and *C′D′*. Place the control sheet over the strip so that *C′* is over *C*, and *C′D′* coincides with *CD*. Trace the ray *C′M′*. Place *D′* over *D*

and orient by the line $D'C'$, and draw $D'M'$, thus locating the position of M' The triangles CDM and $C'D'M'$ are similar. To plot the adjusted position of the point *3*, for example, with D' over D and the adjacent sides of the triangles in coincidence, draw a part of the ray over the point *3* Similarly, draw rays from C' and M' Point *3'*, at the intersection of the three rays, is the adjusted control sheet position of the point *3* Other principal points and wing points may be intersected at the same time.

Any similar triangles will serve, provided the scale ratio is maintained. A sharp, three-ray intersection must be obtained for every point. At least two of the rays should intersect at not less than approximately 60° Points *7*, *8*, and *9* could not be accurately intersected from the three points previously used In such event, select a new auxiliary point at N; if necessary, extend the line between the ground control points to double or any other multiple of its length, as at L. To conserve material the strip sheets may be narrow and the triangles arranged to fit.

In some cases quadrilaterals offer a better solution than triangles The four-ray intersections will not always be sharp, but the best position in proportion to the lengths of the rays can be plotted By this method, the area within the quadrilateral has the least distortion and maintains a proportional ratio with adjoining areas The quadrilateral method is as follows

In Figure 16 let D be the location of that control station on the transparent sheet, and D' the plotted position which is in error Draw a line connecting these two points. Let *1–2*, *2–3*, etc , Figure 16a, represent the principal point traverse on the transparent sheet.

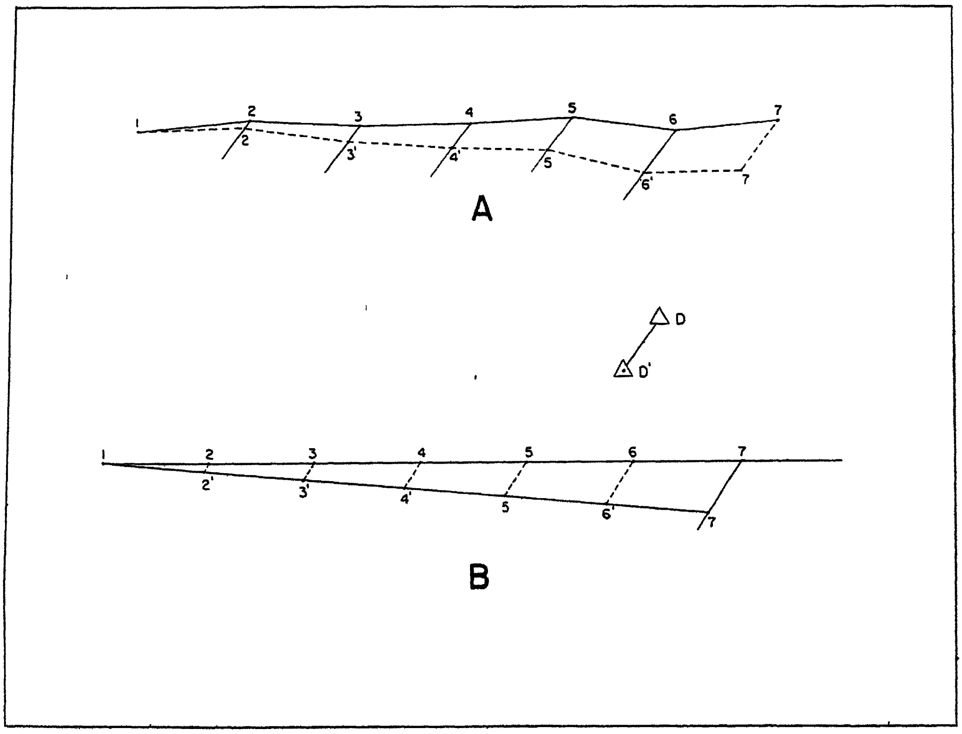


FIG. 16. Quadrilateral closure adjustment.

On a separate transparent sheet, lay out a line *1–7* extended, Figure 16b. On this line, lay out the distance *1–2*, *2–3*, etc , scaled from the plotted positions of these centers on the transparent sheet. Through *7*, Figure 16b, draw a line parallel to *DD′*. The error in position of point *7* may be assumed to be equivalent in distance and direction to the error in control point, *D*. Connect points *1* and *7′*, Figure 16b Draw *6–6′*, *5–5′*, etc., parallel to *7–7′*. On the transparent sheet, draw 7–7′, Figure 16a, parallel and equal to *D–D′*, and in the same direction Through points *6*, *5*, *4*, *3*, *2*, draw lines parallel to *7–7′*, Figure 16a, and lay off the distances *6–6′*, *5–5′*, etc , from Figure 16b. Connect points *1*, *2′*, *3′*, *4′*, *5′*, *6′*, *7′*. This course line will then represent the corrected principal point traverse Wing points may be similarly corrected if considered advisable

It is suggested that the radials be drawn with a thin-lead, red pencil. Very fine lines may be drawn; yet, they will show clearly on the cellulose and may be removed with a damp cloth without affecting any other work on the sheet.

When the radial plot is completed and all adjustments made, the centers of the photographs should be permanently marked on the cellulose sheet, and the number of each photograph placed alongside its respective circle Both the circle and the number are put on with red acetate ink. A point is pricked through all radial intersections and properly identified.

Figures 17, 18, 19, and 20 illustrate the more frequent conditions experienced in the resection of photographs, and are usually encountered in the extension of any radial plot.

8. Selection and Establishment of Detail Control—The following steps in the compilation of maps from aerial photographs will be governed by the amount and character of the detail desired, the character of the terrain, and the closeness of the photographic scale to the scale of the compilation sheet.

After photographic control has been established for the entire plot, the limits of the general area to be delineated on each photograph are outlined. Relief displacement increases in proportion to the distance images are removed from the principal point. Therefore, it is best to divide the area between successive principal points so that only the best portions of each photograph will be used. A sharp wax pencil of any brilliant color may be used for drawing these boundary lines. These should be drawn through image points common to adjacent overlapping photographs. The division between adjoining flight strips can best be accomplished by drawing lines through corresponding radial control points at the sides of each photograph. This procedure may vary in detail between projects. Judgement must be used in selecting the detail areas of each photograph, bearing in mind that the composite planimetry should come from the best areas of each. (See Figure 21.)

On photographs of more or less level terrain the control points selected may be two or more inches apart while in mountainous country it may be necessary to have control one-half inch apart in order to have a satisfactory compilation. All points are encircled as selected. On planimetry which extends from one flight strip into another, it is well to have a detail control point common to both side lap photographs so that a more perfect joint can be made.

Previously selected pass points may be used for detail control where the detail, such as a house, is at the same relative elevation as the pass points and adjacent thereto. Only by using a stereoscope will one be able to judge this

After the detail control has been selected for the entire plot, each photograph should be placed under the projection sheet and oriented according to previous determination, and the radial intersections of the new points obtained.

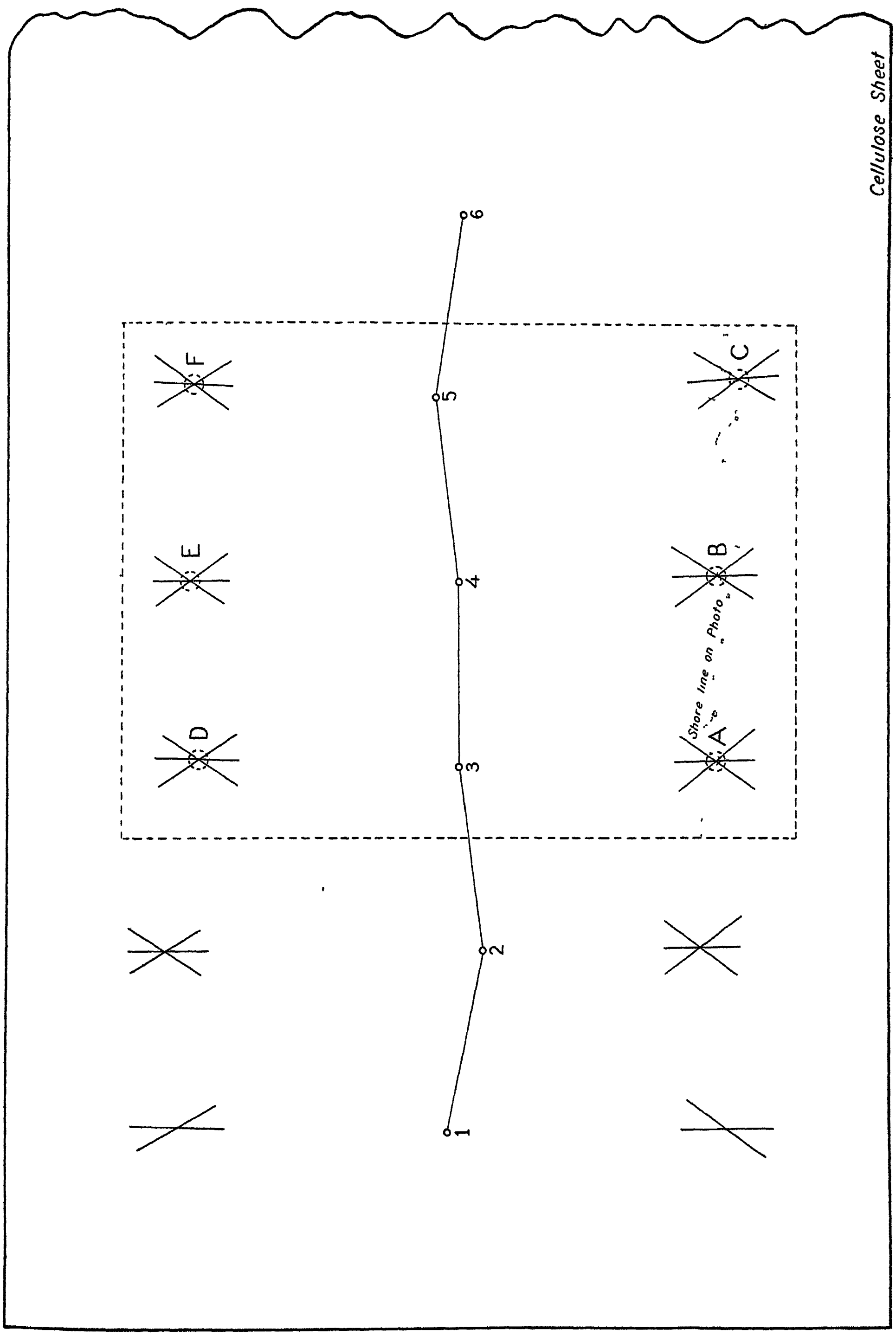


FIG. 17 If there were no tilt or relief in the photograph, and if the scale of the photograph exactly equalled the scale of the sheet, then the intersection of the radials as drawn on the cellulose sheet would be as shown above.

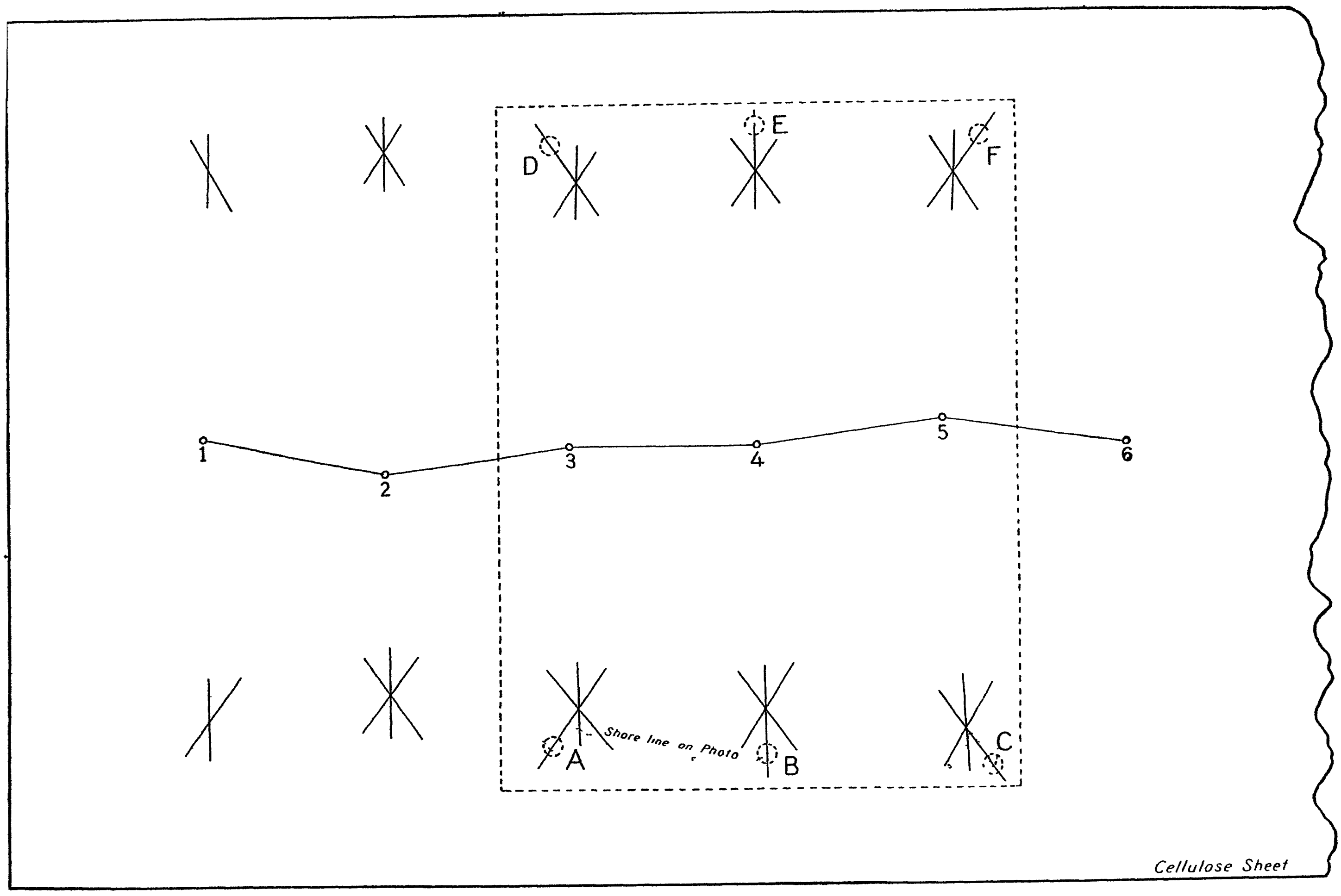


FIG. 18. If the scale of the photograph were larger than the scale of the compilation sheet, and if there were no relief or tilt in the photograph, then the intersection of the radials would be inside the selected points (i e , nearer the principal point of the photograph.) The result would be reversed if the scale of the photograph were smaller than the scale of the compilation sheet

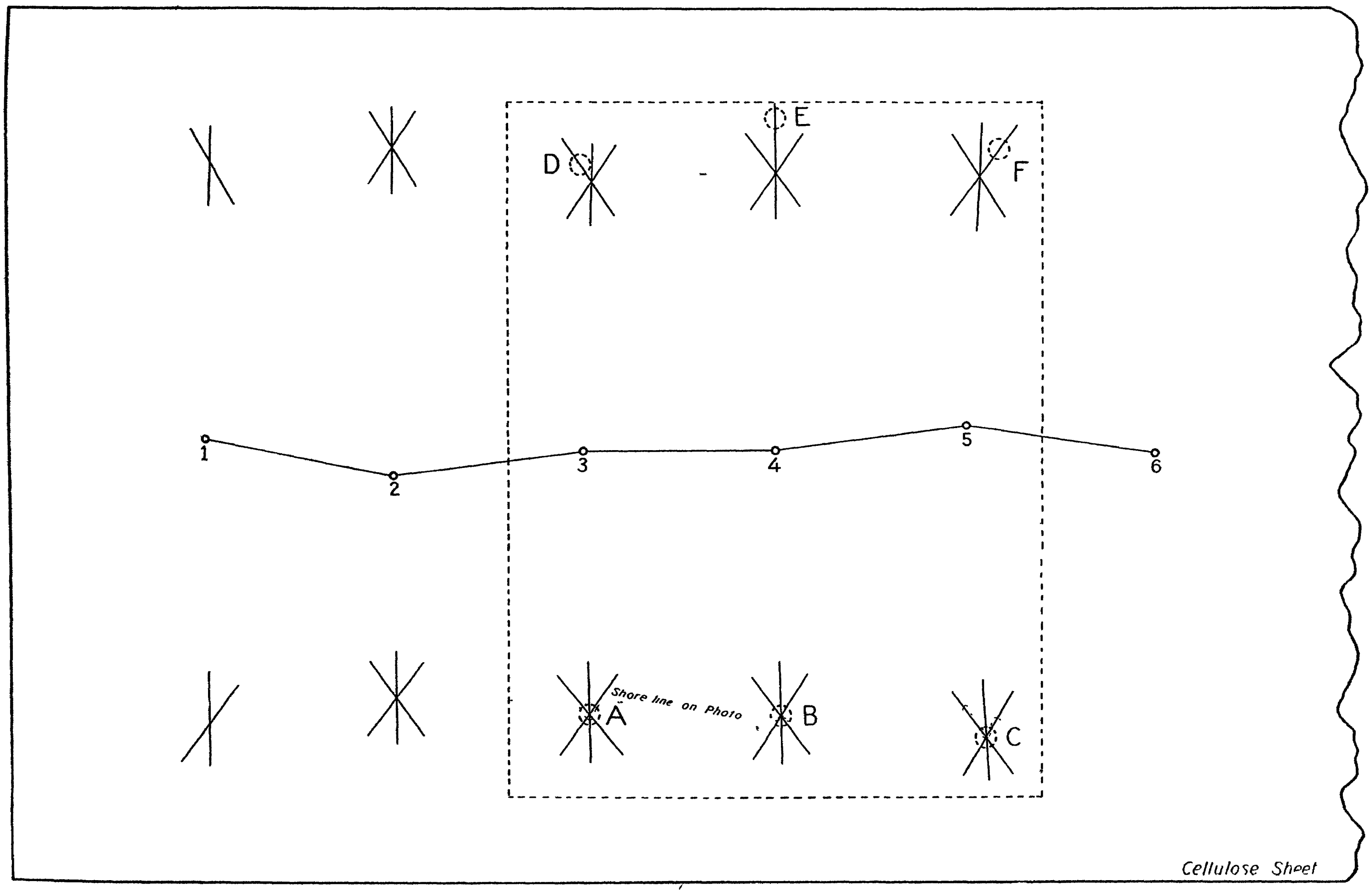


FIG. 19. If there were relief in the photograph, then the intersection of the radials would be as shown Points A, B, and C are along the high water line, and at a scale equal to the scale of the compilation sheet. Points D, E, and F are back from the shoreline and at varying elevations. Therefore, the scale of these points is expressed by $f/H-h$ and the displacement is equal to $(h/H)V'b$. (Figure 2)

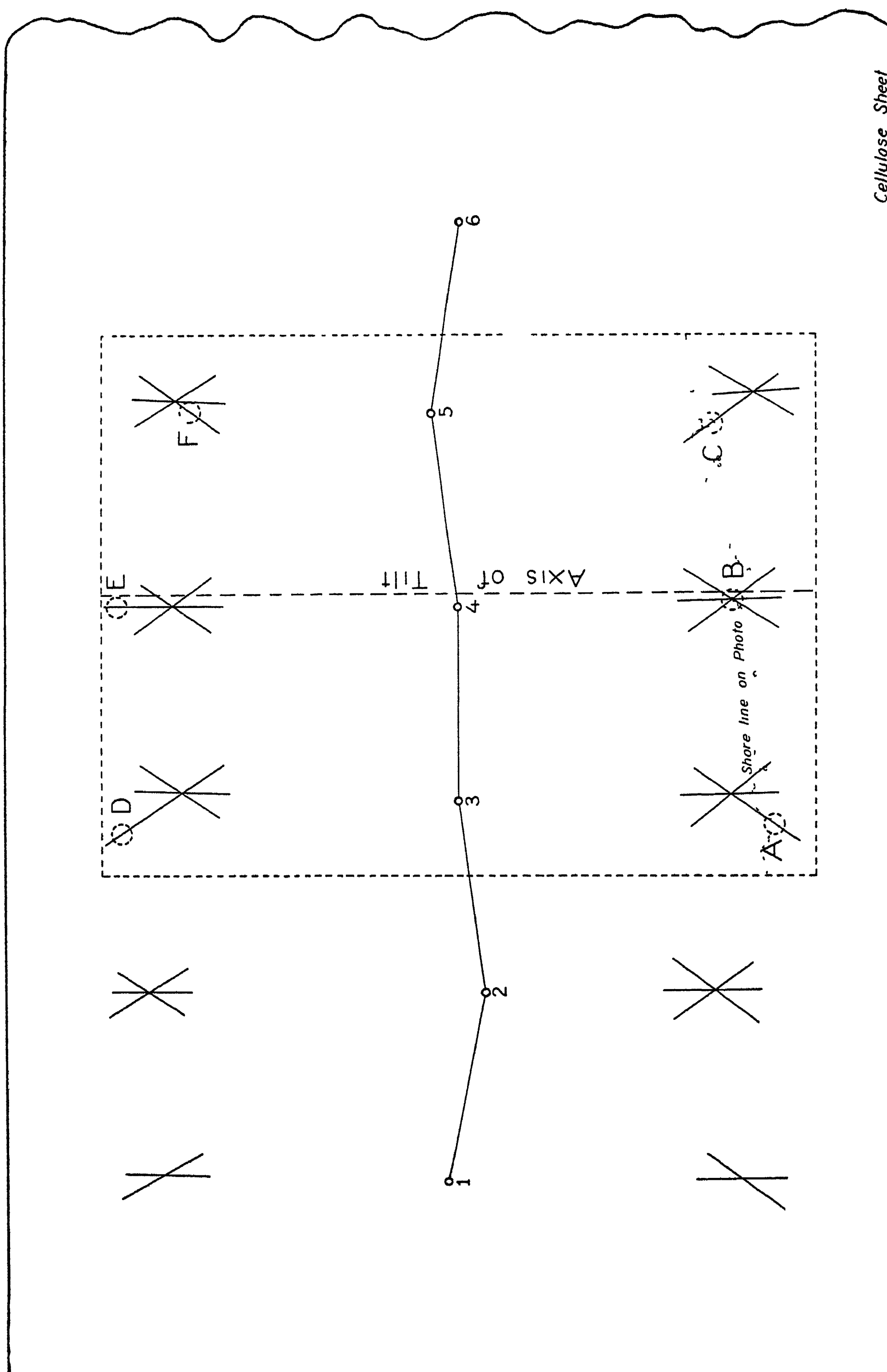


FIG 20. If there were tilt and relief in the photographs, then the intersection of the radials would be as shown. Assuming that the axis of tilt is as illustrated, then the points nearest the axis of tilt would be affected only by relief (points B and E). The other points would be displaced as illustrated

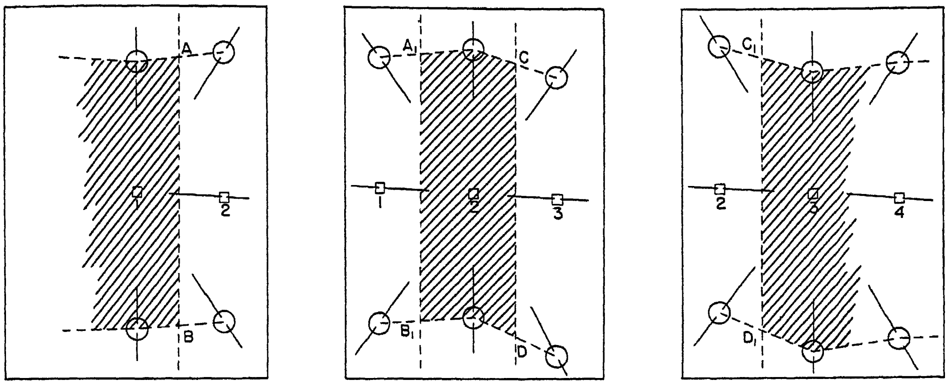


FIG. 21. Delineation Area of each photograph.

9. DELINEATION—After the detail control has been located on the compilation sheet, each pair of over-lapping photographs are returned for study under the stereoscope The detail to be delineated can then be traced, keeping within the proper boundary on each A stereoscope is necessary for giving a three dimensional model of an area, thus permitting a clearer interpretation It is suggested that the procedure for using the stereoscope in Chapter VII, "Orient Your Stereoscope Correctly" be used. A mixture of water color or show card color is recommended for delineating detail The color selection should be made with a desire of providing the greatest contrast to the average tone of the photographs used. Usually, one color is selected for culture and another, preferably blue, for drainage.

After this is done, each photograph in turn is placed under the sheet in its proper orientation The detail is traced by moving the point of intersection on the sheet exactly over the corresponding point on the photograph, and in line with the principal point, and tracing off the detail around that point, gradually moving over to the next intersection. Movement should always be from the point of intersection to the corresponding point on the photograph, out or in as the case may be, radial from the principal point. In this manner, ground features are kept in proper orientation and held to proper scale.

It is readily seen that if the projection sheet is near the scale of the photograph the tracing of detail is expedited Better results can be obtained by the use of reflecting projectors or instruments employing the principles of the camera "lucida." The use of the vertical sketchmaster (based on the principle of the camera "lucida") is described in an article on Tri-Metrogon Mapping in this book. Therefore, in this procedure only the use of vertical projectors will be discussed

The projector consists of an illuminated, optical system whereby images placed at the designated location are projected down on to a standard table for tracing Its flexibility in adjustment is such that an original image may be reduced or enlarged within a wide range. The photograph is placed so that its principal point is in registration with the etched cross on the glass "backplate." This etched mark represents a point in the path of the optical axis of the lens system. Images on the photograph, enlarged or reduced, will be radial from this point. The compilation sheet is placed on the table and oriented. The machine is adjusted to give the proper focus at the desired scale. The detail is

traced by holding each increment of detail to its respective control on the compilation sheet This step is repeated for each photograph used until all planimetry has been transferred.

Some trace directly in ink though it is advisable that inexperienced personnel use a sharp, thin-lead pencil and then ink in afterwards

Summary

Although this article deals primarily with the radial method in its application to overlay compilation, the principles discussed will apply to the slotted templet, hand templet, and metal templet methods.

For the compilation of accurate planimetry from aerial photographs there is no other method surpassing the radial plot method in economy, simplicity, and speed. It is predicated on good photography, accurate and intelligent control selection, and careful drafting The equipment needed is inexpensive, and a decided advantage is offered in the fact that instruction in the technique may be absorbed in a relatively short time The photographs may be either single or multiple-lens verticals. The flight lines should be as straight as possible, and the photography accomplished with a minimum of crab and tilt. (See Chapter IV) The overlap in line of flight should never be less than 55% or more than 65%. The side-lap between adjacent flight strips should be as close to 30% as possible

Each Bureau or office will, of course, set its own standards of uniformity for the marking of photographs. However, listed below are suggestions that are frequently used.

1. Ground control points are circled in green on the photographs along with their proper identification. These points are circled in red on the acetate sheet.
2. Pass points are circled in red on the photographs and in blue acetate ink at their proper location on the acetate sheet.
3. The color of detail control circles on the photographs as well as the colors used for delineation should be chosen with the object of providing the best contrast to the average tone of the photographs especially where a vertical projection machine will be used in the compilation.
4. When a "master sheet" is desired for reproduction purposes, a direct tracing in ink of the photographic compilation sheet may be made Only the needed features are traced, usually the culture and drainage together with the projection lines.
5. Projection lines and control circles are best inked on the reverse side of the compilation sheet They will then be permanent regardless of erasures and corrections occurring in the compilation work.

THE SLOTTED TEMPLET METHOD*

Harry T Kelsh

Introduction

IN THE construction of planimetric maps from aerial photographs, the primary problem is to secure the horizontal control necessary to place the photograph centers in their correct positions on the map grid and to obtain the correct orientation of the photographs, and then, with this accomplished, to eliminate the errors of position of all other points as they appear on the photographs—these errors being the result of differences in scale and the displacement of each ground point on the photographs from its correct position, due to the relief of the ground Of course, in practice, lens distortion, tilt, and paper and film shrinkage are factors which may introduce additional errors

If previously located ground control positions obtained by triangulation or transverse are sufficiently numerous so that several appear on each photograph, then the compilation of planimetric maps from these photographs is a comparatively simple matter, but this ideal situation, unfortunately, seldom exists In fact, in terms of usual single-lens photographs, such control is often 50 to 100 photographs apart

It is, therefore, necessary to break down this ground control spacing to a degree sufficient that a number of points can be located within the area covered by each photograph. This may be done by triangulation, by traverse, or by plane table; but since all of these methods are expensive and slow, and under certain conditions are completely unfeasible, one of the many problems of photogrammetry has been to find a workable method of carrying forward the control through the photographs themselves.

Since the true position of each photograph center must be obtained, effort has been directed to reducing this cost by cutting down the number of photographs required. This may be done in a number of ways

If the same photographic equipment is used and the altitude is doubled, then four times the area will be included in each photograph; but to double the usual flying altitude of about 13,000 feet above the ground requires special flying equipment; and, furthermore, by doubling the altitude, the scale is reduced to one-half and all photographic detail reduced accordingly.

The use of a wide-angle lens in the camera will increase the covering power, and this, of course, decreases the necessary number of photographs to cover a given area With such type of lens, however, there is considerable distortion of physical features near the edges of the resulting photograph, since image displacement, due to relief, increases with the distance from the photograph center. This tends to limit the general usefulness of the wide-angle photograph, particularly in rugged country, to mapping purposes only, in which the increased displacement may be no disadvantage—in fact, it may for some purposes actually be of material benefit, provided there is no masking out of ground detail by the relief.

Multiple-lens cameras furnish another approach to the solution of this problem, but special printing or plotting equipment is necessary to use the type of photograph obtained.

Consideration of the advantages and defects of the various types of mapping

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cameras thus briefly discussed, together with the fact that about 2 million square miles of the United States already have been covered with vertical photographs produced by single-lens cameras with lenses of $8\frac{1}{4}$-inch focal length on film sizes $7''\times 9''$ or $9''\times 9''$ on a scale of 1:20,000; and the desirability, in the interest of practical economy, that photographs should be usable for other than map-making purposes, have tended to direct experimentation toward the solution of the problem of securing the necessary horizontal control for accurate planimetric maps through the development of a system that would advantageously use the existing general purpose photographs.

Figure 1 gives a general idea of the appearance of such photographs General flying specifications require an overlap in line of flight of approximately 60 per-

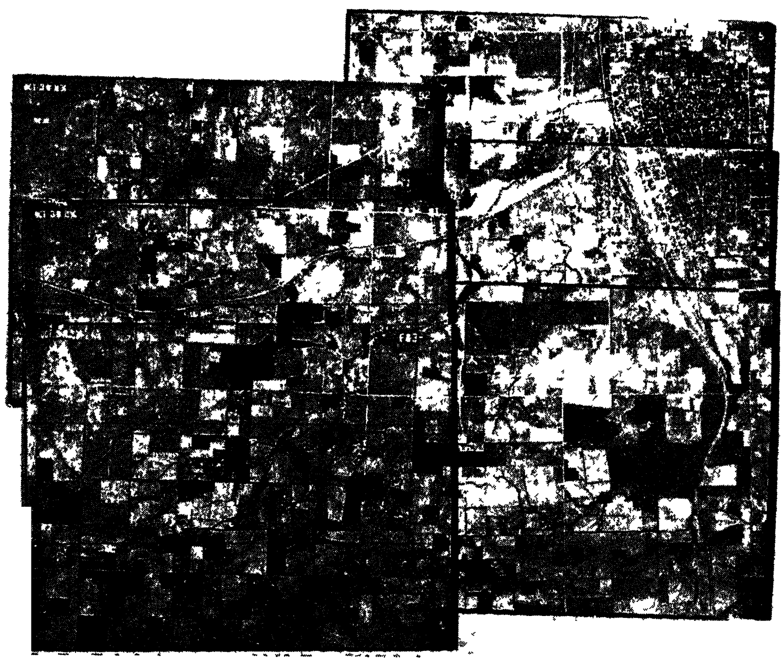


FIG. 1. Average photograph overlap. Note that with this overlap every ground point will appear on at least two photographs in line of flight.

cent and a side overlap of 25 to 30 percent, also that the photographs shall be at approximately the correct scale, and with small allowable departure from the true horizontal plane.

THE HAND-TEMPLET METHOD

Using the type of photograph just described, a very simple method of obtaining actual positions of the horizontal control on such photographs was worked out. It has been called the hand-templet method and is based upon the principle that, provided an aerial photograph is taken with the lens in a true vertical position, all angles radiating from the principal point of the photograph through the image points as identified on the photograph, remain constant irrespective of scale change or relief.

In practice, the method consists of reducing each photograph to a trans-

parent templet on which the picture points are represented by lines radial to the principal point, and then assembling these templets so that all the radials common to each identified position intersect, the intersection being the true location of the point (Fig 2.)

While theoretically completely satisfactory, the limitations of this system soon became apparent They include the fact that templets of transparent cellulose acetate or nitrate are subject to considerable distortion, due to humid-

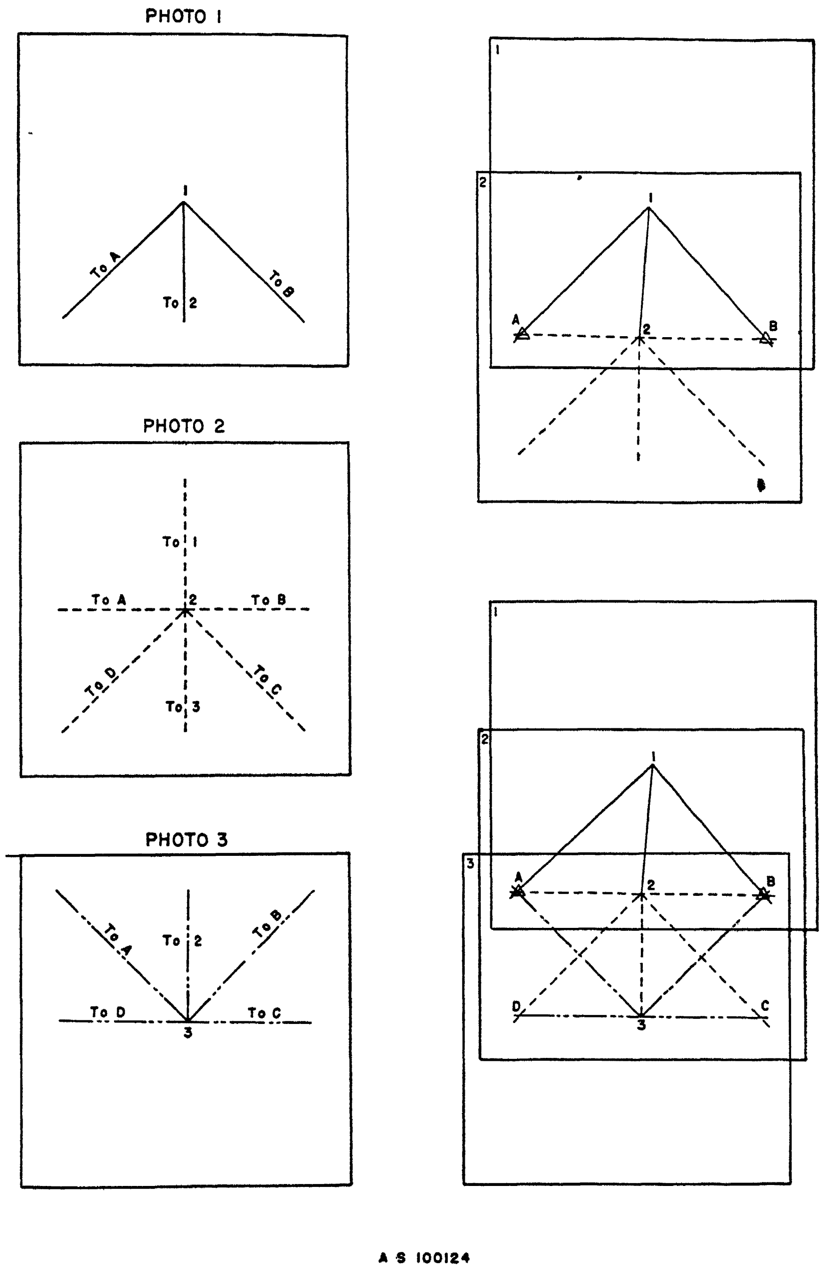


FIG. 2. Diagrammetric sketch of extension of control by radial triangulation.

ity and temperature change Limitations in drafting cause a certain amount of error to accumulate If tilt is present, the radials will not intersect but will form triangles of error, and the elimination of these is a matter requiring considerable experience and, at the best, is largely a matter of personal judgment and compromise

However, if there are a number of control stations on each photograph, then this system will allow very rapid assembly of the templets; but such condition is rare in ordinary practice. The problem in map control is not how to fit a large amount of control to the photographs, but how to use the usually insufficient existing ground control with as little addition in new field work as possible.

Since the slotted-templet method is directly a development from the hand-templet method, although eliminating the features which limited the scope of the latter, a description of the hand-templet method is worth while The steps in its operation, as well as a direct comparison of the two methods, are included herein.

The hand-templet method for the extension of radial control works as follows: The first step in the preparation of the photographs is to locate the principal point on each print This may not be identifiable, and it is better to pick a definite image nearly coinciding with it than to attempt to locate an indefinite principal point, since the point must be transferred to the succeeding photograph. The point selected should never, however, be more than one-eighth of an inch from the true principal point. Where the templet is approximately the same size as the photograph, this variance will introduce no plottable error in the angles between the picture points, as measured from the selected point instead of from the true principal point.

Under a stereoscope, this point is transferred to the succeeding picture and, as ordinarily there is a 60-percent overlap in line of flight, three azimuth points (Fig. 3) will thus appear on each photograph. At least three additional pass points, approximately parallel to the line of flight and opposite these plotted centers, are picked on each side, in the overlap area between the lines of flight. They should be of such character that they can be identified and transferred, stereoscopically, to the photographs in the adjacent flights. This gives at least nine picture points on every photograph. Usually, at least three or four more than this minimum are selected and marked.

In practice, the points are located by pricking fine holes through the photograph and encircling them with small circles, red for the pass points and blue for the azimuth points, approximately three-sixteenths of an inch in diameter. Ordinary drawing ink is quite suitable for this purpose.

In advance of the office work, the ground control points have been identified stereoscopically on the photographs in the field. They are marked by pricking a fine hole with a needle through the photograph and encircling the hole on the back of the photograph with a small penciled circle. It is advisable not only to number these, but to add a brief description of each point, together with a small sketch, at the time of identification. These points are now marked on the face of the photograph with small yellow circles, and are transferred stereoscopically to all overlapping photographs on which they appear, and on which they can be identified.

When all of the photographs are thus prepared, the next step is to make a templet for each photograph. For this purpose clear, transparent cellulose acetate is used. This can be obtained either in sheets or rolls. A thickness of 0.0075 inch is very satisfactory. It should be cut into rectangles approximately 1 inch larger on each side than the corresponding dimensions of the photograph. This

1. Date photographed
2. Time of exposure
3. Contracting agency
4. Scale
5. Symbol.
6. Roll number.
7. Photograph number
8. Radial control point In tersection of fence and road
9. Azimuth point of adjoining photograph. Small bush.
10. Permanent reference points.
11. Photo control point. Point of grass
12. Radial control point Small bush.
13. Radial control point. Fence corner.
14. Geometric center of photograph Located from fiducial marks
15. Azimuth point Small bush.
16. Radial control point. Point of grass
17. Radial control point. Small isolated tree.
18. Azimuth point of adjoining photograph Small bush.
19. Radial control point Small isolated tree.
20. Fiducial marks

N B. Azimuth points, blue Radial control points, red. Ground control points, yellow

FIG. 3. Typical aerial photograph with essential data completed.

allows the photograph to be taped face down on the templet, using removable crepe-paper adhesive tape along the edges. The assembly is then turned over, the center point marked with a small black circle, and radials from that point drawn through the other pass points appearing on the photograph. (See Fig. 4.)

It is preferable to etch the lines, and fill them with an opaque substance such as graphite. In this manner, very fine, distinct lines are obtained

The azimuth lines are extended from the center, but the radials are merely drawn through the pass points, with about one-half inch of line on each side of the actual point. This is long enough to take care of ordinary relief differences of average country, on photographs made with normal-angle lens. It takes care,

as well, of slight scale changes between individual photographs. However, if it is desired to make a map upon a scale differing from the scale of the original photographs, it is necessary to make a larger or smaller templet and extend the radials a proportionate distance out from the center, corresponding to the difference in scale desired.

The picture number is inked on the templet alongside the azimuth point, with the same orientation as on the photograph. (See Fig. 3.)

The photographs are now stripped from the templets, and need not be referred to again in the establishment of the radial triangulation net unless some error is found in the templets Where this occurs, it is generally due to mis-

FIG. 4. Drawing the radials on a transparent hand templet.

identification of points. The templets are stacked by flights in numerical order, and are ready for assembly. A base, upon which to assemble the templets, is constructed.

In order to start the assembly of hand templets, it is necessary to locate definitely the positions of the exposure stations of two successive photographs. This is usually done by a graphical solution of the three-point problem. Two templets in the same flight, on the common overlap of which appear at least three picture ties, preferably so placed so that they approximate an equilateral triangle, are selected. The two templets are then placed upon the base sheet so that the radials to these pass points pass through the plotted positions of the points, and so that the azimuth lines on the two templets coincide; and they are then fastened down to the grid with adhesive tape. These now establish additional positions by a system of radial triangulation, the distance and direction between the positions forming the base from which additional templets can then be laid, carrying the control forward along the flight line.

As each templet is laid it is taped to the grid sheet and the preceding templet

by small pieces of adhesive tape. This procedure is repeated until the flight is tied in to the next control points.

In spite of all possible care, it is seldom that a correct assembly, even of a single strip, will be made the first time. The difference which is found at the end of the line is proportioned through the whole distance by picking up and relaying the templets to a smaller or larger air base (the distance between two consecutive exposure stations). Personal judgment is of great importance, since ordinarily, the first problem which presents itself is how closely to hold to the azimuth lines when triangles of error are found in the intersection of the radials.

When one strip is apparently satisfactorily adjusted, the next strip is laid, when additional adjustments must usually again be made in order to bring points common to both strips into coincidence. From this trial and error procedure, however, an average result is finally arrived at; and the time that it will take depends very largely upon the skill of the operator. As the number of templets increases, the slight errors accumulate and the proportioning out of these becomes increasingly difficult, and beyond a certain point, impossible.

After the assembly has been completed, if a transparent base has been used, the simplest method of transferring the intersections of the radials to the base is to turn the whole assembly over and mark the positions by small black circles on the back of the base. The use of a transparent base is quite advantageous also in the succeeding phase of map compilation, but cellulose acetate is very unsatisfactory from the basis of accuracy, since it is practically impossible, under ordinary working conditions, to prevent a considerable amount of distortion. As a result, it is usual to take off only a small portion of the data at one time, evening up any distortion within each grid rectangle.

The use of any rigid material is, therefore, from this standpoint alone a very distinct advance.

The Slotted-Templet Method

The slotted-templet method represents a simple mechanical solution of a hitherto difficult problem in hand templet practice.

The radial line on the templet is replaced by a radial slot, in which a close-fitting round stud is inserted, free to slide along the longitudinal axis of the slot. Since the slot takes the place of the line, the templet material need not be transparent, and therefore can be made of stiff material not subject to distortion to the same degree as the flexible cellulose sheets used for hand templets. The rigidity of templet material is one of the vital factors in the method as it permits the laying of all of the templets having cuts to the same common point, over the stud representing that point, and since the stud and the templets are of rigid material, motion is possible only along the longitudinal axes of the slots. Thus the templets will tend to adjust themselves into theoretically perfect position.

Preparation of the photographs for making slotted templets is practically the same as for hand templets. It consists of the selection of the center point (a definite image so nearly coinciding with the principal point as to introduce no appreciable error through its use), the selection of the pass points of the density and location required by the particular job at hand, and the addition of all picture ties the geographic positions of which are known. Excluding such picture ties, a minimum of eight slots and a center hole will appear on each templet, two representing the radials to the preceding and succeeding photographs' centers, and six representing the radials (three on each side) to pass points in the overlap area between flights (Fig. 5).

For small areas and limited production, a portable hand slotter ordinarily is

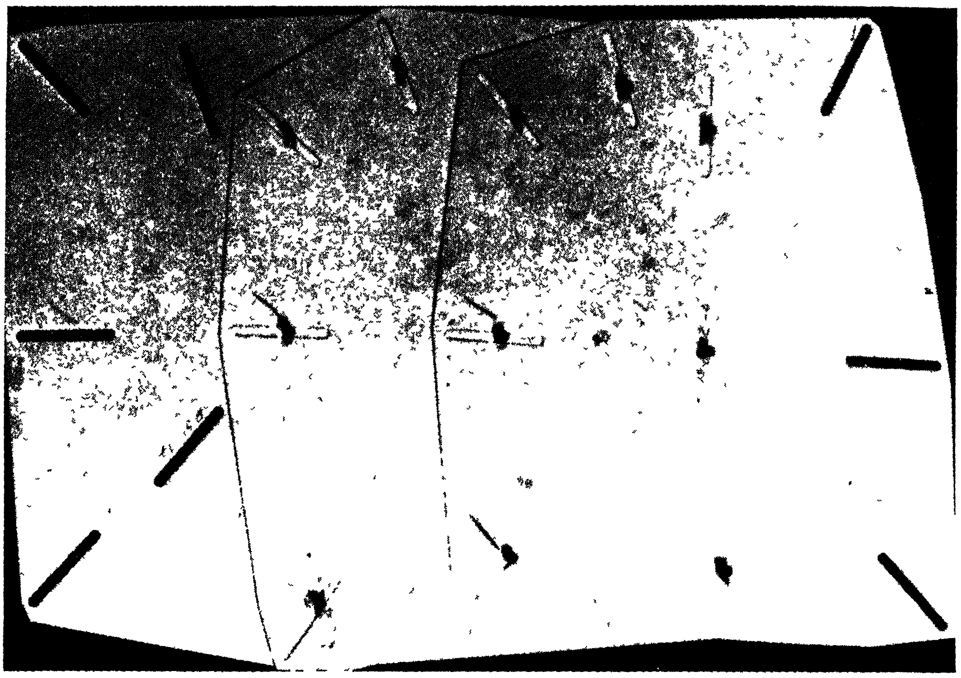


FIG. 5 Slotted templets. The shafts of the studs are pierced, as shown, so that the position of the center of the stud can accurately be recovered.

used. First, photographs are taped, face upward, to the templet material. Four-ply Bristol board is very satisfactory. This is commercially available in sheet sizes up to 22″×30″. With a fine needle or pricker point, the pass points on the pictures are then punched through the photograph onto the templet. The photograph is then removed, the points circled with pencil, and the number of the photograph marked alongside the principal point, with its correct orientation to correspond with the line of flight. If the map is to be laid at approximately the same scale as the photographs, this is all that is necessary; but if the scale is to be materially changed, radials from the center are drawn with fine lines through the points which have been established, and on these lines are set off approximate enlargement or reduction of distances to take care of the desired change.

The center of the templet is then punched out with a small hand punch (Fig. 6) which makes a hole the same size as the diameter of the studs used in assembling the templets. For highest quality, a center punch with a retractable center point should be used. This allows the cutting of a much more accurate center. The templet is then placed over a metal stud of the same diameter, sliding in a groove on the hand cutter, and the templet rotated on this pin and pushed in or out until the center of the die approximates the position of a marked picture point The lever is pressed down (Fig 7) and a slot, whose longitudinal axis should correspond to the radial line from the principal point through the picture point, is punched in the cardboard templet.

There has been developed (Fig. 8) and put in use in the Soil Conservation Service, a templet cutter which materially decreases the cost of the work. With this cutter, the photograph is placed upon a turntable; the templet of cardboard

FIG. 6. Punching center hole

The length of the slot ($1\frac{1}{2}$ to $2\frac{1}{2}$ inches has been used) is sufficient to take care of slight scale differences in the individual photographs as well as relief.

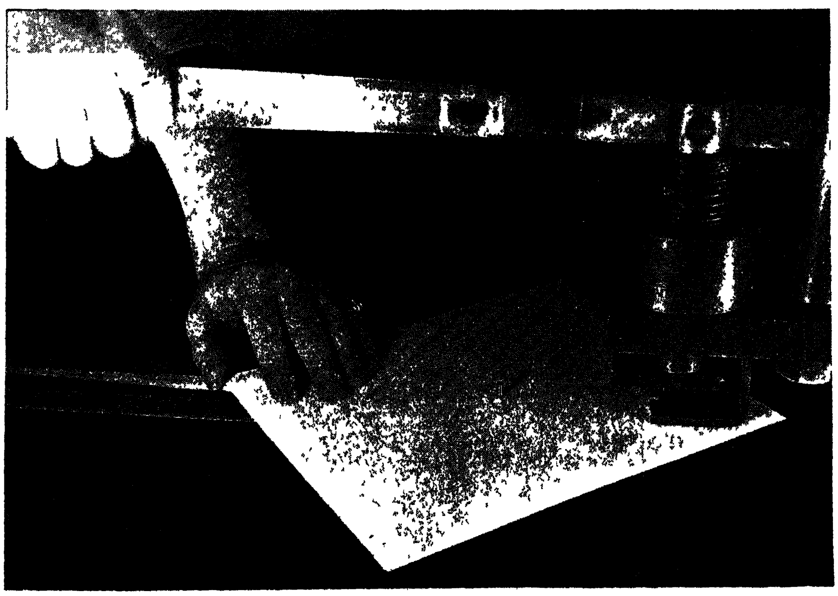


FIG. 7. Cutting templets with hand cutter.

or other material is inserted below the table top; and a small lever, which also cuts the center hole at the same time, clamps the templet in position, so that it rotates with the photograph when the turntable is swung. A small pointer moves in and out from the photograph center, through the use of a small handwheel located on the front of the machine. When the turntable is so swung as to bring a pass point on the photograph into position along the axis of motion of the pointer, and the pointer so placed that it is directly over the pass point, a foot lever is depressed, cutting a slot in the templet.

FIG. 8. Improved templet cutter. This machine can be set automatically to change scale between photographs and templets.

This machine also is so arranged that it can be set for variations in scale, so that a templet from one-half-diameter reduction to two diameters enlargement can be made by setting the ratio scale on the machine. The machine combines the various operations described in the paragraphs on the hand cutter, and not only effects a reduction of approximately 40 percent in the time required, but cuts more accurate templets.

As a map base or grid upon which to assemble slotted templets, aluminum sheets have been found to be very satisfactory. Sheets 5 by 10 feet, which are about as large as can be conveniently handled, are abutted and taped together

with transparent tape, and the outer edges of the sheet junctions soldered together. This has been found to work very satisfactorily The largest assembly of slotted templets that has come under the writer's experience is the one to which reference is made later in this description, an area concisting of approximately 4,400 square miles, assembled on a scale of 1:15,840, and requiring a base approximately 25 by 35 feet. (Fig. 9.)

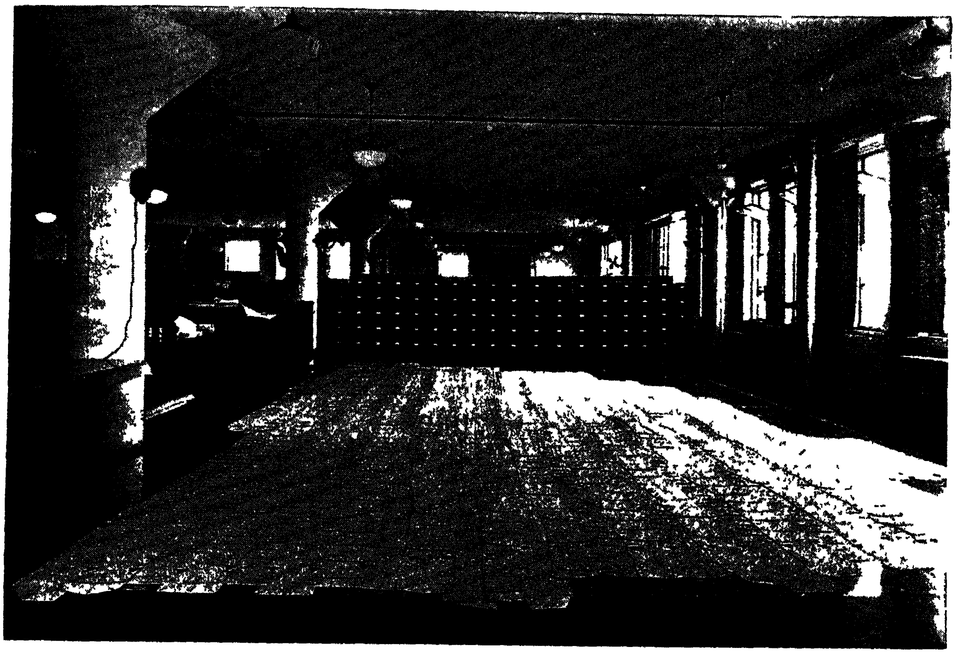


FIG. 9. An area of 4,400 square miles, covered by 2,700 photographs This was laid (for test purposes) with ground control only around the perimeter of the area. Control is indicated by black triangles.

The grid is etched on the aluminum as a very fine line, after which the control positions are carefully plotted, and a metal templet stud cemented to the aluminum sheet at each control point. This is a very simple procedure. The bottoms of the studs are coated with an aluminum cement; the studs placed over a fine pin with the same diameter as the hole through the shaft of the studs; the point of the pin centered over the plotted position (Fig. 10); and the stud then carefully dropped on the aluminum. The operation avoids the necessity of nailing through the base, a decided advantage, as it was found that, even with care, the nails or pins so driven tend to work away from correct position.

As a simple check on the accuracy of placement of the studs, when they are scraped off the aluminum sheet after the assembly is completed, the bottom of the stud will be found to bear a distinct impression of the cross that was etched on the aluminum base to locate the plotted position of the point, and these lines should clearly quartersect the small hole in the stud.

The cement may easily be cleaned from the stud by immersing in an acetone solution for a few minutes.

After making the templets and etching the grid base, the next step is the assembly of the radial triangulation net. It is not necessary to have three known

control positions, as with the hand-templet method The assembly can be started from any single picture tie and with any flight, although it is preferable to pick the flight strip with the most ground control points. Ordinarily, the flight strip will be laid in its entirety, just as is done with transparent templets However, when the next control is reached, if a slight scale adjustment is necessary, this is easily accomplished by squeezing together the whole assembly or stretching it out slightly along the line of flight, as may be required

As stated before, to be theoretically perfect, the slotted-templet system requires perfect fit between the slots and the studs and a minimum of friction both between studs and templets and between the templets, as well as a templet material sufficiently rigid to resist any distortion under the forces applied.

It will be noted in the photographs of the templet assemblies that the templets are not square or rectangular. This is due to the fact that the edges are trimmed off to avoid interference on overlapping flights Waxing the cardboard templets will materially decrease friction, and increase the accuracy of adjustment; as well as decrease amount of possible moisture absorption.

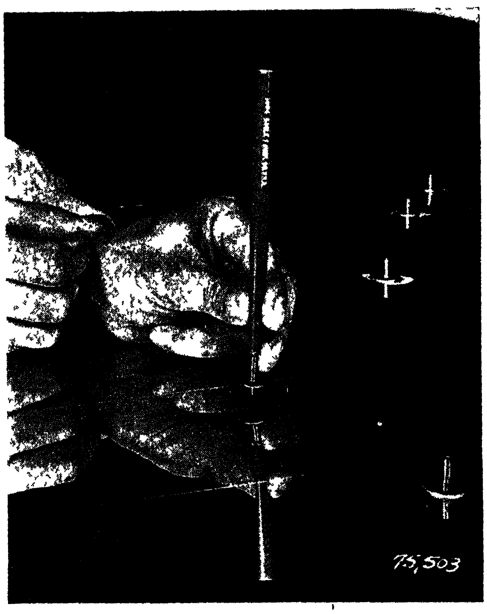


FIG. 10. Placing the control studs in position. This also illustrates size and shape of studs.

If no difficulty is encountered in making the assembly, then the procedure under which each adjacent flight strip is laid in turn, is usually the easiest However, alternate flights may be laid (Fig 11), or the templets may even be built up across the lines of flight. Where difficulty in laying a certain portion of the area is encountered, the work can be carried around this portion, and eventually the templet or templets causing the trouble will be segregated.

Laying the assembly at approximately the same scale as the photographs means that pass points on 9″×9″ photographs are never over 5 inches away from the center. Consequently in average terrain a small amount of tilt will cause no appreciable error in the assembly, since the angular amount that it will displace a picture point at that distance will not ordinarily be beyond the usual tolerance limit of good drafting. A templet from a photograph with a large amount of tilt cannot be laid, unless the attainable accuracy is reduced through the use of slots with too great a tolerance between the diameter of the studs and the width of the slots or unless the material used for templets is of such weak consistency that the studs will indent the side of the slot at the least binding. When a templet will not fit into the assembly, it should be checked immediately for possible construction error. It is easily possible to omit one templet and proceed with the assembly, but where two or more successive templets will not fall into the general assembly without binding or distortion, an analyis should be made of the photographs which these templets represent, for camera tilt. From the analysis, the position of the isocenter is obtained and the use of this instead

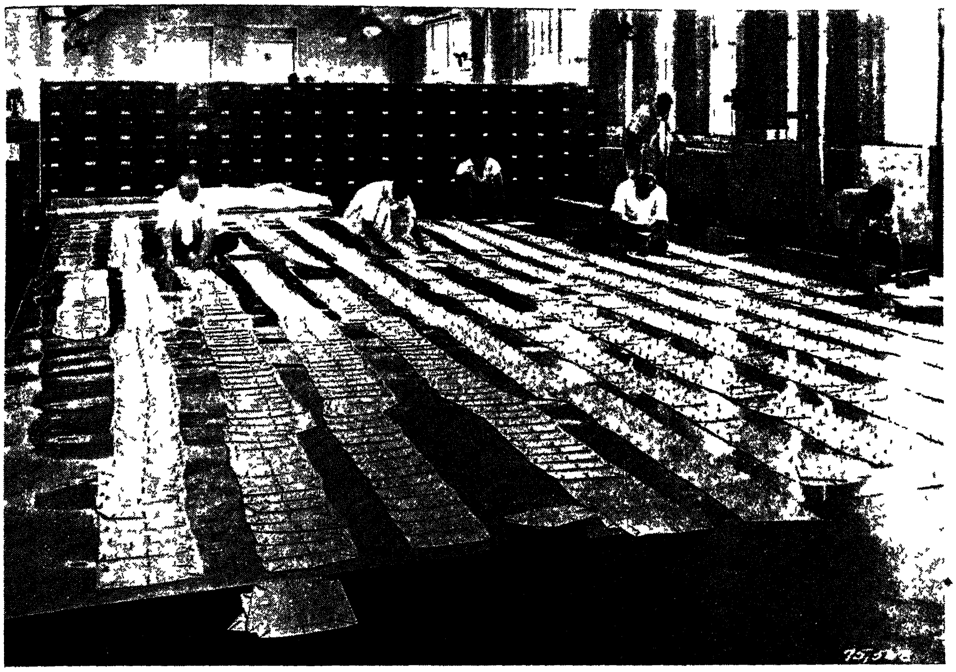


FIG. 11. Laying templets in alternate flights

of the principal point will result in an angular correction to the directions of the slots, which will allow the templet to fall into place.

In practice, the small errors which result in small triangles of error on hand templets and cannot be worked out but must be averaged in, are taken up in the differential between practical accuracy and theoretical perfection.

SLOTTED-TEMPLET TESTS

As an initial proving ground for tests of the slotted-templet system's accuracy, an area was selected which had been covered by aerial photographs of average quality and where an accurate and dense system of control points had been established. This was found at Beltsville, Md., where, in a roughly rectangular area of 155 square miles of rolling country, with a maximum elevation difference of approximately 200 feet, 273 ground control points were available. These points were established for test purposes under the direction of the U. S. Geological Survey with position error not exceeding 1 in 7,500. Two hundred and thirty-three photographs, size 9″×9″, on a scale of 1:12,000, cover the area in 12 flights, averaging 19 photographs each with approximately 60 percent overlap in line of flights and 30 per cent side overlap.

Low-shrink paper was used for the photographic prints. The 273 picture points, which had been located on a set of field prints, were transferred, and the azimuth and radial control points identified and added.

A polyconic projection with 1-minute grid, scale 1:12,000, was constructed upon two sheets of 26-gage aluminum, size 3′×8′ each, abutted, taped, and riveted together with aluminum strips to form a single sheet. The projection was tested for accuracy before and after completion of the work. Upon this projection, the 273 control points established were carefully plotted.

A sheet of opaque paper was then placed over the entire projection, leaving visible only the control points actually held to in each individual test Thus, in the first test in which four control points, one in each corner of the area, were held to, these were the only control points visible, so as to avoid influencing, in any way, the placing of the templets

Four-ply Bristol board was used for templets. More than ordinary care was taken in their handling to avoid damage to the sides of the slots; and the same set of templets was used throughout. At the end of the test, they were still in good condition.

Tests were made in the order as numbered. Diagrammatic sketches, showing the result of each assembly, are included, giving for each point tabulated, the approximate azimuth and distance from the true geographical position A small percentage of points where the intersection of the radials was very flat or where points fell too near the centers of the photographs to be of any use were not recorded

Flights were numbered from west to east beginning at "1."

Discussion of Assembly No. 1

Four control points only, one located in each corner of the area, were used for this assembly (Fig. 12) This gave control at the ends of the first flight with 21 photographs between, and control at the ends of the 12th flight with 19 photographs between.[1]

The distance between control points was approximately 12 miles along the line of flights, and 13½ miles across flights

This test alone demonstrates the value of the slotted-templet method. The assembly was loose and could be moved to an appreciable extent in any direction, except near the control points Nevertheless, an assembly was accomplished with this extremely scant control in which the average error of absolute position was less than 50 feet, which on this scale is one-twentieth of an inch. The relative error in distance between adjacent control-point positions as plotted by this templet assembly averaged 25 feet, or only half as much.

Discussion of Assembly No. 2

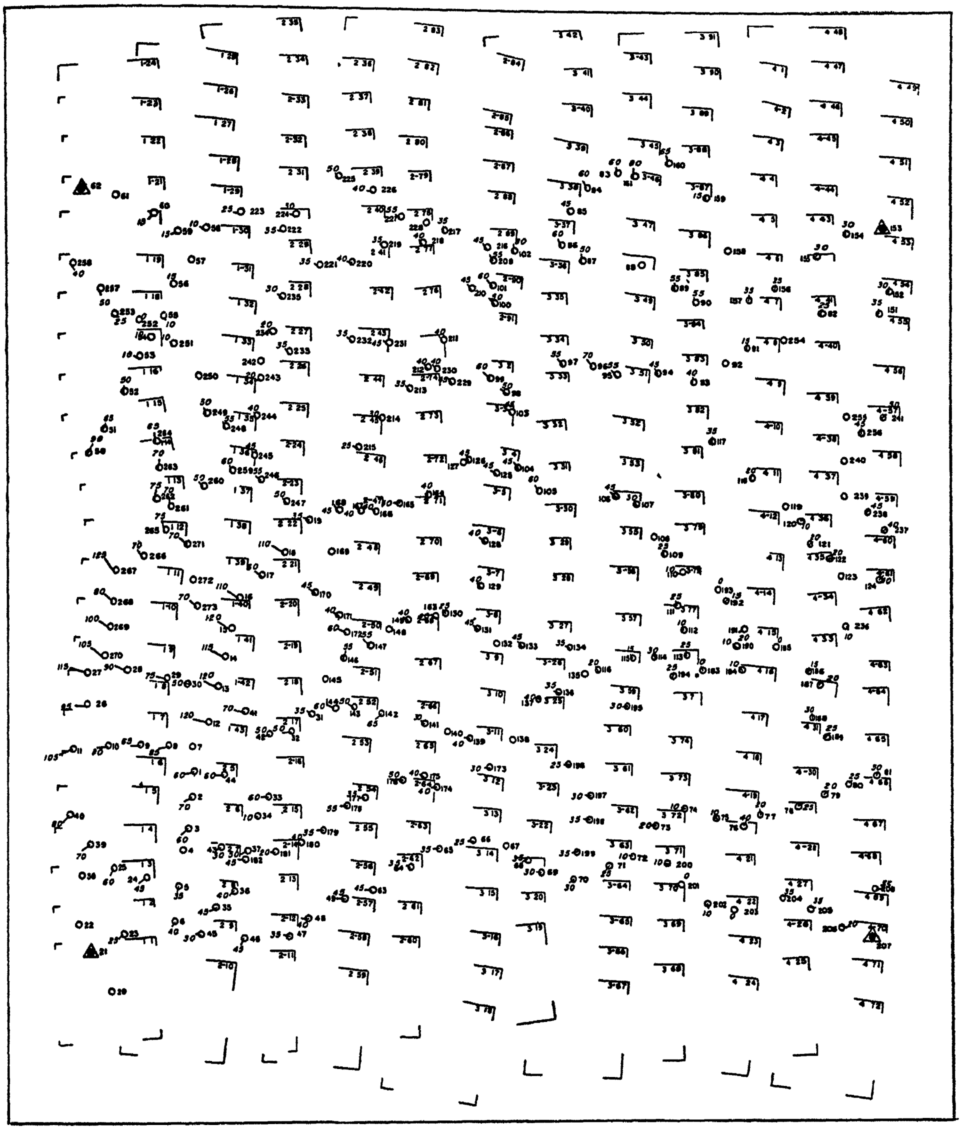
An additional point to the four located in the corners was added in the middle of the area (Fig. 13), thus breaking the square down into four triangles approximately 10 miles on each side, with control on each corner.

The additional point strengthened the assembly only slightly. The assembly could still be moved at the outer edges between control, but the errors were reduced appreciably in the vicinity of the additional control point. The general error of assembly, which had been without definite pattern in assembly No. 1, swung from a northwesterly direction to a more southerly one.

Discussion of Assembly No. 3

Eight control points were held (Fig. 14) This gave control on flights 1, 6, 7 and 12 with additional control points on flights 1 and 12 at points midway on these flights. As located, this reduced the number of photographs between control points to a minimum of 10 and a maximum of 16.

[1] In counting the number of photographs between control on this and subsequent tests, the rule was established to include only the number of photographs between such control positions sufficient that an intersection of two radials would be made on each control point.



Legend

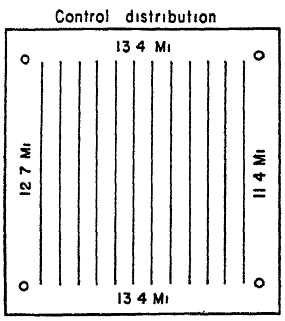
Scale of assembly - 1 inch = 1000 feet

4-30 - picture number and location

O207 picture point position by traverse

▲ - picture point controlling assembly

O-50 - azimuth and distance from geographic position to the position obtained by radial triangulation

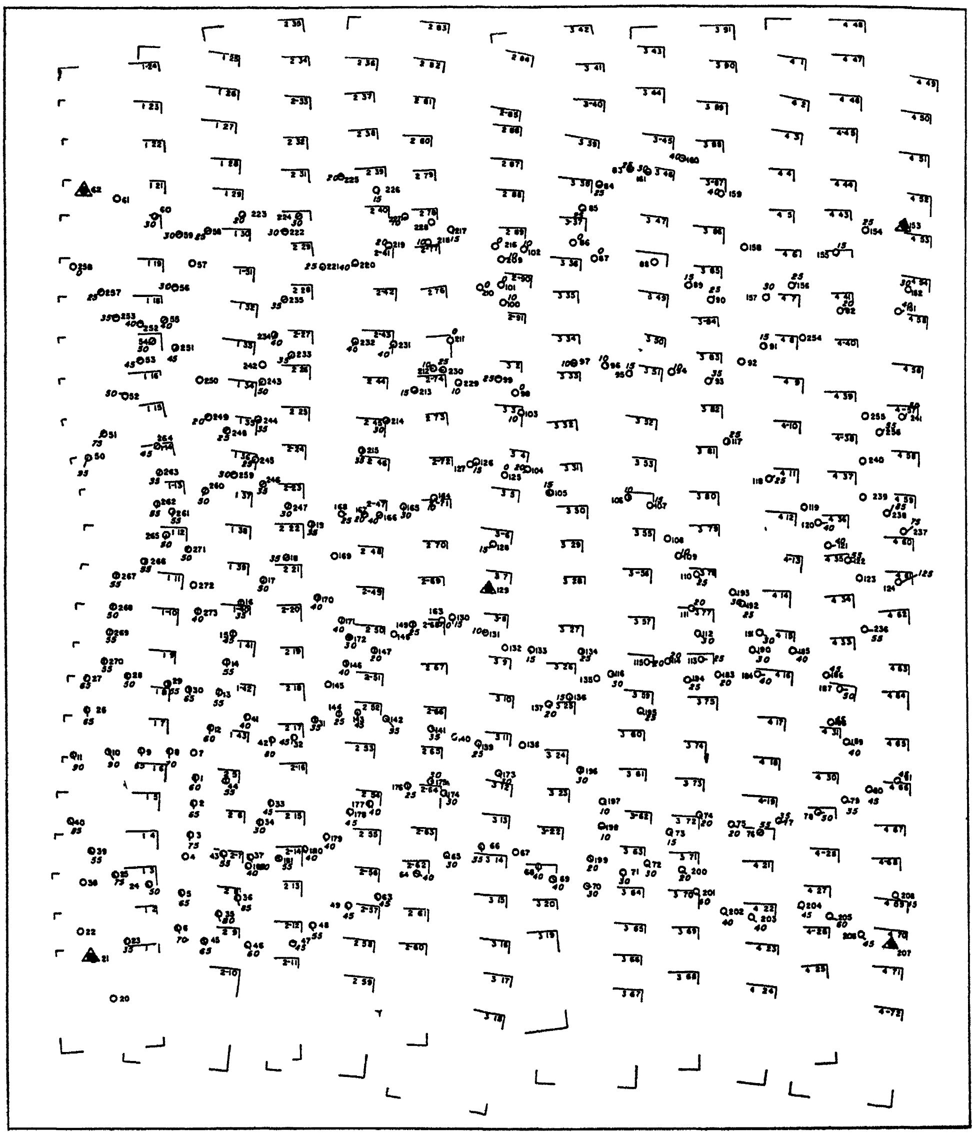


Breakdown of points into error classifications

Error	No of points	Percent	Cum. percent
0	5	2	
10	18	8	10
20	21	9	19
30	36	15	34
40	53	22	56
50	40	17	73
Over 50	64	27	100

Average error of all positions recovered is 43 feet, maximum error is 125 feet

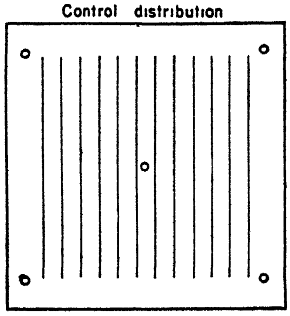
FIG. 12. Results of radial triangulation by the slotted-templet method, assembly No. 1, Beltsville, Maryland, Demonstration Area.



Legend

Scale of assembly—1 inch = 1000 feet
4-30'-picture number and location
○207-picture point position by traverse
▲-picture point controlling assembly
○50-azimuth and distance from geographic position to the position obtained by radial triangulation

Control distribution

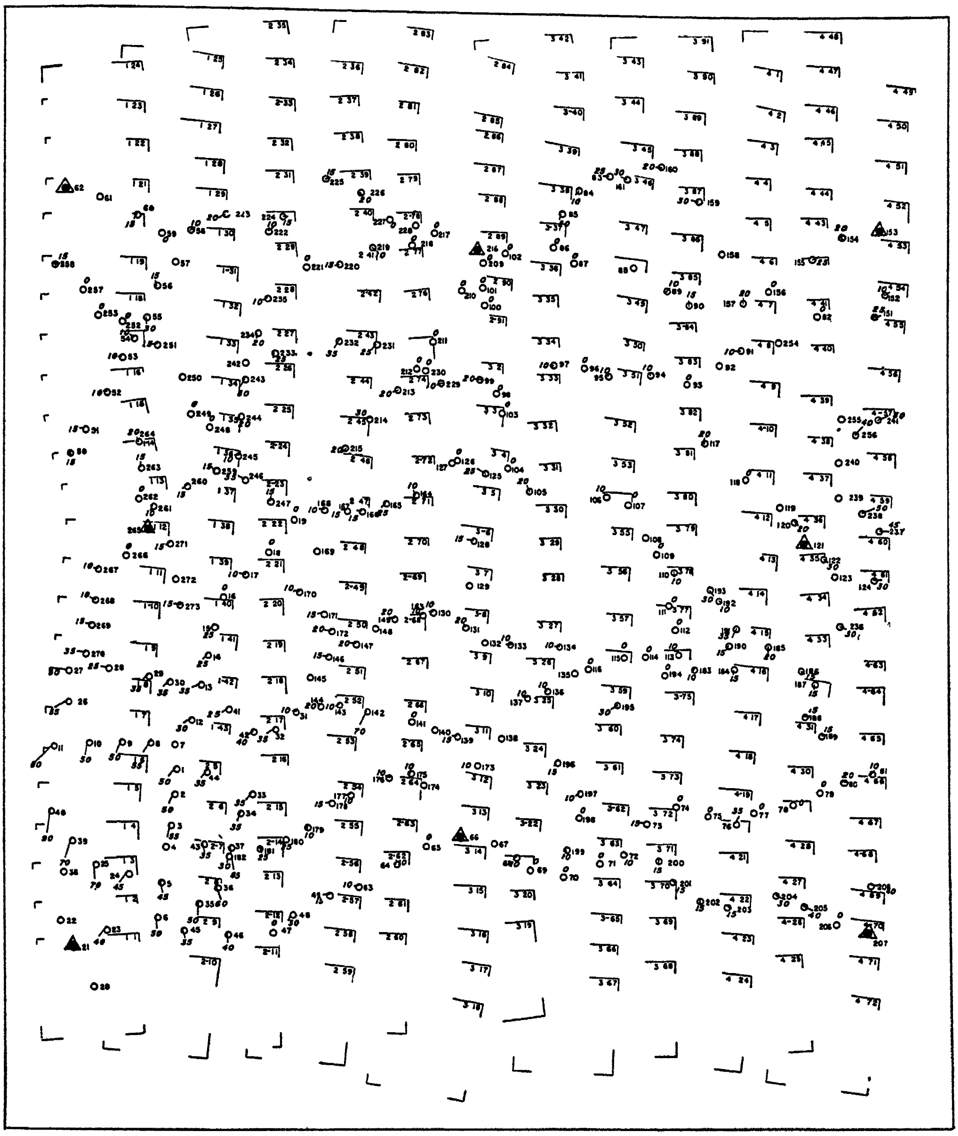


Breakdown of points into error classifications

Error	No of points	Percent	Cum percent
0	9	4	
10	18	8	12
20	32	14	26
30	53	22	48
40	47	20	68
50	31	13	81
Over 50	46	19	100

Average error of all positions recovered is 36 feet; maximum error is 125 feet

FIG. 13. Results of radial triangulation by the slotted-templet method, assembly No. 2, Beltsville, Maryland, Demonstration Area.



Legend

Scale of assembly–1 inch = 1000 feet

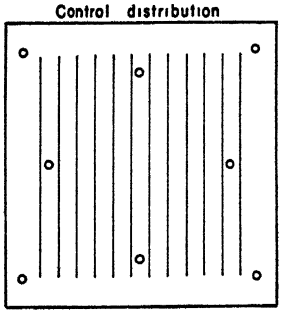
4-30–picture number and location

○207–picture point position by traverse

▲–picture point controlling assembly

⊖50–azimuth and distance from geographic position to the position obtained by radial triangulation

Control distribution



Breakdown of points into error classifications

Error	No of points	Percent	Cum percent
0	55	24	
10	49	21	45
20	59	25	70
30	27	12	82
40	20	9	91
50	12	5	96
Over 50	10	4	100

Average error of all positions recovered is 18 feet, maximum error is 90 feet

FIG. 14. Results of radial triangulation by the slotted-templet method, assembly No. 3, Beltsville, Maryland, Demonstration Area.

This amount of control was sufficient to eliminate any movement in the assembled templets.

Discussion of Assembly No 4

Thirty-one picture points (Fig. 15), one in each corner of the area, one in the common overlap of each flight at each end, two intermediate along flight 1, and three along flight 12, were held.

This test approximates the result that might be obtained by breaking down the original triangulation control into 15-minute quadrangles, and traversing across the ends of the flights from one triangulation station to another Since with such traverses it is necessary to cover all of the flights anyway, it is customary to secure a picture point in the overlap between each flight while making such traverses. Furthermore, it is always desirable to have one or two points along the outer flights to prevent bowing in or out of the templets

While the maximum error was reduced in this test to a considerable extent over test No. 3, the average error was reduced but very slightly

This test indicates clearly the point beyond which the cost of obtaining additional control should be closely balanced against the required accuracy of result.

Discussion of Assembly No 5

It has been found in practice that with the slotted-templet method of control, the accuracy of the system breaks down very rapidly outside of control points. However, conditions might arise in which it would be necessary to extend the mapping to a considerable extent beyond control For this reason, a test was made with control normal to the lines of flight at one end of the desired area, only.

Test No. 5 (Fig. 16) extended the templets for 12 miles from such a base, using control points in the overlap area of each successive flight The maximum error on this assembly was, of course, on the outer fringe of the photographs and amounted to 380 feet. It is probable that this error would have been materially reduced if one or more azimuth lines had been secured from the control base.

Discussion of Assembly No. 6

Assembly No. 5 was repeated except that only seven templets were laid from the control base. The general pattern of error remained the same but the error was greater at the outer edges than at the same distance from the control base in assembly No. 5.

Discussion of Assembly No. 7

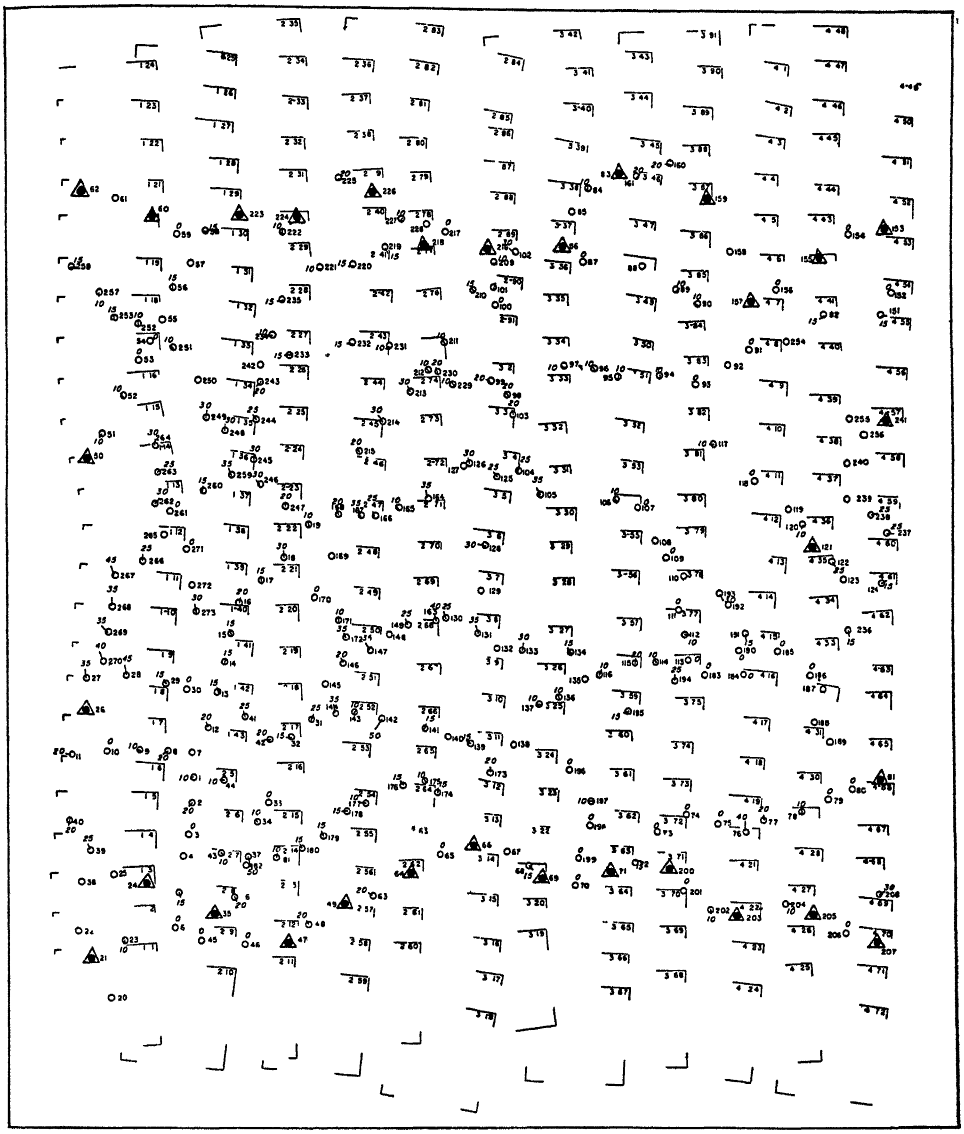
This assembly (Fig. 17) was made with control along an outside line of flight, only. The maximum error was nearly four times as great as on test No. 5.

Comparative Tests Between Hand-Templet and Slotted-Templet Methods

In an attempt to compare the two systems, hand templets were made from the same photographs used for the slotted-templet tests at Beltsville, Md

As test No. 4 had established the point at which very good accuracy could be obtained by the slotted-templet method, the amount of control used in that test was established for this hand-templet test. The original aluminum base sheet was used, and all except the bands of control on the North and South and points on the edges were covered with a sheet of opaque paper.

It became apparent, as soon as the assembly was started, that the number



Legend

Scale of assembly – 1 inch = 1000 feet

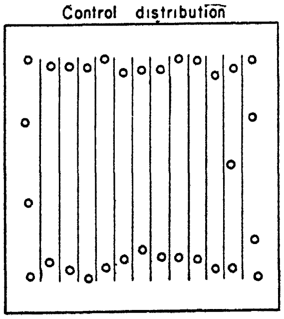
4-30 – picture number and location

○207 – picture point position by traverse

▲ – picture point controlling assembly

⊖50 – azimuth and distance from geographic position to the position obtained by radial triangulation

Control distribution

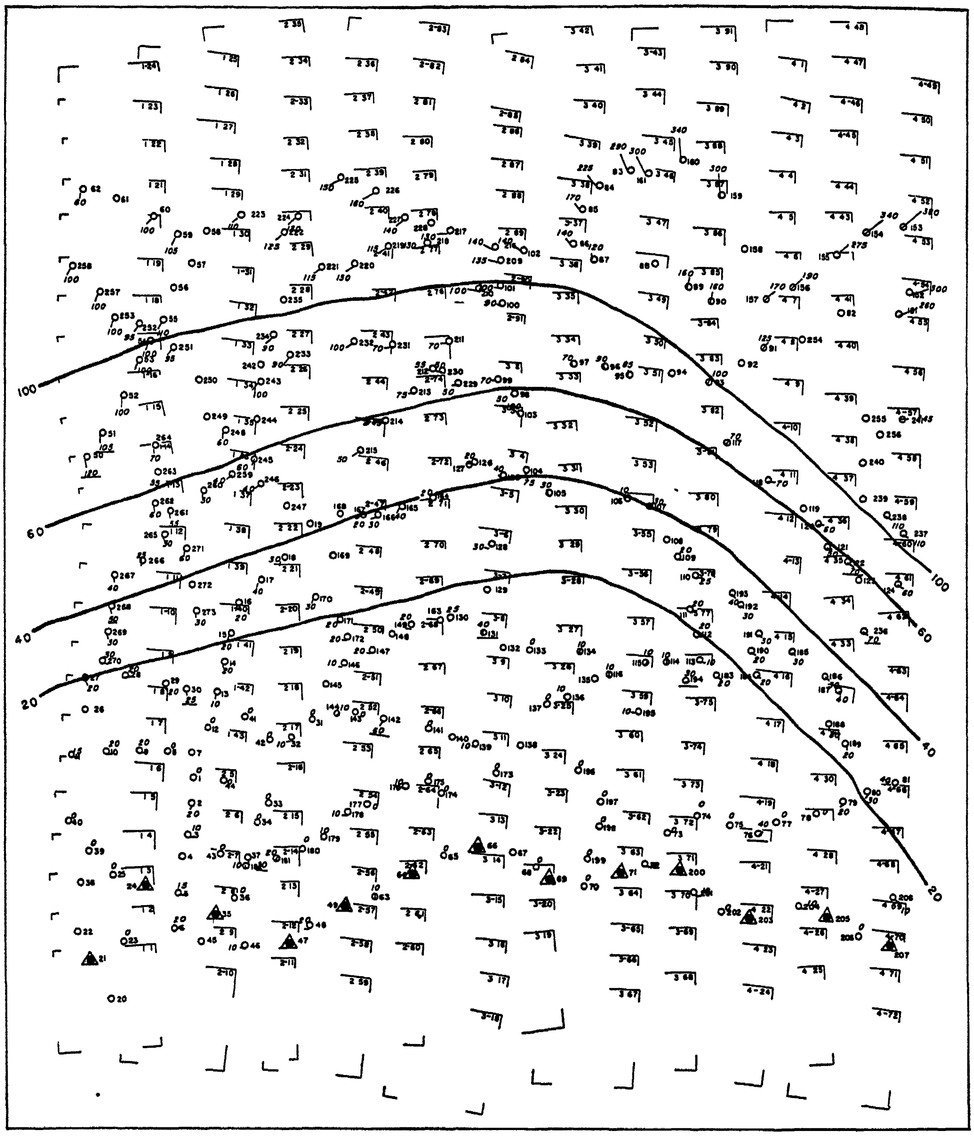


Breakdown of points into error classifications

Error	No. of points	Percent	Cum percent
0	50	24	
10	44	21	45
20	62	30	75
30	32	16	91
40	14	7	98
50	4	2	100
Over 50	0		

Average error of all positions recovered is 15 feet; maximum error is 50 feet

FIG. 15. Results of radial triangulation by the slotted-templet method, assembly No. 4, Beltsville, Maryland, Demonstration Area.



Legend

Scale of assembly–1 inch = 1000 feet

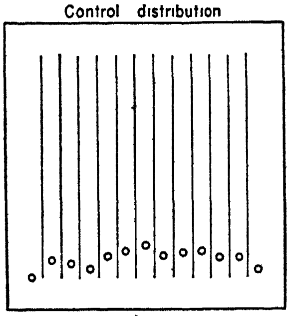
4-30—picture number and location

○207—picture point position by traverse

▲—picture point controlling assembly

⊖50—azimuth and distance from geographic position to the position obtained by radial, triangulation

Control distribution

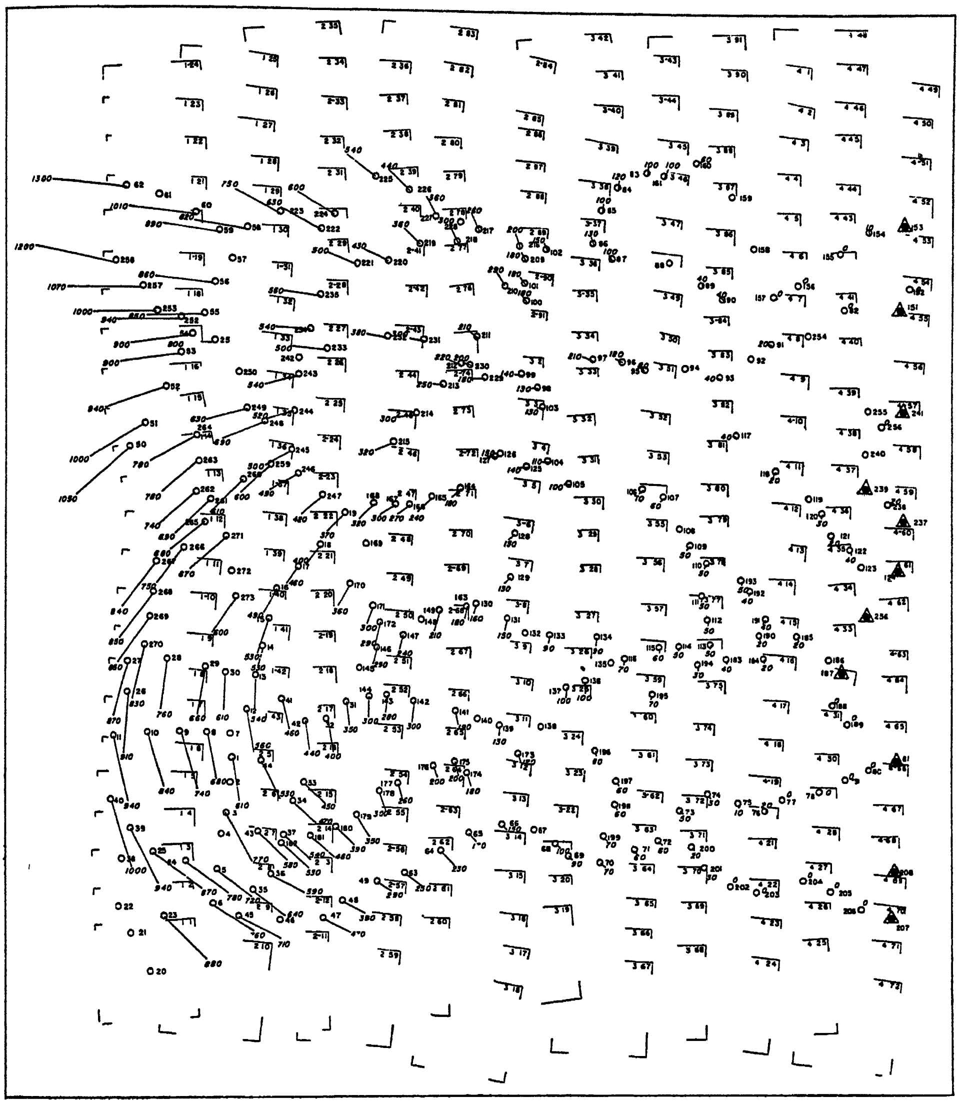


Breakdown of points into error classifications

Error	No of points	Percent	Cum percent
0	38	18	
10	23	11	39
20	30	14	43
30	22	10	53
40	11	5	58
50	7	3	61
Over 50	82	39	100

Average error of all positions recovered is 60 feet, maximum error is 380 feet

FIG. 16. Results of radial triangulation by the slotted-templet method, assembly No. 5, Beltsville, Maryland, Demonstration Area.



Legend

Scale of assembly- 1 inch = 1000 feet

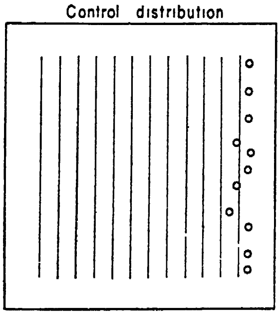
4-301-picture number and location

O207-picture point position by traverse

▲- picture point controlling assembly

O-50-azimuth and distance from geographic position to the position obtained by radial triangulation

Control distribution



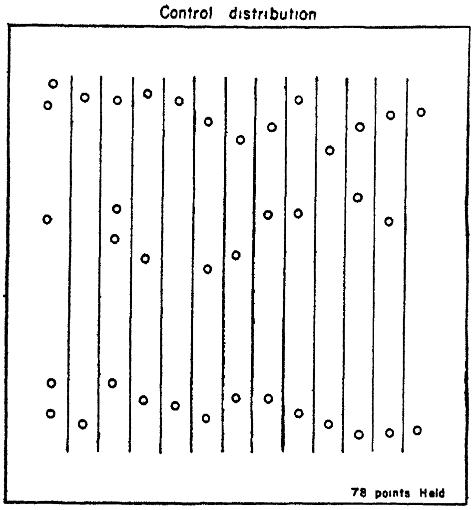
Breakdown of points into error classifications

Error	No of points	Percent	Cum. percent
0	18	8	
10	2	1	9
20	9	4	13
30	4	2	15
40	8	3½	18½
50	8	3½	22
Over 50	178	78	100

Average error of all positions recovered is 350 feet, maximum error is 1300 feet

FIG. 17. Results of radial triangulation by the slotted-templet method, assembly No. 7, Beltsville, Maryland, Demonstration Area.

Control distribution



Breakdown of points into error classifications

Error	No of points	Percent	Cum percent
0			
10	66	40	40
20	42	25	65
30	40	24	89
40	11	7	96
50	7	4	100
Over 50	1		

Average error of all positions recovered is 21 feet, maximum error is 65 feet

FIG 18. Results of radial triangulation by the hand-templet method test, for comparison with slotted-templet assembly, Beltsville, Maryland, demonstration area.

of radials was insufficient for the hand-templet method; and approximately 20 per cent more points were selected on the photographs and cut on the templets.

A diligent effort was then made to lay the templets, but without success; and eventually, eight more control points were added, stretching across the center of the flights. This was in line with previous experience which indicated that under average conditions the maximum gap that could be bridged successfully with hand templets usually would not exceed 10 photographs It must, of course, be understood that this refers to areas and not to a single strip of photographs. The final assembly, with this additional control, was within the usual specifications for templet assemblies and may be classified as good.

The result was not as accurate as that obtained from assembly No. 4, using slotted templets. The summation of percentages of error is shown in Figure 18 so that a direct comparison can be made.

The results of the hand-templet assembly test clearly indicate the limits to which this method may be extended. If the available control is more widely spaced than that used in this test, it usually must be supplemented by additional ground control.

Additional Slotted-Templet Tests

The Beltsville test gave no indication that the limit to which slotted-templet control could be extended, had been reached. The assembly, even with the minimum control used for test No. 1, was easily made. A considerably larger area, therefore, was selected for the next test. This was Carter and Murray counties, Oklahoma, covering an area of 1,200 square miles of comparatively flat country. The area covered is shaped somewhat like a T-square, with an average of 20 photographs between control in the western part of the area and 40 photographs between control in the eastern part. The control held to was spaced around the perimeter of the area. Fifty-five check points gave an average error of 55 feet and a maximum error of 130 feet.

Considerable difficulty was experienced in moving the mass of templets into adjustment, indicating that without further development this was probably the limit to which this method of obtaining control could be extended. However, distances between control stations, as used on this test, are not the maximum

required to bridge the spacing between established triangulation nets in all parts of this country To meet these two conditions, the simple expedient of waxing the templets in order to reduce friction was resorted to in the next larger test.

This test was on an area of 4,400 square miles included in the Bird and Caney watersheds in Kansas and Oklahoma (figs. 9 and 19). Two thousand seven hundred templets on a scale of 1:15,840, made from negatives taken on a scale of

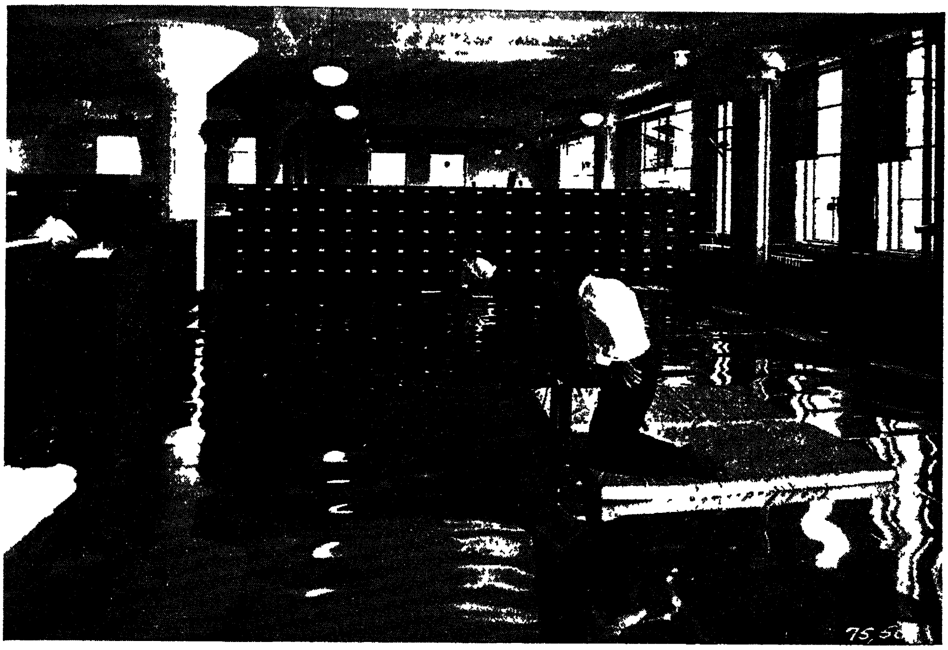


FIG 19. Laying out projection for the 4,400 square mile area in Bird and Caney watersheds in Kansas and Oklahoma referred to in the tests The sponge-rubber pads shown are an important aid in assembling large areas. They can be laid over the studs without disturbing the assembly and can be walked on without discomfort

1:20,000, covered the area. As with the previous test, control was held to only around the edge of the area, thus giving the maximum distance possible between control. The distance along the line of flight averaged 93 miles with 108 templets. Across flights, the average distance was 51 miles, covered by 25 flights Eighty-seven control points were held to as shown in, Figure 9. Table 1 gives the number of check points and percentage of error. The total number of points checked for error was 192 The maximum error was 140 feet and the average error, 45 feet

No difficulties were experienced in making the assembly, and the average and maximum errors, while maintaining the same relative ratio as indicated by previous tests, were considerably less than was anticipated. Waxing the templets made a shifting of the mass of templets possible, so that they could be pulled into adjustment with other flights. In addition, much more correct general adjustment was obtained.

TABLE 1 CHECK POINTS AND PERCENTAGE OF ERROR FOR TEST ON AREA OF 4,400 SQUARE MILES IN BIRD AND CANEY WATERSHEDS IN KANSAS AND OKLAHOMA

Error (feet)	Number of points	Percentage	Cumulative percentage	Error (feet)	Number of points	Percentage	Cumulative percentage
0	11	6	—	70	15	8	83
10	14	7	13	80	17	9	92
20	21	11	24	90	6	3	95
30	23	12	36	100	1	—	95
40	34	18	54	110	7	4	99
50	22	11	65	120	1	—	99
60	19	10	75	140	1	1	100

Test Deductions

In general map work, using 7″×9″ or 9″×9″ single-lens photographs, and with a control situation approximating the conditions of the Beltsville, Md., and the additional tests discussed, that is, bands of reasonably closely spaced control normal to the lines of flight at definite intervals and with a number of control points along the sides of the assembly sufficient to hold the azimuth of the outside flights, we may expect the following

1. The errors of position as a percentage of maximum error will approximate the graph shown in figure 20.

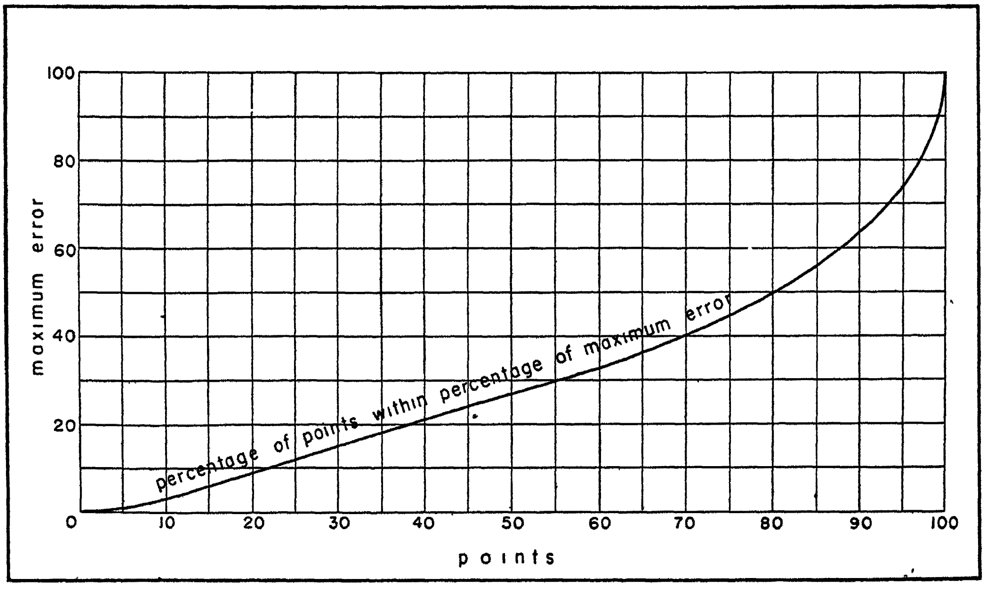


FIG. 20. Error distribution indicated by test templet assemblies.

2. For maps made from photographs taken at a scale of 1:20,000 and compiled at a scale up to 1:15,840, the average error in feet should not exceed the

average number of photographs between control, and (as shown in fig. 20) the maximum error should not exceed three times the average error.[2]

CONCLUSION

The Beltsville, Md , and additional tests reported, it is believed, allow a fair evaluation of the worth of the slotted-templet method.

They demonstrate for example, that the limitations of the system would in no way prevent its use in making a planimetric map of fair accuracy of any portion of the United States, using only the existing ground control, since the spacing of the control used for the Bird and Caney test exceeds the maximum spacing of the triangulation nets already established.

Furthermore, they give an idea of the degree of accuracy that could be expected with such a small amount of control, and indicate the increase of accuracy obtainable by adding additional control, so that, considering the purpose of the desired map, the cost of the additional control can be correctly weighed against the resulting increase in accuracy

Finally, these tests indicate equally clearly the point beyond which additional control—using this method—adds little to the accuracy of the finished map.

[2] The proportional errors in the Bird and Caney test are materially less than this

THE GEOLOGICAL SURVEY RADIAL INTERSECTOR

Application in the Alaskan Branch

John I. Davidson

PHOTOGRAMMETRISTS need no introduction to radial triangulation. The theory, as pointed out by Major Talley[1] has been known at least since the time of Laussedat. Colonel Bagley may well be credited with much of its modern development in the United States The purpose of this paper is not to dwell upon a theory which is so adequately explained by Colonel Bagley[2] but to describe a mechanical method for accomplishing the actual work.

The development of templets by means of which radials could be transferred from aerial photographs to work bases, allowing the user to make necessary adjustments to fit control, was an inevitable development. From hand templets, in which adjustments were painstakingly made upon one templet at a time, to the slotted templet is a direct step but one that was awaited for many years.

The usefulness and accuracy of slotted templets is admirably explained in many publications.[3] With the knowledge of results obtainable with slotted templets and a background of years of field work. Mr. S P Floore, of the Topographic Branch of the Geological Survey, in 1938, started to develop a method of compiling a radial triangulation net in which the underlying work was not covered and in which bulky or expensive templet cutters were not needed An additional incentive was the thought that perhaps greater accuracy could be obtained by the use of metal in place of some of the softer materials used previously.

In 1939, the Alaskan Branch of the Geological Survey, looking for a templet which would make possible the rapid compilation, at a minimum cost, of small-scale reconnaissance maps from photographs taken with the T2A four-lens camera adopted Floore's method

The use of the equipment in the Alaskan Branch varies greatly from that in the Topographic Branch, as more control is available for areas within the United States, and the maps are compiled on a larger scale. The procedure followed by the Topographic Branch is covered by J. L. Buckmaster in another section of this paper.

Although small areas in Alaska have been mapped on scales of 1:62,500 or larger, most of the mapping has been done on reconnaissance scales of 1:250,000 or smaller. No instrumental triangulation has been carried out except in the southeastern portion, in a narrow belt along the International Boundary, and in scattered areas along the Gulf of Alaska. Much of the field work is based on astronomic stations that have never been connected by accurate surveys, and where the stations have been connected, it has been with plane-table triangulation. The result is that large areas of Alaska have been mapped as separate units based on local control. Later, as ties were made, these had to be adjusted into still larger units.

The errors inherent in work of this type have been compensated for by arbi-

[1] Talley, B B , Maj. Corps of Engineers, U. S. Army, Engineering Applications of Aerial and Terrestrial Photogrammetry, pages 366–367. Pitman Publishing Corp., New York and Chicago, 1938.

[2] Bagley, Lt. Col. Ret , U. S. Army, Aerophotography and Aerosurveying, McGraw-Hill Book Co , Inc., New York and London, 1941.

[3] PHOTOGRAMMETRIC ENGINEERING for October, November. December 1936, Bibliography of Photogrammetry.

trary adjustments. This procedure has produced satisfactory reconnaissance maps but has not furnished adequate control for the compilation of planimetry from photographs at scales of approximately 1:20,000.

As field surveys are slow and expensive in extensive river valleys, especially where the vegetation is heavy, aerial photography has been used to cover valley areas that separate areas previously mapped.

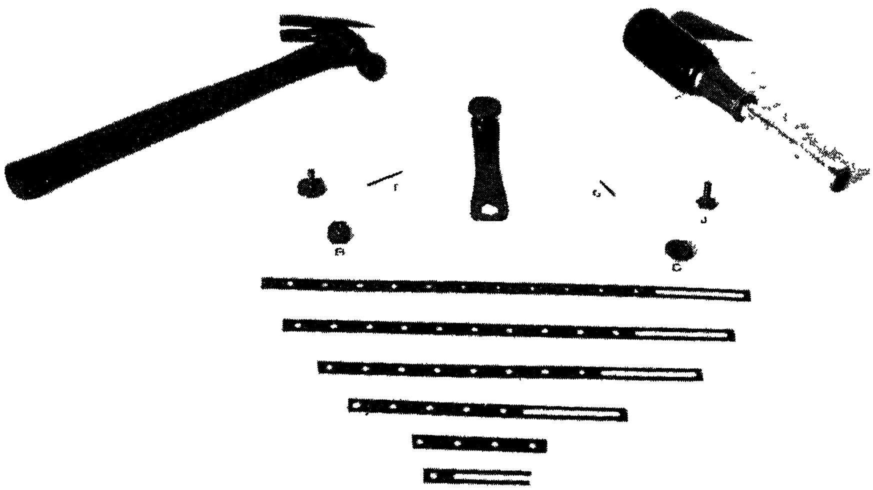


FIG. 1. Parts of the Floore Radial Intersector and tools used in its assembly.

Because of the wide and irregular spacing of control, it has been necessary to compile as units large areas in which the planimetry is relatively correct; these unit areas can then be adjusted between the areas previously mapped with due consideration for the geographic data available. The amount of work necessary to make the same adjustments if flights are worked up in single strips is enormous, and to adjust a templet layout consisting of many flight strips to control that is materially incorrect is impossible

The essential parts of the Radial Intersector are shown in Figure 1 and are described below.

A. Slotted intersector arms, die pressed from .02″×$\frac{3}{8}$″ blue spring steel stock 11″ long Width of slot and diameter of holes .162″±.001″. Arms are cut to required lengths with metal snips

B. Brass Hex nuts 7/16″ threaded 8-40. Used to hold radial arms in position around center bolt J.

C. Fiber washer Used between arms and Hex nut to permit tightening without stripping threads or breaking center bolt

D Tack hammer used in setting pins F & G on picture centers and on pass points when assembling intersector and for marking located points on base when layout is complete.

E Socket wrench used to tighten nut B in intersector assembly.

F, G. Marking pins.

H Tension wrench Holds base of center stud steady when intersector is being assembled

I Floating stud Placed over picture points in assembling intersector and forms tie between arms in layout.

J Center bolt, cold rolled steel, Hex base, 7/16″, Shaft 160″± 001″ threaded 8-40, acts as pivot of any one templet Placed at principal point of vertical photograph

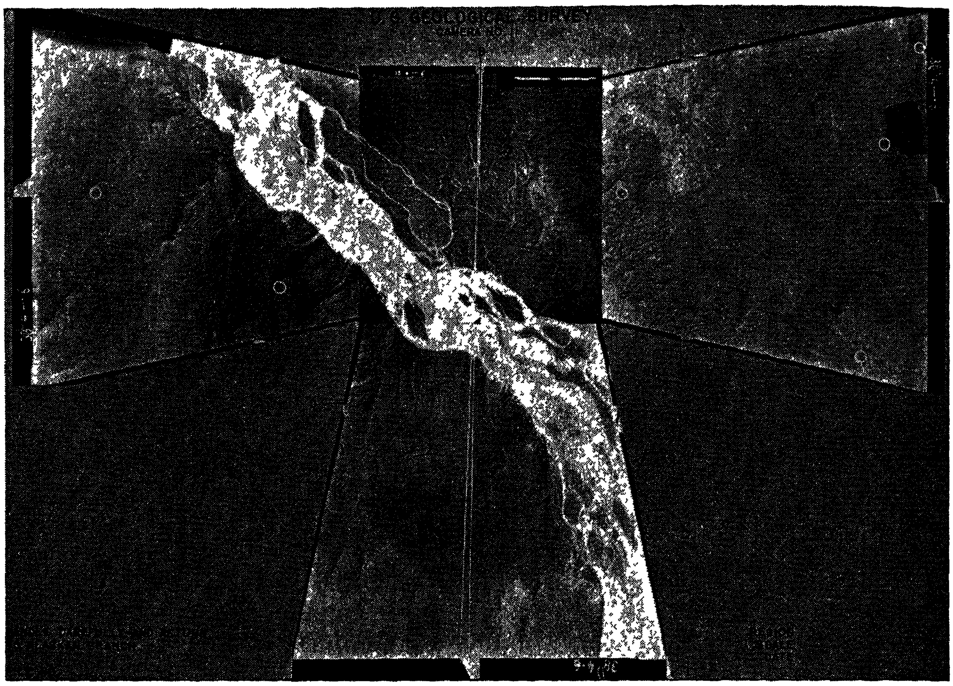


FIG. 2. T2A photograph ready for construction of the intersector Azimuth lines indicated in white. Pass points are circled in white.

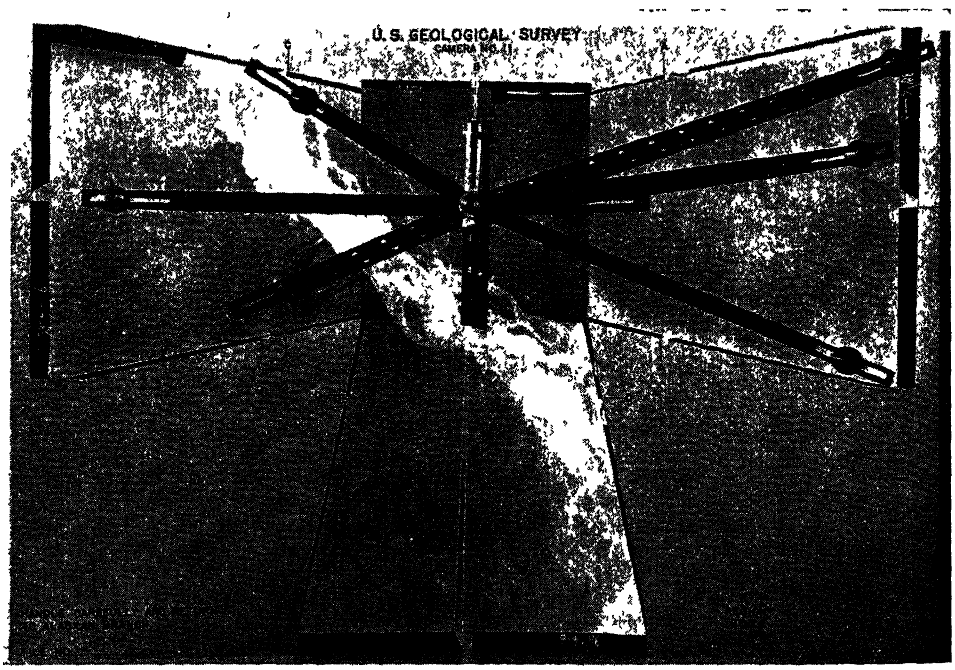


FIG. 3. Radial Intersector assembled on photograph.

The steps leading up to the assembling of the intersector units do not differ radically from those in the more general use of templets with vertical photographs and sufficient control The principal points of the pictures are located from the fiducial marks, and the principal points of adjacent photographs are transferred onto each picture by image points, so that three principal points appear on each photograph. The principal point of the photograph to be worked is connected by a straight line to the principal point of the preceding photograph and similarly to the principal point of the succeeding photograph. These lines are termed azimuth lines and are indicated in white on the photograph shown in Figure 2. Pass points that appear on at least three successive photographs are selected and marked on the photograph (see small white circles on Fig 2). These pass points may also serve to tie adjacent flight strips together and whenever practicable points are selected so that they help to control the location of important topographic or planimetric features such as stream joinings, mountain tops, or lake shores.

When a flight or series of flight strips have had azimuth lines and points indicated on them in this manner, the constucton of the intersector units is begun (see Fig 3) The photograph is placed on a soft pine board or a desk covered with a cork composition; the studs (I, Fig. 1) are centered over the pass points and fixed in position by driving a pin through the center of each stud into each located point Similarly, a center bolt is located at the principal point of the picture Studs are located also on the azimuth lines to the adjoining pictures. Intersector arms are then chosen of such a length that when the hole is placed over the center bolt the stud located at a pass point is approximately at the center, longitudinally, of the slot In choosing arms for the azimuth lines, it will be noted that one contains a slot and the other only holes. This is done so that two studs may be used to control two directions of the succeeding intersector, allowing no motion except in the direction of the azimuth lines A fiber washer and Hex nut are placed on the center bolt, and by means of wrenches (H and E, Fig 1), the whole assembly is tightened up so that the metal arms are fixed in relation to each other.

In the first models center bolts were made of brass, but it was found that the tension required to hold the arms in correct position was such that it caused

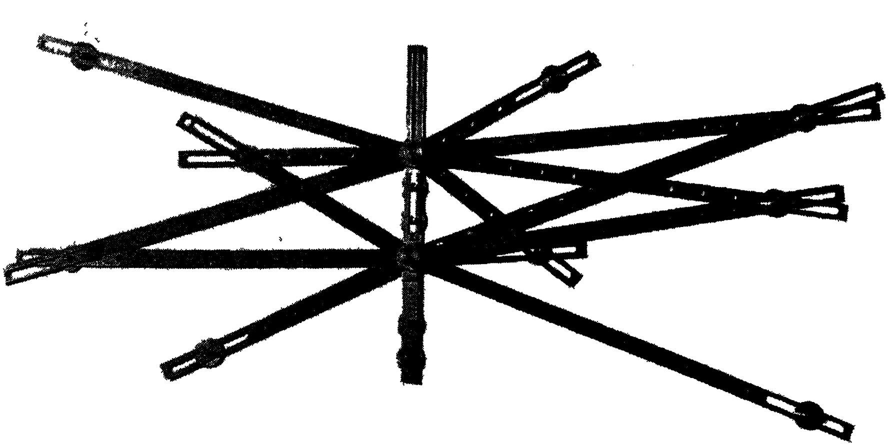


FIG. 4. Photograph showing assembly of intersector units.

stretching and the eventual breaking of these bolts. In the newer equipment cold rolled steel has been substituted for the brass with satisfactory results

The intersector is next lifted from the photograph, the pins are removed from principal points and pass points, and the operator is ready to repeat the process with the next photograph

The intersector layout is made on a base of high-grade celluloid sheeting with a low coefficient of expansion for temperature and humidity changes, stretched over cork cemented to the floor or on soft pine drafting tables

Figure 4 shows assembly of two radial intersector units as they appear in the layout Figure 5 shows one of the largest areas worked up by this method from the tri-lens photographs. It represents seven parallel flights and one cross flight. The scale is 1:30,000 and the area covered is approximately 3,000 square miles

The assembly is tapped gently so that the intersectors will assume a position in which there is the least strain at any intersection point. Pins are then inserted through all studs and center bolts in the assembly and each is tapped with the hammer just hard enough so that the celluloid sheet is marked with a small hole. The templets are then removed one at a time from the base. The locations of the intersected points are marked with a small inked circle The location of the center bolt is marked with a small inked square, beside which is inked the number of the photograph.

If, as is common, the number of pass points is not sufficient to control the

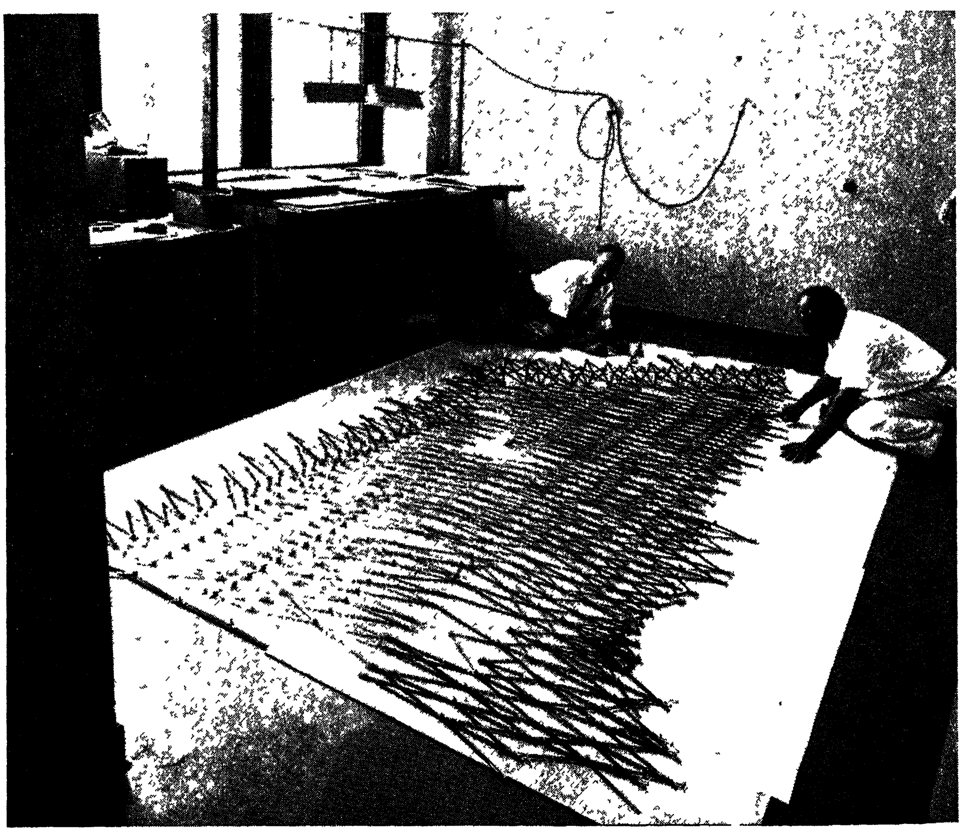


FIG. 5. A Typical intersector layout.

position of all the planimetric detail which it is planned to transfer from the photograph, additional points called minor or sketching points must be picked These are intersected by placing the photograph underneath the celluloid base so that its principal point coincides with the mark that represents the center of the intersector unit on the celluloid base, and so that the locations of the pass points as transferred to the celluloid base fall on a fine radial line scratched on the photograph from the center to the pass point The photograph thus oriented is in position so that radial lines can be drawn on the base to the minor or sketching points Proceeding in this manner through a flight of pictures, all the additional points necessary to control planimetric sketching may be located.

The transfer of planimetric detail from photographs to the base is done in various ways In the Alaskan Branch projects, where the layout is made at picture scale, it has been found that the detail can be traced with lead pencil directly from the photograph onto the base Adjustments are made so that the detail sketched is controlled by the nearby intersected points Great care must be taken by the operator, especially in sketching from photographs of rugged terrain, to use only such points as are near the same elevation as the material to be taken off; for instance, points located on tops of ridges and mountains will not serve as control for streams and other features in bottoms of valleys, and vice versa

It has been found that in tracing the detail from the photograph onto the base a soft lead pencil gives lines of sufficient weight without inking, so that reduction to compilation scales of 1:96,000 or 1:180,000 may be done photographically Final publication is usually on the scale of 1:125,000 or 1:250,000

Application in the Topographic Branch

J. L Buckmaster

THE mapping problems to which the Floore Radial Intersector is applied in the Topographic Branch of the Geological Survey are greatly different from those in the Alaskan Branch The Topographic Branch is engaged primarily in making a complete series of maps in quadrangle form at scales from 1:24,000 to 1:62,500, and these quadrangle maps, if reasonably possible, should meet grade A standards of accuracy This requires a considerable amount of third-order horizontal control Consequently, it is the usual practice when the control consists of transit traverse to have one line of control located near the perimeter and one line across the center of each quadrangle, although considerable variation from this ideal must often be tolerated because of the lack of roads or other suitable routes for traverse In the less frequent cases in which the horizontal control consists of triangulation, a smaller number of points are available, but as these are distributed over the whole quadrangle, the final results will not be greatly different

In the Topographic Branch, therefore, the Radial Intersector system is used chiefly in working up quadrangle units for which a line of control will be available near the perimeter of the sheet and another line across the center Usually the unit is a 15-minute quadrangle, embracing an area about 13 miles wide by 17 miles long, a total of about 220 square miles When $7'' \times 9''$ photographs on a scale of about 1:20,000 are used, as is often the case, about 7 north-south flights are required, with a total of about 140 prints The compilation is made on the approximate scale of the photographs The published maps should have a horizontal accuracy for most points of $0.02''$, and since the publication scale for 15-

minute sheets is usually 1:62,500, the desired tolerance for the radial intersector control on the 1:20,000 scale is about 0 04″ to 0 06″, or about 1/20 of an inch

The detail is adjusted to the intersected base by tracing directly from the photographs by the use of an overhead projector or with a Sketchmaster. Though it is entirely possible to compile the maps on drawing paper, topographic acetate sheets are still used as the base because of the convenience in checking and tracing small detail directly from the photographs

Although the Floore Radial Intersector method is similar in operation to the slotted template method, there are a number of differences which should be mentioned. With the slotted template method, if a control point is plotted in error, if it is omitted, or if a stud is not centered over it, the error is difficult to detect without taking up the templates. With the Radial Intersector method, in which the control sheet is visible, such errors can usually be observed and corrected with little difficulty. One of the chief difficulties of the radial control man is that due to slight errors in plotting, in identifying control, or in the geodetic control itself, adjacent plotted control points may not agree closely enough with the corresponding intersected positions to avoid strains in the system, and small errors may be made by holding to a control point which should be disregarded Large errors are readily apparent, but small ones that are close to the Survey's limit of tolerance for horizontal accuracy can easily be overlooked

In the Geological Survey method each control point is carefully inked with its usual symbol and, in addition, with a blue circle the diameter of the stud pinned to it Before the stud is pinned in position the intersector units in that area are connected in position, and if any of the plotted and intersected points are in disagreement, the error can be detected by the failure of the studs to be superimposed over the control circles without strain in which case further checking should be done A similar advantage in detecting such disagreements could be gained with the slotted template system by using lucite studs, over the control points, which would make their agreement or lack of agreement with the control points below apparent.

Less strain in the units and slightly better radial control adjustments might result if the layout were fixed in position by pins through the centers of the units adjacent to the control rather than through the studs over the control points. Such a system would require that the agreement between plotted and intersected control positions be visible

Interference between arms and units is something of a problem with both the slotted template and the Floore Radial Intersector system In the former system the templates are trimmed to a size as small as possible, and in the latter the units should be made up of metal arms no longer than necessary to reduce the interference In both systems judgment must be used in the selection of pass points to obtain their proper distribution, otherwise interference will result. The Floore system has an advantage in that any such interference will be visible without removing any of the units.

As no slot cutter is required for the Floore system and all arms and parts can be used over and over again, the cost of material is less than for the slotted template system, and the equipment is more portable, a special advantage for military operations.

A less emphasized point of difference between the two systems is that in the slotted template system a tolerance must be allowed for differences between the width of the slots and the diameter of the studs. If the tolerance allowed is too small, strain and deformation of the slots will occur. In the Floore system the slots can be cut to fit the studs with a tolerance as small as machining practice

will permit and the inaccuracies of the unit can be taken up by the elasticity of the arms. It may be reasoned that the adjustment of small and inevitable errors will be more satisfactorily made by elasticity rather than by play in the slots or by deformation, but only considerable research and tests will prove which is better.

The azimuth or center-to-center arms of the Geological Survey's Radial Intersector have not yet reached a completely satisfactory stage of development Various methods have been tried, but the one most favored is to use standard arms short enough to avoid interference between centers and held in alignment with two studs through their overlapping slots It has been found through tests that the azimuth system in a mechanical method is not so important as it is in graphical methods and that only small differences in adjustment will be found if it is omitted entirely, provided perimeter control is used. Other tests indicated that almost identical adjustments will be obtained if only one stud is used to connect the units thus holding them laterally but not in azimuth or orientation. This is not always true, however, as in the case of a layout which lacks control along one side If the center of a photograph cannot be accurately transferred to the adjacent photograph, as when it falls within a body of water, it is much better to omit the azimuth connection.

Preparation of Photograph

In marking conjugate centers and picking the pass points for radial intersector control, the methods used vary slightly with the individual, but all are similar to the usual radial control methods The principal points are transferred by stereoscope, or a definite image point is picked near the principal point and the conjugate substitute centers are located visually. If the area being mapped is of high relief or if there is evidence of considerable tilt in the photographs an approximate tilt analysis should be made.

Pass points are chosen, where possible, so that each is common to three photographs of two overlapping flights. The system works better and with fewer interferences when the number of points used is kept to a minimum. More than six pass points per photograph are not usually necessary or desirable. Eight or

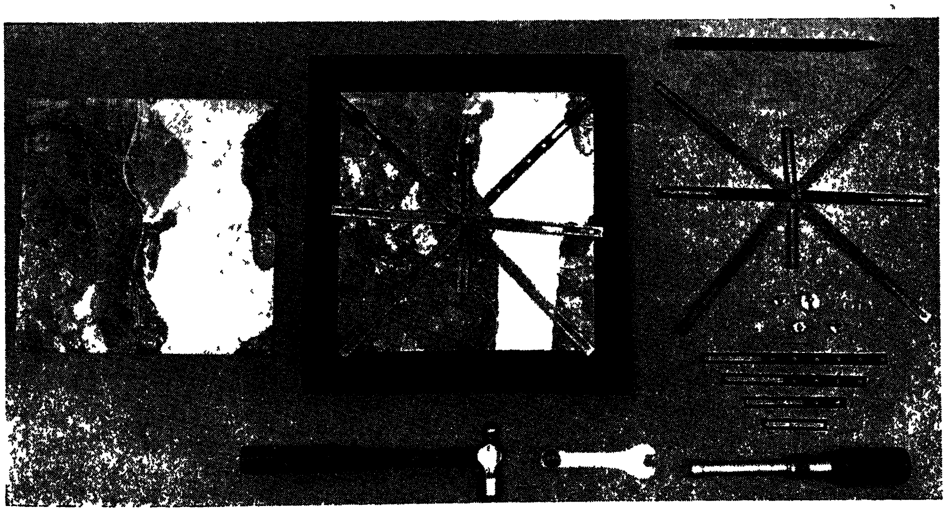


FIG. 6. Radial Intersector Parts and Unit

twelve pass points per intersector unit have been used with no impairment of the resulting layout, but the additional points caused a loss of time in setting up the unit and checking against interference without adding to the accuracy of the photogrammetric control Secondary intersections are made graphically after the primary radial point system is completed. When two flights overlap so that it is not practicable to choose a pass point that will fall on three photographs of each flight, it is usually better to use a single point that falls on three photographs of one flight and two of the other rather than to pick an additional point unless this is required for compilation detail

Contrary to common belief, tests in which only five cuts are used for each intersection show results almost identical with those using the usual six cuts. These tests were made without a special picking of points, merely by disconnecting the sixth arm at each intersection stud in a standard layout The Floore system lends itself especially well to such research, and some surprising results are obtained Usually such tests are made on short notice on some regular job, and more systematic research is needed.

Control points are usually identified on the photographs in the field, prior to the radial triangulation. Often the marking of these points on the photographs is not as accurate as desired, and the characteristic of the Floore system by which a stud can be released from its corresponding control point to see if it will maintain that position without undue strain permits a very valuable check.

Construction of Units

The procedure for making up intersector units here described has been satisfactory, but many details and short cuts might be developed to increase both production and accuracy. Each photograph is first placed over a special cork-composition mounting board, and special pins are driven vertically through the principal point (or substitute principal point) and the pass points. Pins are also driven through the azimuth rays, which should be previously marked on the photographs, at the proper distances to hold the azimuth arms The pins are made of music wire of a diameter such that they will just fit the holes in the studs and bolts. The upper end of the shaft is left the same diameter as the original wire, but the lower end is ground to a smaller diameter so that it will make a much smaller hole when driven through a photograph or the acetate projection sheet. After a bolt has been placed over the pin at the principal point, and studs over the other pins, the metal arms of suitable length are placed in position, as shown in Fig. 6

With the arms in position, a washer is placed on the bolt over the arms, and a nut is applied to the bolt and tightened A special wrench is used to hold the bolt from turning, and care must be taken in the method of tightening the nut to prevent any strain in the arms. No trouble is encountered with slipping of the arms except when inaccuracy in the building up of individual units is so great as to cause serious strains when the units are assembled.

Preparation of Projection Sheet

Because of the inconvenience of determining the average scale of photographs for each quadrangle and the computing and construction of special scale projections, a standard scale is usually adopted for a whole project A determination of the average scale for the photographs of the project is first made and a suitable scale for the whole unit is then adopted. Thus, on one project a scale of 1:20,000 was first planned, but as the photographs varied in scale from

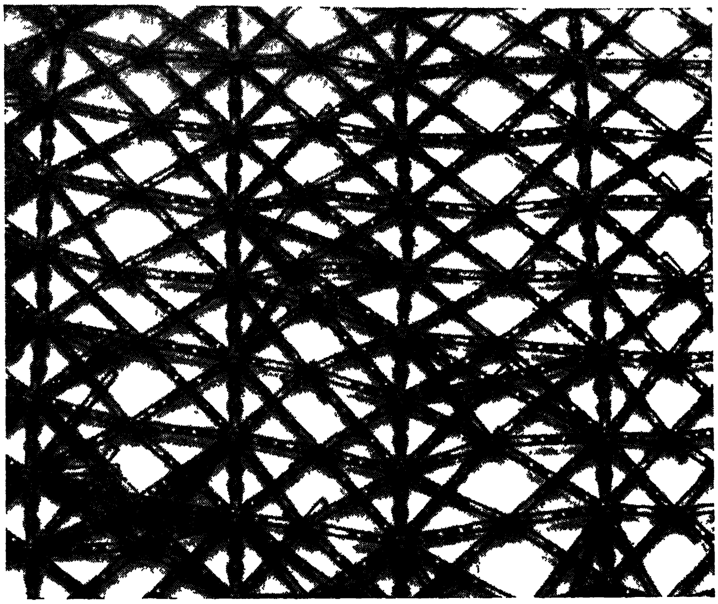


FIG 7 A Section of Assembled Units

1:19,500 to 1:21,500, the standard was later changed to 1:20,500. A metal-mounted regional projection template has been devised from which a $7\frac{1}{2}$-minute projection can be traced for any latitude within the region. If this template is carefully constructed, projections traced on the acetate with its aid are more accurate than if they were constructed individually on the acetate in the usual manner, and there is a substantial saving in time Projections covering larger areas can be made by tracing one $7\frac{1}{2}$-minute unit at a time

PLOTTING CONTROL

The horizontal control, which is usually in geodetic coordinates, is plotted on the acetate sheet by the usual diagonal scale method. The permanent control points are marked with a small triangle, and the intermediate control points are marked with a 0 2″ circle In addition to these symbols a blue circle with the same diameter as that of the studs is centered on each control point.

ASSEMBLING OF UNITS

The projection acetate sheet is usually placed on a large composition-covered table. The units are then assembled on the sheet and held in position by the special pins Studs are pinned first to the control points that are judged to be the more accurately identified on the photographs The intersector units are then connected in proper position with respect to the control beginning with the flight which is best controlled. After the units for several flights have been assembled a check is made to see that the intersected control positions agree with the plotted control positions. If the stud representing an intersected position can be superimposed over the corresponding plotted position with little strain it is pinned exactly to the plotted position. If an appreciable difference in position exists an investigation is made. The error is located and corrected if possible, otherwise that control point is disregarded.

After the units have been assembled and pinned to the accepted control points the arms are tapped and the studs rotated to reduce or eliminate strain and to assist the assembly in reaching its best adjustment, after which pins are driven through a few of the studs to prevent any movement from that position The adjusted positions are transferred to the acetate projection sheet by means of a fine-pointed plotting needle which will just fit the holes in the studs and center bolts, through which it is forced with enough pressure to pierce the acetate The intersector units are then removed, the radial control points circled, and the principal points numbered Any intermediate or secondary radial control points that are required are intersected graphically.

Compilation of Detail

The compilation of detail is similar to that of most planimetric radial compilations. Usually the photographs are inspected in the field prior to the radial control work, at which time all detail to be compiled is inked on alternate photographs with colored ink The use of overhead projectors has frequently been explained. The Sketchmaster has the advantage that it can be adjusted for tilt without detectable loss in focus; though primarily a field instrument, it is well adapted also for office use.

The detail is inked on the acetate sheets with craftint celluloid ink No attempt is made to obtain a finished appearance, as it must be reinked after it has been photographed. The effort is made, however, to compile the detail with sufficient accuracy so that the published map will qualify under grade "A" standards of accuracy.

CHAPTER X

PHOTOGRAPHIC MOSAICS

METHODS OF LAYING DIFFERENT TYPES OF MOSAICS

A. F. Warren

General Discussion

AERIAL photographic mosaics have been of great benefit to map readers in the past. They originated in the First World War and became very popular later because of the enormous amount of information that they furnished. Many private concerns as well as state and federal agencies use them extensively for planning highways, drainage projects, geological investigations, soil conservation, forestry, and mining studies. Although well written articles have been published on mosaics in the past, many writers versed in the theory find it difficult to apply in a practical way.

An aerial photographic mosaic is an assembly of individual aerial photographs forming a *single picture* This is very important to remember in constructing a mosaic for if the finished product appears to be a single photograph its value is greatly increased. Mismatches will be less noticeable and relief will appear more prominent. Mosaics are usually made from overlapping vertical aerial photographs, but it is also possible to make them from rectified oblique photographs provided they have sufficient overlap The ideal condition exists when all aerial photographs for a mosaic map are of the same scale. The longest focal length camera possible should be employed in mosaic photography This calls for a higher altitude in obtaining the desired scale and results in a more constant scale between overlapping photographs and adjoining strips, for any decrease or increase in the altitude of the plane will affect the scale of the photograph less at a higher altitude than it would be with a camera of shorter focal length at a lower altitude. Relief displacement is also less in photographs made from a higher altitude.

A mosaic produces a greater amount of detail than can be shown on a planimetric or topographic map. Every feature produced in an aerial photograph assumes its natural appearance and its dimensions are in direct proportion to the scale. No group of topographic map symbols has yet been devised to show all the true characteristics of different objects as they can be interpreted from aerial photographs The amount of detail to be observed on a mosaic depends on the scale.

The appearance of a mosaic is dependent on several things:

A. *The skill of the compiler.* Mosaic compilation is an art acquired only by long experience and patience. Speed cannot be expected until the operator has acquired adequate experience. For proper assemblage, the compiler should have the necessary equipment and a thorough understanding of the area. It is his skill that largely controls the appearance of the mosaic.

B. *The characteristics of the terrain.* A flat or level area is much easier to compile into a mosaic than rugged, mountainous terrain. A vertical picture covering hills and valleys has various scales within itself, and it is difficult to make one photograph match the adjoining one, without leaving mismatches or double images. Prints of such areas should always be restituted to a common scale, and the remaining errors distributed throughout the strip, holding the mismatches to a minimum.

C. *The color and tone of the photographs.* All photographs used in a mosaic should be consistently printed to the desired contrast. If this is practiced, it will be easier to produce an even tone and color throughout the mosaic. If at all

possible, off-color and off-tone prints should not be laid Sometimes, however, off-color prints can be eliminated, provided there is 60% overlap as the remaining 10% overlap is sufficient for trimming purposes.

Color match can greatly be improved by the skill of the mosaicer in trimming the photographs for the laydown. They should, as much as possible, be trimmed along an even color match to the adjoining print Great improvement can be made by trimming along a fence line or edge of a woods. Do not trim along roads since the slightest mismatch will be very noticeable Difference in color and tone can also be improved by overlapping the lighter print onto the darker one

D. *Distortion and displacement due to relief* Usually the outer edges of an aerial photograph have the most distortions and relief displacements This condition causes a great amount of difficulty in matching the detail between photographs, particularly between adjoining strips In controlled mosaics this is taken care of by restitution, but in uncontrolled mosaics it becomes a delicate task for the compiler to distribute the distortions of the images so that they will match For example, the slope of a hill nearest the axis of the camera is greater in scale than the slope beyond the crest. One of the best methods in partially overcoming scale differences is by trimming the photographs along a line of equal elevation This will not entirely eliminate the mismatches or overlaps but will greatly reduce their magnitude

Much of the distortion and relief displacement may also be eliminated by rectifying the image to a mean photo scale. This is done by stretching or shrinking portions of the photograph. 7×9 inch photographs made on single weight photographic paper will stretch approximately three-tenths of an inch across the grain of the paper, but will not stretch as much with the grain. This stretching effect is made by thoroughly soaking the trimmed photograph in water, and by pulling on each end of the print a slight amount it is expanded in size where the pull has been applied. Care should be taken not to apply too much pull in stretching a wet print as it is liable to tear. Photographs may be shrunk a small amount by squeezing the prints towards their centers during the mounting process To do this, they must be mounted as dry as possible and rubbed down with a dry cloth. Considerable practice in necessary to become efficient at this operation.

Due to the fact that distortion and relief displacement increases toward the outer edges of an aerial photograph, the center portion should always be used This can be accomplished in the following manner·

1. Select the print that is to be laid first and prepare it by feather-edging. Cement it to the base.

2 Overlap the adjoining print and trim off half the overlap, keeping in mind the color and tone match Do not trim much from the lateral edges.

3. Trim half the overlap from all adjoining prints They can be laid as a mosaic throughout the strip.

4. Half the overlap on adjoining strips can likewise be trimmed away and the strip assembled. This leaves the center portion of each photograph, which is the most accurate part of the print, uncovered

E *Difference in scale* Some differences in scale are found between photographs and also between strips This is caused by the plane not maintaining a constant altitude or by changes in ground elevation To correct this, prints must be ratioed to a common scale If this is not possible, the compiler should distribute the errors throughout the strip or strips by stretching the prints and concealing the mismatches

The instructions given under the paragraph on distortion, and displacements due to relief, may also be applied to a certain extent in correcting for difference

in scale. The strips of smaller scale may be stretched to increase the scale and the strips of larger scale mounted as dry as possible in order to maintain an even scale throughout the assembly If there is a greater difference in scale between two strips than can be corrected by this method, the smaller scaled strip should be laid with a small double image between successive photographs throughout its length and the larger scaled strip placed with a small overlap between successive photographs throughout its length These instructions apply only to uncontrolled or partially controlled mosaics where time is more essential than accuracy.

F. *The quality of the reproduction* In order to reproduce a mosaic, a copy negative must be made, which should be at the desired scale and with good printing quality. These can be either enlarged or decreased in scale as much as two diameters if necessary.

It is always best to reduce when copying a mosaic so that errors are reduced. Often the photographs are enlarged during restitution so that a reduction may be made when copying. It is more difficult to lay enlarged prints in a mosaic than contact prints because the inherent errors in the enlargements are increased and the paper expansion factor becomes greater.

Although care has been exercised in trying to keep the mosaic at a uniform tone, individual prints may be noticeable in the reproduction. This can be improved by blending and retouching the copy negative with a small air brush and red dye The red dye is sprayed on the thinnest portions of the negatives (caused by the darkest prints in the mosaic) until a density is attained which is equal to the darkest portions of the negative. The lines caused by the shadows cast from the edges of the overlapping prints, can be eliminated by retouching. Blending and retouching copy negatives, if done satisfactorily, will greatly improve the appearance of the reproduction, making it appear as "one single photograph." The following equipment is necessary in blending and retouching negatives·

1. A small air brush and a pressure controlled source of clean air.
2 A shadow table.
3. Retouching varnish and retouching pencils.
4. Small strips of paper or vellum

The following procedure is suggested in blending and retouching negatives:

1. Prepare a solution of diluted red dye in a small bottle.

2. Connect the air brush to an air source of approximately 30 pounds pressure.

3. Place the negative emulsion side up on the shadow table.

4. Place the paper or vellum on the negative along a print match line and over the densest portion of the negative

5. Spray a small amount of red dye on the thinnest portions of the negative. Increase the density of the dye until the thinnest portion is at an equal density with that of the darkest portion It is best to hold the shading paper with one hand so that it may be easily removed for comparing the density of the dyed part with that of the remainder of the negative.

6. After a negative has been blended a proof print should be made and any spots having too much or not enough dye remedied Excess dye may be removed by swabbing with wet cotton.

7 Apply retouching dope or varnish to remove the shadow lines between print matches on the negative. This is a delicate operation since too much graphite will cause two white lines outside of a black line instead of one single black line on the reproduction. Figure 1 shows a mosaic on easel preparatory to making a copy negative.

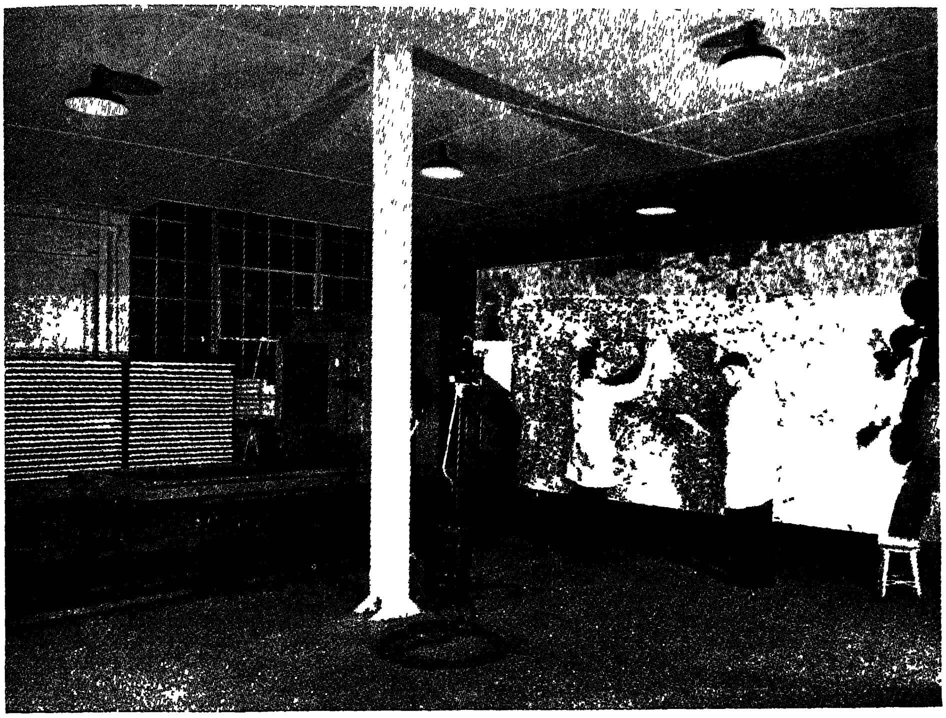


FIG. 1. Copying a controlled mosaic to quadrangle sheet size Camera is capable of making 40×40 inch negatives.

PHOTO INDEXES

Photo indexes are hurriedly compiled mosaics of the area to be surveyed. The individual photographs throughout the area are stapled or fastened together by matching images in the overlap area so that all marginal data on the photographs can be seen This particular type of mosaic assembly is used as an index in determining the number and location of individual photographs in the project and for determining re-flights while the project is being flown Indexes are prepared by placing the prints in their respective flight lines so that the print numbers appear uppermost. Images are matched throughout each strip and adjoining strips are matched to each other Each print is fastened to the mount by staples or tape. This arrangement of prints makes possible a hurried check of the strips for coverage and quality of photography. Both side lap and running overlap coverage can be easily determined Indexes may be prepared as shown in figure 2 when they are to be used for two purposes The smaller numbers represent the location of the individual prints and the larger numbers represent the location of the atlas sheets which are to be made covering the area of the project. The atlas sheets are usually enlargements made to a certain scale.

UNCONTROLLED MOSAICS

Uncontrolled mosaics are laid at picture scale, by matching like images and provide a rapid method of producing a picture of a large area. The best method in making this type of mosaic is to lay the middle print of the mosaic first, thus relegating errors to the outer edges. An approximate azimuth of the flight line

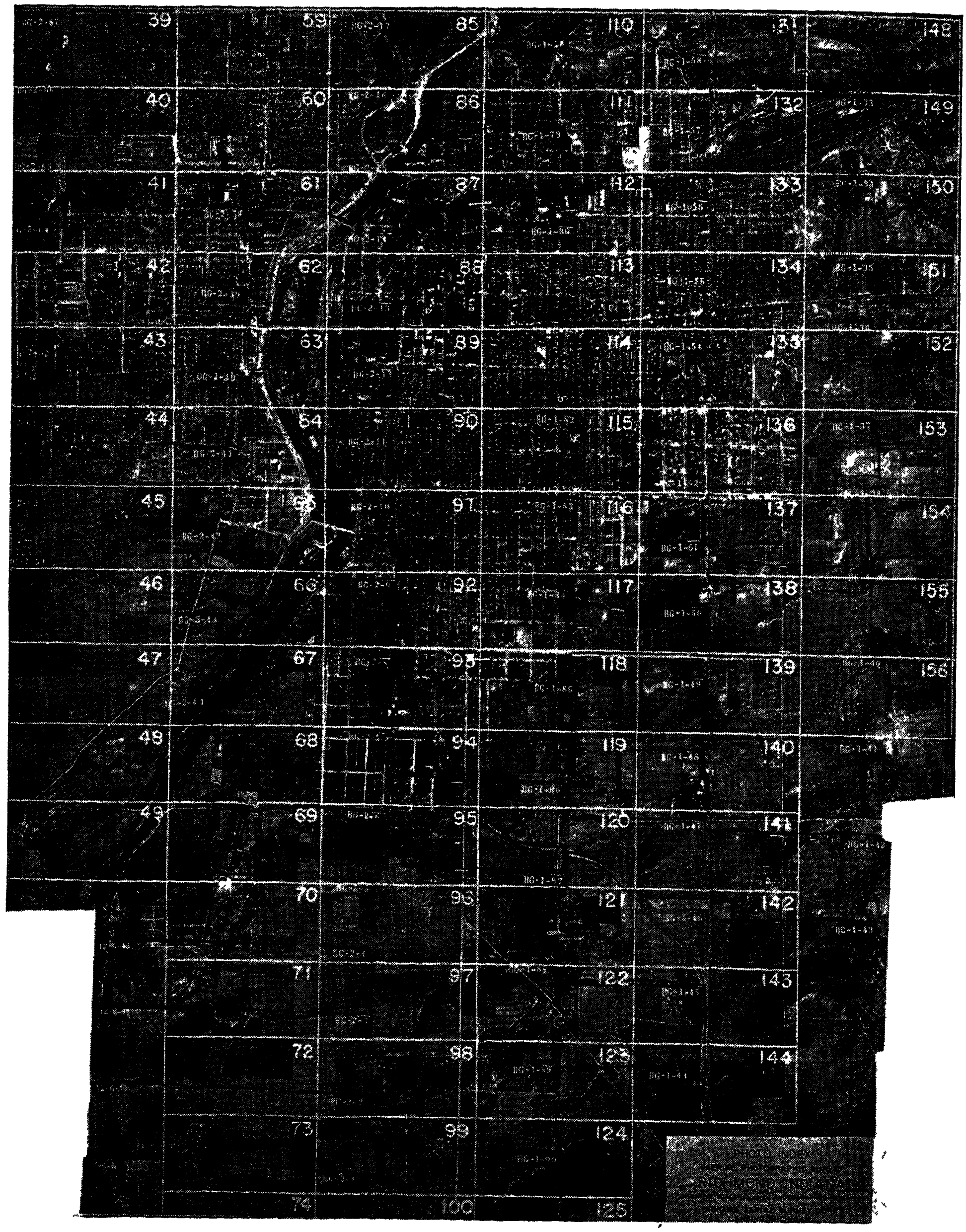


FIG. 2 Portion of a photographic index.

can be determined by laying the middle strip first This gives a better orientation and also distributes the scale error by allowing for a rapid adjustment between adjoining strips before the prints become dry and set.

SEMI-CONTROLLED MOSAICS

Semi-controlled mosaics are those for which directions or distances are given. The prints used in this type of assembly should be restituted for scale,

Common methods used in making a semi-controlled mosaic are described below

A. Straight line control:

1. Railroads or roads which run in a straight line in the area to be mosaiced are drawn as straight lines on the mount and the prints oriented to that line when mosaiced. This will orient the strip or strips of photographs wherever the line is crossed. The detail on the prints must match the lines on the mount.

B. Reconnaissance strip method:

In this method an attempt is made to orient correctly, with respect to each other, the prints in a single strip of overlapping photographs. A strip so oriented may be used in connection with certain types of reconnaissance, for determining average print scale from the ratio existing between map and photo distances, or in some cases, to assemble a strip to which can be joined the remaining strips of an uncontrolled mosaic.

This method is considered very valuable to the mosaicer and since it can be applied to the laying of any type of mosaic, additional details covering this method are explained

1 Locate and mark the principal point of each photograph

2. On a table or desk assemble the photographs in their proper order by matching detail. (Photo image points.) Hold the assembled prints in place with weights or scotch tape

3. Adjust a straight edge on the assembly in such manner that its edge passes as closely as possible to all principal points. Draw a fine line along the straight edge on the end photograph that is uppermost

4. The line drawn on the first photograph is transferred to each of the other photographs in sequence throughout the strip.

(a) On the line on the first photograph and in the area overlapped by the second photo identify and prick two points that can be readily transferred to the second photo. These two points should be as far apart as possible

(b) Identify and prick these two points on the second photo, and draw a fine line through them from edge to edge of the second photo.

(c) On this line on photograph number two, identify and prick two points that can be transferred to the third photo.

(d) In this manner, the straight line is transferred to every photograph in the strip.

5. Draw a straight line of proper length on the base material on which the strip is to be mosaiced. Orient the photographs to this line by placing each photo over the line and adjusting it so that the line on the photo coincides with the line on the base and at the same time maintains a detail match between photographs.

C. Pantograph method:

A pantograph may be used for transferring points or lines from an existing map to the mosaic mount at a desired scale, and the prints can be ratioed and restituted to fit these distances as plotted on the mount. Such a mosaic is as accurate in both distance and direction as the existing map used for the control. By using these horizontal distances throughout the line of each flight, the mean scale of the flight and project can be computed and the prints made to fit those measurements. Then, by using the straight line control method for assembly, a mosaic having the desired scale can be produced.

D. Field measurement method:

This is the most practical method of producing an accurate semi-controlled mosaic. The photographs are taken to the field, and measurements made on the ground between points which have been identified on the photographs. This is done throughout each strip The points are picked on the photographs and dis-

tances between them indicated on the back. Such distances should be long enough to extend at least half the width of the print, and at least one should be made for every fifth print These prints are ratioed and an enlargement factor determined. Prints that do not have measurements are given an adjusted mean ratio determined from those adjoining To determine the ratio factors for enlarging a strip of photographs to a desired scale, see the following table·

Print No	Distance on ground	Distance on print	Scale	Fiducial measurements for 1″=4800″
1–1	1777 5′	4 10″	1:5202	9 59″
1–2			1.5189	9 56″
1–3			1·5176	9 54″
1–4			1.5163	9 51″
1–5	1652 3′	3 85″	1 5150	9 48″

Distance between fiducial marks on negative = 8 85″

To determine the measurement between fiducial marks for making an enlargement at a scale of 1 4800, compute by proportion

$1/5202\ 8\ 85 = 1/4800\ X \qquad 8\ 85/4800 = X/5202 \qquad 4800X = 46037\ 7 \qquad X = 9\ 59''$

By using these horizontal distances throughout the line of each flight, the mean scale of the flight and project can be computed and the prints made to fit these measurements. Then by using the straight-line control method, a mosaic having the desired scale may be assembled. (See figure 3.)

FIG. 3. A semi-controlled mosaic laid to horizontal distances throughout each strip.

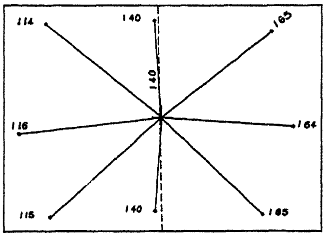
CONTROLLED MOSAICS

Controlled mosiacs are constructed so that points identified on the photographs will be superimposed over the plotted positions of the same points on the mosaic base The plotted positions may be geographic ground control points or photogrammetric control points located by one of the various radial plot methods. Several such methods are described in Chapter IX, "Radial Plotting Methods."

After the control has been established, it is possible to compute the mean ratio factor of each photograph. This is best done on prepared ratio computation sheets such as illustrated in figure 4. Measurements are made from the plotted

RATIO COMPUTATION **Print No.** FM 21-26

PRINT SCALES	BASE SCALES	BASE / PRINTS
2.50	3.50	1.40
2.30	3.80	1.65
1.20	1.96	1.64
3.42	5.64	1.65
2.62	3.66	1.40
3.36	3.86	1.15
2.08	2.41	1.16
3.21	3.75	1.14



Mean Scale 1" = 1400 feet

Ratio Factors

Short Side + 1.16 Long Side + 1.64

Dimensions

Short ______ Long ______

FIG. 4. A ratio computation sheet.

center on the base sheets to all the radial points plotted on the base and entered in the column of the base scales. Corresponding measurements are made on the prints and entered in the column of print scales. The amount of enlargement necessary to make the distances on the print equal those on the base sheet is determined by dividing the base distance by the print distance. After all points have been ratioed, a mean ratio is computed by adding all ratios and dividing the sum by the total number. In the upper right-hand corner of figure 4 is a small figure indicating the approximate location of each radial point and its ratio factor. From this diagram can be determined the degree and axis tilt of the photograph. This sheet is then attached to the print and sent to the laboratory to be used in making the restituted print.

The rectifying projector operator projects the image of each particular negative so that it has a ratio factor equal to that recorded on the computation sheet. The projector has a tilting easel so that corrections for tilt can be made. The image is first projected on the easel at a flat mean scale and then tilted the

amount indicated on the diagram of the computation sheet. All measurements are made between the fiducial marks of the negative so it is necessary to make a conversion template representing different ratio factors in order to properly restitute various prints This template is usually made on cardboard. Two intersecting lines are drawn at right angles across the center of the board. These lines are graduated to computed distances which represent the ratio factor changes necessary for giving the correct measurement between the fiducial marks of a print when it is to be restituted to a certain scale.

One of the chief reasons for employing nine or more points in a controlled mosaic is to provide sufficient control to restitute portions of prints. It is found

FIG. 5 Portion of a controlled mosaic quadrangle sheet

that in rugged mountainous areas the relief will cause great scale changes between control points on each and every photograph. With sufficient control these differences can be overcome by breaking the print up into several triangular parts and restituting a separate print for each triangle formed. By this system the radial points can be made to fall at their correct positions on the base sheet and also maintain detail match. It sometimes requires as many as eight different patches to correctly match the area included in one photograph.

Controlled mosaics are much more difficult to construct than other types. They require patience and time and many remakes are often necessary because of print shrinkage and expansion. The chief advantage in assembling a controlled mosaic is that any photograph can be laid to the base sheet as long as it is made to correct scale.

Any portion of a restituted print is useable in a controlled mosaic as long as the three points of any triangular piece have the same dimensions as those plotted on the base sheet. Prints over two diameters larger than the negative will expand to such an extent that control over position is almost impossible. Due to this expansion of paper it is often necessary to make two prints, each at a slightly different scale, and each with the grain of the paper running in opposite directions Then, by laying the larger scaled print first, the overlap can be covered by using patches from the second print inasmuch as the second printing is at a smaller scale.

The steps for laying a restituted controlled print are as follows

1 Select a restituted print of proper color and tone and transfer all the control from the control print to the selected print by pricking the identical images with a pin or needle and circling with a grease pencil

2. Insert a pin through the pricked center point and into the identical plotted point on the base sheet and orient the print in respect to the control.

3 Likewise, insert pins through the other pricked points and check to see that the pins fall into the proper positions plotted on the base sheet If they do not match with a reasonable tolerance discard that particular print and make another ratio print to fit.

4. Mark the trim line of the print with a grease pencil, including the area within and beyond the points that have pins. The points that do not have pins were not restituted correctly and should not be used in the laydown

5 Remove the print, trim, featheredge, and sandpaper.

6. Apply adhesive to the back of the print and replace in correct position on the base sheet. Reinsert the pins and check positions on all points.

7 Squeeze out the surplus adhesive and remove with a cloth.

8. Special patches may be restituted for those areas that do not properly match the plotted control points.

Figure 5 is a portion of a controlled mosaic quadrangle sheet. Notice the effect that blending and retouching of the negative have on the appearance of the reproduction.

In compiling a controlled mosaic the procedure may be broken down into the following steps—preparation, assembly, determination and placing of title, making negatives preparatory to reproduction.

A. *Procedure for preparing a mosaic.*

1. *Check for coverage* Ask for reflights if necessary. See that no prints are missing.

2. *Select compilation scale desired.* Enlargement up to (2) diameters can be used, without giving much trouble in laying a mosaic

3. *Prepare board or mount.* A mosaic base board should be large enough to cover the area, smooth in surface, but not too highly polished since the adhesive must make a good join with it Vehosote is one of the best prepared boards for mosaics, and Masonite hard board is often used, although it does not allow for easy marking with pins when plotting control points.

4. *Plot projection and control.* All control to be used should be plotted in its proper location and marked on the mount All identified triangulation points should be marked with their proper designations. Control points and projection lines should be identified on an overlay so they can be reclaimed when necessary.

5. *Prepare prints* Restitute prints if accuracy so requires Always use those that are uniform in tone. All known points on all the prints should be identified and circled with a grease pencil If straight lines are used for assembling, they should also be drawn on the prints. Prints should be neatly trimmed, feather-

edged and sandpapered on all overlapping edges. To improve the appearance of the finished job, care should be taken in trimming and featheredging mosaic prints. The following steps and precautions should be taken:

(a) Mark the trim line, preferably with a grease pencil

(b) Lean the cutting tool away from the portion of the print to be used in the assembly and cut along the trim line, just through the emulsion.

(c). Bend the print back along the trim line and break with the thumb and forefinger.

(d). Lay the print emulsion down on a flat surface and hold the print with the thumb and forefinger so that the discarded portion of the print can be removed

(e). Tear the print back along the trim edge and towards the usable portion of the print so that the edge has a feathered appearance.

(f). Sand the feathered edge with fine sandpaper until the emulsion begins to appear dark along the edge. The edge of the print will then have a sharp edge.

B. *Assembly* After prints are properly trimmed, the adhesive is applied to the back of the print and each print is oriented to its position on the mount All surplus adhesive is squeezed from underneath the print and removed with a sponge. Care must be taken that lumps and wrinkles are not left under the print. If too much of the adhesive is removed, the print, when dry will not be attached to the mount and a blister will be formed. Always keep the assembly as clean as possible of surplus adhesive, for it it drys on the print, cracked emulsion will result.

C. *Title.* All mosaic maps should have a title and necessary marginal data giving the location and type of survey made, by whom and when compiled, and some method of determining the scale Scale can be represented by a numerical fraction or by a graphic (bar) scale. A graphic scale is desirable as it represents the true scale of the mosaic regardless of whether the mosaic has been increased or decreased in size by reproduction.

D *Making copy negatives.* Copy negatives can be made to any scale desired. They should be made of good density and if necessary blended and retouched. They should be made to a definite scale so that the detail on the edge of one section will exactly match other sections. A film and paper shrinkage factor should be taken into consideration. See figure 1 for camera and mosaic set up for copying.

E *Reproduction.* Can be either photographic or lithographic. In large projects they represent only a portion of the total area and some method of indexing should be set up to indicate their location in respect to the other sheets.

Adhesives

A. Gum arabic is one of the best and most commonly used adhesive in making mosaics. It allows for easy adjustment of the prints and permanently seals all joints. Its chief disadvantage is that it uses water as a solvent, thus expanding and changing the scale of the prints. This causes considerable trouble in controlled mosaics as the paper expands more across the grain of the paper than it does with it

GUM ARABIC FORMULA

Salicylic acid	$3\frac{1}{2}$ oz.
Gum arabic tears	5 lbs
Glycerine	10 oz.

Dissolve tears in 1 gallon water (120° F) and add salicylic acid and glycerine.

B Rubber cement is also used as an adhesive for mosaics and eliminates expansion of the prints Its disadvantage is that it does not permit easy adjustment of the print to its proper position and is not a permanent cement

C. Various other adhesives are on the market which can be used for mosaics but they often contain various chemicals which cause discoloration in the prints. These adhesives usually require water which causes excessive print expansion.

MATERIALS REQUIRED FOR LAYING A MOSAIC

1 Assembly board
2 Prints of uniform color and scale
3 Cutting tool
4. Sand paper
5 Cloth
6. Tray of clean water
7 Adhesive
8 Squeegee (small)
9. Sponge
10 Masking tape
11 Straight edge
12 Ruling pen
13 Printed lettering stickup or lettering pens.

LAYING A CONTROLLED PHOTO MOSAIC

Wm. H. Meyer, Jr.

1 General Discussion

WHILE the use of single vertical photographs of terrain has been most extensive there are many needs for photographic studies in map form of areas which are greater in extent than areas covered by a single photograph The uses and demands for assemblies of photographs has been so great that engineering practices and techniques have made the assembly of vertical photographs a practical product of photogrammetry which is produced with map accuracy and therefore may be classified as a mosaic map.

Mosaics of aerial photographs have been made from the very early days when the first vertical aerial photographs were successfully made in the summer of 1915. These were a series of pictures taken of Fort Sill. A few of the vertical aerial photographs obtained at that time were pieced together and were published as a reproduction in the *Field Artillery Journal* in January, February, March, 1916. From those early beginnings mosaic maps have experienced many ups and downs At first many people accepted the pictorial representation secured from vertical aerial photographs as a true map of the ground only to find to their sorrow that all that showed promise and glistened was not gold. As was to be expected the pendulum began to swing the other way and for a period of time mosaics were considered useless, inaccurate and not to be trusted, especially by such engineering firms who may have at one time believed that all of their mapping difficulties had been solved with the advent of the aerial mosaic which showed such a wealth of detail.

It is a well known fact today that in judging the accuracy of controlled mosaics more than the appearance and the image match at the edge of photographs must be considered. A series of photographs if very carefully matched at the edges so as to render a composite of good appearance may be far more inaccurate than a mosaic map which was constructed on the basis of centering each photograph and not shifting from the true position because of slight mismatches at the edges, therefore, often a pictorially poor mosaic may be a more accurate compilation. Image displacement due to change in ground elevation is greater toward the edge of the photographs, than it is nearer the center and unless a large number of prints are used it is often difficult, if not impossible, to keep the center of the photograph in its true position and yet find some means of matching the detail at the edges. As the knowledge of aerial photography progressed more information became available concerning displacement of images due to relief, straight line methods as a means of orienting prints, the use of existing maps as a source of control and many other steps which gradually improved the quality and dependability of mosaics so that since their early unfortunate introduction to the engineering profession, the gradual improvement of the work warrants greater faith in the product. It is still difficult to determine what should be classified as a controlled, a semi-controlled or an uncontrolled mosaic because many factors enter into the consideration such as scale, nature of the terrain, image detail which may or may not be helpful, and the size and shape of the area In deciding what should constitute adequate control, very serious consideration should be given to the purpose for which the mosaic map is being compiled and the need for accuracy in relation to the studies being made.

Many mosaic maps have been constructed by various organizations over a period of years and these were used by many commercial organizations, cities,

counties and other government agencies as an aid in solving their engineering problems. Probably the first recognition in Government circles of the mosaic map as a basis or equivalent of a planimetric map was by the Soil Conservation Service, Department of Agriculture, when they contracted and wrote specifications for the production of mosaic maps with strict specifications as to scale and position

A repetition of these specifications at this point would be pertinent.

"The entire mosaic shall be mounted on a complete radial control skeleton, compiled at a scale of 1:31,680, with at least twelve (12) well-distributed radial control points plotted within the area covered by each negative. For each negative at least six (6) of the radial points used in the radial control assembly shall be so plotted that they fall in the overlapping area of adjoining flights All radial points shall be intersected by radial lines on all prints upon which they appear. All of the twelve (12) required radial control points per picture shall appear on at least three (3) pictures, in line of flight, and three (3) of the six (6) points required to be in the overlapping portion of adjacent flights must be on each side of the azimuth line. Of these three (3) points on each side of the azimuth line two (2) must be corner points so located as to be clearly identifiable on all prints to which they are common. By a corner point, it is meant, a point approximately opposite the centers of adjacent photographs in line of flight The material used for templets shall be approved by the Soil Conservation Service prior to the extension of the radial control One (1) templet shall be prepared indicating the center, the twelve (12) radial lines, and the number of each print which it represents. These templets shall be adjusted in accordance with the control as hereinafter specified so that all plotted centers and radial control points of intersection will be within one-twentieth (1/20) of an inch of their true geographic positions at a scale of 1:31,680. In each fifteen (15) minute quadrangle ninety-five (95) per cent of the plotted points of intersection shall be within point zero one five (.015) inch from all radial lines common to the respective points, and the remaining five (5) per cent of the plotted points of intersection shall be within point zero two five (.025) inch from such radial lines. If at any radial point the contractor can show to the satisfaction of the Soil Conservation Service that a combination of distortion due to tilt and differences in elevation in pictures common to the point have introduced errors in excess of the amount heretofore stated, which it is impractical to correct by adjustment to the isocenter, special allowances will be made with respect to the radial point in question. In any event the amount of tilt shall not exceed the conditions in paragraph 8 of the attached specifications."

(This limit of tilt not in excess of 5 degrees and to an average of 2 degrees in any ten mile section of flight line, nor more than 1 degree for the entire job.) The mosaic maps made for the Soil Conservation Service, based on the specifications, served as base maps.

Recently mosaic maps have been made for the Army as the means of furnishing it with tactical maps of various areas. At first there was a question as to whether mosaics could be made to a degree of accuracy necessary for their use in military operations. It was decided to use some test areas and the specifications written by the Army Map Service covering their requirement of accuracy are as follows:

"ACCURACY—All control points which have been utilized in the radial control layout shall be located on the mosaic within .02 inches of their true geographic position. At least 90% of all identifiable picture points and photographic images of any features which can be identified by examination on the ground and the

pictures will appear within .05 inches of their true geographic position and no identifiable picture points shall be in error by more than 0.10 inches." This was the specification used for maps made to a scale of 1:20,000 and there were other stipulations in the specifications as to means of assembling the mosaic, of the basic control to be used, the projection, grid and other data required according to Army requirements

The uses to which a well controlled mosaic may be put are many and varied Careful consideration should be given to the detail and information which it is necessary to secure from the mosaic as an aid in planning the work as to the scale of the original photography in the various steps in its making Military uses of accurate mosaics are obvious, in that the wealth of detail enables the operations officer to readily locate himself on the map and plan his movements with knowledge as to the conditions Artillery fire can be directed at definite objectives when advancing in unknown territory. This is especially true in modern warfare when it is so vitally important to plan movement of troops and equipment so as to afford maximum cover from aerial observation and bombardment.

Planning and zoning problems in cities or counties are most adequately served by the use of satisfactory mosaic maps in that the relationship of the various congested areas, the existing roads, cultivated and uncultivated areas, streams, lakes, recreational facilities and the type of data which is most important in formulating a satisfactory long range plan, can readily be seen. The locating of resources such as timber, oil or minerals, and study of geological data is greatly expedited by the use of mosaic maps since large areas can be studied at one time. The relationship of the resources to existing transportation or means of access to the territory can be determined as the importance of such information is vital in determining whether or not resources are of economic value. Flood control projects, reservoir sites and problems concerning drainage are again more readily studied when an adequate controlled mosaic of the area is available.

A large scale mosaic of a congested area is the most advantageous way of making a thorough study of traffic conditions, necessity for street widening or changes to eliminate congestion and bottlenecks caused by inadequate facilities. Engineers confronted by right-of-way problems, planning transmission lines, long distance telephone lines or pipe lines, through highways and parkways, have accepted controlled strip mosaic maps as a means of securing the most satisfactory answer to their problem. The need of numerous preliminary surveys in the field is almost entirely eliminated if an adequate strip mosaic map is available because each time a line is plotted across the map between identifiable points it is the equivalent of running a preliminary field survey. Furthermore, by having a strip of territory a mile or two wide the most satisfactory location can be made, whereas with ground investigation usually a compromise is arrived at due to limited information secured by the field engineer

2. Preparation for Laying Mosaic

It can be readily appreciated that since the mosaic map is only one in a series of products of a complete aerial photographic mapping project it is essential that the planning of the project on the whole be well performed in order that the final product will be at the correct scale to supply sufficient detail and yet be economical. Mosaics can be and are compiled from all types of vertical photography, taken at various scales. The ideal would be reached when camera and altitude are selected to give proper negative scale with due regard for the copying proc-

esses and the scale desired in the finished product Mosaic maps when laid by experienced personnel are usually most effectively made if the scale of the ratio prints is between 1 to 1 and 1 to $1\frac{1}{2}$ times the scale of the original negative. In instances reduced ratio prints have been used in the preparation of mosaics and the claim has been made that inexperienced mosaic layers can more readily handle small prints than larger ones. However, experience indicates that it is better to use a slightly larger print than the original exposure in order to secure the best type of mosaic map The amount of control that should be plotted on the board with relation to the scale and the number of flight strips and their length is a subject which will be more adequately covered in another part of this *Manual*, and should be seriously studied in order to be assured of securing the final accuracy which is desired. Excess control is both expensive in the laboratory work as well as requiring considerable expensive field work, but on the other hand inadequate control will very definitely limit the usefulness of the mosaic if accuracy is an important factor.

Suitable hard surface composition boards should be laid on a floor or platform covering the extent of the entire project or if this is not possible, a large part or section of the project On this board a suitable projection is drawn and the control stations accurately plotted and checked for position. Any system of grid which is to appear on the map such as a state or military grid or both should also be plotted. The sheet layout having been decided upon, this too should be indicated on the master layout. The satisfactory aerial exposures having been secured, either contact prints on double weight or low shrink paper should be made depending on the final accuracy required.

Selection of radial points as well as the identification of control points by image, having been accomplished, the necessary templets are made and assembled on the master layout board When this assembly has been satisfactorily completed the location of the radial intersections secured by the templets are marked on the large board. Transparent acetate overlay sheets the size of the small boards which are to be used in the making of the mosaic are now placed in the proper position over the master layout in accordance with the sheet layout which was previously indicated on the large board. All of the points located by the intersections of radials and the centers of the prints are transferred to this transparent overlay sheet. In addition to including the territory shown as a net area on the small board the overlap on all sides should also be included as a means of carrying forward from one small board to another. Due to the new Aero Service Corporation break-away system developed for making mosaics it is possible to use individual small boards corresponding to the final sheet layout without difficulty concerning overlap from one small board to another, so that as mentioned, the transparent overlay sheet is used to transfer the positions of the radial intersections from the master laydown board to the small individual board which might correspond to an Army map size sheet or $7\frac{1}{2}$ minute quadrangle. After intersected points have been transferred from the transparent overlay to the small boards, preliminary work prior to actually laying the mosaic itself has been completed.

3. Necessary Material

A hard press board with a smooth surface on one side similar to masonite is a satisfactory base for mosaic mapping; if the board is too hard there will be difficulty in marking the intersections which locate the points, some difficulty will be encountered in that the adhesive may not hold It is important that temperature change and humidity conditions have a minimum effect on the dimensions of the board. Plywood has been used but in many instances the grain of the ply-

wood shows through the print, thereby giving a design or pattern which is undesirable. An adhesive must be selected which is suitable for the temperature and humidity conditions under which the work is being performed.

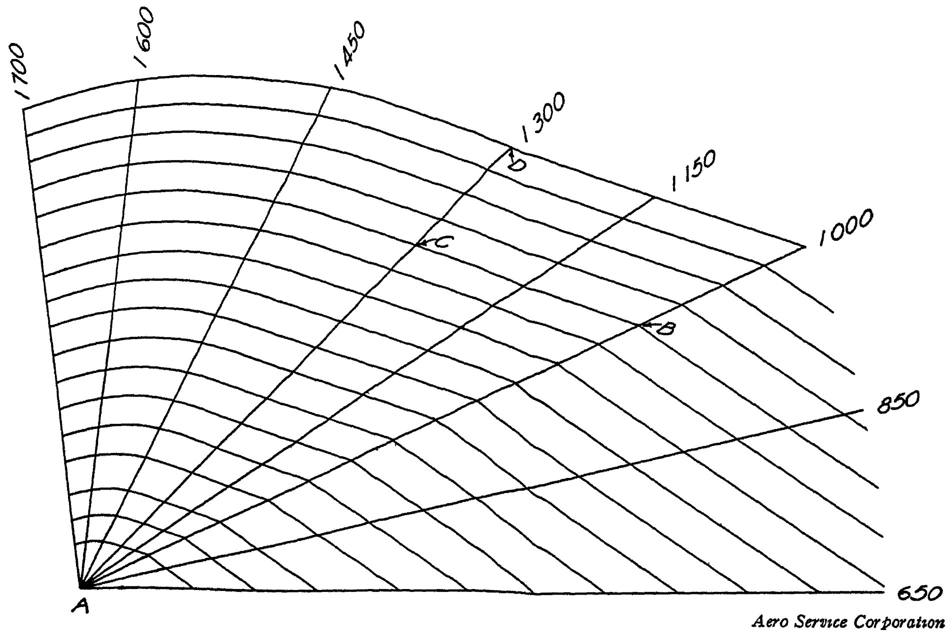
Ratio prints rectified to compensate for the displacement due to relief in the terrain should be made on single weight glossy paper. There are various opinions as to whether sufficient time and effort should be taken in making the ratio prints to maintain a uniform tone match between the various assembled prints or whether it is more economical to make ratio prints with only a fairly satisfactory tone match and then compensate for the variations by retouching or dying the copy negative prior to the final reproduction of the mosaic map. It is a good practice to secure the best possible original tone match in the prints so that only a minimum amount of blending will be necessary, since all blending operations have a tendency to hide or confuse the minute detail which is found in good aerial photography. Experience of the Aero Service Corporation indicates that the most satisfactory ratio prints are obtained by use of a horizontal rectifying camera using arc lights with a sufficiently high light intensity so that the lens may be stopped down to a very small aperture The focal depth is increased in this manner and the sharpest detail obtained. If the rectified prints are made in flight strips by well qualified and experienced dark room operators the uniform tone from print to print and strip to strip will appear on the completed mosaic like one large photograph.

4. Control and Computation to Rectify the Individual Photographs

In view of other chapters which deal with photogrammetric control and the distribution of ground control necessary to secure various degrees of accuracy, the assumption is made that adequate photogrammetric control has been established by an accepted templet method. For maintenance of accuracy the best prints to use for photogrammetric compilation are low shrink contact prints having the image points which are to be used for radial intersections marked and circled to aid in checking. These points should be picked with the aid of a stereoscope in order to assure accuracy. As previously mentioned the templet control to determine the intersections of the radials is marked on a large board so that the entire area or a large part of it may be assembled at one time. A transparent overlay sheet is made which shows the intersected points as plotted on the large assembly board. The transparent overlay sheets can be approximately the size of the small mosaic boards which are to be prepared and will be oriented by grid lines or sheet lines which are marked on the master assembly board.

As a means of expediting the work, the relationship between the radial points as selected on individual prints and the position of the same image points as determined by the radial plot (the intersection of 3 or more radials to the same image point), the contact print is oriented beneath the transparent overlay which shows the radial intersections as this data was transferred from the large master board. After the prints have been oriented in the proper position a pin prick mark is made on the print showing the intersected position. After this procedure has been completed for all of the 8 or more points we now have the contact print marked and a ready comparison can be made between the picture image point and the intersected position of the same image point. Ratios between those points may be computed by various methods, one of which is the special ratio computing chart of the Aero Service Corporation (Fig. 1). The first operation is to locate and pin the chart point A at the center of the contact print, then ro-

tate it until the intersected position conforms with the 1 to 1 line The position where the plotted point falls on the line is held by a needle point or fine pencil (*B* on Figure 1). The entire chart is now rotated until the picture point corresponds with the same traverse line as indicated in the figure at point C. By following the line originating in point *A* that is nearer the intersection at Point *C*, to the edge of the chart, the ratio that will be necessary to bring the image point to the intersected point is determined as shown at point *D* on the figure This same routine is carried out for all the other points and ratios are marked on the print for each of the image points used in the radial plot.



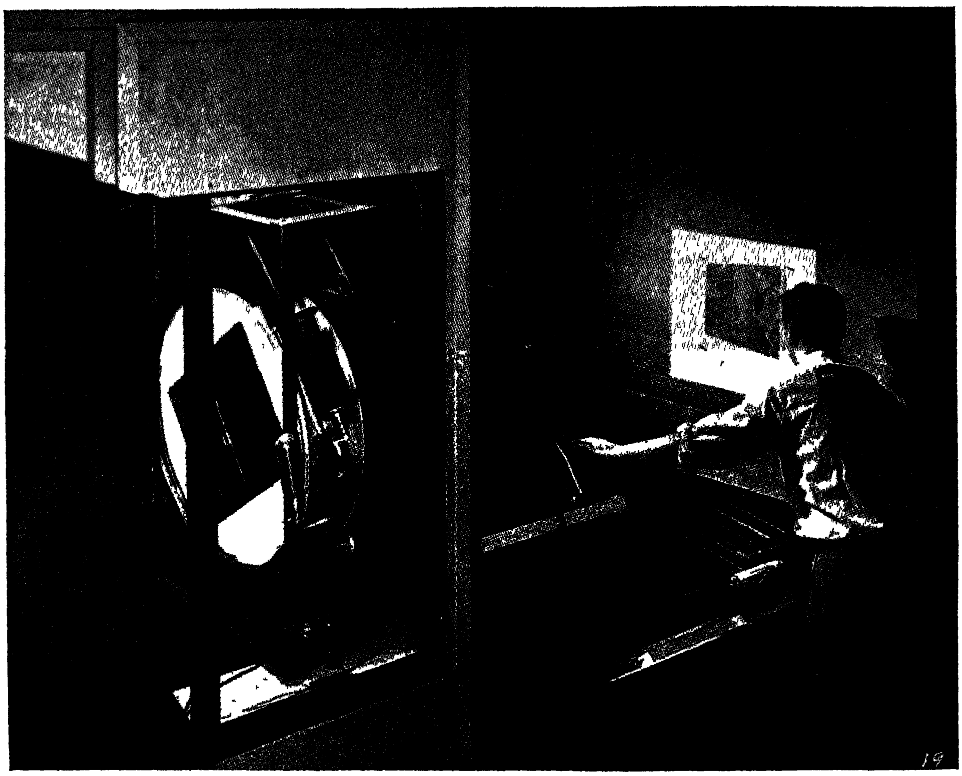
Aero Service Corporation

FIG. 1 Chart used to compute print ratios. 1 Pin *A* at center of print. 2. Rotate till radial line intersection as transferred to print is on 1 000 line as at point *B*. 3. Note transverse line as at *B* and rotate until this transverse line is over image point on print used to secure radial line intersection position as at *C*. 4. Read along the line to determine ratio as at *D*.

By examining the nature of the terrain on the print as well as the various ratios, a line is selected which while going through the center of the print will also connect two points requiring an equal ratio. This line is assumed to be the axis of tilt but it should be remembered that it is not necessarily the true axis of tilt as recorded at the instant of exposure of the aerial camera but a compromise which corrects or compensates for the aerial camera tilt at the time of exposure and for the displacement due to ground relief. It is a line along which the print may be tilted to bring the image points as near as possible to the true geographic position of the same points determined by the radial plot. In order to determine the tilt angle necessary to compensate for points on either side of the assumed tilt axis, a graphic chart should be prepared for each rectifying camera used. By placing a grid in the negative holder of the camera and introducing arbitrary enlargement factors and various tilt angles, a chart may be plotted based on the resultant movement of certain points which have been selected, usually about 3 inches from center of the grid corresponding to the negative size. After the

chart has been prepared showing these amounts of variation, the angle of tilt necessary to bring about the correct movement of the points other than those along the assumed axis of tilt can be determined. In other words, if the points along the axis require a 1 2 diameter enlargement the difference between the ratio of those points and one 3 inches from this line is 0 005 inches, then approximately 1 degree of tilt would have to be added for each such five thousandth change. This is just an arbitrary example; of course, the actual variation will depend upon the rectifying camera lens and therefore each camera should be calibrated in order to determine the amount of tilt, in or out, which is necessary Also some differences may be required between the swing angle of the negative holder and the easel for enlargements greater than one to one. In many instances it may be necessary to make more than one rectified print in order to secure the proper position of the points when laying the ratio prints into the completed mosaic Proper allowance must also be made for variations in the ratio prints caused by processing. This can best be determined by trial and is a definite factor which must be taken into consideration when preparing ratio enlargements.

After the ratio prints have been made and before they are cut and trimmed for laying into the mosaic, it is advisable to check the ratios with the overlay sheets to determine whether or not the various image points will correspond with the intersected points of the same features. This check will enable corrections to be made before the operation of mosaicing the prints is undertaken, thereby saving time and trouble in laying the mosaic.



Aero Service Corporation

FIG. 2. Rectifying camera showing roll negative holder and paper easel—various scales.

5 Rectifying Printers

Experience indicates that horizontal rectifying printers are more serviceable and convenient to handle than other types. In a specially constructed unit (Figure 2) a high intensity arc light is used as a source of illumination which permits the stopping down of the lens to a very small aperture thereby increasing the sharpness of the image as well as reducing distortion when tilts are introduced. The focal length of the lens generally used in the rectifying camera is about 12 inches.

The film holder is so constructed that it will accommodate either roll film or cut film and by changing the adapter the film may be rotated to correspond with the tilt axis selected on the print. The angle of rotation is usually marked on the print to facilitate operation and in addition it may be swung to the degree required. A graduated scale is supplied to facilitate making the settings indicated by the instructions marked on the print The easel for holding the paper is arranged so that it may be swung and the graduated scale furnishes the operator with the correct position to correspond with the swing setting of the negative holder. The enlargement factor required may be set by a scale both at the lens board and the easel. All of the scales are illuminated with shielded lights to facilitate setting under dark room conditions. These scales eliminate the necessity for any measuring of the projected image by the dark room operator to determine the enlargement factor and also gives assurance that the prints will be properly focused to sharpness since the scale setting for the lens board and the corresponding setting for the easel accomplish this result. A swing up to 40 degrees is possible although rarely is that much swing introduced because of resultant angular changes which take place. It is found in practice that beyond 20 degrees it is more satisfactory to make additional prints from sections of the original negative for use in the mosaic rather than endeavor to correct too large a portion of the print by introducing one large angle of swing. The skill of the dark room operation in producing satisfactory tone match in prints is extremely important and cannot be over emphasized.

6. Step by Step Assembly of the Mosaic

The general use of gum arabic as an adhesive for mosaic work is not recommended in all cases as other adhesives are also satisfactory. It is difficult to name one satisfactory adhesive for all conditions since changing conditions of humidity and temperature make it necessary to approach the problem differently. If the locality has a relatively low humidity, far greater difficulty will be experienced in laying a satisfactory mosaic than if the humidity is normal, 55% to 65% relative humidity. Some adhesives have the advantage that the print may be laid dry. Under favorable humidity conditions very little water need be added to such paste and therefore the print is not subject to expansion because of moisture. If the proper scale ratio and tilt correction has been applied so that the prints fit the plotted position of the points on the mosaic directly, the adhesive with a small amount of water added will enable the mosaic operator to mount the print in its proper position with the least amount of difficulty. On the other hand, if it is necessary to expand prints for proper scale and position then the print should be wet or moistened to cause the necessary expansion before the paste adhesive is applied for mounting on the board.

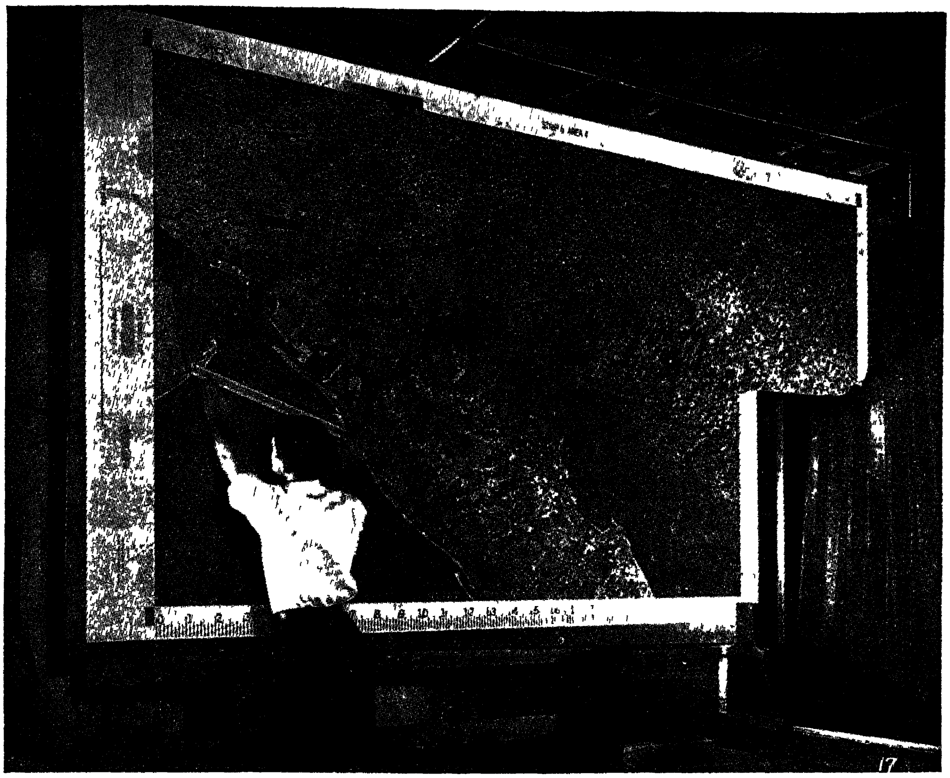
One point which is extremely important in selecting an adhesive is to be sure that it is free of acids or chemicals which might react with any free hypo remaining in the print and cause rapid discoloration Proper washing of the ratio prints

Aero Service Corporation

FIG. 3. Aero Service Corporation breakaway system of laying mosaic. (Note)—Tape on incompleted section. Strip cut and removed to give perfect match and overlap for new section. Necessary tools. Operator removing tape and separating assembled strip from tape.

is important, but adhesives with any appreciable amount of free acid will cause discoloration. Usually mosaic maps are copied before discoloration is extensive enough to cause much loss of detail or change in tone.

In most instances mosaic maps are laid on large boards, either 6×8 feet or dimensions of that order, depending on the number of units which are required to cover a given area. However, a method known as the break-away system has been developed by Aero Service Corporation which enables mosaics to be laid on boards of smaller sizes corresponding in size with the final units to be delivered. When this method is used masonite boards of the proper dimensions are prepared so that the board extends about 6 to 8 inches beyond the net area for overlap. Assuming that the ratio prints have been made and that the intersected position of the image points, as plotted on a transparent overlay sheet, have been transferred to the small mosaic board the operator is now ready to start work. A strip of drafting tape approximately 6 inches wide is laid along the edges of the net area wherever this will extend into adjacent sheets. The image points on the contact prints are marked on the ratio prints and should correspond with the intersected positions as marked on the mosaic board. With the print held on the plotted control an inspection is made where it overlaps the adjoining mounted prints to determine a line where the new print best matches the existing work.

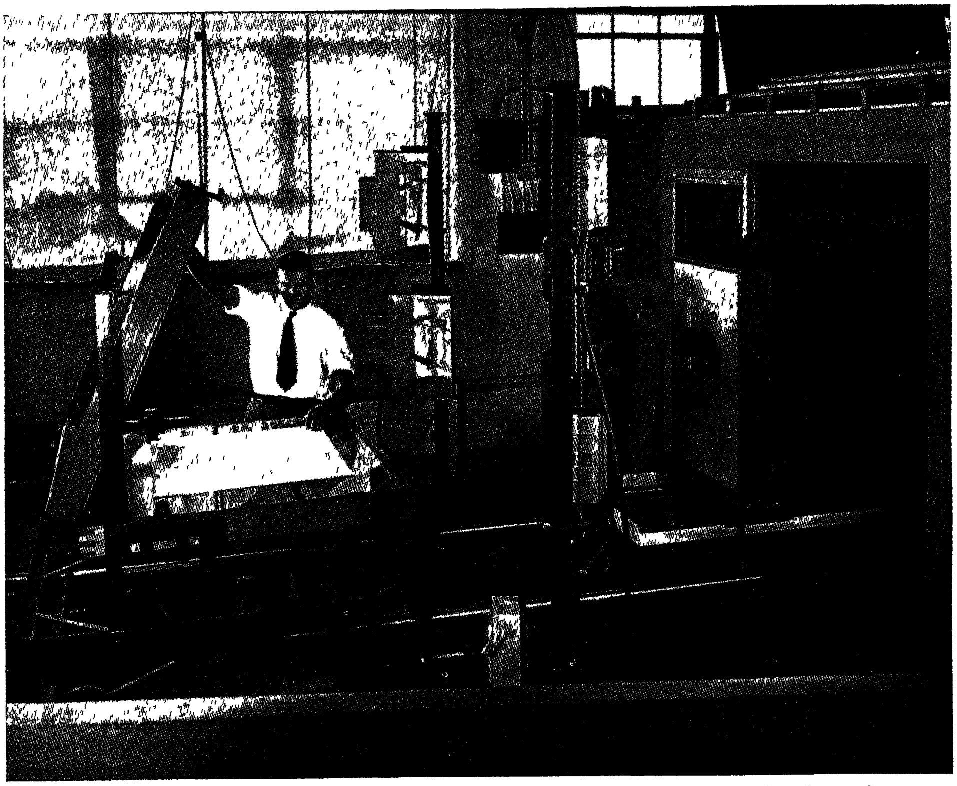


Aero Service Corporation

FIG. 4 Large mosaic in position for copying on single negative.

This line will usually fall along points of equal elevation common to the two prints and is the match line decided upon. This line is not necessarily a line which follows any given feature as a means of hiding the joint but a line which gives the best match between the prints. The print is then cut on the emulsion side just enough to cut through the emulsion but not through the paper and by tearing back along the cut a beveled edge can be secured. This is further beveled by sanding. Care should be taken that this edge is not sanded so fine that the emulsion becomes transparent but sufficiently thin to permit it to lay closely when placed down with the adhesive.

After all of the net area and the matched area has been completely covered with prints, the entire assembly is checked for accuracy against the transparent overlay to assure that all prints have been placed on their proper control. A sharp cut is now made along the net line of the one particular section that has been completed and the scotch tape, including the mounted prints, is removed from the mosaic board. After this operation has been completed the assembled mosaic prints are removed from the scotch tape as one continuous strip. This strip is then transferred to the adjacent board and serves as a perfect match from which to continue on when assembling the prints in the next area. By this method it is possible for a larger number of operators to work on a given area, because after the first section has been completed within the center of a large area, four adjacent boards are immediately ready for further work, and as the mosaic operation progresses, more and more boards become available for indi-



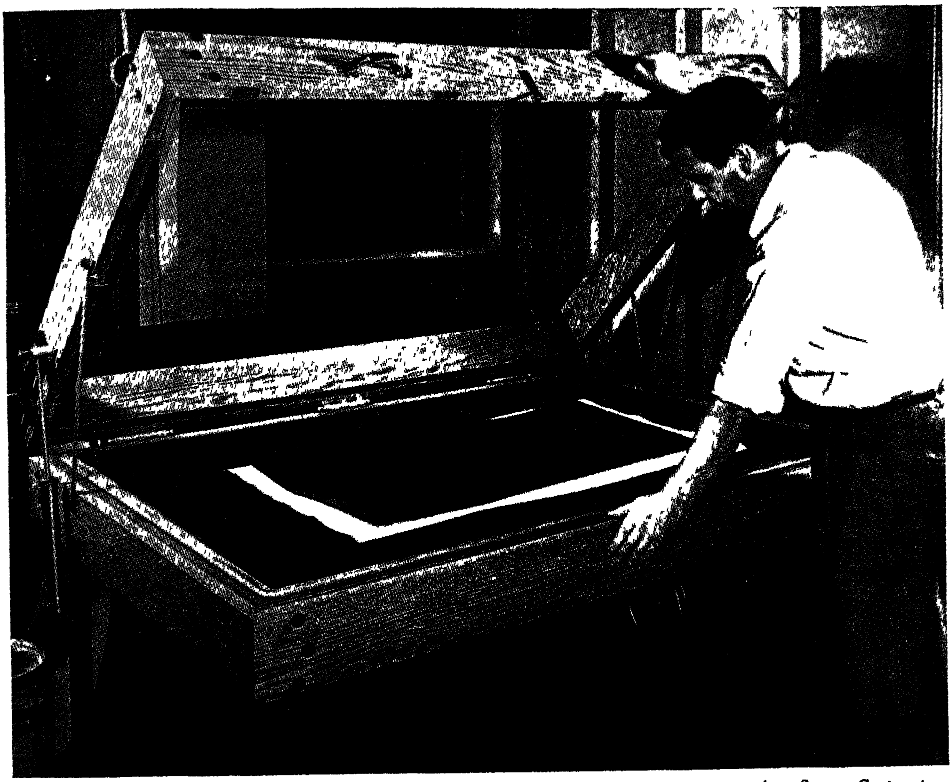
Aero Service Corporation

FIG. 5. Copy camera with vacuum frame Negative capacity 40"×40".

vidual operators to work upon The ease with which mosaic work can be performed, when working on a small board, as compared with a large and unwieldy board is apparent. The need for using a border mask while copying from a large mosaic is eliminated by using the individual units which are identical in size to the material for final delivery. The main or master control board on which all of the radial control is plotted as a unit may be maintained as a unit for purposes of checking as the mosaic is laid in small individul units. Special sections of any part of the entire project may be more readily compiled, completed and delivered because greater freedom of action is possible by the use of these smaller units and the break-away system. Copying of the individual unit is faster because a standard set-up is made and the units can be readily centered and difficulties as to border mask and changing of data from one section to another is eliminated. Experience indicates that the break-away system is a very important contribution to the technique of compiling mosaics.

7. Checking the Scale of the Completed Mosaic

With the use of small units in mosaicing the checking of the accuracy is materially facilitated because of the transparent overlay sheet which has been made This sheet which has marked on it the position of all intersected points and centers of the various prints as well as the sheet corners, grid lines and other data, can be directly laid over the completed section of the mosaic and image



Aero Service Corporation

FIG 6. Vacuum backed contact printing frame Capacity 40″×60″.

points checked as to their positions in reference to the intersected positions of the same points The positions of grid lines and net lines along the edges of the mosaic also will necessarily correspond with those same lines as marked on the overlay sheet

8 REPRODUCING THE MOSAIC

The reproduction of a mosaic is an extremely important step because the actual copies that are used are reproductions, photographic or otherwise, from the original mosaic boards (Figures 4 and 5). Selection of the proper method of copying and reproducing is important and should be decided on the basis of the amount of detail clarity required to obtain adequate use of the material. The most satisfactory though perhaps more expensive method, is to photograph the mosaic area on a 1 to 1 scale and make copy prints from a copy negative by the contact method (Figure 6). This insures the minimum loss of detail In some instances, however, where it is necessary to produce the final copy at a scale larger than the original, copy prints are made by projection methods. When this is undertaken it is especially important that both the film and the paper be held flat and that the dark room technique be adequate to produce the best possible results.

Blending of the original mosaic or the copy negatives is an important step in producing satisfactory reproduction of mosaics. Experience indicates that the most satisfactory results can be secured by endeavoring to get the best possible

tone match of the ratio prints used in the mosaic and that some of the slight blendings remaining should also be done on the original mosaic prior to copying. A black dye, which may be diluted in water to obtain the proper tone of gray, is required. This grey toning material can be readily used in parts of the mosaic that need toning down and because it is in black or gray the final result can be more readily judged than if a different color is used. If used sparingly the loss in detail is not discernible. If additional work is necessary, this blending with the same material can be performed on the copy negative and again should not be applied in any large amount. It appears that, by blending a small amount on the mosaic and if necessary, an additional small amount on the copy negative, the resulting copy print is better than if all of the blending is done on the copy negative.

9. Editing

A completed mosaic map requires a considerable amount of editing on both the mosaic proper as well as on marginal information. Printed names and numbers may be prepared on a transparent medium coated with an adhesive, so that when the material, including the printed information is placed on the body of the mosaic, only the printed information is visible and the detail of the mosaic may still be copied without appreciable loss. Names of towns, cities, geographical features, highway numbers, railroad designations, are usually required on mosaics. In addition, extensive data is required in the margin, which may be readily applied if the break-away system is used and white mounting paper is framed around the edges of the mosaic upon which the marginal data is applied, again by the use of printed gum back white material. Grid line numbers, declination diagrams, road destinations, scale, title information and other data as may be required, is applied prior to making the copy negatives or lithographic plates if it is to be a printed reproduction.

CHAPTER XI

STEREOSCOPIC MAPPING METHODS

ELEMENTARY ELEVATION DETERMINATION FROM AERIAL PHOTOGRAPHS

Revere G. Sanders

WHEN a novice in the art of photogrammetry becomes fairly expert in the use of the simple stereoscope, it is only natural that a desire to take the next step forward arises The intention of this article is to start off the novice photogrammetrist on the most interesting and certainly important phase of the use of aerial photographs. The determination of elevations from aerial photographs can become complex and deeply involved in the eventual attempt to achieve the ultimate in accuracy. However, to become capable of determining elevations precisely for map work of a high order of accuracy, the novice must begin at the beginning—and this article constitutes one good way of beginning. Furthermore, although the beginner may never aspire to stereoscopic elevation determination work of the highest order of accuracy, he can rather quickly become proficient at determining elevations to a lesser degree of accuracy for a multitude of highly important purposes.

This article will not attempt to explain the reason why certain procedures are followed or why certain formulae are used. No theoretical explanations will be given It is possible for the beginner to learn quickly how to determine elevations without a thorough knowledge of the theoretical background. Naturally there is a point beyond which one cannot go without an understanding of the theory of stereoscopic measurements. For a complete treatise on stereoscopic measurements the student is referred to Chapter VII, "Elements of Stereoscopy" and "Practical Application of the Stereocomparagraph" in this Chapter.

Simple Equipment

This article will deal only with the simplest form of stereoscopic equipment available for elevation determination. The novice cannot expect to learn overnight how to operate the more complex stereoscopic plotting machines which are available for mapping from aerial photographs to a high order of accuracy. However, the fact that the plotting equipment discussed in this article is referred to as simple equipment does not mean to infer that the equipment cannot do valuable work; nor does it mean to infer that a person capable only of operating the simplest equipment cannot do very necessary and effective work. As a matter of fact the simple equipment described herein has several outstanding advantages.

a) The equipment being simple, a novice can become rather proficient in its use for elevation determination within a period of only a few weeks.
b) The stereoscopic principles involved in this simple equipment are similar to those involved in the more elaborate plotting machines. Consequently, the simple plotting equipment can be utilized in the training of operators for the more elaborate instruments
c) In a large percentage of cases where elevation determination is necessary, the degree of accuracy which can be tolerated is much lower than that which can be attained by the elaborate instruments Consequently the simple devices become adequate and, in fact, economically desirable
d) The low cost of the simple instruments together with the fact that new operators can be quickly trained makes it possible to put a large number of men and instruments to work on a given project to complete the project in a short period of time.

Limitations of Simple Equipment

A word of caution is necessary so that the novice will not become too enthusiastic over what can be accomplished with simple plotting devices. In the

past, unfortunately, enthusiastic advocates of the simple plotting equipment have encouraged potential users of aerial photographs or of the simple plotting instruments to believe that a high order of accuracy can be attained However, the exercise of common sense, completely disassociated from any photogrammetric knowledge, should be sufficient to convince anyone that simple plotting devices costing only a few hundred dollars at the most, have a far lower potential accuracy than the more elaborate plotting machines costing many thousands of dollars.

Actually, under optimum conditions and with a reasonably expert operator, and with good control available, spot elevations can be determined with the simple equipment to an order of accuracy of $\pm 1/400$th or 1/500th of the flight altitude. That is to say, for an airplane flying at 10,000 feet the accuracy to be expected under best possible conditions would be somewhere between ± 25 feet and ± 20 feet. When it is desired to actually sketch contours (more correctly called form lines) with the simple equipment, the possible accuracy under best conditions falls off to approximately $\pm 1/250$th of the flying altitude. Thus, for an airplane flying at 10,000 feet the accuracy of the form lines would be in the neighborhood of ± 40 feet As the conditions depart from the ideal with the introduction of tilt, tip and difference in elevation between successive photographs, the accuracy possible with the simple equipment drops rapidly below that mentioned

It is also necessary to have a certain amount of ground control (points whose elevations have been determined by ground surveying methods) in each stereoscopic pair of photographs The more control available, the greater will be the accuracy of the elevations determined with the simple plotting equipment and the more able will be the operator to compensate for tilt and other errors. Six well-distributed control points throughout the area of overlap is considered a desirable minimum. However, when the situation demands, very effective elevation information can be gained with as little as one ground control point from which to start.

Equipment Required

The elementary elevation measuring device, herein described, consists of four main parts (Fig. 1).

a) There must be a stereoscope to give a three dimensional view of the stereoscopic pair of photographs
b) Either attached to the stereoscope or separate from it is a floating mark system.
c) There should be a drawing attachment which can be connected to the floating mark system when desired.
d) Whenever the drawing attachment is used, the floating mark-drawing attachment combination should be attached to a parallel motion protractor.

The floating mark system consists usually of two glass discs, each of which has a very minute dot in the center. Sometimes these floating marks are incorporated in the stereoscope itself, whereupon the assembly becomes known as the Stereocomparagraph, as shown in Figure 2. In another case the floating marks are made a part of a measuring bar which is called a Parallax Bar and which is independent of the stereoscope as shown in Figure 3. Actually the Parallax Bar is of no value by itself, but must be used in conjunction with a stereoscope as shown in Figure 3.

The Stereocomparagraph is preferable where an instrument is needed continuously for the purpose of determining elevations or delineating form lines. The Parallax Bar is more desirable where the stereoscope is used primarily for visual inspection or interpretation of stereoscopic pairs of photographs and

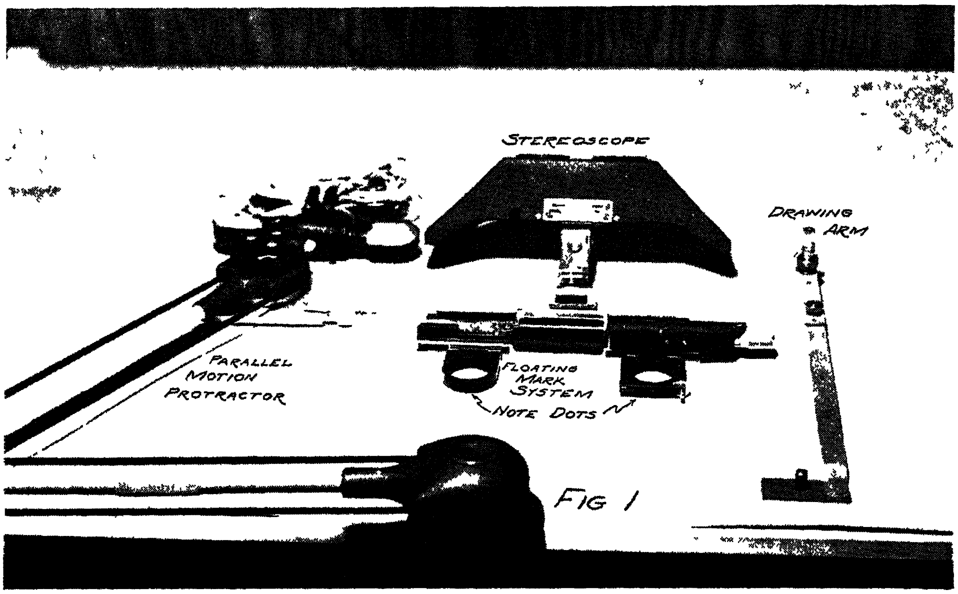

STEREOSCOPE
DRAWING ARM
PARALLEL MOTION PROTRACTOR
FLOATING MARK SYSTEM
NOTE DOTS

FIG 1

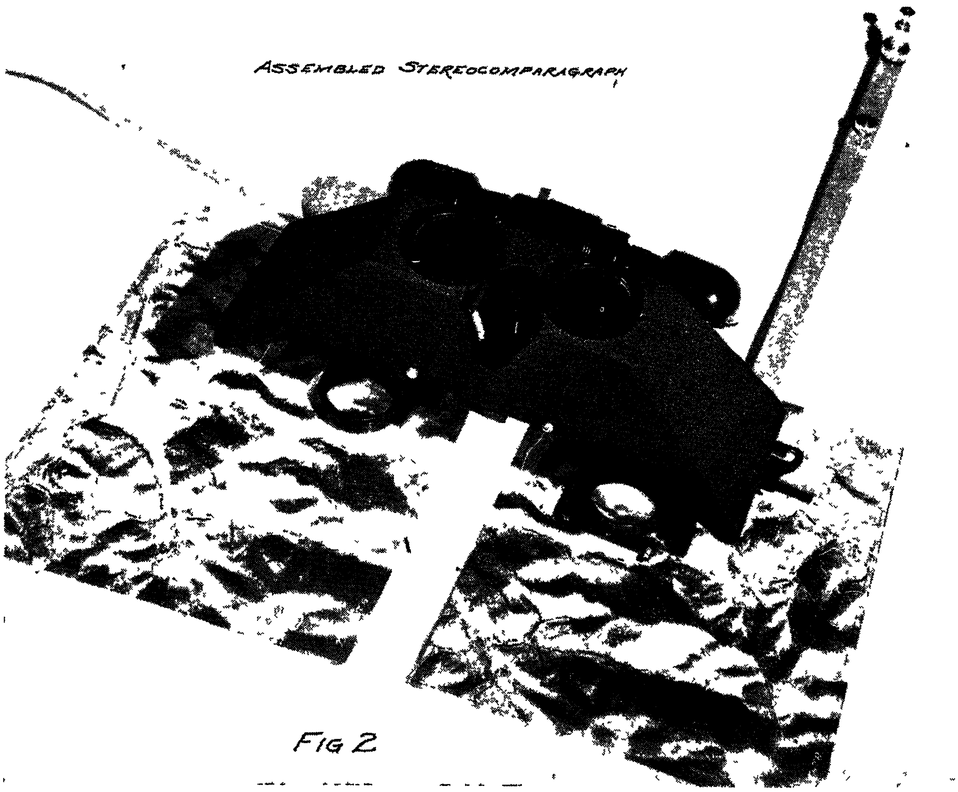

ASSEMBLED STEREOCOMPARAGRAPH

FIG 2

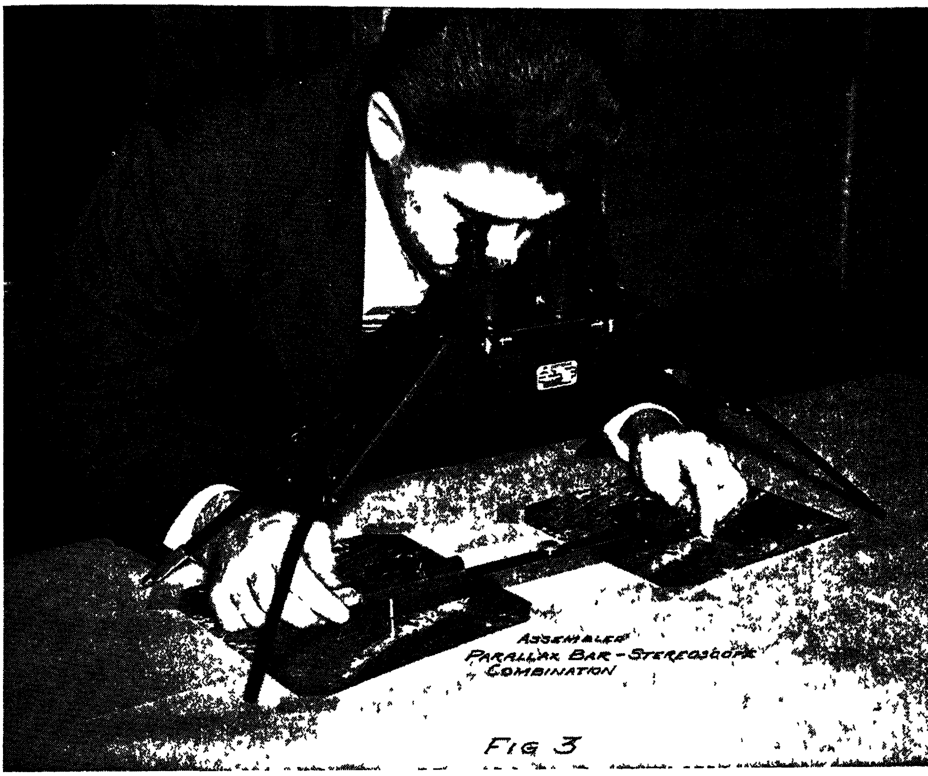


Fig 3

where only occasionally it is necessary to determine elevations or to delineate form lines. The accuracy and ease of operation of the two generalized types of equipment are essentially the same.

Floating Mark

Whether the simple plotting device is of the Stereocomparagraph type or of the Parallax-Bar-Stereoscope type, the determination of elevation is entirely dependent upon an understanding of, and ability to use the floating mark. The floating mark consists actually of two so-called "half" marks which were described above as being two circular or rectangular pieces of glass, each of which has a very fine dot at the center (Fig. 1). By the design of the Stereocomparagraph or by the design of the Parallax Bar, when the photographs are properly oriented and the stereoscope or Stereocomparagraph is in place for three dimensional viewing, the "half" marks are lying approximately on top of the same bit of image detail in each of the photographs. When the operator looks through the stereoscope or Stereocomparagraph, he sees only one dot. The reason he sees only one dot when actually there are two dots in the field of view of the stereoscope is the same as the reason for the fact that he sees only one photograph when he looks through the stereoscope at a pair of photographs. The one view which results from the pair of photographs is in three dimensions. Likewise, the one dot that he sees which is the floating mark is also seen three dimensionally.

An important part of a Stereocomparagraph or of a Parallax Bar is the micrometer which moves the right hand half mark either toward or away from

the left hand half mark. As this is done, the single floating mark which is seen by the operator looking through the stereoscope appears to rise up or to sink down in the field of view The mark can be made to rise up to the point where it is resting on top of a hill, or it can be made to sink down until it lies in the bottom of a valley

Generally speaking, to determine the difference in elevation between the hill top mentioned and the valley below, the operator causes the floating mark to rise up and, to the best of his ability, come to rest on the top of the hill. The micrometer reading can then be noted. The operator then causes the floating mark to descend until, to the best of his ability, it appears to lie on the bottom of the valley. Again he reads the micrometer. The difference in micrometer readings can be found by subtraction and is called the parallax difference. With the value thus found, tables can be entered from which a knowledge of the difference in elevation between those two points can be determined.

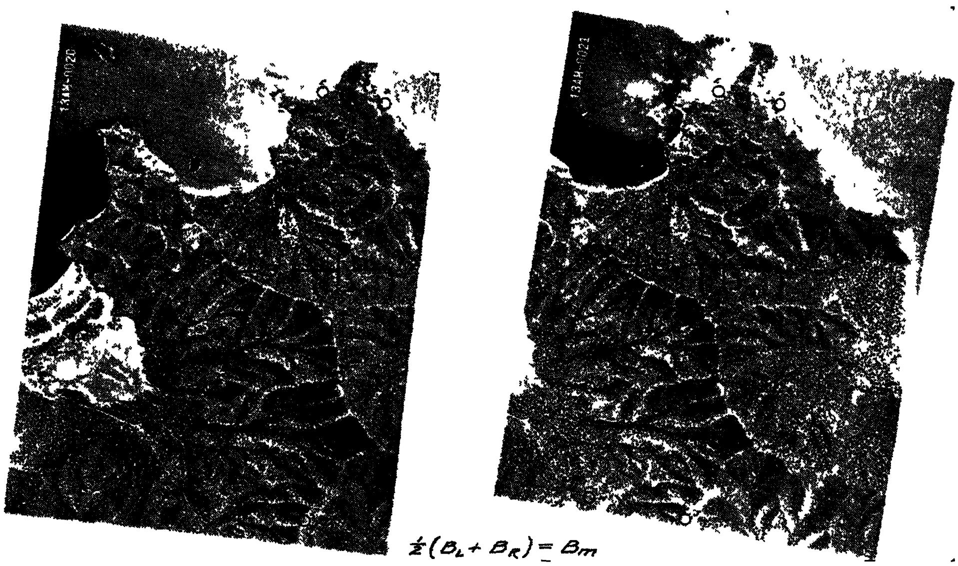


FIG. 4

The judgment as to whether or not the floating mark is lying exactly on the ground at any given point depends largely upon the operator's eyesight and experience. The micrometer which moves the right hand half mark can be read directly to hundredths of a millimeter and can be estimated to thousandths. After a few weeks' training, a novice should be able to read the parallax measurements on any given point in the stereoscopic view within $\pm .05$ millimeters consistently. With prolonged experience, an operator with particularly good eyesight and stereoscopic judgment can reduce this to approximately $\pm .03$ millimeters. Consequently, for the beginner, a good practice problem is to select six or eight points at various elevations and in various portions of the stereoscopic view and to measure the parallax of each one in succession. Go around the circuit several times marking the readings down independently and after the third or fourth time around, check to see how closely the parallax readings on each point agree with one another.

Parallax Tables

As mentioned previously, the difference between the micrometer readings of two points in the stereoscopic view constitutes a measure of the difference in elevation. It does not give the elevation in terms of feet. It is necessary to convert the difference in micrometer readings, which is really the difference in parallax, into difference in elevation With certain known factors to begin with, formulae and computations can be resorted to in order to convert the parallax difference into difference of elevation as expressed in feet. However, where a lot of elevations are being determined it is slow and cumbersome to perform a computation each time.

For many years a book of tables known as "parallax tables" has been published by the British government and has been used. However, in recent years it has been impossible to obtain any of these parallax tables. Furthermore it was felt that the tables could, in the light of present day knowledge, be set up in a more simple form. Parallax tables are now available in this country and are incorporated in "Practical Application of the Stereocomparagraph" and in the War Department publication, *Topographic Drafting*, T.M. 5-230 Incidentally this particular publication is one of the most practical treatises on basic photogrammetric mapping available and every photogrammetrist should have a copy.

The compilation of parallax tables constitutes quite a problem because a number of factors have to be taken into consideration Since these factors change according to the different conditions under which the photographs are taken, it becomes necessary to make certain assumptions. One assumption made in the parallax table is that the length of the stereoscopic base (symbol B_m) is 100 millimeters.

Point	*Elevation*	$(H-h)$	$\Sigma\Delta p$	Δp	$\Delta p \times \frac{B_m}{100}$	*Micrometer Reading*
			From "E" to "L"			
E	0	10,500 ft	86 750 mm			10 84 mm *
L	433 ft	10,067 ft.	90 958 mm	4 21 mm	2 35 mm	13 19 mm.
			From "D" to "L"			
D	0	10,500 ft	86 750 mm			10 95 mm *
L	413 ft	10,087 ft	90 760 ft	4 01 mm.	2 24 mm	13 19 mm.

Average Elevation of "L"—423′

* Should be identical except for observational, tip, tilt and altitude errors.

FIG. 5

For the purpose of this article the stereoscopic base can be considered as the distance from the principal point of one photograph to the transferred principal point in that same photograph as shown in Figure 4. It is quite unlikely that the stereoscopic base length of each photograph of the stereoscopic pair will be identical. Therefore the average stereoscopic base should be used Actually, if one finds a photograph with a stereoscopic base of exactly 100 millimeters, it would be by strangest coincidence. However, the effect of change in length of stereoscopic base is one of direct proportion. Therefore, if one measures the

stereoscopic base and finds it to be, let us say, 53.5 millimeters in length, each value of parallax shown in the table must have the ratio of 53.5/100 applied to it. This is a very easy and simple computation. The assumption of $B_m = 100$ millimeters upon which the parallax tables are based does not compromise the accuracy of the tables in any way.

Procedures to Determine the Elevation of a Point

The following is a step-by-step explanation of the procedure to determine the elevation of one or more points in the stereoscopic view.

A stereoscopic pair of photographs and the Stereoscope or the Stereocomparagraph should be set up in strict conformity to the instructions contained in Chapter VII, "Orient Your Stereoscope Correctly."

As mentioned previously, there must be a certain number of points of known elevation in the stereoscopic view known as ground control points. The position of these points should be accurately marked on both photographs of the stereoscopic pair. These points can be marked by a fine pin prick surrounded by a circle of approximately $\frac{1}{4}''$ diameter (Fig. 4). For ease in keeping track of the data, the control points should be lettered consecutively and it is sometimes desirable to actually write on the photograph adjacent to the control point, the elevation of the point. In like manner the points concerning which it is desired to determine the elevation should be indicated with a pin prick surrounded by a circle. Likewise, these points, the elevation of which is to be determined, should be given an identifying letter as shown in Figure 4.

Sample Problem

Assume that it is desired to find the elevation of the hill top point L in Figure 4. Points E and D are at mean sea level and provide starting points from which the elevation of point L can be determined. Actually to get the best determination of elevation of point L a determination should be made, based upon point E, and then an independent determination made, based upon point D. If the two determinations do not agree exactly, which they probably will not, the average should be taken.

a) The first step is to measure the stereoscopic base length on each photograph of the stereoscopic pair. As mentioned previously, the stereoscopic base length on the two photographs of the pair will undoubtedly differ, which is the case in the actual problem under consideration here, where the average stereoscopic base length works out to be 55.85 mm.
b) The altitude of flight of the aircraft from which the photographs were taken must also be known. In this particular case, the altitude of flight was 10,500 feet above mean sea level.
c) The next step is to make up a form in which to insert the various data as shown in Figure 5. Fill in this table with all the known information concerning the points D and E. Inasmuch as the point E is at sea level, the elevation is obviously o. The quantity $(H\text{-}h)$ must next be figured. H refers to the altitude of flight of the aircraft while h refers to the elevation of the point being dealt with at the time Consequently for the point E we have (10,500–0) which leaves us 10,500 feet to insert in the $(H\text{-}h)$ column.
d) It now becomes necessary to refer to the parallax table found in "Practical Application of the Stereocomparagraph" page 464, which for convenience, is reproduced herein (see Fig. 6). Give your attention to the right hand set of three columns and in the column headed $(H\text{-}h)$ find the value 10,500 Opposite that value and in the column headed $\Sigma\Delta p$ find the value 86.750 millimeters and insert this value in the form which has been prepared as shown in Figure 5.
e) The next step is to read the parallax of the point E by the use of the Stereo-Comparagraph or the Stereoscope and Parallax Bar combination. This is done by so moving the instrument and by so adjusting the micrometer that the floating mark appears to lie exactly on the ground at point E. This reading is found to be 10.84 millimeters and this value is inserted in the column headed "Micrometer Reading" in Figure 5. Actually the 10.84 millimeter value is not a single reading but is the average of four or five readings.

$(H-h)$ ft	Δp mm	$\Sigma\Delta p$ mm.	$(H-h)$ ft	Δp mm	$\Sigma\Delta p$ mm	$(H-h)$ ft.	Δp mm	$\Sigma\Delta p$ mm.
14580	0 137	53 922	60	0 150	62 661	40	0 165	72 237
60	0 137	54 059	40	0 150	62 811	20	0 165	72 402
40	0 137	54 197	20	0 150	62 961	12100	0 165	72 567
20	0 138	54 335	13300	0 150	63 111	12080	0 165	72.732
14500	0 138	54 472	13280	0 150	63 261	60	0 166	72 898
14480	0 138	54 610	60	0 151	63 412	40	0 166	73.054
60	0 138	54 749	40	0 151	63 563	20	0 166	73.230
40	0 138	54 887	20	0 151	63 714	12000	0.167	73 397
20	0 139	55.026	13200	0 151	63 866	11980	0.167	73 563
14400	0 139	55 164	13180	0 152	64 017	60	0 167	73 730
14380	0.139	55 303	60	0.152	64 169	40	0 167	73.896
60	0 139	55 443	40	0.152	64 321	20	0.168	74.065
40	0 139	55 582	20	0.152	64 473	11900	0.168	74 233
20	0 140	55 721	13100	0.153	64 626	11880	0.168	74.402
14300	0 140	55 861	13080	0.153	64 779	60	0.168	74 570
14280	0 140	56.001	60	0.153	64 932	40	0.169	74 739
60	0 140	56 141	40	0.153	65 085	20	0.169	74 908
40	0 140	56 282	20	0 153	65 239	11800	0.169	75 077
20	0 141	56 422	13000	0 154	65.392	11780	0.170	75.247
14200	0 141	56 563	12980	0 154	65 546	60	0.170	75 417
14180	0 141	56 704	60	0 154	65 700	40	0.170	75.587
60	0 141	56 845	40	0 154	65.855	20	0.170	75 758
40	0 141	56 986	20	0.155	66 010	11700	0 171	75 928
20	0.142	57.128	12900	0 155	66.164	11680	0.171	76.099
14100	0 142	57.270	12880	0 155	66.320	60	0.171	76.271
14080	0.142	57.412	60	0.155	66.475	40	0.172	76 442
60	0 142	57.554	40	0.156	66.631	20	0 172	76 614
40	0.142	57 696	20	0.156	66.787	11600	0.172	76.787
20	0 143	57 839	12800	0 156	66.943	11580	0.173	76.959
14000	0 143	57.982	12780	0.156	67.099	60	0 173	77 132
13980	0.143	58 124	60	0.157	67.256	40	0 173	77.305
60	0 143	58.268	40	0.157	67 413	20	0.173	77.479
40	0 143	58 411	20	0 157	67 570	11500	0 174	77 652
20	0 144	58 555	12700	0.157	67 727	11480	0 174	77 827
13900	0 144	58.698	12680	0 158	67.885	60	0 174	78.001
13880	0 144	58.842	60	0.158	68 042	40	0.175	78 176
60	0 144	58 987	40	0 158	68.201	20	0.175	78 351
40	0 144	59 131	20	0 158	68 359	11400	0.175	78.526
20	0 145	59 276	12600	0 159	68 518	11380	0 176	78 701
13800	0 145	59 420	12580	0 159	68 676	60	0 176	78 877
13780	0 145	59 565	60	0.159	68 836	40	0 176	79 054
60	0 145	59.711	40	0.159	68.995	20	0 177	79.230
40	0 145	59.856	20	0.160	69 164	11300	0 177	79 407
20	0 146	60.002	12500	0 160	69 314	11280	0 177	79 584
13700	0 146	60 148	12480	0.160	69.474	60	0 177	79.761
13680	0 146	60 294	60	0.160	69.635	40	0.178	79.939
60	0.146	60.440	40	0 161	69.796	20	0.178	80.117
40	0.147	60 587	20	0.161	69 956	11200	0 178	80.296
20	0.147	60.733	12400	0.161	70 118	11180	0 179	80.474
13600	0.147	60.880	12380	0 161	70 279	60	0.179	80.654
13580	0.147	61.027	60	0 162	70.441	40	0 179	80.833
60	0.147	61 175	40	0.162	70.603	20	0.180	81 013
40	0.148	61 322	20	0.162	70 765	11100	0.180	81 193
20	0.148	61.470	12300	0 162	70 927	11080	0 180	81 373
13500	0.148	61 618	12280	0.163	71 090	60	0 181	81.554
13480	0.148	61.767	60	0.163	71.253	40	0.181	81.735
60	0.148	61.915	40	0.163	71.416	20	0.181	81.916
40	0.149	62.064	20	0.164	71.580	11000	0.182	82.097
20	0.149	62.213	12200	0.164	71.744	10980	0 182	82.280
13400	0.149	62.362	12180	0.164	71.908	60	0.182	82.462
13380	0.149	62.511	60	0.164	72.072	40	0.183	82.645

FIG. 6

$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft.*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*
20	0.183	82 828	50	0 101	93 140	40	0 108	99 533
10900	0.183	83 011	40	0 102	93 241	30	0 108	99 641
10880	0.184	83 194	30	0 102	93 343	20	0 108	99 750
60	0 184	83 378	20	0 102	93 445	10	0 109	99 858
40	0.184	83 563	10	0 102	93 547	9200	0 109	99 967
20	0.185	83 747	9800	0 102	93 649	9190	0 109	100 075
10800	0.185	83 932	9790	0 102	93 751	80	0 109	100 184
10780	0 185	84 118	80	0 102	93 853	70	0 109	100 293
60	0 186	84 304	70	0 102	93 955	60	0 109	100 402
40	0.186	84 490	60	0 102	94 058	50	0 109	100 512
20	0 186	84 676	50	0 103	94 160	40	0 109	100 621
10700	0 187	84 863	40	0 103	94 263	30	0 109	100 730
10680	0 187	85 050	30	0 103	94 366	20	0 110	100 840
60	0 187	85 237	20	0 103	94 468	10	0 110	100 950
40	0 188	85 425	10	0 103	94 571	9100	0 110	101 060
20	0 188	85 613	9700	0 103	94 674	9090	0 110	101 170
10600	0 188	85 802	9690	0 103	94 778	80	0 110	101 280
10580	0.189	85 991	80	0 103	94 881	70	0 110	101 390
60	0.189	86 180	70	0 103	94 984	60	0 110	101 500
40	0.190	86 369	60	0 103	95 088	50	0 110	101 611
20	0 190	86 559	50	0 104	95 191	40	0 111	101 721
10500	0 190	86 750	40	0 104	95 295	30	0 111	101 832
10480	0 191	86 940	30	0 104	95 399	20	0 111	101 943
60	0 191	87.131	20	0 104	95 503	10	0 111	102 054
40	0 191	87 323	10	0 104	95 607	9000	0 111	102 165
20	0.192	87 514	9600	0 104	95.711	8990	0 111	102 276
10400	0 192	87 707	9590	0 104	95 815	80	0 111	102 387
10380	0.192	87 899	80	0 104	95 919	70	0.111	102 498
60	0 193	88 092	70	0 104	96.024	60	0.112	102.610
40	0 193	88 285	60	0 105	96.128	50	0.112	102 722
20	0 194	88 479	50	0 105	96.233	40	0 112	102 833
10300	0.194	88 673	40	0 105	96 338	30	0 112	102 945
10280	0.194	88 867	30	0.105	96.443	20	0 112	103 057
60	0 195	89 062	20	0 105	96 548	10	0 112	103 170
40	0 195	89 257	10	0.105	96.653	8900	0 112	103 282
20	0 196	89 452	9500	0 105	96.758	8890	0 112	103 394
10200	0 196	89 648	9490	0 105	96 863	80	0 113	103 507
10180	0.196	89 845	80	0 105	96 969	70	0 113	103 620
60	0 197	90 041	70	0 106	97 074	60	0 113	103 732
40	0 197	90 238	60	0 106	97 180	50	0 113	103 845
20	0 197	90 436	50	0 106	97 286	40	0 113	103 958
10100	0 198	90 634	40	0 106	97 391	30	0 113	104.072
10080	0 198	90 832	30	0 106	97 497	20	0 113	104 185
60	0 199	91 030	20	0 106	97 604	10	0 113	104 298
40	0 199	91 229	10	0 106	97.710	8800	0 114	104 412
20	0 199	91.429	9400	0 106	97.816	8790	0 114	104 526
10000	0 200	91.629	9390	0.106	97.923	80	0 114	104.639
9990	0.100	91 729	80	0 107	98.029	70	0 114	104.753
80	0.100	91 829	70	0 107	98 136	60	0 114	104 867
70	0 100	91 929	60	0 107	98 245	50	0 114	104 982
60	0.100	92.029	50	0.107	98 349	40	0 114	105 096
50	0.100	92 130	40	0 107	98 456	30	0 114	105 210
40	0.101	92 230	30	0 107	98 564	20	0.115	105.325
30	0 101	92.331	20	0 107	98 671	10	0 115	105 440
20	0 101	92 432	10	0 107	98 778	8700	0.115	105 555
10	0.101	92 533	9300	0.107	98 886	8690	0 115	105 670
9900	0 101	92.634	9290	0 108	98.993	80	0 115	105 785
9890	0 101	92.735	80	0 108	99 101	70	0.115	105.900
80	0 101	92 836	70	0 108	99 209	60	0 115	106.016
70	0 101	92 937	60	0.108	99.317	50	0 116	106 131
60	0 101	93 038	50	0 108	99 425	40	0 116	106.247

FIG. 6—continued.

f) The next step is to determine the parallax reading for the unknown point (point L in this case) This is done by causing the floating mark to rest exactly on the point L as in the case of point E previously The resulting parallax measurement amounts in this case to 13 19 millimeters, which value is also the average of four or five different readings This value is also inserted in the column under "Micrometer Readings" (Fig 5)

g) The difference in parallax as found by the difference between the two micrometer readings mentioned immediately above amounts to 2 35 millimeters This value is inserted in the column headed $\Delta p \times \frac{B_m}{100}$ in Figure 5

h) It was explained previously that the parallax tables are based upon photographs having a stereoscopic base length of 100 millimeters However, the difference in parallax of 2.35 millimeters just mentioned above is based upon measurements taken from photographs having an actual stereoscopic base length of 55 85 millimeters In order to convert this parallax difference so that it will be on the same basis as the tables, a proportion must be worked out as follows.

$$2\ 35 \text{ mm} \cdot 55\ 85 = X\ 100$$

X in this case is actually Δp on the basis of the tables and works out to be 4 21 millimeters. This value, 4 21, is then inserted in the column Δp (Fig. 5)

i) By inspection of the stereoscopic view, it can be seen that point L is higher than point E. Therefore, the value of 4 21 millimeters in the Δp column is added to the value 86.750 millimeters in the $\Sigma\Delta p$ column for point E to obtain a $\Sigma\Delta p$ value for point L of 90.958 millimeters which is inserted in its proper place (Fig 5)

j) Refer again to the parallax tables (Fig 6) and under the column headed $\Sigma\Delta p$ look for the value closest to 90 958 millimeters This value can be seen to lie somewhere between 10,080 feet and 10,060 feet. By a process of interpolation such as is commonly resorted to in work with logarithms, it is found that the value of $(H\text{-}h)$ for a $\Sigma\Delta p$ of 90 958 is 10,067 feet This value is inserted in its proper place in the form as shown in Figure 5.

k) There now exists the equation $(H\text{-}h) = 10{,}067$ feet. It is known that H is the altitude of flight of the aircraft, namely 10,500 feet Therefore h solves out to equal 433 feet This is inserted in the elevation column opposite point L (Fig 5). Thus the elevation of point L when computed from point E is 433 feet above sea level.

l) Since point D is equally close to point L and can be used as a control point, it is desirable to run through the above described computation, but working from point D instead of from point E. Fig. 5 shows the computation involved It can be seen that the elevation of point L works out to be 413 feet in this case The reason for the discrepancy between the two determinations of the elevation of point L can be due to many factors among which the most important are the presence of tilt and difference in elevation between successive photographs The two values averaged give an elevation for point L of 423 feet

Situations may arise where neither parallax bars nor Stereocomparagraphs are available. The lack of these instruments hampers elevation determination but does not preclude it for the ingenious photogrammetrist. With the photographs oriented and mounted as explained in Chapter VII, "Orient Your Stereoscope Correctly," much can be accomplished in the way of elevation determinations with a finely divided scale. A millimeter scale is preferable but an English scale can be used provided the values determined are converted into metric units.

The procedure consists of measuring the distance from a control point on the left hand photograph to the same control point where it appears in the right hand photograph. The scale should be read with great care, using a high power magnifying glass, and recorded. The same procedure should be followed with respect to the point whose elevation is to be determined. The difference between the two distances is the parallax difference of the two points. With this value known, the computational and tabular procedure outlined above can be resorted to for the elevation of the point.

At best, this expedient results in considerably lower accuracy than that possible with the Stereocomparagraph or parallax bar.

General Remarks

The same procedure as explained above can be used in any part of the photograph where reference can be made to actual ground control points. The closer the point to be determined is to ground control, the higher will be the accuracy of the determination; and conversely, the farther away the point is from ground control the more error will be introduced into the determination. It should always be attempted to determine important elevations with reference to two or more control points if available. In averagıng the result, a good approximation is to give the greatest weight to the determination as derived from the nearest control point if the linear distances to the control points are not approximately equal.

CONTROL POINT DATA FOR GRAPH CONSTRUCTION

Point	*Elevation*	*Parallax*
C	1,013 ft.	14.70 mm.
E	0	10 84 mm.

FORM LINE DATA TAKEN FROM GRAPH

Elevatıon	*Parallax*
0	10 84 mm
50	11.03
100	11.22
150	11.41
200	11.60
250	11.79
300	11.99
350	12.18
etc.	etc.

Fig. 7

Contouring a Stereoscopic Pair of Photographs

The term "contour" is frequently used rather loosely in connection with stereoscopic plotting with simple parallax devices. A contour is a line on a map joining points of equal elevation. The contours on some maps are more accurately placed than those on other maps. However, the term contour can be used so long as the degree of accuracy of the location of the contour is the same throughout the map regardless of whether the accuracy be high or low. When the degree of accuracy of the position of the contour is not consistent but varies in different portions of the map, the line cannot be called a contour but should be referred to as a form line.

In the previous description of the procedure of determining the elevation of a given point, only one point was considered, namely the point L. However, if a person so desires, the elevation of a great number of points in the stereoscopic view can be determined in the same way. These points, for example, should be located along the ridges of all the various hills showing in the area of overlap and also along the valleys and also at all points where the general slope of the land changes. In a stereoscopic view showing very gently rolling land, not a great number of spot elevations are required. In very rough terrain, hundreds of points might be required along the ridge lines, valleys and breaks in slope. However, if the large number of points required are all very carefully determined as explained in the foregoing procedure, a topographic map can be made in which the lines joining points of equal elevation can be termed contour lines. These contour lines can be plotted by the usual mapping procedure of interpolating the contours between points of known elevation.

Form Lines

The determination of contour lines according to the procedure briefly mentioned above is exceedingly long and tedious. It is much faster and easier, although less accurate, to use the Stereocomparagraph or the Parallax Bar as a sketching device to draw form lines in a continuous fashion.

The drawing of form lines can be based on any number of control points from one on up. As a matter of fact, under conditions of absolute necessity, some success can be attained even in the face of the complete lack of ground surveyed control points. In that case, the best possible guess or estimate is made of the elevation of some one point in the stereoscopic view and that elevation becomes a control point from then on. Where the operator is so fortunate as to have a bit of shore line showing in the stereoscopic view, the estimation of the sea-level zero elevation will be reasonably accurate. The estimation of inland elevations is apt to be considerably in error and any elevation determined from such points will likewise be in error proportionately. However, the intent of form lines is not to show relative elevation, but to show the form of the hills and valleys for the many uses which can be made of such information.

If only one control point is known, the elevation of some other point in the stereoscopic view, either considerably higher than the control point or considerably lower, should be determined by the method explained previously. (See Fig. 5.) The second point, whose elevation thus becomes known, can be considered as a second control point.

A table can be made up, such as the one shown in Figure 7, giving the elevations of the two control points E and C (Fig. 4) and the parallax readings obtained from the Stereocomparagraph or from the Parallax Bar on those points. After having tabulated in the table (Fig. 7) the elevations and parallax readings of the two control points, a graph can be made as shown in Figure 8.

The vertical axis of the graph can be the elevation axis, whereas the horizontal axis of the graph can be the parallax axis. Scales should be selected for the graduations which will make it possible to read the elevation and parallax to the desired accuracy. It is important that the parallax axis be so graduated to permit the parallax to be plotted to .01 millimeters.

Knowing the elevations of the two control points, the operator can, by studying the stereoscopic view, estimate roughly the range of elevation from highest to lowest in the stereogram. This obviously gives the extent of the elevation axis of the graph which has to be plotted.

The form line interval to be used is found by dividing the altitude of flight of the aircraft by a value ranging between 200 and 300. Where control is at a minimum and where serious tilt is likely to be present in the aerial photographs, the altitude should be divided by not more than 200. As the amount of control increases and the general conditions of the work improve, a greater divisor can be used. However, in the example shown in Figure 8 the 200 figure is used which gives the nearest even form line interval to be 50 feet.

Plot the elevation of the two control points with respect to the parallax values in the conventional manner as shown in Figure 8. Draw a straight line through the two points to complete the graph. Actually the parallax curve, as the resulting straight line is called, should be slightly curved. However, the curve is so slight and the possibilities for considerable errors in form line work are so great that the difference between the curve and the straight line would not appreciably effect the result. If more than two control points are available, they should also be plotted in the same manner. In that case it is likely to be found that the three or more points will not lie in the same straight line, due to

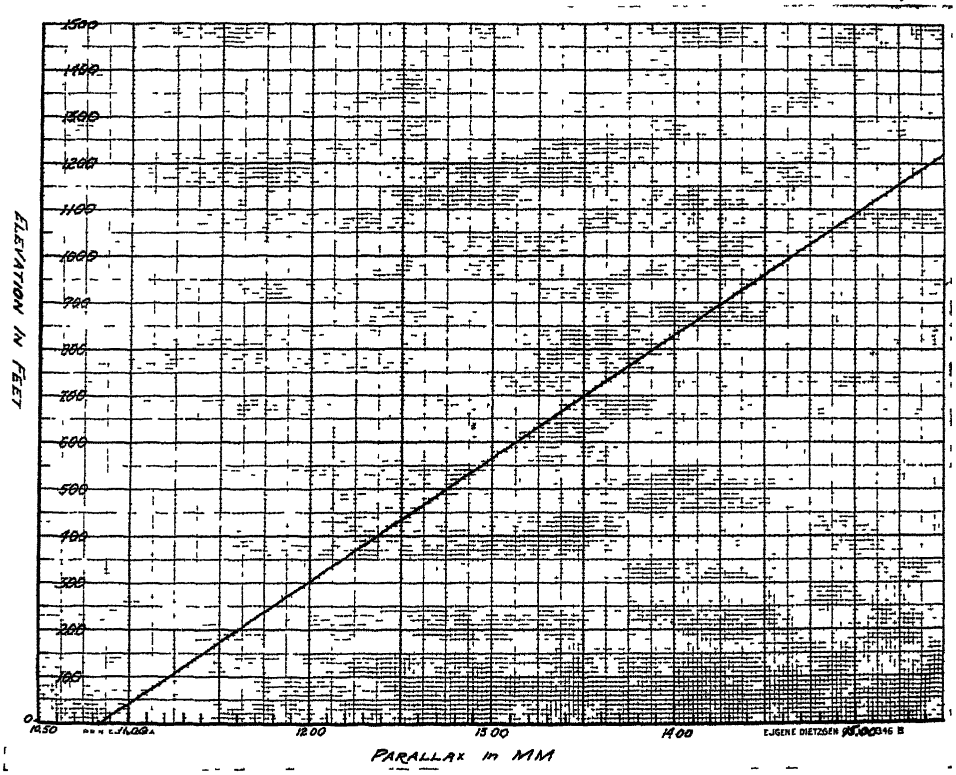


FIG. 8

errors of observation, errors due to tip and tilt of the photographs, and errors due to difference in elevation between the succeeding photographs. In that case, the straight line should follow an average path through the plotted control points.

With the graph now completed, the table in Figure 7 can likewise be completed by reading off from the graph the parallax readings for each 50 foot form line.

For form lining, the Stereocomparagraph or the Parallax Bar should be attached to a Parallel Motion Protractor and should have a drawing attachment in place

A sheet of paper of suitable size should be affixed to the drawing table so that when the Stereocomparagraph or Parallax Bar is moved to any portion of the photograph to be viewed, the pencil will still be on the drawing paper. With the operator looking through the stereoscope of the plotting device, whether it be Stereocomparagraph or Parallax Bar, the road system, shore line, rivers, streams, lakes and all other features of a similar nature should be drawn first. This is done most conveniently by turning up the right hand floating mark assembly of the Stereocomparagraph. When looking through the Stereocomparagraph the remaining mark appears always in contact with the ground regardless of the apparent depth of the three-dimensional image. The operator can then cause the mark to move along the roads, streams, fence lines, etc., while the pencil attachment delineates the features traced. In the case of the

parallax bar where the floating marks cannot be lifted up, the left hand mark can be removed completely for the tracing of planimetry.

After all the features mentioned above have been drawn in, the form lining can begin. If the parallax bar or Stereocomparagraph is in the planimetry tracing condition (only one mark in place) the instrument must be restored to its normal condition with both half marks in position. It is customary to start by drawing the lowest form line. Scan the area for what appears to be the lowest point and cause the floating mark to rest on that point Read the micrometer, thereby obtaining the parallax measurement of that point Look in the table (Fig. 7) for the next highest value of parallax which corresponds to an even 50 foot form line elevation Then adjust the micrometer of the instrument to read the value of parallax of that form line elevation By means of the small locking lever provided, the micrometer can be locked so that it cannot accidentally be changed due to an accidental brushing of the hand against the micrometer, or other cause. With the micrometer locked at that particular setting, the operator moves the instrument throughout the stereoscopic view wherever it can be moved, always maintaining the floating mark in contact with the ground. This will result in a line starting at one edge of the photograph and proceeding by an irregular course to some other edge of the drawing paper, or will result in a line starting somewhere within the drawing area and after proceeding along an irregular course, closing the figure by coming back to the starting point. After all parts of the photograph have been covered for that one form line, the operator can refer to the table in Figure 7 for the setting which must be set on the micrometer for the next highest 50 foot form line elevation. The micrometer is then adjusted to that value and the same procedure is followed. Eventually the sketch drawn by the pencil attachment takes on the appearance of a topographic map with the form lines superimposed over the highway and drainage features.

Other pairs of photographs can be form lined in a similar fashion. The several perspective sketches resulting can be reduced and combined by a radial line plot, pantagraph, or projector to give a form line map of a larger area than that shown in any one stereoscopic view.

Conclusion

Many refinements can be resorted to, to improve the accuracy of stereoscopic elevation measurements. These refinements are omitted from this article purposely to avoid confusing the beginner The refinements are discussed in "Practical Application of the Stereocomparagraph" and will be rather easily understood after the student has attempted and mastered the simple fundamental procedure outlined herein.

PRACTICAL APPLICATIONS OF THE STEREOCOMPARAGRAPH

Albert L. Nowicki

INTRODUCTION

THE impetus given to the field of aerial mapping during the last decade has brought to the fore many complex photogrammetric instruments and equipment. The development of machines of extreme precision such as the aerocartograph, the stereoplanigraph, or the multiplex has been, to a large extent, responsible for the rapid progress made in the field of photogrammetry. Instruments of this type have found their greatest use in the compilation of accurate, large scale, contour maps. Speed of compilation has been another important asset especially in the case of the multiplex machine. However, the difficulty and high cost of manufacture, the difficulty and time involved in training operators, the complexity, size and relative immobility of equipment, and the large amount of

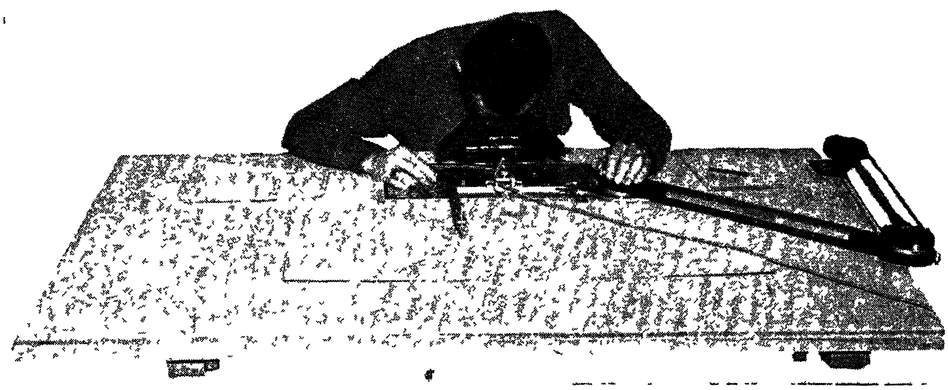


FIG. 1. Operator using the Stereocomparagraph and drafting machine to compile a templet sheet.

space required for efficient operation are elements all of which tend to limit their use on a widespread scale.

It would appear, therefore, that for certain mapping conditions, stereoscopic plotting machines, of a less complex nature than the ones mentioned above, would be much more desirable. This would be especially true where accuracy can be sacrificed in favor of speed of compilation and mobility of equipment and personnel.

At present, two such relatively simple stereo plotting instruments are available, namely, the Abrams' Contour Finder and the Fairchild Stereocomparagraph. Both of these instruments are capable of measuring differences in elevation by means of micrometer screw adjustments which may be read to the nearest 0.01 mm. The former utilizes a lens stereoscope, whereas the latter makes use of a mirror stereoscope, for stereoscopic perception. Differences in elevation are measured by means of two "floating dots," or "grids," placed on transparent glass plates that are located in the plane of the photographs. The dots are called floating dots because when set apart at the proper distance they will fuse into a single dot which will appear to be "floating" beneath the eyepiece of the stereoscope. If the dots are moved further apart the fused dot will appear to be lowered

vertically through space away from the observer, and if the dots are brought together, the floating dot will appear to rise, relative to the landscape. Hence, by varying the spacing between the dots, the fused image can be made to approach, recede from, or actually touch a given portion of the ground.

Since both instruments are essentially alike in principle and in operation, only one will be discussed in detail, namely, the Stereocomparagraph.

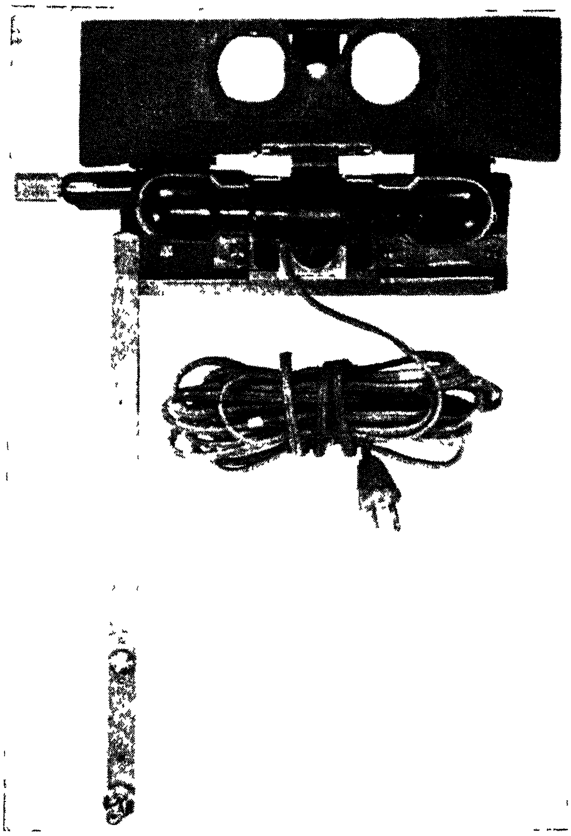


FIG. 2. General assembly view of Stereocomparagraph.

I. DESCRIPTION OF EQUIPMENT

(a) *General*

The Stereocomparagraph is, primarily, an instrument consisting of a stereoscope, a measuring device, a drawing attachment, and a parallel motion mechanism. Aside from the above features it also includes an attached lighting system, and comes in a convenient carrying case with a few assessories and spare parts (Figures 1, 2, & 3).

(b) *The Stereoscope*

The stereoscope (Figure 3) is of the reflecting type with matched lenses for magnification. To restitute for differences in interpupillary distances a screw adjustment is provided which changes the angles of the reflecting mirrors, thus allowing for easier fusion of images and floating marks.

(c) *The Measuring System*

The measuring system of the Stereocomparagraph consists of two marks, or dots, called the "left hand mark" and "right hand mark," respectively (Figure 3). A screw-type micrometer adjustment, attached to the right hand mark (Figure 4), may be used to change the spacing of the marks. This micrometer adjustment is graduated from 0 to 25 millimeters and may be read directly to one one-hundredth of a millimeter. One complete revolution of the screw moves the micrometer head one-half of a millimeter. The long marks below the line on the barrel indicate whole millimeters while the short marks above the line indicate one-half millimeters. The circumference of the vernier screw is divided into 50 units, each equal to 0.01th of a millimeter. For contour work the microm-

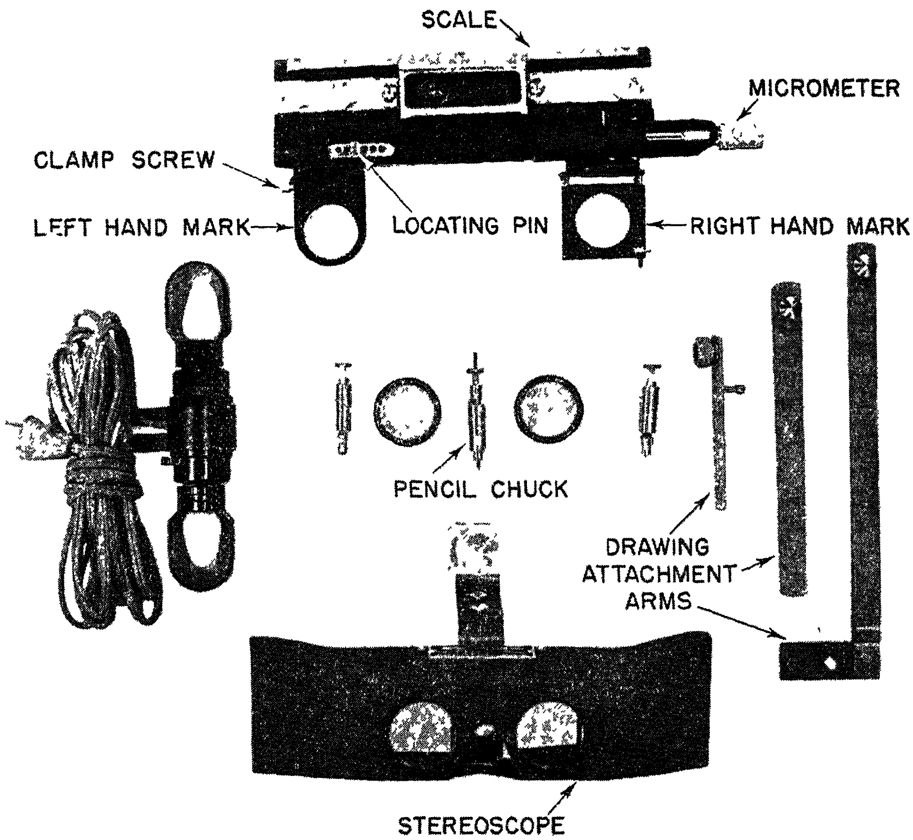


FIG. 3. View showing integral parts of the Stereocomparagraph set.

eter may be clamped to a given reading by means of a friction lock (Figure 4). In addition to the micrometer adjustment, a "B-Y" adjustment is available which may be used to facilitate proper fusion of the marks as the instrument is moved about during the process of contouring. This, however, does not affect the micrometer reading since the motion is in a direction at right angles to the micrometer adjustment (i e , the "B-X" direction).

The left hand mark, which may also be moved through an interval of twenty-five millimeters, but only by increments of five millimeters at a time, is held in place by the locating pin (Figure 3). If it should become necessary, after operation has begun, to move the left hand mark, the corresponding movement should be added to the micrometer reading if moved to the right, and subtracted if moved to the left.

In order to facilitate better inspection of the photographs, both the left hand and the right hand floating mark assemblies may be lifted, or tilted up, from the plane of the photographs.

A metric scale graduated in $\frac{1}{2}$-millimeter divisions is scribed on the beveled edge at the rear of the stereoscope base (Figure 3). This scale is laid out parallel to the "B-X" direction of movement of the marks, or micrometer adjustment, and may be used in measuring the stereoscopic base of the photographs, in lieu of an ordinary millimeter ruler.

(d) *Drawing Attachment*

The drawing attachment consists of a pencil attachment located at the end of a drawing arm which in turn is rigidly fastened to the base of the Stereocomparagraph (Figures 2 and 4). A ball-pointed rest is located near the end of the arm in order to keep the arm at the proper height above the drafting templet sheet. The latter consists of a piece of heavy paper onto which the planimetry and contours are compiled. The pencil attachment, at the end of the drawing attachment, consists of a pencil chuck into which may be inserted a pencil lead Provision is made whereby the pencil may be raised when it is desired to clear

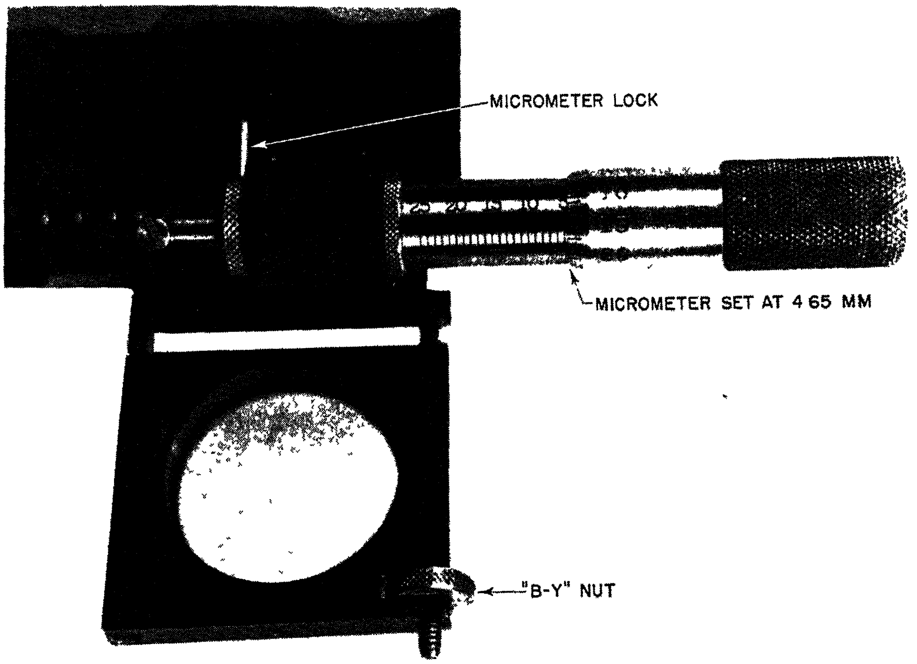


FIG 4 Enlargement of the "B-X" micrometer screw and "B-Y" motion nut.

the drafting templet sheet. A pricker, or stylus, may be inserted in place of the lead pencil when center points and control points are transferred from the photographs to the drafting sheet.

(e) *Alignment Mechanism*

The Stereocomparagraph may be attached to any of the standard types of drafting machines, or parallel motion devices. This insures the continuity of adjustment of the instrument once it has been established and provides for a movement at all times parallel to the flight line (i.e., "B-X" direction). Attachment to the drafting machine is made in the same manner as in the case of a straightedge.

II. STEREOSCOPIC ADJUSTMENT

The first step in the operation of the Stereocomparagraph, as in the case of other similar stereoscopic contouring instruments, is the proper alignment of the

stereoscopic base of the photographs with that of the stereoscopic base of the instrument itself. This adjustment is considered to be correct when the stereoscopic base of the photographs is made parallel to the stereoscopic base of the stereoscope, and the photographs are adjusted to the proper interpupillary distance of the operator's eyes to facilitate easy fusion of the images.

Procedure from this point on is as follows: Locate the principal points of the photographs at the intersection of the two lines drawn through the fiducial, or collimating marks. These centers should be marked with either a circle or a cross and labeled accordingly Each center should then be transferred to the adjoining photograph (Figure 5). If no tilt occurs, the principal point and the plumb point of each photo will coincide. When all four plumb points (or principal points, on untilted photos) are aligned in a straight line the photographs are considered to be oriented to their stereoscopic base (Figure 5). The next problem is to adjust this base to that of the stereocomparagraph. This may be accomplished in several ways, the simplest of which can be done as follows:

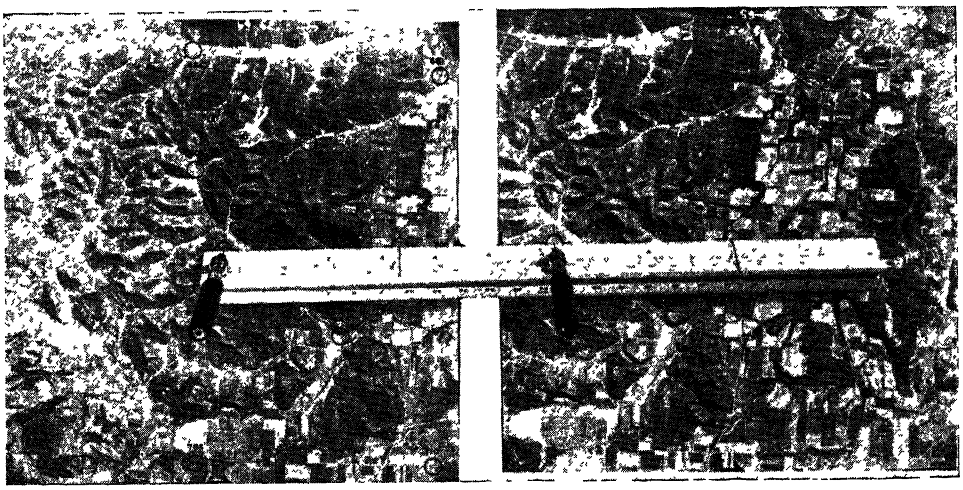


FIG. 5. Stereo-base method of orienting photographs for use with the Stereocomparagraph.

(a) After the principal points are pricked and transferred, fasten the left hand photograph to the drafting board, after aligning the two points with the stereoscopic base of the instrument by using the beveled edge at the back of the instrument, as a straightedge. Hold the photo in place with scotch cellulose or masking tape.

(b) For average terrain, adjust the left hand mark by setting the locating pin at the 15.00 mm. position (third hole from left), and set the micrometer screw at 12.50 mm. The distance between the marks will then be 6.10 inches, or 155 mm. If the locating pin had been set in the second hole (20.00 mm.) and the micrometer at 12.50 mm., the distance between the marks would be 6.30 inches, or 160 mm.

(c) Shift the Stereocomparagraph until the left hand mark coincides with the plumb point of the left hand photograph.

(d) Place the right hand photograph alongside of the other photo so that the right hand mark rests upon the plumb point that has been transferred from the

FIG 6. Left hand photo of a stereo pair showing plumb points and vertical control points.

left hand photograph. This procedure spaces the photographs at the proper interval, called the "stereo index distance."

(e) Align the base of the right hand photograph with both that of the instrument and the left hand photograph by having all four plumb points (or principal points, as the case may be) on both photographs, in a straight line, parallel to the beveled edge of the base of the stereoscope. Fix the right hand photograph in place with scotch cellulose tape.

III. OPERATION IN CONNECTION WITH MASTER PARALLAX TABLES

A. Control Points

The exact position of the vertical control points should be located, and the points pricked, on the left hand photograph. These points should be circled and marked with properly designated numbers and elevations for easy identification. Although the number of control points to be used is optional, it is advisable to

use a minimum of nine or more well distributed points per photo. As a general rule, the more vertical control points used, the greater will be the accuracy of the final work Likewise, it may be stated that for the same degree of accuracy, the more rugged the country that is encountered, the greater the number of control points necessary In all cases, these points should be so picked as to include positions at critical changes in elevation as well as positions at the four corners of the area being contoured. Such positions would be similar to the "spot" elevations used in "logical contouring" problems. The determination of the elevations of the plumb points is also desirable, wherever possible.

B. Form Sheet

For each stereoscopic pair of photos a micrometer reading "form sheet" has to be compiled (Figure 7). In filling out the form, the elevation of the camera station above sea-level (in feet), and the stereoscopic base (in millimeters), must be known The camera station elevation may be obtained from the flight record, whereas the stereoscopic base may be determined, approximately, by measuring the distance between plumb point and transferred plumb point of both photographs, and averaging the two values

Example

$$
\begin{aligned}
B_m \text{ of Photo 1} &= 85\ 00 \text{ mm} \\
B_m \text{ of Photo 2} &= 86\ 00 \text{ mm.} \\
\text{Average } B_m &= 2\underline{)171\ 00} \\
&\quad\ 85.50 \text{ mm.}
\end{aligned}
$$

The average B_m is the converting factor used in adapting the Master Parallax Tables, included at the end of this article, to a usable table for the stereo pair concerned.

The table shown in Figure 7 may be compiled as follows·

(1) Determine the approximate maximum and minimum elevations of the area concerned; enter in column (4) all of the multiples of (Δh) (*i.e.*, contour interval) starting at the lowest contour level and going to the highest contour level that may be encountered.

(2) Enter in the column headed (H-h), the difference between the figure in column (4) and the flight elevation above sea level (H).

(3) Enter the corresponding values of $\Sigma\Delta p$ in column (2) as they appear in the Master Parallax Tables starting with $\Sigma\Delta p$ for (H), which corresponds to the elevation of the camera station above sea level.

(4) The values of (Δph) are derived by subtracting the value of $\Sigma\Delta p$ for (H) from the value of $\Sigma\Delta p$ for (H-h).

Example:

$$
\begin{aligned}
\Sigma\Delta p \text{ for } 18{,}380' &= 30.761 \text{ mm.} \\
\Sigma\Delta p \text{ for } 19{,}000' &= \underline{27.444} \text{ mm} \\
\Delta ph &= \ \ 3\ 317 \text{ mm.}
\end{aligned}
$$

(5) The values of (Δph) for the stereo pair concerned are derived by multiplying the values of (Δph) in column (3) by $B_m/100$ and are entered in column (5).

Example:

PROJECT FRANKLINTON
QUADRANGLE DOW
PHOTOGRAPH NOS. 64-65
OPERATOR J T G
PHOTO. SCALE 1 38,000
DATE JAN 29, 1943
FLIGHT ALTITUDE (H) 19,000
TIME 8 ½ HRS
B_m 85 5
CAMERA T-5

1	2	3	4	5	6	7
H-h	$\sum\Delta p$	Δ ph	h	$\frac{\Delta ph \times Bm}{100}$	Micrometer reading	Mic reading to sea level
19,000	27 444		0			
18,306	31.164	3 720	694	3 18	6 70	3 52
18, 319	31 093	3 649	681	3 12	3 90	0 78
18,250	31 471	4 027	750	3 44	6 12	2 68
18,195	31 772	4.328	805	3 70	7 07	3 37
18,300	31 197	3 753	700	3 21	5 97	2 76
18,210	31 690	4 246	790	3 63	6 54	2 91
18,355	30 897	3 453	645	2 95	6 20	3 25
18,215	31 662	4 218	785	3 61	5 75	2 14
18 290	31 253	3 809	710	3 26	7 70	4 44

19,000	27 444		0			
18,500	30.110	2 666	500	2 28		
18,460	30 327	2 883	540	2 46		
18,420	30 544	3 100	580	2 65		
18,380	30.760	3 317	620	2 84		
18,340	30 979	3 535	660	3 02		
18,300	31 197	3 753	700	3 21		
18,260	31.416	3 973	740	3 40		
18 220	31 635	4 191	780	3 58		
18,180	31 855	4 411	820	3.77		
18,140	32 075	4 631	860	3 96		
18,100	32 296	4 852	900	4 15		
18,060	32 517	5 073	940	4 34		
18,020	32 739	5 295	980	4 53		
17,980	32 961	5 517	1020	4 72		
17,940	33 184	5 740	1060	4 91		
17,900	33 407	5 963	1100	5 10		
17,860	33 631	6.187	1140	5 28		
17,820	33 855	6 411	1180	5 48		

FIG. 7. Table showing typical values of micrometer readings for corresponding elevations appearing on left hand photo.

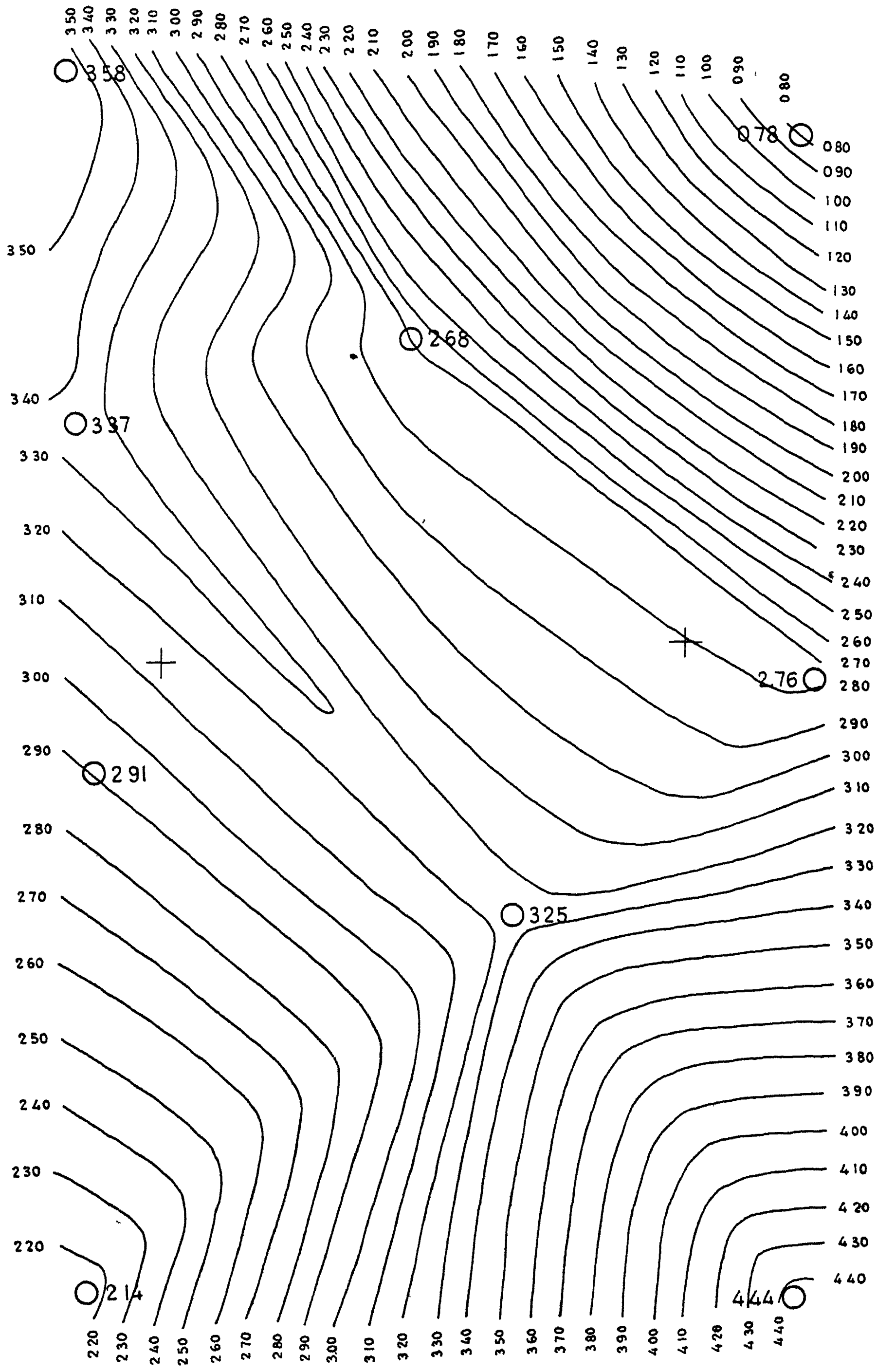


FIG. 8. Datum plane correction graph that has been used to plot the templet sheet shown in Figure 10.

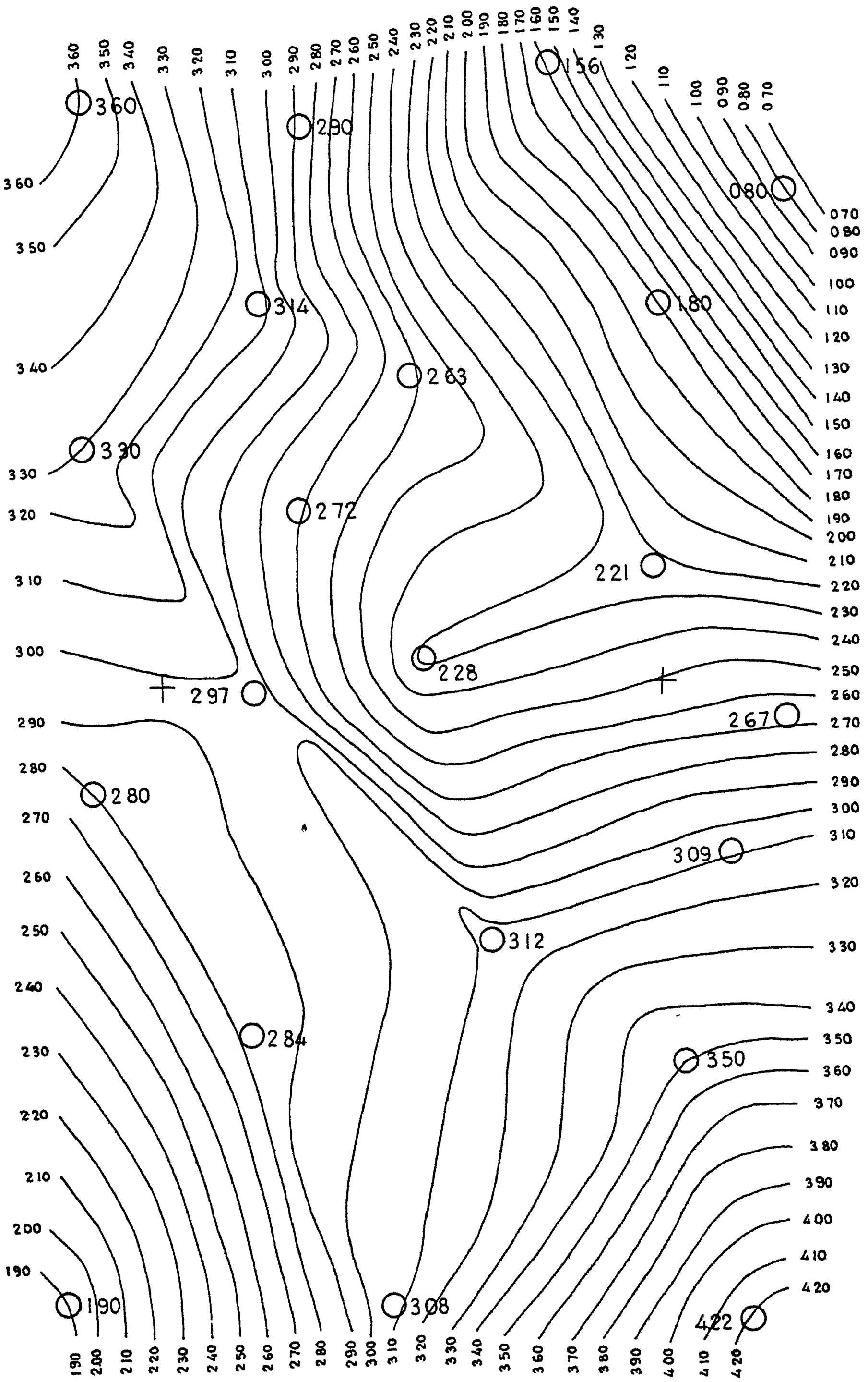


FIG. 9. Correction graph of left hand photo showing effect of inserting additional control points. Compare with Figure 8.

$$B_m = 85.50 \text{ mm.}$$

$$\Delta ph \text{ for } 700' \text{ contour} = 3\ 753 \times \frac{85.50}{100}$$

$$\Delta ph \text{ for } 700' \text{ contour} = 3.21 \text{ mm}$$

C. Datum Plane Correction Graph

The datum plane correction graph is an important link in the preparation of contour maps since by its use compensations are made for distortions due to tip or tilt, irregular shrinkage of negatives and prints, variations in flight altitude, vacuum plate failure, lens distortion, and also distortions arising from errors existing in the present widely-used Parallax Tables. Corrections for the latter errors will be discussed later. Quite evident, then, is the fact that the accuracy of the final contoured templet sheet depends to a great extent on the care with which the correction graph is constructed.

The graph may be derived as follows:

(1) In the upper half of the form appearing in Figure 7, enter the elevations of the given vertical control points, in column 4. Values of column 1 are derived in the same manner as the values of (H-h) were derived in the preceding description.

(2) Values of $\Sigma\Delta p$ appearing in column 2 are determined also as previously outlined except that the values will have to be interpolated since values of (h) will most always fall between contour intervals.

(3) Δph values appearing in column 3, as before, are derived by subtracting the value of $\Sigma\Delta p$ of (H), the flight altitude, from the $\Sigma\Delta p$ values of (H-h).

(4) Values in column 5 are derived by multiplying the ratio of Δph by the stereoscopic base (B_m) divided by 100. The division is necessary because the tables are made out for $B_m = 100$ mm.

(5) Column 6 includes the actual micrometer readings of various vertical control points as measured from the two photographs.

(6) Values in column 7 are derived by subtracting the value of (Δph) from the micrometer reading for the different values of (h), (i.e., from the elevations of the given vertical control points). If all the values appearing in column 7 came out the same, no graph would be necessary, but such a condition is highly improbable because of the many distortions inherent in the photos.

(7) Place a piece of tracing paper over the left hand photograph and upon it mark the positions of the vertical control points, including their elevations Draw straight lines from one point to another until a network of triangles exists. Mark the control points with their corresponding values found in column 7.

(8) Using the graph index values appearing at each control point, and considering them as if they were spot elevations, interpolate between points for 0.05 mm. intervals, and sketch in the graph lines similar to the process of logical contouring. Should the graph lines be very close, an interval of 0.10 mm. may be used instead of 0.05 mm., as was done in the graphs shown in Figures 8 and 9.

(9) Either transfer the graph directly to the left hand photograph or trace the lines onto a clear sheet of celluloid acetate, and use this tracing as an overlay. The latter method is preferable if it is desirable to preserve the photographs for further use. With the graph lines superimposed upon the left hand photograph, actual contouring can start.

The accuracy of the correction graph, as was previously indicated, is mainly dependent upon the number of vertical control points used. Comparing Figures 8 and 9, it will be noted that though the patterns of the two graphs are similar there are major differences in some areas simply because more points were used

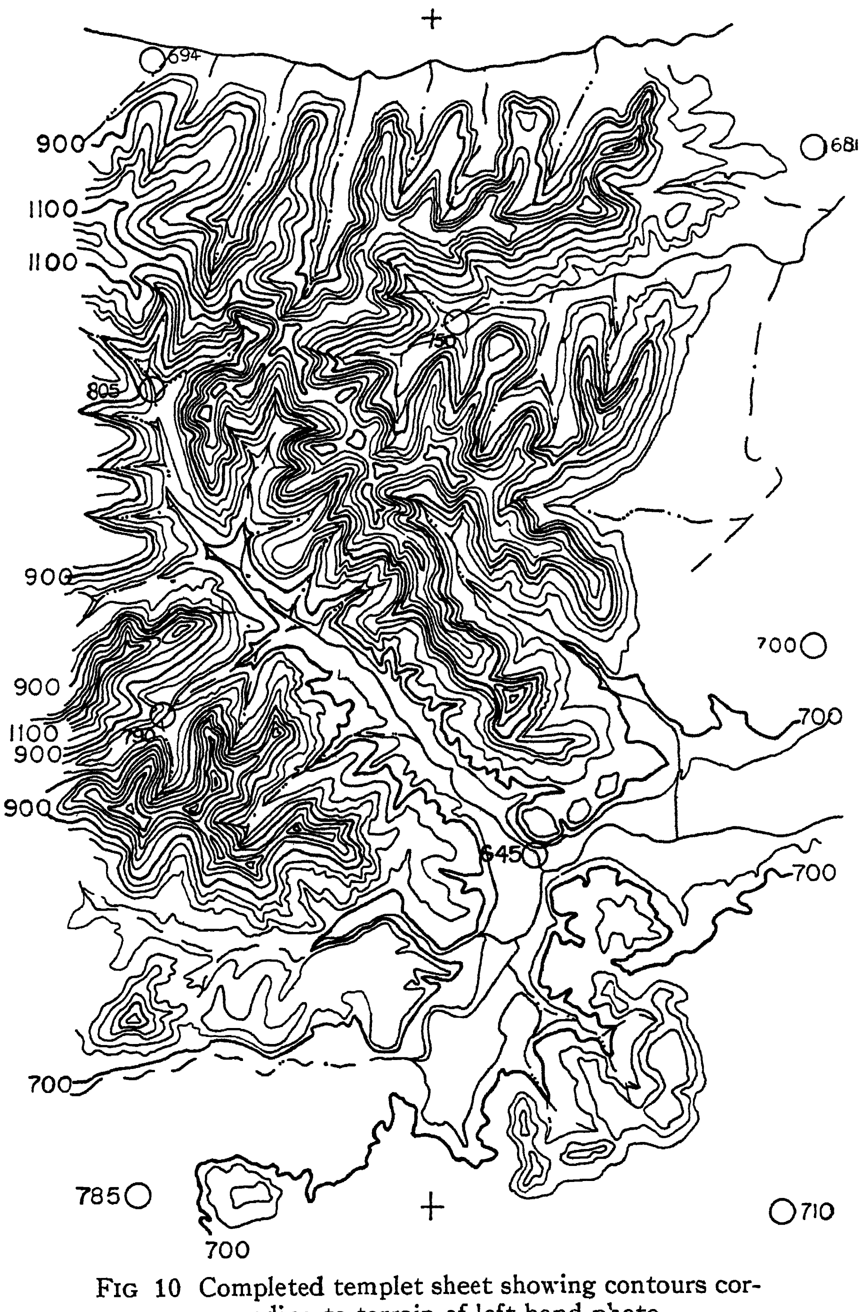


FIG 10 Completed templet sheet showing contours corresponding to terrain of left hand photo.

in Figure 9 than in Figure 8. It appears that the graph in Figure 8 was easier to construct, but in the final analysis the contours will generally be more accurate by using the graph in Figure 9, since the extra points locate other areas of local distortion for which the first graph does not compensate.

IV. Contouring Operation

Contouring should be preceded by inspection of the entire overlap area to familiarize the operator with the characteristics of the terrain. This may be done easily by raising the two "floating dot" assemblies, and will insure unimpaired observation through the stereoscope. The following procedure is suggested.

(a) Using only the left hand mark, trace the drainage of the entire area to the contour templet sheet and ink in blue. Likewise, transfer the vertical control points and plumb points to this same templet sheet.

(b) Determine the approximate location of the lowest contour; add algebraically the correction graph reading for that point to the parallax reading for the corresponding elevation which appears in column 5, of Figure 7, and set the micrometer at this value.

Example:

$$\text{Graph Reading} = 2.75 \text{ mm.}$$

$$\frac{\Delta ph \times B_m}{100} \text{ for } 620' = 2.84 \text{ mm.}$$

$$\text{Micrometer Reading} = 5.59 \text{ mm.}$$

(c) Lower the right hand mark to the plane of the right hand photo and move the instrument until the two dots are fused and appear to be touching the surface of the ground. Move the instrument so that the fused dots always appear to be touching the ground, at the same time increasing or decreasing the micrometer reading, as the case may be, an amount equal to the graph-line interval as the lines are crossed. Other contours may be traced in a similar manner by setting the index marks at the proper interval and locking them in position by means of the locking screw shown in Figure 4. If, in the course of the operations, the two floating dots do not fuse (due to displacements caused by tilt displacement), it will be necessary to use the "B-Y" motion adjustment until the dots are fused.

The position of the micrometer screw is so located that the observer can see it while looking through the stereoscope. Hence, he is able to observe the motion of the fused dot as it crosses the graph lines of the correction graph and at the same time is able to observe the movements and readings of the micrometer screw when he makes the necessary adjustments. In this way it is possible for him to trace the entire contour line in the overlap area without raising his eyes away from the stereoscope. See Figure 1.

(d) Trace the roads and culture data onto the templet sheet in the same manner as the drainage, and ink in red.

(e) The templet should be numbered with the numbers of both photographs, name of the operator, date completed, the (B_m) value, and project name should also be recorded.

V. Compilation

After all of the templet sheets have been contoured and the culture transferred to them, they are ready for the final compilation. Each templet represents the perspective view of the left hand photo of a stereo pair and not the ortho-

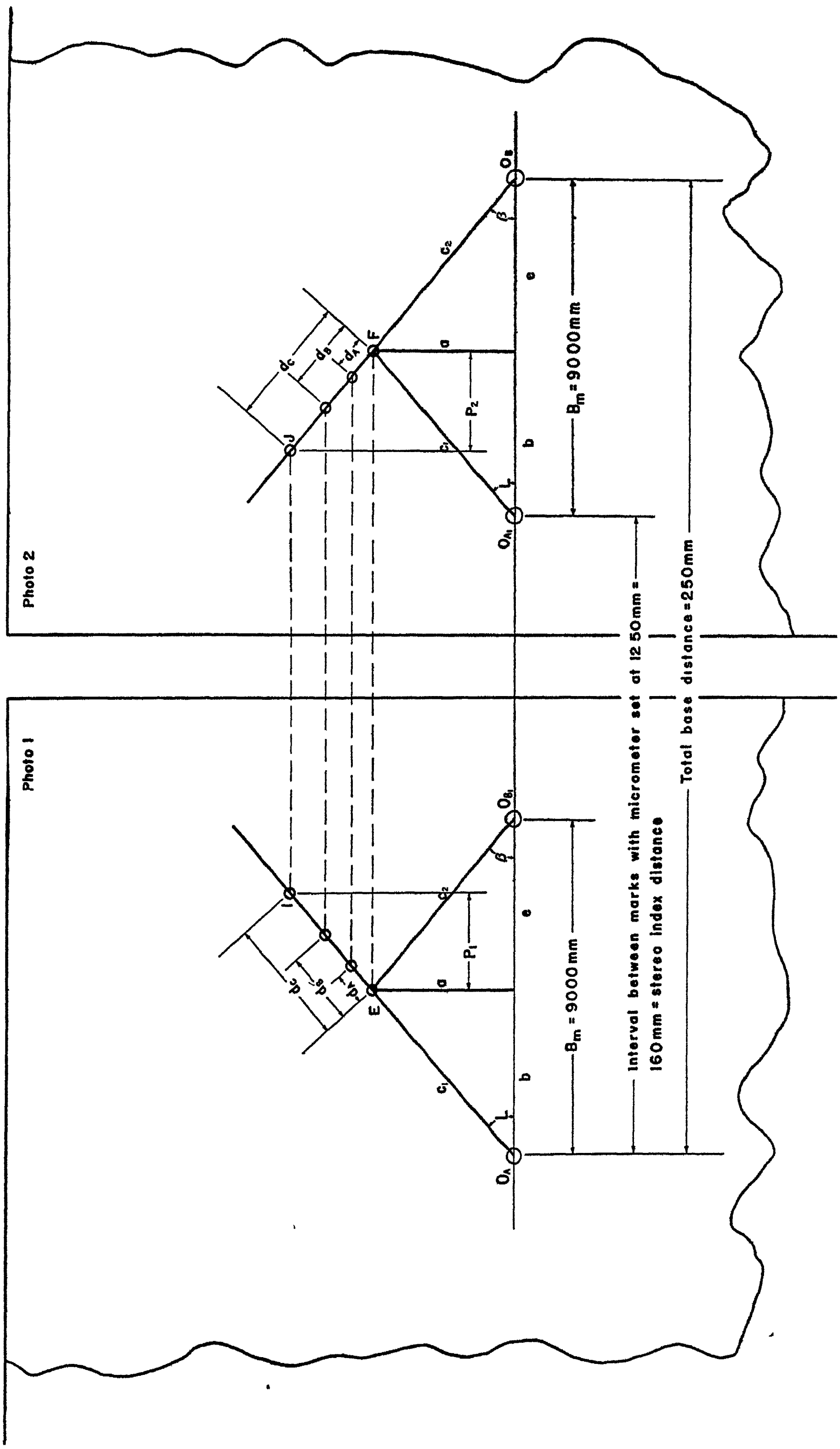


FIG. 11. Principles utilized in the Stereocomparagraph method of measuring differential parallax distances from a stereo pair of photographs.

graphic projection of the area included in the overlap area. All the errors due to relief displacement are still inherent in the templets and must be further corrected. Any one of the many radial control methods can be used for this purpose, such as the slotted templet, the metal slotted arm, or the celluloid templet method.

Radial control points as well as primary ground control points must be located on the transparent base sheet that is to be used for the final compilation. A suitable map projection, such as the Polyconic Projection, must also be plotted on the base sheet to a scale approximating the mean scale of all the photographs falling within the given quadrangle.

Sufficient secondary control points (i.e., topography points) should also be cut in from successive templets to insure a network of control points on the base sheet with a density of about one in every square inch of area (more if the country is rugged). Vertical projectors can then be used for transferring the planimetric and topographic features to the base sheet. Where no projectors are available, the detail may be traced by shifting the templets under the base sheet until the control points, in a localized area, coincide with the corresponding points on the base sheet. Where excessive relief occurs, critical points (i e. road crossings, stream junctions, spot elevations, etc.) may have to be "cut in" from successive photos as in the case of the location of the radial control points

Where it is desirable to transfer the contours directly to the photograph, instead of to a map sheet, the templet sheet can be eliminated. In this case, a copy of the left hand photograph is substituted for the templet sheet and the contours traced directly onto it. Or, if it is desired, the information from the templet sheet could be transferred to the left hand photo, directly, by means of carbon paper.

VI. Differential Parallax Displacement Equations

Inherent errors present in the basic equations used to construct the master parallax tables result in measurable discrepancies in the computations for differential parallax displacement. These errors may be so large as to make the tables impracticable. Figure 11 will be used as a reference in the derivation of parallax formulas which may be used to correct these errors.

Point (E) on photo 1 represents the sea level location of the same point (F) on photo 2. Points (O_A) and (O_B) are the sea level positions of the plumb points of photos 1 and 2, respectively. Since both these points (and the transferred plumb points) are all at sea level, the stereo base distance (B_m), in each case, will be the same. In the example, B_m is assumed to be 90 millimeters. Let the distance $(O_A\text{-}E)$ be equal to (C_1), or 10 mm., and the angle (L) equal to 45°. The triangles $(O_A\text{-}E\text{-}O_{B1})$ and $(O_{A1}\text{-}F\text{-}O_B)$ are identical since none of the points in the sea level datum plane will have any relief displacement from their true horizontal positions. Parallax relief displacements, for equal increments of elevation above sea level, are represented by the letters (d_A), (d_B), (d_C), etc.

Figure 12 shows the relationship between the elevation of a point above sea level and its corresponding relief displacement distance on the photo. Thus: for any height (h_n) above sea level:

$$\frac{D_n}{h_n} = \frac{W}{H - h_n}.$$

And:

$$\frac{W}{C_1} = \frac{D_n}{d_n}; \qquad W = \frac{C_1 \cdot D_n}{d_n}$$

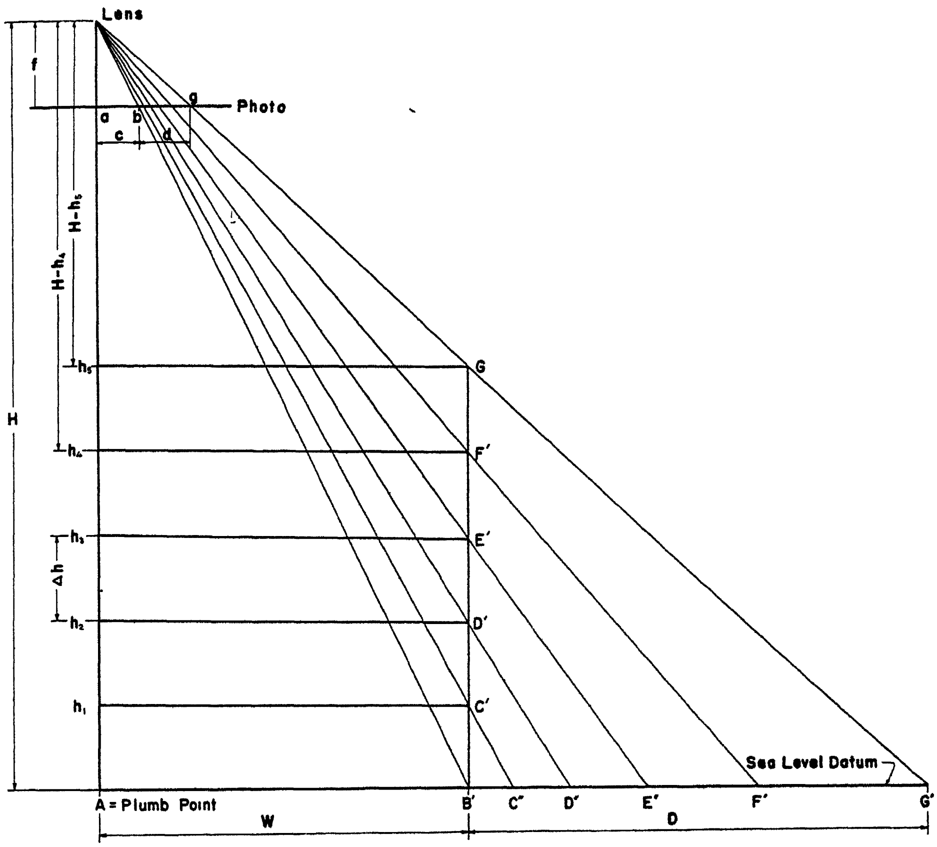


FIG. 12. Relief displacement for equal increments of elevation above sea level datum.

Substituting:

$$\frac{D_n}{h_n} = \frac{C_1 D_n}{d_n (H - h_n)}.$$

Therefore:

$$d_n = \frac{C_1 h_n}{(H - h_n)}. \tag{1}$$

The horizontal component (i.e. measured parallel to the flight line, or "B-X" direction) would be

$$P_1 = d_n \times \cos L = \frac{C_1 (h_n) \cos L}{(H - h_n)}.$$

In like manner the horizontal component of the relief displacement for a similar height (h_n) on photo 2 is

$$P_2 = \frac{C_2 (h_n) \cos \beta}{(H - h_n)}.$$

By definition: "differential parallax" is equal to

$$(p) = \text{distance } (\overline{EF}) - \text{distance } (\overline{IJ}) = P_1 + P_2 = \frac{C_1\ (h_n)}{(H-h_n)} \cos L + \frac{C_2 \cdot (h_n)}{(H-h_n)} \cdot \cos \beta$$

$$p = \left(\frac{h_n}{H - h_n}\right)(C_1 \cos L + C_2 \cos \beta).$$

But:

$$\cos L = \frac{b}{C_1}; \quad \text{and} \quad \cos \beta = \frac{e}{C_2}; \quad \text{and} \quad b + e = B_m.$$

Therefore:

$$p = \left(\frac{h_n}{H - h_n}\right)\left(C_1 \times \frac{b}{C_1} + C_2 \times \frac{e}{C_2}\right) = \left(\frac{h_n}{H - h_n}\right)(b + e).$$

Or:

$$p = B_m\left(\frac{h_n}{H - h_n}\right). \tag{2}$$

This equation may also be expressed in terms of the camera base width (W) as follows:

$$\frac{W}{B_m} = \frac{H}{f}; \qquad B_m = \frac{f \cdot W}{H}$$

$$p = \frac{f}{H} \cdot W \cdot \frac{h_n}{H - h_n}. \tag{3}$$

In the above equations express (p) in the same units as (f), the focal length, or in the same units as (B_m), namely, millimeters.

It is seen from the above relations that all points which have the same differential parallax displacement, regardless of their positions on the photographs, will have the same elevation above sea level datum. Or, conversely, all points at the same height above a given datum plane will have the same parallactic displacement.

If it is desirable to obtain the difference in parallactic displacement (Δp) between two points of a slightly different elevation, the following derived equations may be used:

$$\Delta p = p_H - p_L = B_m \cdot \left[\frac{h_H}{H - h_H} - \frac{h_L}{H - h_L}\right].$$

Where: h_H = elevation above sea level of the higher point.
h_L = elevation above sea level of the lower point.

$$\Delta p = B_m \cdot \left[\frac{H \cdot (h_H - h_L)}{(H - h_L)(H - h_H)}\right]. \tag{4}$$

But:

$$\Delta h = h_H - h_L; \qquad h_H = h_L + \Delta h;$$

Therefore:

$$\Delta p = B_m \left[\frac{\Delta h \; H}{(H - h_L) \; [H - (h_L + \Delta h)]}\right].$$

In general:

$$\Delta p = \frac{B_m \; H \; \Delta h}{(H - h_L)(H - h_L - \Delta h)}. \tag{5}$$

A similar equation may also be derived by the "delta" process of differential calculus as follows:

Given:

$$p = B_m \left(\frac{h}{H - h}\right) \qquad \text{See equation 2.}$$

For a slight increase (Δh) in (h) there will result a corresponding increase (Δp) in (p).

Thus:

$$p + \Delta p = B_m \cdot \left[\frac{(h + \Delta h)}{H - (h + \Delta h)}\right]$$

$$\Delta p = B_m \cdot \left[\frac{(h + \Delta h)}{H - (h + \Delta h)} - p\right]$$

$$= B_m \cdot \frac{[(h + \Delta h)]}{[H - (h + \Delta h)]} - B_m \cdot \left(\frac{h}{H - h}\right).$$

Simplifying and combining terms:

$$\Delta p = \frac{B_m \cdot H \cdot \Delta h}{[H - h][H - h - \Delta h]}. \tag{6}$$

For extremely small values of Δh;

$$\Delta p = \frac{B_m \cdot H \cdot \Delta h}{[H - h][H - h]}; \qquad = \frac{B_m \cdot H \cdot \Delta h}{(H - h)^2}. \tag{7}$$

The values of (B_m) in the above formulas refer to the photo stereo base, as measured at the "sea level" position of both plumb points. In most cases this value of (B_m) will not be available from the photograph. However, if the elevation of the transferred plumb point (i.e., right hand photo plumb point) is known, the stereo base, as actually found on the left hand photograph, can be readily expressed in terms of (B_m). For instance, let (h_x) = the elevation of the plumb point of the right hand photo:

And: (B_x) = stereo base as measured on the left hand photo.

Then from Fig. 12:

$$\frac{W}{H} = \frac{B_m}{f}; \quad W = \frac{H \cdot B_m}{f}; \quad \text{also} \quad \frac{W}{H - h_x} = \frac{B_x}{f}$$

$$\frac{B_x \cdot (H - h_x)}{f} = \frac{H \cdot B_m}{f}; \qquad B_m = \frac{B_x \cdot (H - h_x)}{H}. \tag{8}$$

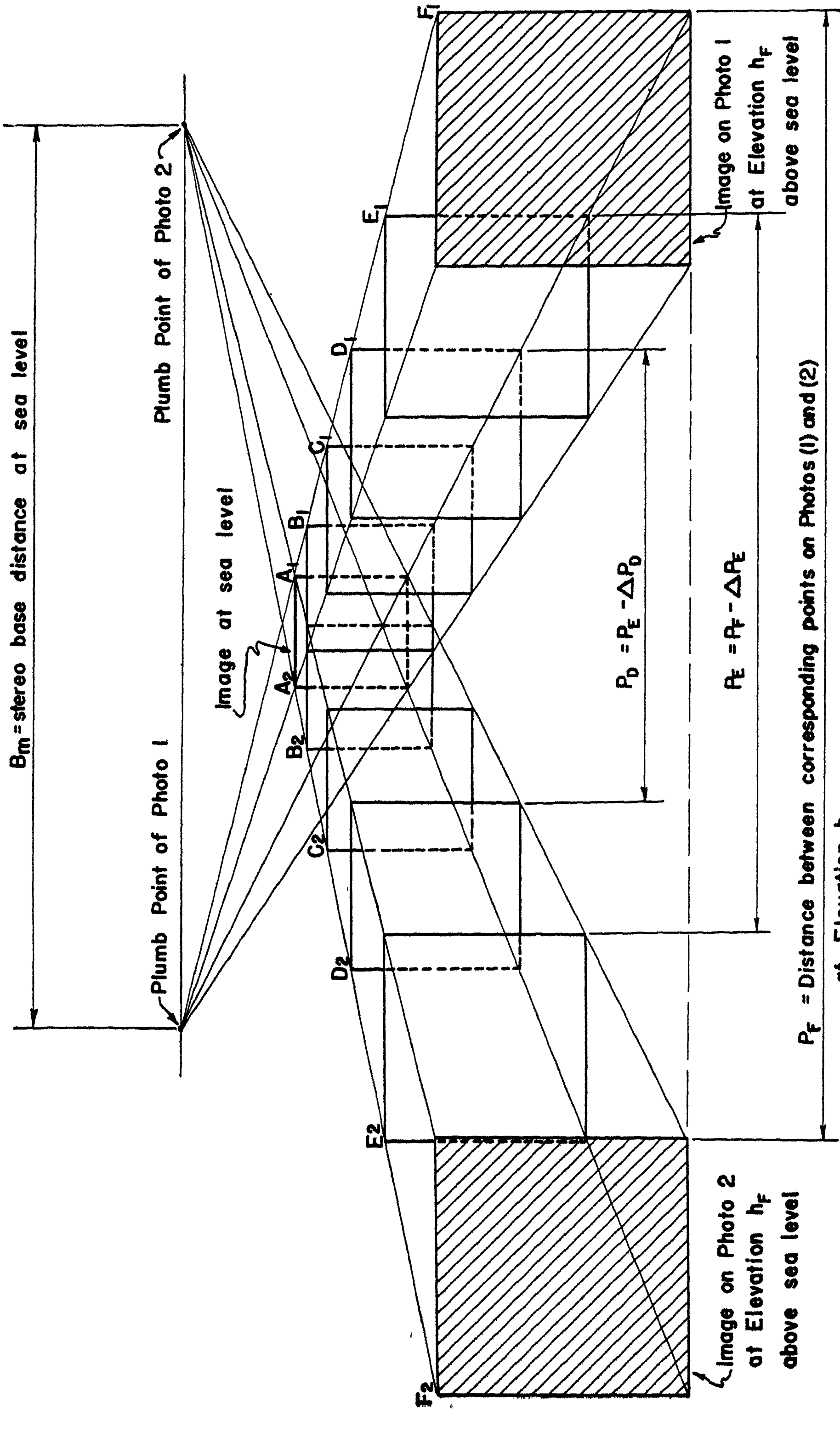


FIG. 13. Relative displacement of image (square) due to differences in elevation above sea level datum as measured from the plumb points of a stereo pair of photographs.

Hence:

$$p = \frac{B_m\ (h)}{(H - h)}, \quad \frac{B_x\ (H - h_x)\ (h)}{H\ (H - h)} = p. \tag{9}$$

And:

$$\Delta p = \frac{\Delta h\ H \cdot B_m}{(H - h)(H - h - \Delta h)} = \frac{\Delta h\ H\ (B_x)\ \dfrac{(H - h_x)}{H}}{(H - h)(H - h - \Delta h)}$$

$$\Delta p = \frac{B_x\ \Delta h\ (H - h_x)}{(H - h)(H - h - \Delta h)}. \tag{10}$$

Applications of the above parallax equations to simple geometric figures in order to substantiate their correctness, and also practical methods of utilizing the Stereocomparagraph in mapping work, are indicated in the paragraphs which follow.

VII. Application of Parallax Equations to Simple Geometric Figures

The effect of relief displacement of objects at different elevations above sea level is diagrammatically illustrated in Figure 13 Radial lines are drawn from the plumb points of a stereo pair of photos, separated at sea level by the distance B_m, through the image (square) that is assumed to be located in the sea level datum plane.

Formula (1), namely,

$$d_n = \frac{C_1 \cdot h_n}{(H - h_n)}$$

is used to calculate the relief displacements for simultaneous increments of (h). Reference to the figure will show that the displacements due to relief (such as the distances A_2B_2, B_2C_2, etc.) increase proportionately as the distance (h) above sea level is increased, as does the size of the image. However, it is of interest to note that the area of the image on the left hand photo is exactly the same as on the right hand photo—at any given height above sea level. For instance, at a height (h_F) above sea level, the shaded square (F_2) is equal in shape and size to that of the shaded square (F_1) although it is a greater distance away from its plumb point than the square (F_1) is away from the plumb point of the left photo. Likewise, it is apparent that distances such as P_F, P_E, P_D, etc , or their corresponding differences, namely, ΔP_E, ΔP_D, etc. may be used as an index to the heights of the images above sea level. It is the measurement of these same distances (ΔP_E, ΔP_D, etc.) which are attempted by the Stereocomparagraph in the determination of the elevations above sea level of the points under consideration. In other words, all four corners of the shaded image (F_2) will be at the same distance (P_F) from the corresponding corners of the image (F_1). The same will hold for every other pair of images located in the same plane above sea level. Furthermore, it can be shown that the sum of the horizontal components of the distances $(\overline{A_2\ F_2})$ and $(\overline{A_1\ F_1})$ is equal to the "differential parallax distance" as measured with the Stereocomparagraph. This latter sum would be a measure of the elevation of the image (F) above sea level

Figure 14 represents the images of a simple geometric figure (i.e , a pyramid) that have been divided into four equal increments of height. Displacements due

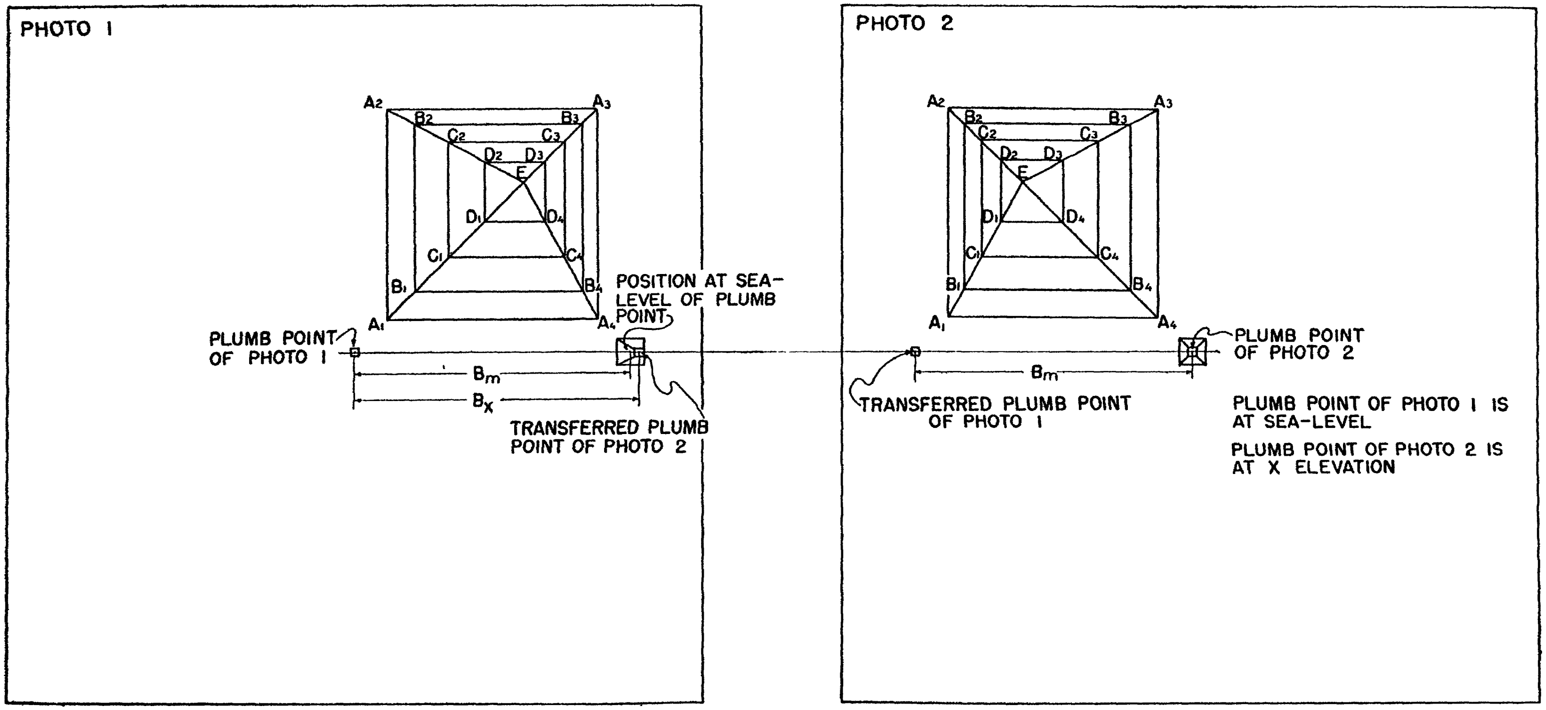


FIG. 14. Displacements due to relief of equal increments of elevation above sea level datum of an image (Pyramid) on a stereo pair of photographs.

to relief affect the two images as indicated. Distances measured between corresponding points on the two images can likewise be used as an index of the height of the points above sea level datum.

In other words, Figures 11, 13, and 14 all tend to prove the fact that points which have the same differential parallax displacement, regardless of their positions on the photographs, will have the same elevation above datum. This leads to another conclusion that whenever micrometer readings of vertical control points do not correspond with the values as calculated by the formula for the

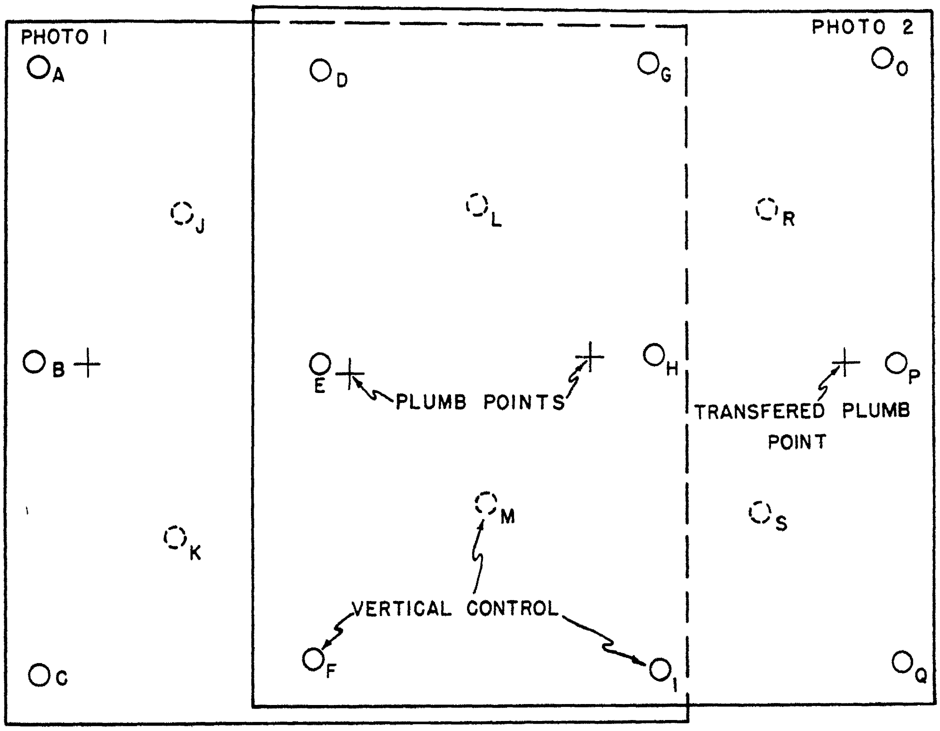


FIG. 15. Optimum arrangement of vertical control points for use with correction graph.

differential parallax displacement distance, the errors must be due to some other factor than elevation (i.e., tilt, paper distortion, etc.). Hence, any correction graph that is found to be necessary as a result of using the formula $p=B_m\cdot(h)/(H-h)$ must be due to the presence of tip or tilt, unequal levels of camera stations, etc. About nine control points, distributed per photograph as shown in Figure 15, would be all that would be necessary to construct such a graph. However, if local distortions were present due to lens distortions, paper film distortion, etc., then additional points, as shown by the dotted circles, would be preferable.

On the other hand, if the Master Parallax Tables are used, a much greater number of points is necessary, especially in rugged country, in order to take care of the elevation errors introduced in the graph as a result of errors in the tables. As indicated before, control points would have to be located at all critical breaks in the topography to compensate for the latter errors. In either case it would be

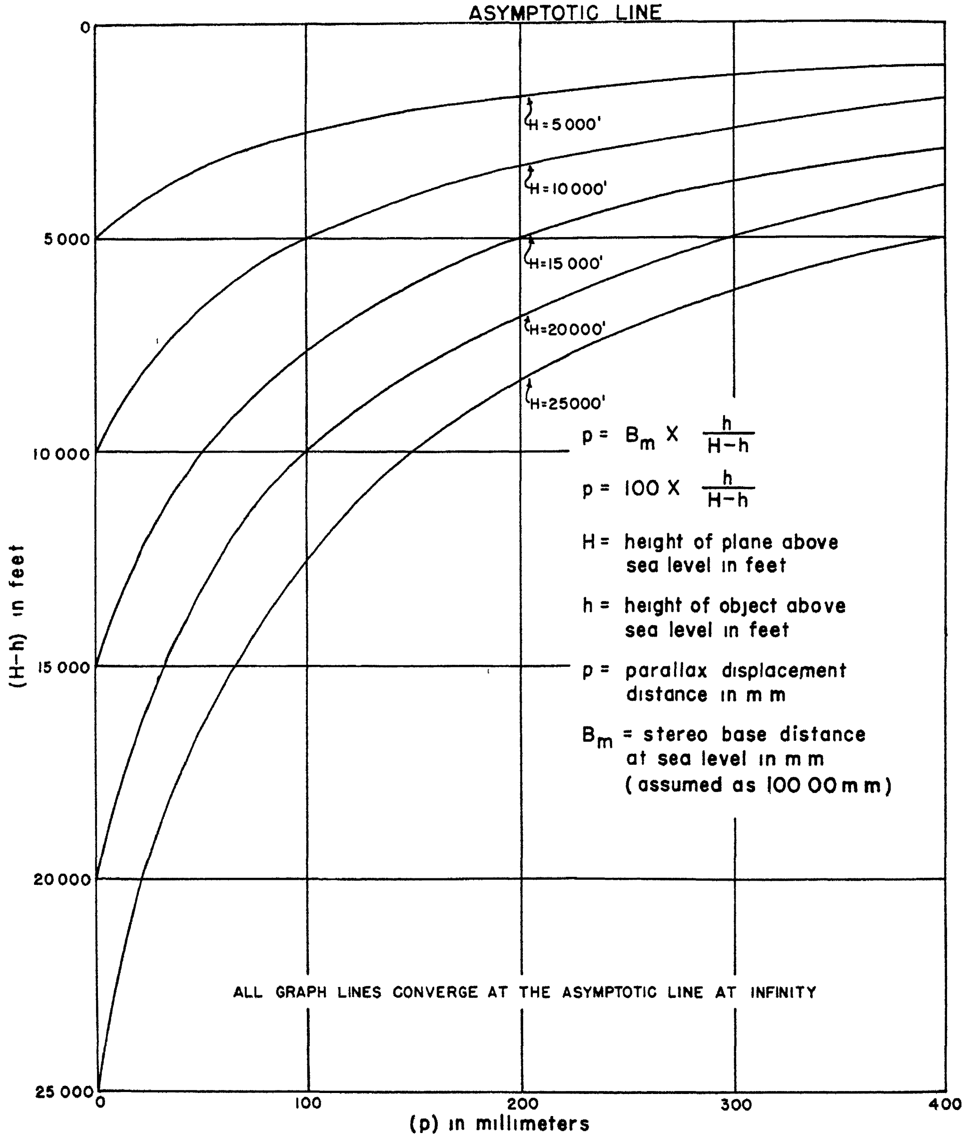


FIG. 16. Graphs showing relation between differential parallax distances and $(H-h)$ for various heights of camera stations (H).

desirable to have the elevations of the plumb points so as to obtain the sea level values of (B_m). If the Stereocomparagraph work was being done in connection with multiplex extensions, such elevations could be readily obtained. An optimum arrangement for mass production work would be one whereby the multiplex projectors would be used only for extensions, in order to obtain the control points, and the stereocomparagraph was used for compiling the topography.

Another discrepancy arising from the use of the Master Parallax Tables is that the value of $(\Sigma\Delta p)$, for a given $(H\text{-}h)$ value, is measured from a datum plane of $H=25{,}000$ feet below the camera station level. These tables, therefore, should

not be used for flight elevations other than $H=25,000$ feet except where the elevations of the points being contoured are very little above sea level.

Figure 16 shows a series of graphs for various heights of camera station above sea level. Each was computed from the correct basic formula for differential parallax distance, namely, $p=B_m(h/H\text{-}h)$ A comparison of these graphs illustrates the necessity for knowing the correct value of (H) in each stereo pair of photos being compiled. In other words, corresponding differences in values of (p) are not the same on the different graphs as is assumed in the case of the construction of the Master Parallax Tables. For instance, a thousand-foot difference in elevation, between the ordinates $(H\text{-}h)$ equals 8000 feet and $(H\text{-}h)$ equals 7000 feet, would give values of (Δp) as follows:

on the $H=25,000'$ graph: $\Delta p=257\ 0-212\ 5=44\ 5$ mm.
on the $H=20,000'$ graph: $\Delta p=185.8-150.0=35.2$ mm.
on the $H=15,000'$ graph: $\Delta p=114.2-\ 87.5=26.7$ mm.
on the $H=10,000'$ graph: $\Delta p=\ 42\ 8-\ 25\ 0=17.8$ mm.

It is apparent that if the parallax tables are used instead of the correct basic formula the errors will become objectional especially if the points to be measured are considerably above sea level

Additional proof substantiating the correctness of the basic parallax equation is shown in Figure 17 which indicates a table compiled from information shown in Figure 11. The purpose of this table is to show that in all cases (assuming that no tilt or other distortions are present) the theoretical values of the difference in differential parallax should be equal to those computed from the equation:

$$\Delta p = B_m \frac{\Delta h}{(H - h)} \frac{H}{(H - h - \Delta h)} .$$

Where the differences in micrometer readings, for two given control points, do not equal these computed values of (Δp), then it must be inferred that an error exists due to tilt, paper, or other distortion and must be corrected by use of a correction graph, similar to those shown in Figures 8 and 9.

Figure 11 shows the sea level position of a point (E) on the left hand photo and its corresponding position (F) on the right hand photo. Displacements due to relief, for equal increments of $(\Delta h)=200$ feet, are indicated by the values of d_A, d_B, d_C, etc. It is to be noted that the values of these displacements will not be the same on the two photographs because the points (E) and (F) are not the same distance from their respective plumb points Values of (d_1) as found on the left hand photo, for different values of (h), are shown in column 3 of Figure 17 Formula. $d_1=C_1\ (h_1)/H-h_1)$ was used in this case. Column 4 represents the horizontal component (i.e component parallel to the stereo flight base connecting the two photo plumb points) of the (d_1) values found on the left hand photo. These values are designated by the letter (P_1) and are equal to $(d_1\times\text{Cos } L)$. In like manner, column 5 refers to values of (P_2) and is equal to $(d_2\times\text{Cos } \beta)$. By trignometric relations it is seen that $P_2=e/C_1\times d_1=8\ 2929\times d_1$ Angle (L) was assumed to be 45°, $C_1=10\ 00$ mm., $B_m=90\ 00$ mm., and $H=20.000'$. This resulted in values of (b) of 7.071 mm and $(e)=82.829$ mm.

Column 6 is equal to P_1+P_2 and indicates values of differential parallax distance for a corresponding elevation (h) above sea level datum, when $H=20,000$ feet. Column 7 shows values of (Δp) for differences in (Δh) of 200 feet and is obtained by subtracting successive values in column 6

The value of (Δp) in column 7 will be found to be exactly the same as those derived from the basic formula

$$\frac{B_m \cdot \Delta h \cdot H}{(H - h)(H - h - \Delta h)} = \Delta p.$$

Values in column 8 were compiled for corresponding $(H-h)$ values as found in the Master Parallax tables, and should be compared to corresponding values of (p) in Column 6. It is apparent that as the value (h) increases, the difference between the two values of (p) becomes greater. Elevations near sea level (i e.

1	2	3	4	5	6	7	8
$H-h_1$	h_1	d_1	P_1	P_2	$p=P_1+P_2$	Δp	$\frac{\Delta ph \times Bm}{100}$
20000	0	—	—	—	—		—
						0 90909	
19800	200	0 10101	0 07143	0 83766	0 90909		0 9045
						0 92763	
19600	400	0 20408	0 14431	1 69241	1 83672		1.818
						0 94680	
19400	600	0 30928	0 21869	2 56483	2 78352		2 7477
						0 96651	
19200	800	0 41667	0 29463	4 45540	3 75003		3 6738
						0 98665	
19000	1000	0 52632	0.37217	4 36471	4 73688		4.617
						1 00802	
18800	1200	0 63830	0 45135	5 29335	5 74470		5 5692
						1 02951	
18600	1400	0.75269	0 53223	6 24198	6 77421		6 5313
						1 05183	
18400	1600	0 86956	0 61487	7.21117	7.82604		7 5042
						1 07505	
18200	1800	0.98901	0 69934	8.20175	8 90109		8.4879
						1 09890	
18000	2000	1.11111	0 78568	9 21431	9.99999		9 4814
						1.12365	
17800	2200	1 23596	0 87396	10.24968	11 12364		10 4886
						1.14912	
17600	2400	1 36364	0 96424	11.30852	12.27276		11.5056
						1.17549	
17400	2600	1.49425	1.05660	12.39165	13 44825		12.5334
						1 20294	
17200	2800	1.62791	1.15111	13 50008	14 65119		13 4738
						1.23111	
17000	3000	1.76470	1 24784	14.63446	15.88230		14.6268
						1.26054	
16800	3200	1.90476	1.34687	15.79597	17 14284		15 6924
						1.29087	
16600	3400	2.04819	1.44830	16.98541	18 43371		16 7597
						1.32237	
16400	3600	2.19512	1 55219	18.20389	19 75608		17 8605
						1.35504	
16200	3800	2 34568	1.65865	19.45247	21.11112		18.9648
						1.38888	
16000	4000	2.50000	1.76778	20.73222	22.50000		20.0826
						1.42407	
15800	4200	2.65823	1.87966	22.04441	23.92407		

FIG. 17. Tables showing computations for differential parallax distances as obtained from Figure 11 and as calculated from the Master Parallax Tables

up to about 500 feet above sea level) would have values of (p) nearly alike. However, these differences will be proportionately greater if the camera station elevation is less than 20,000 feet.

VIII. Operation of the Stereocomparagraph Without Use of the Master Parallax Tables

Figure 18 shows a cross-section of a portion of the earth's surface that is to be contoured by means of the Stereocomparagraph Scale (A) represents the

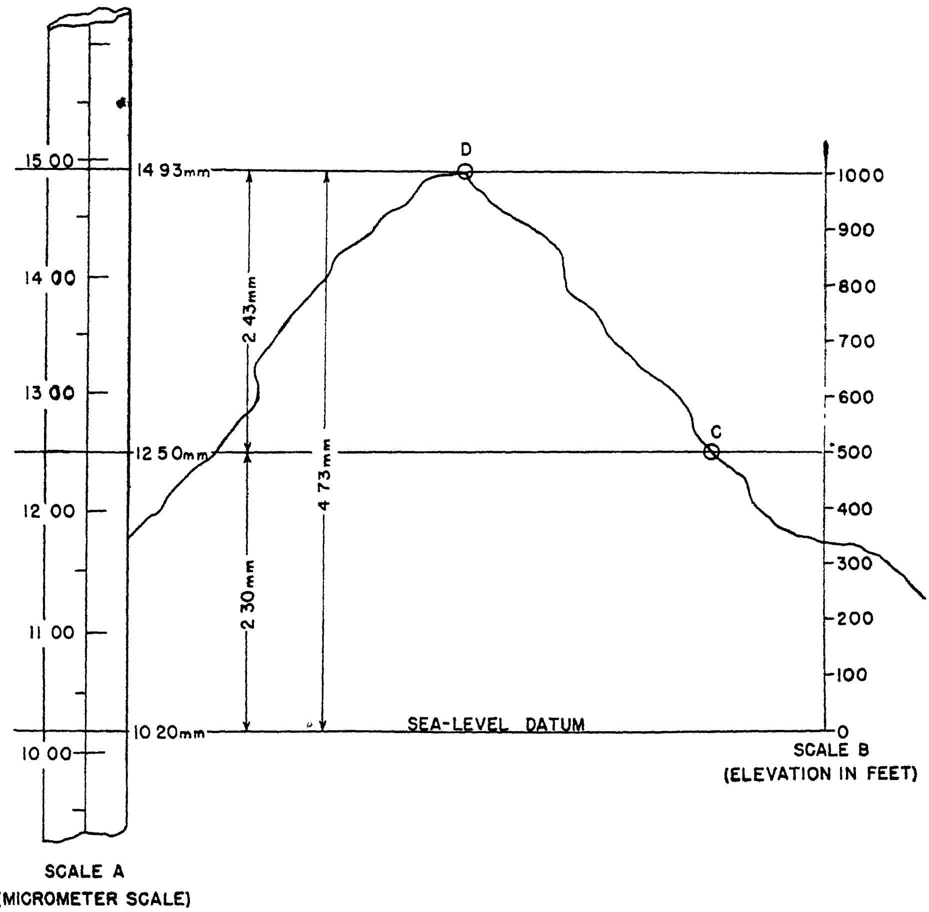


FIG. 18. Diagrammatic view showing the relation between the micrometer scale readings and the corresponding elevations of points above sea level datum.

micrometer adjustment screw, reading in millimeters, whereas scale (B) indicates elevations above sea level datum, in feet. Point (C) is a given ground control point whose elevation has been previously established at 500 feet. Point (D) is another point on the surface of the ground whose elevation is to be determined. It is assumed that the photos are free from tilt or distortion.

In setting up the instrument the following procedure would be carried out: In order to avoid eye strain, the operator should first adjust the mirror stereoscope so that the left hand and right hand floating dots will fuse when the mi-

crometer is set at a value about midway along the scale (i e. about 12.50 mm.). Such an arrangement will allow for movements in either direction without running off the scale. Use the "B-Y" adjustment, if necessary.

Fasten the left hand photograph of the stereo pair to a suitable board so that the machine has a "B-X" motion parallel to the direction of flight (i.e. line connecting the plumb points). Move the right hand photo along this line so that the stereo index distance, which is the distance between the plumb point of the left hand photo and the corresponding plumb point as it appears on the right hand photo, will be equal to the distance between the floating marks. This latter distance will register on the micrometer scale as 12.50 mm.

If, by chance, the two plumb points are located at sea level (which they rarely are) then the micrometer reading of 12.50 mm would actually correspond to the sea level datum plane elevation. All other points at higher elevations would thus have micrometer readings proportionately higher than 12.50 mm. Also, all points at sea level would have a reading of 12 50 mm., provided no other distortions are present.

Should the plumb point of the right hand photo be at some other elevation than sea level, resort can be made to equation (8) to obtain the correct sea level value of the stereo base distance (B_m)

Thus·

$$B_m = \frac{B_x\ (H - h_x)}{H}$$

Where: B_x = the actual stereo base distance as measured on the left hand photo.

h_x = elevation above sea level of the right hand plumb point

For example (B_x) = 100 mm.; H = 20,000 feet; h_x = 2000 feet.

Then:

$$B_m = \frac{100\cdot(20{,}000 - 2{,}000)}{20{,}000} = 90 \text{ mm.}$$

The setting of the micrometer reading for the stereo index distance, as measured between the plumb points as they actually exist on the photographs, would be $(12.50 + (B_x - B_m) = 12\ 50 + 10.00 = 22.50$ mm In other words:

$$\Delta p_x = B_x - B_m = B_m\left(\frac{h_x}{H - h_x}\right) = 90\ 00\cdot\left(\frac{2{,}000}{18{,}000}\right) = 10.00 \text{ mm.}$$

Similar micrometer readings of 22.50 mm. would be read for all other points whose elevations were exactly 2000 feet above sea level. Elevations less than 2000 feet would have micrometer readings varying from 22.50 to 12.50 mm.

From the above example it is seen that the value of 12 50 mm., which was chosen for the reading at sea level, is an arbitrary one. Any other setting could have been chosen for sea level datum without changing the micrometer reading for corresponding differences in elevation. Care must be taken, however, to readjust the mirrors of the stereoscope whenever a new sea level micrometer reading is assumed.

The same results could have been obtained if a vertical control point was available whose elevation was at sea level. (Note: Many such points would be

available if mapping work was being conducted along the coastline of a country bounded by one of the oceans.) In such cases the orientation of the right hand photograph could be accomplished by setting the two index marks on the corresponding sea level control points instead of the plumb points. The micrometer reading would again read 12.50 mm. for all points at sea level since the stereo index distance would be exactly the same as before. Vertical control points at sea level elevation are of great importance in connection with the use of the Stereocomparagraph inasmuch as the stereo base distance (B_m), or the elevation of the plumb points, need not necessarily be known.

Still another method of orienting the stereo pair of photos for contouring with the Stereocomparagraph is the one in which one or more vertical ground control points are available, but no plumb point elevations are known. See Figure 18. In this case, the left and right photos are first oriented so that the plumb points and the transferred plumb points lie on a line parallel to the "B-X" motion of the Stereocomparagraph. The interval between the photos is established by orientation of the two index marks over the corresponding images of the given vertical control point (C), of Figure 18, and noting the micrometer base reading corresponding to that elevation The micrometer reading for another elevation such as the point (D) can be obtained by adding the base reading to the computed difference in "differential parallax distance" (i.e , Δp) between the control point elevation (C) and the elevation of the new point (D) If this latter point happens to be at an elevation corresponding to that of one of the contour lines, another value of (Δp) can be computed equal to the contour interval and added to the micrometer reading of the point (D). Other values of (Δp) can be computed similarly to obtain readings of additional contour lines

For example: in Figure 18, the point (C) has an elevation of 500 feet and has a corresponding micrometer base reading of 12.50 mm. The value of (Δp) between sea level and 500 feet (for a stereo base distance of (B_m) = 90.00), is equal to

$$p = B_m\left(\frac{h_{500}}{H - h_{500}}\right) = 90\left(\frac{500}{20,000 - 500}\right) = 2.30 \text{ mm}$$

Hence, the sea level reading of the micrometer would be 12.50 − 2.30 = 10.20 mm The micrometer reading of some point at some other elevation such as the point (D) whose elevation may be assumed as 1000 feet, would be

$$\begin{aligned} 10\ 20 + P_{1000} &= 10.20 + \left(B_m\left(\frac{h_{1000}}{H - h_{1000}}\right)\right) \\ &= 10.20 + 90\left(\frac{1,000}{19,000}\right) = 10.20 + 4.73 \\ &= 14\ 93 \text{ mm.} \end{aligned}$$

The micrometer reading of the point (D) could have been obtained likewise by using the formula for finding the value of (Δp) between points (C) and (D). Thus:

$$\Delta p = \frac{B_m \cdot H \cdot \Delta h}{(H - h_{500})(H - h_{500} - \Delta h)}$$

where:

$$\Delta h = D - C = 1000 - 500 = 500'$$

$$\Delta p = \frac{90 \times 20,000 \times 500}{(20,000 - 500)(20,000 - 500 - 500)} = 2.43 \text{ mm.},$$

which checks the computed difference of

$$14.93 - 12.50 = 2.43 \text{ mm.}$$

IX. Conclusion

From the discussions above it may be concluded that satisfactory contour maps can be compiled by use of the Stereocomparagraph provided sufficient control is located on each photo and provided, further, that a datum plane correction graph, based upon information gathered from those points, is also utilized. It must be remembered, however, that the resulting templet sheet will be in the form of a perspective view of the left hand photo and not an orthographic projection of the surface of the ground. For that reason, the separate templet sheets must be tied to each other by methods incorporating the principles of either the radial line, slotted template, celluloid template, or metal slotted arm process. Final compilation can then be carried out by means of reflecting projectors or by cutting in points from adjacent photographs.

If the master parallax tables are used, a relatively large number of vertical ground points, located at critical changes in topography, must be provided. Less points can be used if the values of (Δp), as obtained from the tables, are multiplied by the expression

$$\left[\frac{H}{H - h - \Delta h}\right].$$

If the values of differential parallax distances are computed from the formula $p = B_m \cdot (h)/(H-h)$ a relatively simple correction graph will result. Fewer points would be required and they would not necessarily have to be chosen at critical breaks in the topography. Less use of the "B-Y" motion screw would also result, especially if the photographs are oriented accurately along the stereo base line.

The author wishes to acknowledge his appreciation to Mr. Nelson L. Peach who has so generously helped with the compilation of the tabular data and the graphical illustrations.

TABLES OF STEREOSCOPIC PARALLAX

For

$$\Delta p = \frac{B_m \Delta h}{H-h}$$

between the limits of $(H-h) = 25{,}000$ feet to $(H-h) = 10{,}000$ feet, when $B_m = 100$ millimeters and $\Delta h = 20$ feet, and between the limits of $(H-h) = 10{,}000$ feet to $(H-h) = 5{,}000$ feet, when $B_m = 100$ millimeters and $\Delta h = 10$ feet.

$(H-h)$ ft.	Δp mm.	$\Sigma\Delta p$ mm.	$(H-h)$ ft	Δp mm.	$\Sigma\Delta p$ mm	$(H-h)$ ft	Δp mm	$\Sigma\Delta p$ mm
25000	0	0	60	0.083	4 249	20	0 087	8.687
24980	0 080	0 080	40	0.084	4.333	22900	0 087	8.774
60	0 080	0.160	20	0 084	4 416	22880	0 087	8.861
40	0 080	0.240	23900	0 084	4 500	60	0 087	8.949
20	0 080	0 321	23880	0 084	4.583	40	0 088	9 036
24900	0 080	0 401	60	0 084	4 667	20	0 088	9 124
24880	0 080	0 481	40	0 084	4 751	22800	0 088	9.212
60	0 080	0 562	20	0 084	4 835	22780	0 088	9 299
40	0 080	0 642	23800	0.084	4 919	60	0 088	9.387
20	0 081	0 723	23780	0 084	5 003	40	0 088	9 475
24800	0 081	0 803	60	0 084	5.087	20	0 088	9.563
24780	0 081	0 884	40	0 084	5 171	22700	0 088	9 651
60	0.081	0.965	20	0 084	5 256	22680	0 088	9.739
40	0 081	1 045	23700	0 084	5.340	60	0 088	9 827
20	0 081	1.126	23680	0 084	5 425	40	0 088	9 916
24700	0 081	1.207	60	0 084	5.509	20	0 088	10 004
24680	0 081	1 288	40	0 085	5 594	22600	0 088	10 093
60	0 081	1 369	20	0 085	5 678	22580	0 089	10 181
40	0 081	1 450	23600	0.085	5.763	60	0 089	10 270
20	0 081	1 532	23580	0 085	5.848	40	0 089	10.358
24600	0 081	1.613	60	0 085	5 933	20	0 089	10.447
24580	0.081	1 694	40	0.085	6.017	22500	0 089	10.536
60	0.081	1.776	20	0 085	6 102	22480	0.089	10 625
40	0 081	1.857	23500	0.085	6.188	60	0 089	10.714
20	0.082	1.939	23480	0 085	6 273	40	0 089	10 803
24500	0 082	2.020	60	0.085	6.358	20	0 089	10 892
24480	0 082	2 102	40	0 085	6 443	22400	0 089	10.981
60	0 082	2 184	20	0.085	6 529	22380	0.089	11 071
40	0 082	2 265	23400	0.085	6.614	60	0 089	11 160
20	0 082	2 347	23380	0.086	6 700	40	0.089	11.250
24400	0.082	2.429	60	0.086	6 785	20	0 090	11.339
24380	0.082	2 511	40	0.086	6 870	22300	0 090	11.429
60	0 082	2.593	20	0 086	6 956	22280	0 090	11 519
40	0 082	2.675	23300	0 086	7 042	60	0 090	11 608
20	0 082	2.758	23280	0.086	7.128	40	0.090	11.698
24300	0 082	2.840	60	0 086	7 214	20	0 090	11 788
24280	0.082	2 922	40	0 086	7.300	22200	0 090	11 878
60	0.082	3.005	20	0 086	7.386	22180	0 090	11 968
40	0.082	3 087	23200	0.086	7.472	60	0 090	12 059
20	0 083	3.170	23180	0.086	7.559	40	0 090	12 149
24200	0 083	3.252	60	0.086	7.645	20	0.090	12 239
24180	0 083	3.335	40	0 086	7.731	22100	0 090	12.330
60	0.083	3.418	20	0 086	7.818	22080	0 091	12 420
40	0.083	3.501	23100	0 087	7.904	60	0 091	12 511
20	0 083	3.583	23080	0.087	7.991	40	0 091	12 602
24100	0 083	3.666	60	0 087	8.078	20	0.091	12 692
24080	0 083	3.749	40	0 087	8.164	22000	0.091	12.783
60	0 083	3.832	20	0 087	8.251	21980	0.091	12.874
40	0.083	3 916	23000	0 087	8.338	60	0 091	12.965
20	0.083	3.999	22980	0.087	8.425	40	0.091	13.056
24000	0.083	4.082	60	0 087	8.512	20	0.091	13.148
23980	0.083	4.166	40	0.087	8.599	21900	0.091	13.239

$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*
21880	0.091	13 330	20680	0 097	18 971	60	0 103	25 051
60	0 091	13 422	60	0 097	19 068	40	0 103	25 154
40	0 092	13 513	40	0 097	19 164	20	0 103	25 257
20	0 092	13 605	20	0 097	19 261	19400	0 103	25 360
21800	0 092	13 697	20600	0 097	19 358	19380	0 103	25 463
21780	0 092	13 788	20580	0 097	19 456	60	0 103	25 567
60	0.092	13 880	60	0.097	19 553	40	0 103	25.670
40	0 092	13.972	40	0 097	19 650	20	0 103	25 773
20	0 092	14 064	20	0 097	19 747	19300	0 104	25 877
21700	0.092	14.156	20500	0 098	19 845	19280	0 104	25 980
21680	0 092	14 249	20480	0 098	19 943	60	0 104	26 084
60	0.092	14.341	60	0.098	20 040	40	0 104	26 188
40	0 092	14 433	40	0 098	20 138	20	0.104	26 292
20	0 092	14 526	20	0 098	20 236	19200	0.104	26 396
21600	0 993	14 618	20400	0 098	20 334	19180	0.104	26.501
21580	0 093	14 711	20380	0 098	20 432	60	0 104	26 605
60	0 093	14 804	60	0 098	20 530	40	0 104	26 709
40	0.093	14 896	40	0 098	20 629	20	0 105	26 814
20	0 093	14 989	20	0 098	20 727	19100	0 105	26 919
21500	0 903	15 082	20300	0.098	20 825	19080	0 105	27 023
21480	0 093	15 175	20280	0 099	20.924	60	0 105	27 128
60	0 093	15 268	60	0 099	21 023	40	0 105	27 233
40	0 093	15 362	40	0 099	21 121	20	0 105	27 338
20	0 093	15.455	20	0 099	21 220	19000	0 105	27.444
21400	0 093	15 548	20200	0 099	21 319	18980	0 105	27 549
21380	0.094	15.642	20180	0.099	21.418	60	0 105	27 654
60	0 094	15 736	60	0 099	21 517	40	0 106	27 760
40	0 094	15 829	40	0 099	21 617	20	0 106	27 865
20	0 094	15 923	20	0 099	21 716	18900	0 106	27 971
21300	0 094	16 017	20100	0 099	21 815	18880	0 106	28 077
21280	0.094	16.111	20080	0.100	21 915	60	0 106	28.183
60	0.094	16.205	60	0 100	22 015	40	0 106	28.289
40	0 094	16 299	40	0 100	22 114	20	0 106	28 395
20	0 094	16 393	20	0.100	22 214	18800	0 106	28 502
21200	0 094	16.487	20000	0.100	22 314	18780	0 106	28 608
21180	0 094	16 582	19980	0 100	22.414	60	0 107	28 715
60	0 094	16.676	60	0.100	22.514	40	0 107	28 821
40	0.095	16.771	40	0 100	22 615	20	0 107	28.928
20	0 095	16 865	20	0.100	22 715	18700	0.107	29.035
21100	0 095	16 960	19900	0.100	22 815	18680	0.107	29 142
21080	0 095	17 055	19880	0.101	22.916	60	0.107	29 249
60	0 095	17.150	60	0.101	23 017	40	0 107	29.356
40	0.095	17 245	40	0.101	23 117	20	0 107	29.464
20	0.095	17.340	20	0.101	23.218	18600	0 107	29 571
21000	0 095	17 435	19800	0.101	23 319	18580	0.108	29.679
20980	0.095	17 531	19780	0.101	23.420	60	0 108	29 787
60	0 095	17 626	60	0.101	23 521	40	0 108	29 894
40	0 095	17.721	40	0.101	23 623	20	0.108	30.002
20	0 096	17.817	20	0.101	23.724	18500	0 108	30 110
20900	0 096	17.913	19700	0.101	23 826	18480	0 108	30 219
20880	0.096	18 008	19680	0.102	23.927	60	0 108	30 327
60	0.096	18.104	60	0.102	24.029	40	0.108	30.435
40	0.096	18.200	40	0.102	24.131	20	0 109	30 544
20	0.096	18 296	20	0.102	24.232	18400	0 109	30 652
20800	0 096	18.392	19600	0.102	24 334	18380	0.109	30.761
20780	0 096	18.488	19580	0.102	24.437	60	0 109	30.870
60	0 096	18 585	60	0.102	24 539	40	0.109	30.979
40	0 096	18.681	40	0.102	24.641	20	0.109	31.088
20	0.096	18.778	20	0.102	24.743	18300	0.109	31.197
20700	0.097	18.874	19500	0.103	24.846	18280	0 109	31 307
			19480	0.103	24 949	60	0.109	31.416

$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm.*	$(H-h)$ *ft.*	Δp *mm*	$\Sigma\Delta p$ *mm.*	$(H-h)$ *ft.*	Δp *mm*	$\Sigma\Delta p$ *mm.*
40	0 110	31.526	20	0 117	38 448	15800	0.126	45 886
20	0.110	31.635	17000	0 118	38 566	15780	0 127	46 013
18200	0 110	31.745	16980	0.118	38 684	60	0 127	46 140
18180	0 110	31 855	60	0 118	38 802	40	0 127	46 267
60	0 110	31 965	40	0 118	38 920	20	0.127	46 394
40	0 110	32 075	20	0 118	39 038	15700	0 127	46 521
20	0 110	32 186	16900	0 118	39 156	15680	0 127	46 649
18100	0 110	32 296	16880	0 118	39 274	60	0 128	46 776
18080	0 111	32 407	60	0 119	39 393	40	0 128	46 904
60	0 111	32 517	40	0 119	39 512	20	0 128	47 032
40	0 111	32 628	20	0 119	39 631	15600	0 128	47 160
20	0 111	32 739	16800	0 119	39 750	15580	0 128	47 288
18000	0 111	32 850	16780	0 119	39 869	60	0 128	47 417
17980	0 111	32 961	60	0 119	39 988	40	0.129	47 546
60	0 111	33 073	40	0 119	40 107	20	0 129	47 674
40	0 111	33 184	20	0 120	40 227	15500	0 129	47 803
20	0 112	33 296	16700	0 120	40 347	15480	0 129	47 932
17900	0 112	33 407	16680	0 120	40 466	60	0 129	48 062
17880	0 112	33 519	60	0 120	40 586	40	0 129	48 191
60	0 112	33.631	40	0 120	40 706	20	0.130	48 321
40	0 112	33 743	20	0 120	40 827	15400	0 130	48 451
20	0.112	33.855	16600	0 120	40 947	15380	0 130	48 580
17800	0 112	33.968	16580	0 121	41 068	60	0 130	48 711
17780	0 112	34 080	60	0 121	41 188	40	0 130	48 841
60	0 113	34 193	40	0.121	41 309	20	0 130	48 971
40	0 113	34 305	20	0.121	41 430	15300	0 131	49 102
20	0 113	34 418	16500	0 121	41 551	15280	0 131	49 233
17700	0 113	34 531	16480	0.121	41 673	60	0.131	49.364
17680	0 113	34 644	60	0 121	41 794	40	0.131	49 495
60	0.113	34.757	40	0.122	41 916	20	0.131	49 626
40	0.113	34 871	20	0.122	42 037	15200	0 131	49.758
20	0.113	34.984	16400	0 122	42 159	15180	0 132	49 889
17600	0 114	35.098	16380	0.122	42 281	60	0 132	50.021
17580	0 114	35.211	60	0.122	42 403	40	0.132	50.153
60	0 114	35 325	40	0.122	42 526	20	0 132	50 285
40	0 114	35.439	20	0.122	42 648	15100	0.132	50.418
20	0.114	35 553	16300	0.123	42 771	15080	0 133	50.550
17500	0 114	35.667	16280	0.123	42 894	60	0 133	50.683
17480	0 114	35.782	60	0 123	43 017	40	0 133	50.816
60	0 114	35.896	40	0.123	43 140	20	0 133	50 949
40	0 115	36.011	20	0 123	43 263	15000	0 133	51 082
20	0 115	36 126	16200	0.123	43 386	14980	0.133	51.216
17400	0 115	36 240	16180	0 124	43 510	60	0 134	51 349
17380	0 115	36.355	60	0 124	43.633	40	0 134	51 483
60	0 115	36 471	40	0 124	43.757	20	0 134	51 617
40	0.115	36.586	20	0.124	43 881	14900	0 134	51 751
20	0 115	36.701	16100	0.124	44.005	14880	0.134	51 885
17300	0 116	36 817	16080	0 124	44.130	60	0 134	52 020
17280	0.116	36 932	60	0 124	44 254	40	0 135	52 155
60	0 116	37 048	40	0 125	44.379	20	0 135	52.290
40	0.116	37 164	20	0 125	44 504	14800	0.135	52.425
20	0 116	37.280	16000	0 125	44.628	14780	0.135	52.560
17200	0 116	37.396	15980	0 125	44 754	60	0.135	52 695
17180	0.016	37 513	60	0.125	44.879	40	0.136	52.831
60	0.116	37 629	40	0 125	45.004	20	0.136	52.967
40	0.117	37 746	20	0 126	45 130	14700	0 136	53.103
20	0.117	37.863	15900	0.126	45.255	14680	0.136	53.239
17100	0.117	37.980	15880	0 126	45.381	60	0.136	53 375
17080	0.117	38.097	60	0 126	45.507	40	0.137	53 512
60	0.117	38.214	40	0.126	45.633	20	0.137	53 648
40	0.117	38 331	20	0.126	45.768	14600	0.137	53.785

$(H-h)$ *ft.*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm.*
14580	0 137	53 922	60	0 150	62 661	40	0 165	72 237
60	0 137	54 059	40	0.150	62.811	20	0 165	72 402
40	0 137	54 197	20	0 150	62.961	12100	0 165	72 567
20	0 138	54 335	13300	0.150	63 111	12080	0 165	72.732
14500	0 138	54 472	13280	0 150	63 261	60	0 166	72 898
14480	0 138	54 610	60	0 151	63 412	40	0 166	73 064
60	0 138	54 749	40	0 151	63.563	20	0 166	73 230
40	0 138	54 887	20	0 151	63 714	12000	0.167	73 397
20	0 139	55.026	13200	0 151	63 866	11980	0 167	73 563
14400	0 139	55.164	13180	0 152	64 017	60	0 167	73 730
14380	0.139	55 303	60	0 152	64 169	40	0 167	73 896
60	0 139	55 443	40	0 152	64 321	20	0 168	74 065
40	0 139	55 582	20	0 152	64 473	11900	0 168	74 233
20	0 140	55 721	13100	0 153	64 626	11880	0 168	74 402
14300	0.140	55 861	13080	0 153	64 779	60	0 168	74 570
14280	0 140	56 001	60	0 153	64 932	40	0 169	74 739
60	0 140	56 141	40	0 153	65 085	20	0 169	74 908
40	0.140	56 282	20	0 153	65 239	11800	0 169	75 077
20	0 141	56.422	13000	0 154	65 392	11780	0 170	75 247
14200	0 141	56.563	12980	0 154	65 546	60	0 170	75 417
14180	0 141	56 704	60	0 154	65 700	40	0 170	75 587
60	0 141	56 845	40	0 154	65.855	20	0 170	75 758
40	0 141	56 986	20	0 155	66 010	11700	0 171	75 928
20	0 142	57 128	12900	0 155	66 164	11680	0 171	76 099
14100	0 142	57 270	12880	0 155	66.320	60	0 171	76 271
14080	0 142	57 412	60	0 155	66.475	40	0 172	76 442
60	0 142	57 554	40	0 156	66 631	20	0 172	76.614
40	0 142	57 696	20	0 156	66 787	11600	0 172	76 787
20	0 143	57 839	12800	0 156	66 943	11580	0 173	76 959
14000	0 143	57 982	12780	0 156	67 099	60	0 173	77 132
13980	0 143	58 124	60	0 157	67 256	40	0 173	77 305
60	0 143	58 268	40	0 157	67 413	20	0 173	77 479
40	0 143	58 411	20	0 157	67 570	11500	0 174	77 652
20	0 144	58 555	12700	0 157	67 727	11480	0 174	77 827
13900	0 144	58 698	12680	0 158	67 885	60	0 174	78 001
13880	0 144	58 842	60	0 158	68 042	40	0 175	78.176
60	0 144	58 987	40	0 158	68 201	20	0 175	78 351
40	0 144	59 131	20	0 158	68 359	11400	0 175	78 526
20	0 145	59.276	12600	0 159	68 518	11380	0 176	78 701
13800	0 145	59 420	12580	0 159	68 676	60	0 176	78 877
13780	0 145	59 565	60	0 159	68 836	40	0 176	79 054
60	0 145	59 711	40	0 159	68 995	20	0 177	79 230
40	0 145	59 856	20	0 160	69 164	11300	0.177	79 407
20	0 146	60 002	12500	0 160	69 314	11280	0 177	79 584
13700	0 146	60 148	12480	0 160	69 474	60	0 177	79 761
13680	0.146	60 294	60	0 160	69 635	40	0 178	79.939
60	0.146	60.440	40	0 161	69 796	20	0 178	80 117
40	0 147	60 587	20	0 161	69 956	11200	0 178	80 296
20	0 147	60 733	12400	0 161	70 118	11180	0 179	80 474
13600	0 147	60 880	12380	0 161	70 279	60	0 179	80 654
13580	0 147	61 027	60	0 162	70 441	40	0 179	80 833
60	0.147	61 175	40	0 162	70 603	20	0 180	81.013
40	0.148	61 322	20	0 162	70 765	11100	0 180	81.193
20	0 148	61 470	12300	0 162	70.927	11080	0 180	81 373
13500	0 148	61.618	12280	0.163	71 090	60	0 181	81 554
13480	0 148	61.767	60	0 163	71.253	40	0 181	81 735
60	0 148	61 915	40	0.163	71 416	20	0 181	81 916
40	0.149	62 064	20	0.164	71 580	11000	0.182	82 097
20	0.149	62.213	12200	0 164	71 744	10980	0 182	82.280
13400	0 149	62.362	12180	0 164	71.908	60	0.182	82 462
13380	0.149	62.511	60	0 164	72 072	40	0 183	82.645

$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft.*	Δp *mm.*	$\Sigma\Delta p$ *mm*
20	0 183	82 828	50	0 101	93 140	40	0 108	99 533
10900	0 183	83 011	40	0 102	93.241	30	0 108	99 641
10880	0 184	83 194	30	0 102	93 343	20	0 108	99 750
60	0 184	83 378	20	0 102	93 445	10	0 109	99 858
40	0.184	83 563	10	0 102	93 547	9200	0 109	99 967
20	0 185	83 747	9800	0 102	93 649	9190	0 109	100 075
10800	0 185	83 932	9790	0 102	93 751	80	0 109	100 184
10780	0.185	84.118	80	0 102	93 853	70	0 109	100 293
60	0 186	84.304	70	0 102	93 955	60	0 109	100 402
40	0.186	84 490	60	0 102	94 058	50	0 109	100 512
20	0.186	84 676	50	0 103	94.160	40	0 109	100 621
10700	0.187	84 863	40	0 103	94 263	30	0 109	100 730
10680	0 187	85 050	30	0 103	94 366	20	0 110	100 840
60	0 187	85 237	20	0 103	94.468	10	0 110	100 950
40	0 188	85 425	10	0 103	94 571	9100	0 110	101 060
20	0 188	85 613	9700	0 103	94 674	9090	0 110	101 170
10600	0 188	85 802	9690	0 103	94 778	80	0 110	101 280
10580	0 189	85 991	80	0 103	94 881	70	0 110	101 390
60	0 189	86 180	70	0 103	94 984	60	0 110	101 500
40	0 190	86 369	60	0 103	95 088	50	0 110	101 611
20	0 190	86 559	50	0 104	95 191	40	0 111	101 721
10500	0 190	86 750	40	0 104	95 295	30	0 111	101 832
10480	0 191	86 940	30	0 104	95 399	20	0 111	101 943
60	0 191	87.131	20	0 104	95 503	10	0 111	102 054
40	0 191	87 323	10	0 104	95 607	9000	0 111	102 165
20	0 192	87.514	9600	0 104	95.711	8990	0 111	102 276
10400	0 192	87 707	9590	0 104	95 815	80	0 111	102 387
10380	0 192	87 899	80	0 104	95 919	70	0 111	102 498
60	0 193	88 092	70	0 104	96 024	60	0 112	102 610
40	0 193	88 285	60	0 105	96.128	50	0 112	102 722
20	0 194	88 479	50	0 105	96.233	40	0 112	102 833
10300	0 194	88 673	40	0 105	96.338	30	0 112	102 945
10280	0 194	88 867	30	0 105	96.443	20	0 112	103 057
60	0 195	89 062	20	0 105	96 548	10	0 112	103 170
40	0 195	89 257	10	0 105	96.653	8900	0 112	103 282
20	0 196	89 452	9500	0 105	96.758	8890	0 112	103 394
10200	0 196	89 648	9490	0 105	96.863	80	0 113	103 507
10180	0 196	89 845	80	0 105	96.969	70	0 113	103 620
60	0.197	90 041	70	0 106	97.074	60	0 113	103 732
40	0 197	90 238	60	0 106	97.180	50	0 113	103 845
20	0 197	90 436	50	0 106	97.286	40	0 113	103 958
10100	0 198	90 634	40	0 106	97 391	30	0 113	104 072
10080	0 198	90 832	30	0 106	97 497	20	0 113	104 185
60	0 199	91.030	20	0 106	97 604	10	0 113	104 298
40	0 199	91.229	10	0 106	97 710	8800	0.114	104 412
20	0 199	91.429	9400	0 106	97 816	8790	0 114	104 526
10000	0.200	91 629	9390	0 106	97 923	80	0 114	104 639
9990	0 100	91.729	80	0 107	98 029	70	0 114	104 753
80	0 100	91 829	70	0 107	98 136	60	0 114	104 867
70	0 100	91 929	60	0 107	98 245	50	0 114	104.982
60	0 100	92.029	50	0.107	98 349	40	0 114	105 096
50	0 100	92 130	40	0.107	98 456	30	0 114	105 210
40	0 101	92.230	30	0 107	98 564	20	0 115	105.325
30	0 101	92 331	20	0.107	98 671	10	0 115	105.440
20	0 101	92.432	10	0 107	98 778	8700	0.115	105 555
10	0.101	92.533	9300	0.107	98 886	8690	0.115	105.670
9900	0 101	92 634	9290	0.108	98 993	80	0.115	105.785
9890	0 101	92 735	80	0.108	99 101	70	0.115	105.900
80	0 101	92.836	70	0 108	99 209	60	0.115	106 016
70	0.101	92 937	60	0.108	99 317	50	0.116	106 131
60	0.101	93 038	50	0.108	99.425	40	0.116	106 247

$(H-h)$ ft.	Δp mm	$\Sigma\Delta p$ mm	$(H-h)$ ft	Δp mm	$\Sigma\Delta p$ mm	$(H-h)$ ft.	Δp mm	$\Sigma\Delta p$ mm.
30	0 116	106 363	20	0 125	113 693	10	0 135	121 603
20	0 116	106.478	10	0 125	113 818	7400	0 135	121 739
10	0 116	106 595	8000	0 125	113 943	7390	0 135	121 874
8600	0 116	106 711	7990	0 125	114 068	80	0 135	122 009
8590	0 116	106 827	80	0 125	114 193	70	0 136	122 145
80	0 116	106 944	70	0 125	114 318	60	0 136	122 281
70	0 117	107 060	60	0 126	114 444	50	0 136	122 417
60	0 117	107 177	50	0 126	114 570	40	0 136	122 553
50	0 117	107 294	40	0 126	114 696	30	0 136	122 689
40	0 117	107 411	30	0 126	114 822	20	0 137	122 826
30	0.117	107 528	20	0.126	114 948	10	0 137	122 963
20	0 117	107 645	10	0 126	115 074	7300	0 137	123 100
10	0 117	107 763	7900	0.126	115 201	7290	0 137	123 237
8500	0 118	107 880	7890	0.127	115 327	80	0 137	123 374
8490	0 118	107 998	80	0 127	115 454	70	0 137	123 511
80	0 118	108 116	70	0 127	115 581	60	0 138	123.659
70	0 118	108 234	60	0 127	115 708	50	0 138	123 787
60	0 118	108 352	50	0 127	115 836	40	0 138	123 925
50	0 118	108 470	40	0 127	115 963	30	0 138	124 063
40	0 118	108 589	30	0 128	116 091	20	0 138	124.201
30	0.119	108 707	20	0 128	116 218	10	0 139	124 340
20	0.119	108 826	10	0 128	116 346	7200	0 139	124 479
10	0 119	108 945	7800	0 128	116 475	7190	0 139	124 618
8400	0 119	109 064	7790	0 128	116 603	80	0 139	124 757
8390	0 119	109.183	80	0 128	116 731	70	0 139	124 896
80	0 119	109 302	70	0 129	116 860	60	0 140	125 036
70	0 119	109 422	60	0 129	116 989	50	0.140	125 176
60	0 120	109 541	50	0 129	117 118	40	0 140	125 316
50	0 120	109 661	40	0 129	117 247	30	0 140	125.456
40	0 120	109 781	30	0 129	117 376	20	0 140	125 596
30	0 120	109 901	20	0 129	117 505	10	0 141	125 737
20	0 120	110 021	10	0 130	117.635	7100	0 141	125.878
10	0 120	110 141	7700	0 130	117 765	7090	0 141	126 019
8300	0 120	110 261	7690	0 130	117 895	80	0 141	126 160
8290	0 121	110 382	80	0 130	118 025	70	0 141	126 301
80	0 121	110.503	70	0 130	118.155	60	0 142	126 443
70	0 121	110.624	60	0 130	118.286	50	0 142	126 585
60	0 121	110.745	50	0 131	118 416	40	0 142	126.727
50	0 121	110 866	40	0.131	118.547	30	0 142	126 869
40	0 121	110.987	30	0.131	118 678	20	0 142	127 011
30	0 121	111.108	20	0 131	118 809	10	0 143	127 154
20	0 122	111.230	10	0.131	118 941	7000	0.143	127 296
10	0 122	111 352	7600	0 131	119 072	6990	0.143	127 439
8200	0.122	111 474	7590	0 132	119.204	80	0 143	127.582
8190	0 122	111 596	80	0 132	119.336	70	0 143	127 725
80	0 122	111 718	70	0.132	119.468	60	0 144	127 869
70	0 122	111 840	60	0 132	119.600	50	0.144	128 013
60	0 122	111.963	50	0.132	119.732	40	0 144	128 157
50	0 123	112 085	40	0.133	119 865	30	0.144	128.301
40	0 123	112 208	30	0.133	119 997	20	0.144	128.445
30	0 123	112 331	20	0 133	120.130	10	0 145	128.590
20	0 123	112.454	10	0.133	120 263	6900	0 145	128 735
10	0 123	112 577	7500	0 133	120.397	6890	0.145	128 880
8100	0 123	112 701	7490	0 133	120 530	80	0 145	129 025
8090	0 124	112 824	80	0.134	120 664	70	0.145	129 170
80	0.124	112 948	70	0 134	120 797	60	0.146	129.316
70	0.124	113 072	60	0 134	120 931	50	0 146	129 462
60	0.124	113 196	50	0 134	121 065	40	0.146	129 608
50	0 124	113 320	40	0 134	121.199	30	0 146	129 754
40	0.124	113.444	30	0.134	121.333	20	0 147	130 901
30	0.124	113 568	20	0 135	121.468	10	0.147	130.048

$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*	$(H-h)$ *ft*	Δp *mm*	$\Sigma\Delta p$ *mm*
6800	0 147	130 195	6190	0 161	139 593	80	0 179	149 968
6790	0 147	130 342	80	0 162	139 755	70	0 179	150 147
80	0 147	130 489	70	0 162	139 917	60	0 180	150 327
70	0 148	130 637	60	0 162	140 079	50	0 180	150 507
60	0 148	130 785	50	0.162	140 241	40	0 180	150 687
50	0 148	130 933	40	0 163	140 404	30	0 181	150 868
40	0.148	131 081	30	0 163	140 567	20	0 181	151 049
30	0 148	131 229	20	0 163	140 730	10	0.181	151 230
20	0 149	131 378	10	0.164	140 894	5500	0 182	151 412
10	0 149	131 527	6100	0 164	141 058	5490	0 182	151 594
6700	0 149	131 677	6090	0 164	141 222	80	0.182	151.776
6690	0 149	131 826	80	0 164	141 386	70	0 183	151 959
80	0 150	131 976	70	0 165	141 551	60	0 183	152 142
70	0 150	132 126	60	0 165	141 716	50	0 183	152 325
60	0 150	132 276	50	0 165	141 881	40	0 184	152 509
50	0 150	132 426	40	0 165	142 046	30	0 184	152 693
40	0 150	132 576	30	0 166	142 222	20	0 184	152 877
30	0 151	132 727	20	0 166	142 378	10	0 185	153 062
20	0 151	132 878	10	0 166	142 544	5400	0 185	153 247
10	0 151	133 029	6000	0 167	142 711	5390	0 185	153 432
6600	0 151	133 180	5990	0 167	142 878	80	0 186	153 618
6590	0 152	133 332	80	0 167	143 045	70	0.186	153 804
80	0.152	133 484	70	0.167	143 212	60	0 186	153.990
70	0.152	133 636	60	0 168	143 380	50	0 187	154 177
60	0 152	133 788	50	0.168	143 548	40	0 187	154 364
50	0 153	133 941	40	0 168	143 716	30	0 187	154.552
40	0 153	134 094	30	0 168	143 884	20	0 188	154.739
30	0 153	134 247	20	0 169	144 053	10	0 188	154 928
20	0 153	134 400	10	0.169	144 222	5300	0 188	155 116
10	0 153	134.553	5900	0.169	144 392	5290	0 189	155 305
6500	0 154	134 707	5890	0 170	144 562	80	0 189	155.494
6490	0 154	134.861	80	0 170	144 732	70	0 190	155.684
80	0 154	135.015	70	0 170	144 902	60	0 190	155.874
70	0 154	135 169	60	0 170	145 072	50	0 190	156 064
60	0 155	135 324	50	0 171	145.243	40	0 191	156 255
50	0.155	135 479	40	0.171	145.414	30	0 191	156 446
40	0.155	135 634	30	0 171	145 585	20	0 191	156.637
30	0.155	135 789	20	0.172	145 757	10	0 192	156 829
20	0.156	135 945	10	0 172	145.929	5200	0 192	157.021
10	0.156	136.101	5800	0.172	146.101	5190	0 192	157.213
6400	0.156	136 257	5790	0.173	146 274	80	0 193	157 406
6390	0.156	136.413	80	0 173	146 447	70	0.193	157 599
80	0.157	136 570	70	0.173	146 620	60	0 194	157.793
70	0.157	136.727	60	0.173	146.793	50	0 194	157.987
60	0.157	136 884	50	0.174	146.967	40	0.194	158.181
50	0.157	137.041	40	0.174	147.141	30	0.195	158.376
40	0 158	137.199	30	0 174	147 315	20	0 195	158.571
30	0.158	137.357	20	0 175	147 490	10	0.196	158 767
20	0.158	137.515	10	0 175	147.665	5100	0.196	158.963
10	0.158	137 673	5700	0.175	147.840	5090	0 196	159.159
6300	0 159	137.832	5690	0.176	148 016	80	0 197	159 356
6290	0 159	137 991	80	0 176	148.192	70	0 197	159.553
80	0.159	138.150	70	0 176	148 368	60	0 197	159.750
70	0.159	138.309	60	0 177	148.545	50	0.198	159.948
60	0.160	138.469	50	0 177	148 722	40	0.198	160.146
50	0.160	138.629	40	0 177	148 899	30	0.199	160.345
40	0.160	138 789	30	0.177	149 076	20	0.199	160.544
30	0 160	138 949	20	0 178	149 254	10	0.199	160.743
20	0 161	139.110	10	0.178	149.432	5000	0 200	160.943
10	0 161	139.271	5600	0.178	149.610			
6200	0 161	139 432	5590	0 179	149.789			

THE MULTIPLEX INSTRUMENT AND ITS USE

T. P. Pendleton

PHOTOGRAMMETRIC instruments of the multiplex type have advantages that lend themselves to mapping of regions of moderate to rugged relief. The instrument is simple in principle and construction and has the great advantage that it permits adjustment of a long series of over-lapping photographs as a single unit All other types of photogrammetric instruments are confined to the consideration of a single pair of photographs as a unit and are without provision for adjustment of a series of adjacent stereoscopic models as a group.

The multiplex instrument is so designed that it will project several overlapping photographs in the form of diapositive plates in such manner that they can be viewed stereoscopically on a table or small viewing screen The vertical projectors used for this purpose must be in such position and orientation as to reproduce exactly the conditions existing at the moment of photography. The relative position of image points in the area common to two adjacent photographs in a series will be defined in space by the intersection of corresponding rays from both projectors when the two are properly oriented The integration of all such points represents an optical model similar in all respects to the area photographed When the two correctly oriented projectors are separated on the supporting bar by a distance $1/n$th the distance between the two exposure points of the aerial camera the two sheaves of rays of the stereoscopic pair will intersect each other and form an optical model having a scale of $1/n$. To make the plastic effect of this model perceptible it is necessary to arrange that the observer shall see one of the photographs with one eye and the other photograph with the second eye This is accomplished by projecting the images in complementary colors and observing them through correspondingly colored glasses. The optical model can then be perceived and measured in three dimensions with a floating mark. Thus all necessary provision is made for the drawing of either planimetric or topographic maps.

The vertical projectors consist of a miniature camera of two essential parts, one with a lens, a cone and focal plane and the other of a lamp house with the necessary condensing lenses. Provision is made for moving the entire assembly in the X, Y and Z directions as needed and for tilting and orienting it as may be necessary. Inasmuch as all rays must be reprojected in directions parallel to the direction of the original rays it is essential that all critical dimensions of the aerial camera be represented in the camera portion of the projector in the ratio of f/F, where f and F are the principal distances of the projector and the aerial camera respectively. For the same reason the original negative must be reduced for reprojection by the same ratio before it is inserted in the focal plane of the multiplex vertical projector. Instruments of this type were designed originally for use of 18×24 centimeter aerial negatives made with a lens of 210 millimeter focal length. Later the projectors were modified for use with photographs made with wide angle cameras to reduce the number of photographs to be handled and the cost of operation as well. In the latest and most improved models the ratio f/F closely approximates 30/132 thus making it necessary to reduce 24×24 centimeter negatives to 54×54 millimeters for reprojection. These small positive plates are known as diapositives.

Another advantage of the multiplex instrument will be found in the fact that it is of a fixed focus type and thereby free of all the constructional difficulties that arise in instruments employing the large original aerial negatives without reduction. The fixed focus feature is accomplished by equipping the projectors

with short focus lenses and a diaphragm of such diameter that the projected image is in reasonably sharp focus through a great range of depth This depth of focus is sufficiently great to yield stereoscopic models satisfactory for mapping at a scale of 1/10,000, those areas having differences of elevation not exceeding 5000 feet. Areas having differences of elevation of 8000 feet can be mapped if the scale of the stereoscopic model is reduced to 1/15,840 and maps at still smaller scales will make it possible to embrace even greater differences of relief should they be encountered.

The design of multiplex projectors can be modified if need be so that the axis of the projectors can be set at an angle to the vertical thus making it possible to employ this type of instrument with oblique as well as vertical aerial photographs.

One of the superiorities of the multiplex design is its facility of spanning long distances between points of established position Advantage can be taken of this "bridging" process when maps of great excellence must be dispensed with on account of remoteness of the area and lack of the normal amount of map control. Under this condition mapping can be facilitated by using a horizontal supporting bar with tables 14 or more feet in length on which many vertical projectors can be suspended simultaneously. Utilizing wide angle aerial photographs exposed at an altitude of 30,000 feet, as is possible with military airplanes, and employing a 14-foot bar it is possible to "bridge" a distance of 66 miles in a single span and provide a stereoscopic model of 600 square miles. When maps of good quality are desired it is customary to use a shorter bar and provide horizontal control at intervals of 8 to 10 miles and elevations in specified positions in every stereoscopic model. With this amount of control the resulting map should be extremely satisfactory as to accuracy in the horizontal sense and should comply with the requirement that the elevation of 90% of the tested points shall not be in error more than one-half the contour interval. Maps for publication at a scale of 1/24,000 or smaller, in country of moderate relief, usually show 20-foot contour lines. Ordinarily these maps are made from photographs exposed at a flight altitude of approximately 12,000 feet indicating that the precision of measurement of elevation for the contour lines must be better than 1 in 1200 to comply with the specifications The precision of measurements of elevation on well defined points under the same conditions is of the order of 1 in 4000

The construction of topographic maps from aerial photographs requires a different work routine than the one followed with the planetable method, as stereoscopic mapping methods differ so greatly from those employed with the older method. It is believed that the following description of the work program for the multiplex instrument will give a general picture of the routine to follow in planning and conducting any stereophotogrammetric mapping project.

Preliminary Planning

It is important that careful study be given in advance to such matters as map accuracy, map scales and a contour interval suitable to the region to be mapped Preliminary planning should give consideration to the type of aerial camera to be used, the determination of its optical characteristics and constants, and other details relating to the actual completion of the aerial photography, such as careful examination of the contractor's camera and an investigation to determine whether his laboratory equipment is capable of developing the film in such manner as to avoid detrimental distortions.

The camera and its constants require very close scrutiny as the success of a

mapping project depends to a great degree on obtaining negatives capable of being satisfactorily employed in the preparation of the diapositive plates to be used in photogrammetric instruments The principal distance (the measured distance along the lens-axis, between the rear nodal point of the lens and the focal plane) must be checked with extreme care, as this value must be used in determining the amount of reduction to apply to the aerial negatives.

The amount of overlap of photographs in the line of flight as well as between parallel flights must be carefully considered as this has a marked effect on map accuracy Accuracy is decreased when distances between exposures in the line of flight (air bases) are too short, and cost is increased when the parallel flight lines are too close to one another. Irregular side laps and badly crabbed photographs augment the cost of mapping operations as these irregularities necessitate a larger amount of control

The influence of flight altitude, contour interval and scale of negative is shown (Table 1) for four types of commonly employed aerial mapping cameras. The relative value of these cameras for mapping purposes is clearly indicated assuming 1) that a constant flight altitude of 13,750 feet is employed; 2) that the flight height is such as to permit drawing of 20-foot contour lines to the usual precision; and 3) that the flight height is such as to yield negatives of a scale of 1/20,000 in each case. To better compare the value of the various cameras the total number of stereoscopic models required to cover an arbitrary area of 480 square miles has been computed and shown. When all four types of cameras are used at an altitude of 13,750 feet the camera of 100 mm focal length will require only 98 stereoscopic models to cover the area for which 225 stereoscopic models are necessary with the 210 mm cameras. It will also be noted that the 100 mm camera photographs will be entirely satisfactory for drawing 23 foot contour lines whereas the 210 mm. camera photographs will yield an accuracy sufficient only for 41-foot contours.

Assuming again that all cameras are employed at altitudes that will enable the resulting photographs to be utilized in drawing 20-foot contours with sufficient precision to meet the usual standard of accuracy the table shows that 128 models made with the 100 mm. camera will cover the same area as 936 models made with the 210 mm camera.

When the photographs are made at altitudes such that all have a scale of 1:20,000 the camera listed in Table 1 which has the shortest focal length will require about twice the number of models to cover the given area, but the resulting precision of the contouring will be approximately four times as great.

Cameras of short focal length are generally superior for topographic mapping to those of longer focal length both in respect to the number of stereoscopic models required to cover the given area and in the precision with which elevations can be measured. These results have a decided effect in the resulting cost of a topographic map as cost will vary approximately as the number of models to be employed. It is of interest to note too that for a given contour interval (20 feet in Table 1) that the 132 mm. camera is much superior to the 152 mm. camera.

It has been found by repeated tests of maps made by the multiplex instrument that under favorable conditions the following empirical equation holds true:

$$H = 765\ Kc \tag{1}$$

where H is the altitude above mean ground level, K the ratio of the air base to the flight altitude with normal overlapping of photographs and c the contour

TABLE 1. PERFORMANCE OF FOUR AERIAL CAMERAS

Focal Length	Negative Size	Scale of Negative	Flight Altitude	Ground Covered by Net Models	Area of Net Model	Stereo. Area Outside of Net Model	No of Strips* (East-West)	Models per Strip*	Total Models*	Contour Interval†
mm	cm		ft	ft	sq mi.	sq. mi				ft.
				Flight Altitude 13,750 Feet						
100	18×18	1 42,000	13,750	Forward 10,950 Lateral 13,220	5.2	5 4	7	14	98	23
132	24×24	1 31,800	13,750	Forward 10,430 Lateral 13,350	5 0	5 5	7	15	105	24
152	24×24	1 27,500	13,750	Forward 8,330 Lateral 14,520	4 3	4.4	7	18	126	30
210	24×24	1 20,000	13,750	Forward 6,030 Lateral 10,560	2 3	2 0	9	25	225	41
				Contour Interval 20 Feet						
100	18×18	1 36,600	12,000	Forward 9,540 Lateral 11,520	3 9	4.1	8	16	128	20
132	24×24	1 25,900	11,200	Forward 8,490 Lateral 10,870	3 3	3.6	9	18	162	20
152	24×24	1.18,000	9,025	Forward 5,450 Lateral 9,500	1.8	1.4	10	28	280	20
210	24×24	1: 9,700	6,680	Forward 2,930 Lateral 5,120	0.54	0.47	18	52	936	20
				Negative Scale 1 20,000						
100	18×18	1·20,000	6,560	Forward 5,210 Lateral 6,290	1.2	1 3	15	29	435	11
132	24×24	1.20,000	8,650	Forward 6,560 Lateral 8,390	1 8	2.0	11	23	253	15
152	24×24	1·20,000	10,000	Forward 6,060 Lateral 10,560	1 7	1.7	9	25	225	22
210	24×24	1.20,000	13,750	Forward 6,060 Lateral 10,560	2 0	2 3	9	25	225	41

* Area is 2–15′ quadrangles 480 sq. miles 91,000×149,800 feet.
† Computed values.

interval. The numerical value of the constant in the equation has been so chosen that tests of accuracy of maps drawn with the resulting photographs will show that 90% of the tested points in favorable terrain will have errors less than one-half the contour interval, the customary measure of map accuracy.

It is apparent that having made a selection of a contour interval appropriate for the terrain to be mapped that the altitude at which the flights must be made will be determined by the type of camera employed as this governs the air base-

flight altitude ratio, and other things being equal, the map accuracy. The following tabulation indicates typical values of K for aerial cameras commonly employed for topographic mapping at the present time. In this tabulation the principal distance of the camera is listed as the equivalent focal length (*E F L.*) of the camera lens employed.

E.F L. mm	K
100	.785
132	.740
152	.590
210	437

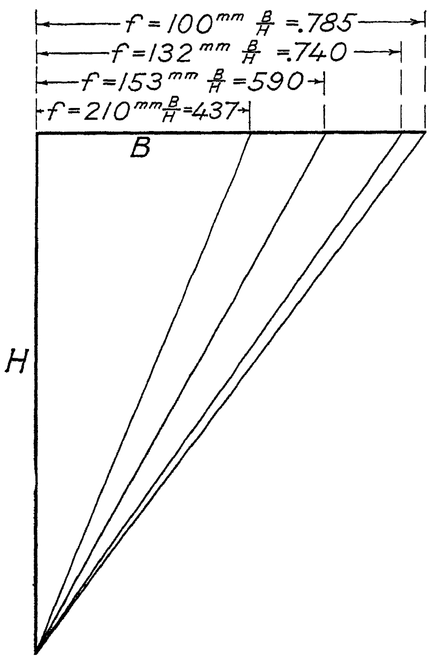


FIG. 1. Air base-flight altitude ratio (K) for aerial cameras

The more favorable value of K for the shorter focal lengths (Fig. 1) have the advantage of permitting higher flight altitudes with a consequent greater area per photograph thus materially reducing the number of photographs to be made and handled. Having selected the contour interval it becomes a simple matter to calculate the flight altitude to be employed from Equation 1 and to determine the length of the air base to be specified from the relation

$$B = HK \tag{2}$$

where B is the air base with other factors as before.

The ability of a skillful operator to secure satisfactory results with the multiplex instruments depends in great part on reprojection by each of the multiplex projectors of a cone of rays identical in all respects to the cone which was effective in recording the image on the aerial negative. If, for example, the effective field of view of the aerial camera is 90° then the field of the projected cone must have the same angular value. This being true, it becomes a simple matter to determine the ratio of reduction to be used in making the diapositive plates when the principal distances of both the aerial camera and the multiplex projectors have been carefully determined. Projectors and aerial cameras most commonly employed by the U. S. Geological Survey have principal distances of 30 and 132 *mm* respectively, consequently, the ratio of reduction to be employed is 132/30 or 4.4 diameters.

This value must be calculated for each camera employed using principal distances determined to the nearest 1/100 millimeter if possible. The relation of critical values in the aerial camera, the multiplex reduction printer and the projectors is shown schematically in Figure 2. The relation of the diapositive, the aerial negative and the optimum projection plane to the flight altitude, will be seen to vary directly as their distances from the point of perspective (Fig. 3).

The scale at which a map should be plotted on the multiplex table depends

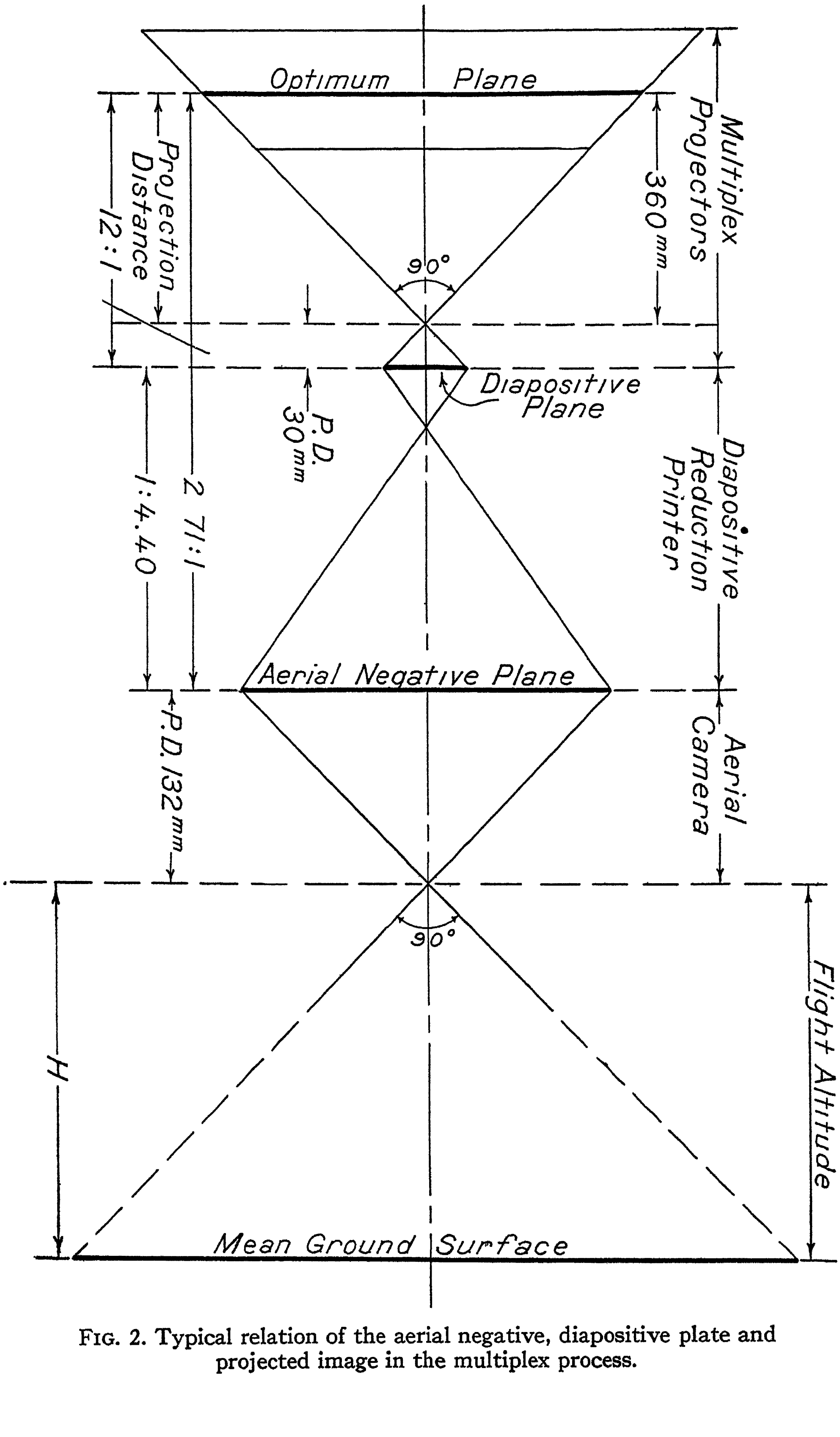


FIG. 2. Typical relation of the aerial negative, diapositive plate and projected image in the multiplex process.

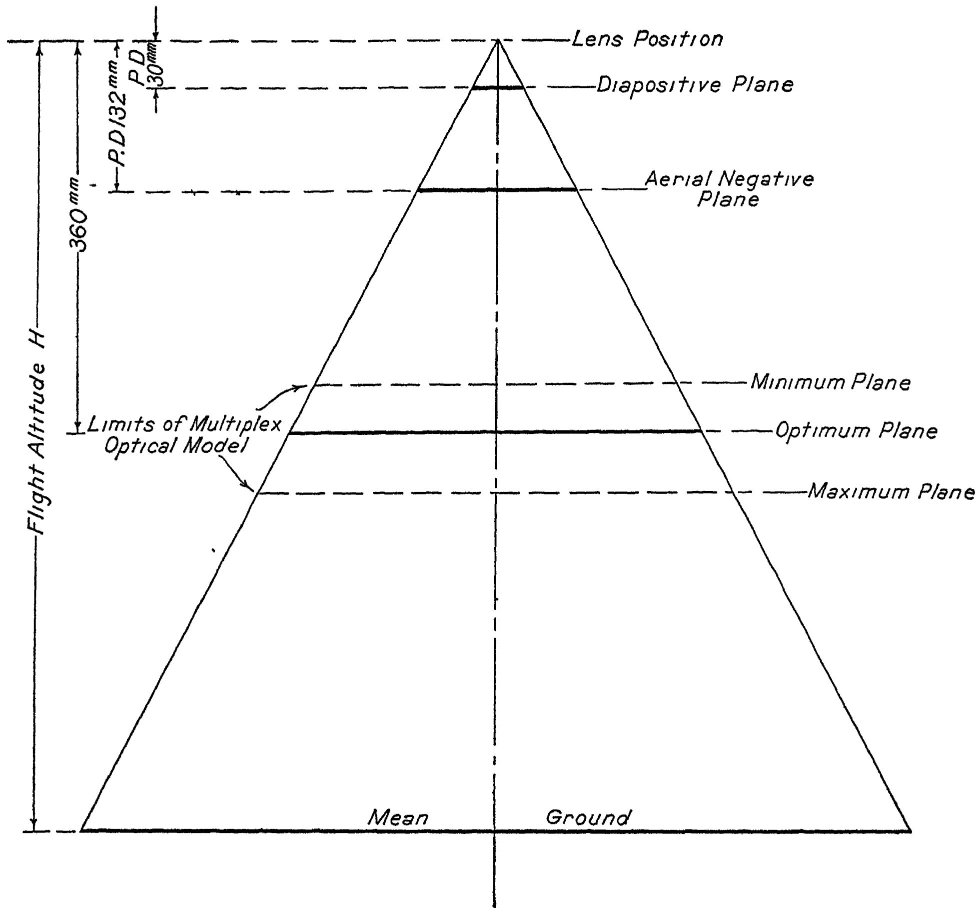


FIG. 3. Sketch showing direct relation of principal planes in the multiplex process.

on the flight altitude at which the aerial photographs were made. This altitude can be determined, knowing the type of camera to be employed and the contour interval to be shown on the map, by application of Equation 1. Simple relations derived from Figure 1 will enable one to calculate the aerial negative scale as approximately 1:36,000, the diapositive scale as 1:22,000 and the original map scale as closely approximating 1:10,000, assuming that a camera of 100 mm focal length is used at a flight altitude of 12,000 feet.

The location of the flight lines that are most desirable can be definitely indicated on a map, if a reliable one is at hand, as this assures a control of the amount of side overlap by specifying in the pilot's instructions the maximum permissible departure. This method has the great advantage that it prevents flights being made along junction lines common to two adjacent maps which are difficult to control and impose unnecessary work on the photogrammetric staff. The mean altitude of flight above mean ground level should also be indicated for each flight line.

CONTROL PLANNING

The control planning operation includes not only a study of the triangulation, traverse and level lines necessary to a good distribution of strong control

throughout the area of the map, but also the general location and type of supplemental control points that may be required in controlling the aerial photographs Supplemental control is additional to control ordinarily available and must be provided if the projected photographs are to be brought into the desired relation to the map surface It consists in part of horizontal control for points so distributed in position as to satisfy the requirements for bringing the series of stereoscopic models to the desired scale in proper orientation and vertical control for use in adjusting the model to the map plane in the vertical sense. The number of supplemental control points needed in any particular case is determined largely by the map accuracy specifications, the design of the aerial camera, and the skill of the airplane pilot. This type of control should always be planned after the photographs are made, as the proper disposition of the control points on the photograph is very important and can not be anticipated before the overlaps of all photographs are known.

Control planning involves a careful study of the forward and side overlaps of the photographs. The points for which data is desired may be indicated on an index map on which are shown graphically the overlap of the photographs on each other or else on a suitable photo index. The control points should be located in positions that will make them available for use on the maximum number of photographs. If it is not feasible to indicate the exact point for which an elevation or position is desired, a small area can be indicated in colored crayon on the face of the photograph to limit the area in which the engineer is to select a point in the field.

Map Sheet Preparation

The map sheet preparation consists of constructing the projection lines that define the limiting parallels and meridians of the map, plotting the basic and supplemental horizontal control points thereon and assembling all the other data that will be needed by the operator. The order in which the drawing operations should proceed is determined largely by the disposition of the supplemental control, and this should be included in the instructions prepared for the multiplex operator.

Diapositive Plate Preparation

The most important element affecting the accuracy of work performed with the multiplex instrument is the quality and size of diapositive plates which are made from the original negatives. These plates, which are utilized in the vertical projectors, must be made to an exact size and have a quality so excellent that they can be enlarged twelve or more diameters without sufficient loss in sharpness to influence the drawing of the map by the multiplex operator. Error in the size of the diapositive will result in errors of elevation in the map and the lack of sharpness will be the cause of unsatisfactory stereoscopic models which will prevent the operator from accurately delineating the contour lines.

The diapositive plates are made by means of a special reduction camera (Fig. 4) which is precisely adjusted to make diapositives of the proper size and of the desired quality from the aerial negatives that are to be utilized in the mapping operation. Owing to the very small size of the diapositive plates and the great influence of their quality of image on the accuracy of the resulting map, it is necessary that the settings of the reduction printer be given close attention. It should be noted that aerial negatives to be utilized in preparing diapositive plates should be used for this purpose soon after the negatives are made and before the negative film has become distorted by age. Any distortion of the negative arising from this cause will be carried over into the stereoscopic model and thus result in maps of a low degree of accuracy. It is a fundamental requirement

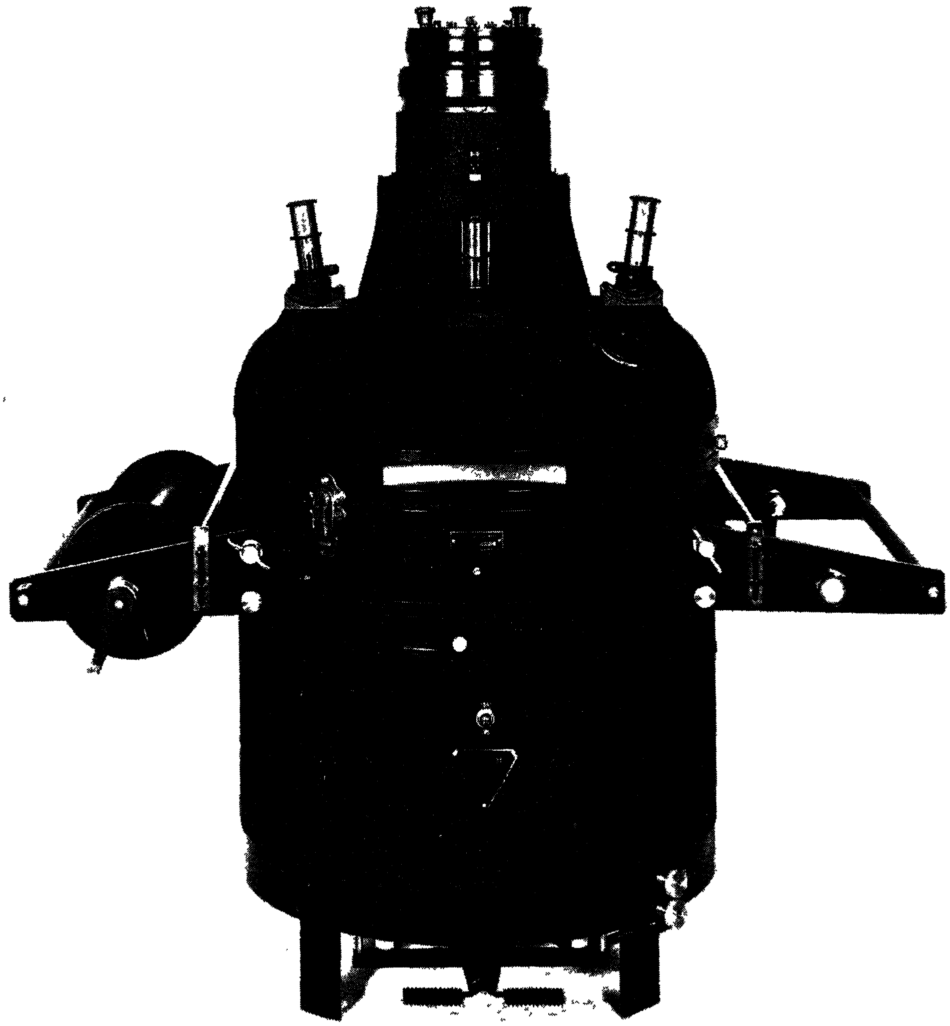


FIG. 4. Multiplex reduction printer, Geological Survey type.

of the multiplex process that the inter-relation of all rays from points in the terrain being photographed which are brought to focus by the aerial camera lens in the focal plane of the camera must be exactly reproduced by projection of the diapositive plate through the lens of the vertical projector. If this angular relation is to be attained it is essential that the original negative be reduced in the ratio of the principal distance of the aerial camera to the principal distance of the multiplex projector. If both of these quantities are known precisely, the ratio can be readily computed and used in calibrating the reduction printer. Proper reduction ratios must be computed for each aerial camera that is employed and the reduction printer settings must be altered whenever a change in the film roll so requires.

The reduction printer is provided with a glass support for the aerial negatives, a special projection lens and a support for the diapositive plates. It is

important that the lens and the diapositive plate be placed in such relation to the aerial negative that a diapositive of a proper size and a maximum degree of sharpness will be obtained. If an undistorted stereoscopic model is to result it is necessary that the reduction printer be fitted with a lens specially designed to compensate for the distortion which was introduced into the original negatives by the aerial camera lens. If this precaution is not taken, the stereoscopic models formed by projection of the diapositive plates will not be exact replicas of the physiographic features of the earth's surface thus introducing errors into the map

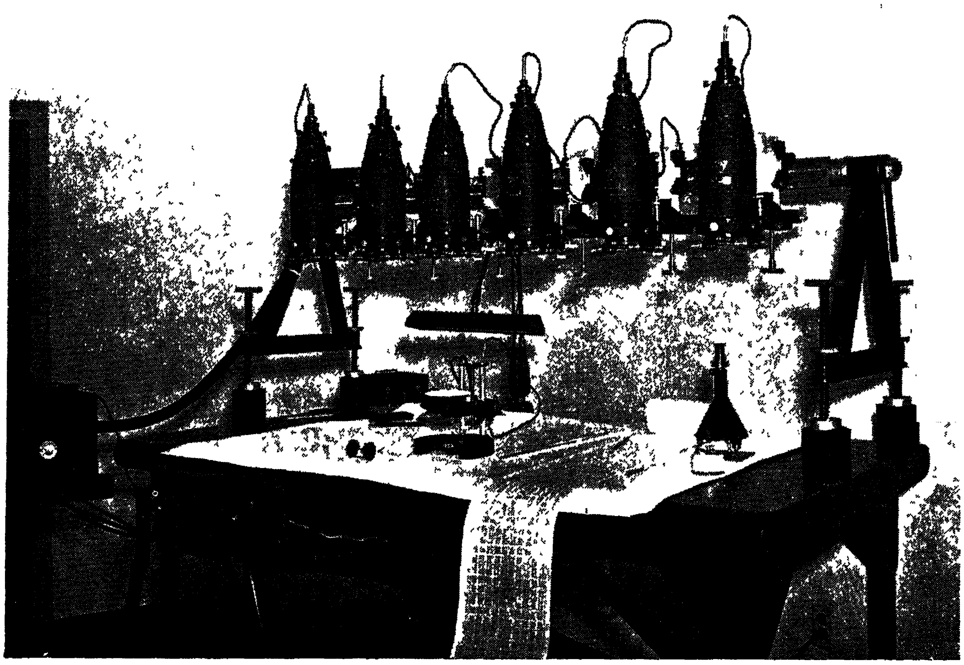


FIG. 5. Multiplex instrument, Geological Survey type.

that may be drawn therefrom. Mechanical means must be provided in the printers to permit such adjustments of the lens and the two focal planes as will be necessary to obtain diapositive plates that will be of a satisfactory quality and size. Failure to obtain the correct size of the diapositive will cause errors in elevation in the map which cannot be readily detected while the map is being drawn, and lack of sharpness in the image will seriously handicap the operator and cause him to make errors that would otherwise be avoided.

Mapping Operation

The multiplex instrument (Fig. 5) consists essentially of a horizontal supporting bar on which are suspended a number of small projection cameras. Each projection camera is so constructed that it is adjustable in three directions with respect to the supporting bar and may be tilted or rotated to bring the small

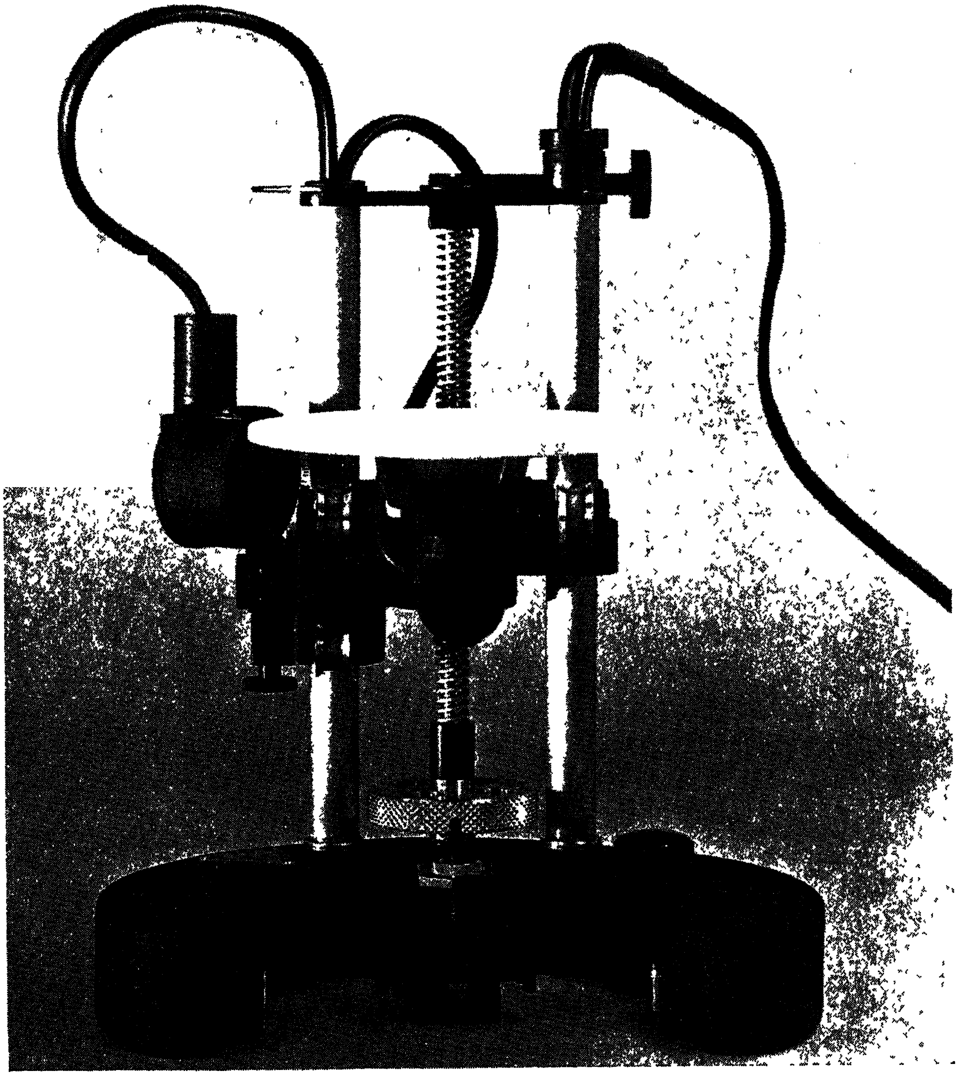


FIG. 6. Multiplex tracing table. (Photo courtesy of Bausch & Lomb Optical Company.)

diapositive plate into the desired position. It contains a platform on which the diapositive is supported, a light source, a light condensing system, a projection lens, and a light filter to color the projected image as desired. The supporting bar is mounted on a rigid table with a plane surface which is used as the reference plane for the mapping operation. The drawing paper with the projection lines and the horizontal control drawn thereon are placed on this table top beneath the supporting bar. A light-weight movable device with a measuring mark on the screen on which the images are cast by the projectors, and a pencil for drawing the map complete the essential parts of the apparatus.

The diapositive plates must first be placed on their supporting platforms in proper sequence, each one being so adjusted as to bring the principal point of

each diapositive plate into coincidence with the principal point of its projector, thus completing what is known as the inner orientation. The stereoscopic model, on which the necessary measurements are made, is the result of proper relative and absolute orientation of the projectors. The relative orientation brings each projector into the same position with respect to neighboring projectors as existed in the air at the moment the photographs were made, and thus makes possible a correct stereoscopic view of the terrain. The absolute orientation has as its purpose such adjustment of the model and projectors as a unit with respect to the map plane as will result in the desired scale and orientation with all points on the model having their proper relation to sea level

The drawing of the map is accomplished by moving the drawing device (Fig. 6) in such manner that it traces out all cultural features such as roads, railroads, trails, transmission lines and visible property lines, as well as residences and all other buildings which from their size or importance merit representation. The representation of the visible property lines can be accomplished quickly and should always be done, not only for the information it makes available in the record, but because these lines are of great help in the completion survey which must follow. Following the drawing of cultural features, the operator will take up the representation of drainage features of the map, which includes the outlines of all water bodies and the courses of all rivers and small water courses. The delineation of wooded regions will follow and finally the contour lines will be drawn showing the shape and position of all topographic features.

Inasmuch as this great amount of detail when represented in pencil on the original map sheet will soon smudge and become poor copy for reproduction, it is necessary that it be inked to increase legibility and to preserve the copy. For this reason, cultural, drainage features and the contour lines are inked roughly as rapidly as the construction of the map will permit.

Multiplex Instruments of Geological Survey Design

Reference has been made in earlier paragraphs of this paper to the effect that many changes have been made in the multiplex instrument as originally designed. It is considered advisable to describe these changes briefly for the benefit of those who have not had an opportunity to become familiar with this instrument, as the improvement in design has had a great deal to do in demonstrating the quality of work of which they are capable.

The Geological Survey received its first multiplex instrument in 1935 and immediately initiated an experimental program to determine whether an instrument of this type could profitably be employed in preparing topographic maps comparable in accuracy to those made with the plane-table. After a single year's experience with multiplex instruments of foreign manufacture it became evident that this particular type of stereoscopic mapping instrument did possess many qualities that recommended it for topographic and planimetric mapping in the United States.

Four years additional experience indicated quite definitely that although the principle and the general design were satisfactory many features of the instrument as originally constructed were susceptible of improvement. It was with the thought of correcting those faults that had become evident, and to introduce certain improvements, that the Geological Survey decided to modify the design of additional multiplex equipment that it was proposed to purchase. Instruments procured at that time and others ordered at a later date have definitely indicated that these changes have all been beneficial.

Carl Zeiss narrow-angle and wide-angle multiplex projectors utilize diapositive plates measuring 45 by 60 millimeters in size, not all of which is necessarily image area. The principal distances of these projectors are 46 and 22 millimeters respectively. The narrow-angle projectors are so designed that the diapositive images are enlarged about 7 8 diameters in the plane of sharpest definition of the projected image; in the case of the wide-angle projectors this enlargement exceeds 16 diameters. The quality of the stereoscopic models obtained from these two types of projectors is such that the multiplex operator obtains the greatest sense of sharpness from the model secured from the smaller projector. Appearance of the model alone should not be allowed to influence one's judgment unduly as further comparison shows that the orientation is stronger and that differences of elevation can be measured more precisely when wide-angle projectors with their more favorable "air base-flight altitude" ratio are employed. The cost of operation with the wide-angle multiplex projectors is lower, also, due to the much larger area covered by each individual model.

Consideration of these facts indicated that new equipment might be so designed as to retain all the advantages of the Zeiss instruments yet result in an appreciable improvement in the quality of the stereoscopic model and eliminate certain mechanical faults. Consequently, it was decided to increase the diapositive plates from 45 by 60 millimeters to 64 by 64 millimeters in size and increase the principal distance from 22 to 30 millimeters. These changes would not necessitate any variation of base-altitude ratio but would bring about a reduction in the amount of enlargement given the diapositive and materially improve the definition of the stereoscopic image.

It was also decided that an improvement of far reaching effect would be obtained if the American manufacturer would design a multiplex reduction printer lens having distortion characteristics of such degree as to compensate for the distortion inherent in wide angle lenses of the Metrogon or Topogon types. The effect of distortion of the Topogon lens on the precision of stereoscopic measurements of altitude has been described at length elsewhere.[1] Consequently, the specifications for the construction of the reduction printer carried the stipulation that its projection lens should compensate for the camera lens distortion, if it were possible to do this without serious sacrifice of other desirable characteristics.

The change in principal distance of the projectors with the resulting increase in size of the diapositive plates and the change in the distortion aberrations of the reduction printer lens were the only changes that were specified that had an important bearing on the optical features of the new equipment. However, there were many changes made in the design of mechanical features, and in the materials employed in the construction of the instruments, for the purpose of correcting mechanical faults of the original design. No attempt will be made here to describe all these modifications but reference to the most important changes will be made in the following comments on the new vertical projectors, tracing tables, and reduction printers, which are essential instruments used in the multiplex process.

The Vertical Projector

The decision to increase the principal distance of the vertical projectors necessitated an increase in size of the diapositive plates to 64 by 64 millimeters. This use of larger diapositive plates carried with it several serious implications, one being that the condensing lens system would necessarily have to be much

[1] Source and Correction of Errors Affecting Multiplex Mapping. Russell K. Bean, "Photogrammetric Engineering," Vol. VI, No. 2.

larger in diameter, and this in turn would greatly increase the diameter of the vertical projector and the weight of each unit. A great increase in diameter might be very troublesome in practice, particularly when working at small map scales, as this would prevent the use of photographs taken with short air bases. However, the solution to this problem as suggested by the Geological Survey and worked out in detail by the Bausch and Lomb Optical Company lay in the re-design of certain mechanical features of the projectors By employment of a supporting bracket of different design from that used by Zeiss it has been possible to make the overall diameter of the new vertical projector 10 millimeters less than the old design, notwithstanding the fact that the new condenser housing has a diameter about 28 millimeters greater than that of the Zeiss model. This was accomplished by reshaping the supporting bracket and reducing the clearance between the lower surface of the condensing lens system and the diapositive stage.

The increased size of the projectors and the heavier materials used in their construction naturally increased their weight. The new supporting bracket and the condenser lens housing weighs approximately 24 pounds as compared with a weight of 8 pounds for the corresponding parts of the Zeiss projector. The light weight of these Zeiss units is due in large part to the alloy of aluminum which is used extensively but which is so soft that repairs to it can be made only with great difficulty. On the other hand, heavier materials such as bronze and brass are freely used in the heavier projectors.

The Zeiss projectors, in common with all other multiplex projectors manufactured to the present, are provided with a glass diapositive supporting platform on which is a fine black dot to indicate the position of the principal point of the projector. The use of glass plates for this purpose has been found rather troublesome, particularly when diapositive plates having slightly warped surfaces are encountered. When the retaining spring pressure is applied to warped diapositives supported on a plane glass surface there is a tendency for the diapositives to take on a different degree of warp, or to change position in the projector due to the effect of temperature changes, particularly those changes that are caused by the alternate heating and cooling which occurs when the projectors are allowed to cool off after the days work. The glass diapositive platforms will pick up a number of fine scratches in time no matter how careful the operator may be to prevent their occurrence, and being in the plane of the diapositive emulsion, will project to the multiplex table with great sharpness. The fixed glass stages also collect dust on the lower surfaces which cannot be removed without taking out the diapositive platform and consequently disturbing the calibration of the projector. The glass plate absorbs some light also but this has not been considered sufficiently serious to call for correction.

To overcome the disadvantages of the glass diapositive platform the new projectors have been designed without such plates, the diapositive being supported by four small bosses. This entirely eliminates the possibility of occurrence of scratches or the dulling of the lower surface by the gradual collection of dust thereon. The elimination of the glass stage plate necessitates the use of some other method of completing the inner orientation. This has been accomplished in a rather novel way which is believed to have several advantages. A three-legged microscope, having a magnification of 15 diameters, is placed over the diapositive platform, with its legs supported by three bearing plates so constructed as to bring the reticule of the microscope to such a position that it will indicate always the principal point of the vertical projector. The adjustment of this microscope can be accomplished by means of a special calibrating fixture. (Fig. 7.) When the diapositive plate and the microscope are in position in the

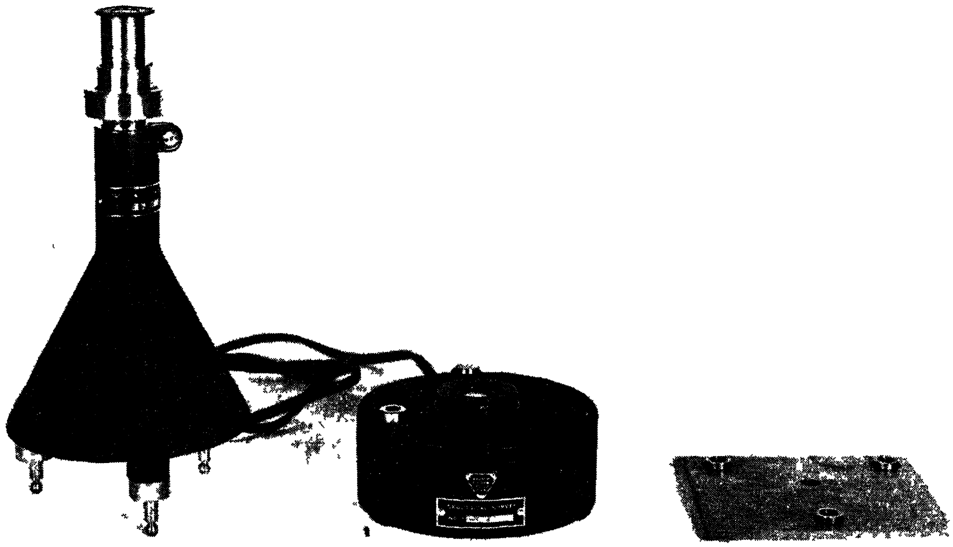


FIG. 7. Principal point microscope and testing fixture

vertical projector, it is possible to accomplish the inner orientation by bringing the principal point of the diapositive plate to coincidence with the cross hairs of the microscope by means of the diapositive push screws which are opposed by spring pressure. This method of indicating the position of the principal point of the vertical projector has the further advantage that it permits the position of this point to be readily tested and adjusted on an optical bench should this be necessary. No provision is made for correcting errors in the location of the principal point of projectors as ordinarily constructed.

As previously stated the diapositive plate is supported by four small bosses and is held in position thereon by pressure devices which apply vertical pressure directly above the four bosses, thereby eliminating any tendency to bend thin plates if such should be employed. As will be seen later, four similarly spaced bosses are used in the reduction printer. Consequently, the diapositive will be supported at only four points of its surface throughout its use.

It is important that the distance between the diapositive platform and the rear node of the projector lens (the principal distance) be the same in all vertical projectors and provision for accomplishing this adjustment has been provided. The single elements of the vertical projector lens are centered with special attention in order to assure that the vertical axis closely approximate a straight line, thus facilitating the calibration of the projectors. The lens barrel is supported in the projector in a sleeve in such manner that the lens can be moved along the axis of the sleeve by opposed clamping rings. The adjustment required to bring all principal distances of the projectors to the desired amount can then be readily accomplished by rotation of the lens clamping rings, the amount of rotation required in any case being readily determined from the pitch of the thread of the clamping rings

The principal distance will be determined indirectly by means of calibrating device (Fig. 8) which has been constructed by the Geological Survey. The Measurement will be accomplished by placing in the projector to be tested, a

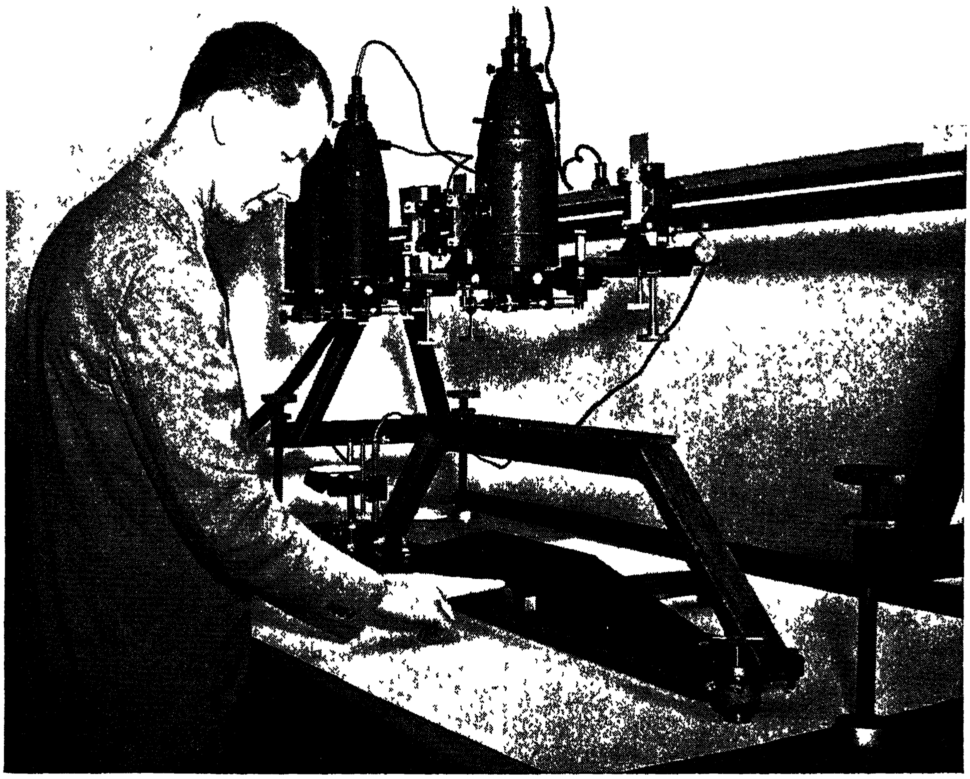


FIG 8 Principal distance determination.

glass plate on which an interval of known-value is engraved. The projector is leveled approximately and centered over the calibrating device and the engraved plate projected in the normal manner onto two horizontal scales of the device which are separated vertically by a known distance. Knowledge of this separation of the scales and the interval subtended on each by the engraved plate in the projector under test permits one to calculate the principal distance by means of a very simple formula Adjustments of the principal distance can be made only with great difficulty in the Zeiss projectors. It has been necessary in the past to shim up the lenses or place them in a lathe in order to take off the necessary small amount of metal from the flange to accomplish this, in each case the desired interval being obtained only after repeated trials. It is believed that the removal of the fixed glass plate from the projectors and the provision for adjusting the lens position and the diapositive platform as a whole will make it a relatively simple matter to calibrate the projectors as closely as may be necessary.

The design of the mechanical parts by means of which the vertical projectors are brought to position in space with the proper degree and direction of tilt has been entirely changed. It is believed that the new design will correct most of the disadvantages that have shown up in the corresponding parts of the instruments now in use.

As might have been expected, the reduction of the enlargment in the optimum plane of the projectors from 16 to 12 diameters has had the effect of im-

proving the illumination on the model. An approximate increase of 25 per cent in the average intensity of transmitted light has been obtained and it is expected that a further material increase will result from the use of a special coating that will be applied to the condenser lens surfaces to reduce the loss of light by reflection that would otherwise occur.

A casual comparison of the two types of projectors available to the Geological Survey will not give one a good idea of the many changes that have been incorporated in the new equipment. The following tabulation lists the most important improvements that have been introduced in the vertical projectors, for the benefit of those who may be interested in comparing the two designs.

1. The new design of the projector lends itself readily to collimation and calibration in the first instance and at later times, should this ever be necessary.
2. The definition has been improved as a result of lower magnification of the image.
3. The illumination has been increased due both to the optical design and the use of a nonreflecting coating on the condenser surfaces.
4. Errors introduced by lack of flatness of the diapositive plates will be eliminated, or at least reduced, by use of glass plates of 3 millimeter thickness if necessary.
5. The method of supporting and adjusting the diapositives to proper position is such that this adjustment can be readily accomplished, and the plate will be firmly held in proper position thereafter.
6. The ease of adjusting the light source results in a better distribution of light over the diapositive.
7. All adjustments of the projector for position can be made with greater smoothness and ease and once accomplished can be maintained. A further advantage results from the ability to change any one of the settings without disturbing others.
8. The relocation of the various tangent screws has increased the free space between the projectors and thus reduced the danger of their accidental disturbance and at the same time improved working conditions for the multiplex operator.
9. The stage plate with its disadvantages has been removed.

The Reduction Printer

The decision to increase the size of the diapositive plates necessitated a corresponding change in dimensions of the diapositive supporting platform as well as the reduction ratio of the printer. The necessity of securing new printers made it possible to incorporate several other ideas in their design to correct objectionable features of the Zeiss reduction printers. It has been the experience of the Geological Survey that the distortion of the projection lenses provided with the Zeiss printers had been gradually reduced over a period of several years until they were without other than a small degree of distortion. This gave a diapositive which contained the entire effects of the distortion of Topogon camera lenses of 100 mm. focal length and, as a result, stereoscopic models formed at a scale of 1:10,000 with an air base-flight altitude ratio of 0.785, were depressed by 0.4 millimeter at their center points. This is equivalent to more than 14 feet of relief in nature and greatly exceeds the error that can be tolerated in a 20-foot contour map constructed in accordance with commonly accepted standards of map accuracy. As a consequence, it was necessary to take from specially prepared charts corrections appropriate to the part of the model being mapped, and apply these to the direct stereoscopic measurements to secure a satisfactory

degree of map accuracy. To avoid loss of time and certain errors that are associated with the use of these correction charts, it was decided to side-step, as it were, these sources of error and delay by deliberately introducing into the reduction printer lens a degree of distortion that would offset these adverse effects of the aerial camera lens, providing this could be done without serious sacrifice of other desirable features.

The mechanical construction of printers made in the past has been such that their reduction ratio could be changed only with difficulty. To alleviate this condition and make possible small changes in the reduction ratio such as would be necessary were an aerial camera of slightly different focal length employed, or if serious film shrinkage had occurred, it was necessary that the diapositive platform and the projection lens be movable with respect to each other and with respect to the negative plane.

Other changes considered desirable related to the matter of adjusting the principal point of the negative supporting platform to its proper position and to the ease and accuracy with which each individual negative could be brought to coincidence with the principal point prior to printing. These ideas have all been incorporated in the new reduction printer with considerable success. The new printer differs materially in appearance from the printers used heretofore.

The body of the Zeiss reduction printer is made of thin sheet metal and is without any provision for adjusting the reduction ratio. Consequently, negatives requiring slightly different reductions for proper use, due either to small differences in the focal length of the cameras with which they were made or to shrinkage of the film base itself, can be employed only by absorbing in the map the error that will result. The negative is placed in position in the Zeiss type of printer by viewing it through a lens giving a magnification of some ½ diameters. The 45 by 60 millimeter diapositive plate is supported by a continuous bearing surface around the perimeter of the plate, and is held in contact therewith by several light-pressure springs. In addition a vertical pressure on the center of the plate is obtained from a spring within the light tight cap which is placed over the diapositive during the exposure. Consideration of this design reveals that any lack of flatness in the diapositive plate may cause it to take up an undesired position when the pressure of the central spring is applied. This spring may also cause thin diapositive plates to bend slightly.

The newly designed reduction printer, as constructed for the Geological Survey by the Bausch and Lomb Optical Company, has a double micrometer device in the head by means of which the settings necessary to secure a given reduction ratio can be made. These micrometers, having a least count of 1/100 millimeter, make it possible to vary readily the conjugate distances between the diapositive plate and the lens and between the lens and the negative plate. It should be understood that this adjustment has sufficient range to care for only very slight differences in the reduction factor and in itself has not sufficient latitude to adjust the printer when a great change in the reduction ratio is necessary.

The diapositive plates are supported in the new printer at four points only and are held in place by spring devices which apply vertical pressure over the four supporting bosses and consequently no bending of the plate can result from this pressure. To eliminate the errors that might be caused by lack of flatness in the glass plates, or their tendency to bend, it was decided to increase the thickness of the diapositives from approximately 1 millimeter as now used, to 3 millimeters if necessary, to secure sufficiently plane surfaces.

Four push screws have been provided for the purpose of moving the negative supporting plate laterally a small amount in order to bring its principal point

into the optical axis of the printer lens, and other adjustments are available to move the diapositive supporting platform in a similar manner. These facilitate the original calibration of the printer.

The diapositive platform of the reduction printer has been made identical with that of the vertical projectors in many respects and this modified form has been adopted as an innovation that will remove some of the disadvantages of the

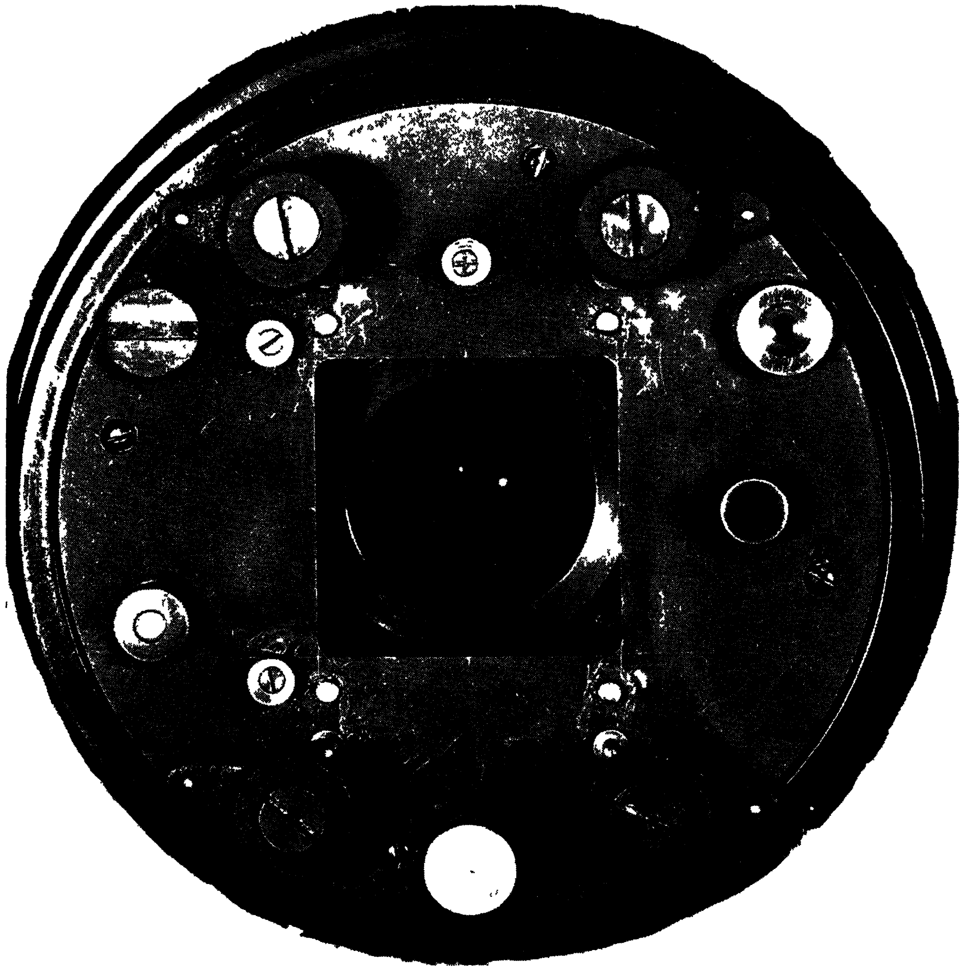


FIG. 9. Reduction printer diapositive platform.

existing type. The platform carries the four bosses that support the diapositive plate and the necessary springs to hold the plate in position against the stops. It also has three points of support for the self-centering microscope (Fig. 9) which have a fixed relation to the four bosses that support the diapositive plate both in the reduction printer and in the vertical projector. These microscopes are useful in checking the adjustment of the reduction printer and are essential in centering the diapositives in the vertical projectors.

One of the most satisfying and useful features introduced into these reduc-

tion printers has been the compensating printer lens. It has been stated before that the Zeiss projector and reduction printer lenses were without distortion, to a degree at least that the multiplex operator had difficulty in detecting its effect by any method readily available to him. Consequently, the full effect of the camera lens distortion was carried into and affected seriously the perfection of the stereoscopic model. The distortion typical of the wide-angle Topogon lens causes the stereoscopic model to be depressed in the center as previously mentioned. This effect is readily apparent in models containing irregular lakeshore lines or rivers of low gradient which pass through the center of the model. Elevation measurements along these shore lines serve as a good direct test of the effect of distortion of the aerial camera lens. Prior to applying such a test it was necessary to determine whether any serious amount of distortion existed in the new vertical projector lenses. This test was made by forming a stereoscopic model from two carefully engraved reseau plates, properly adjusted in a pair of vertical projectors, which when viewed anaglyphically showed that any distortion that might exist in the projector lenses was so small that it could be considered negligible Any considerable distortion of these lenses would have had the effect of causing the net of grid lines to appear as if stretched over a warped surface rather than lying in a plane

The test for the compensation of distortion secured by the new printer lens was made by inserting into a pair of vertical projectors the diapositive plates showing the shore line of a large reservoir, and properly adjusting them to form the best model possible. The average of a number of measurements made by several skilled multiplex operators throughout the area of the model indicated that the water surface was represented as flat to ± 0.05 millimeter, except in one place where the error was -0.2 millimeter. These errors, with the exception of the maximum reading mentioned, are as small as could be detected under the conditions that applied in this case. The occurrence of the -0.2 millimeter reading in only one place in the model, and that far out near the edge of the field, casts some doubt on its significance, as it is to be expected that the effect of camera lens distortion would be symmetrical in the model.

As a consequence of the very satisfactory degree of compensation obtained, the multiplex operator can work with the same freedom as if the lenses of the aerial camera, the reduction printer and the vertical projector were all distortion free. This is a great step forward in multiplex work but it is necessary to add a word of caution regarding the indiscriminate use of reduction printer lenses of this type. The particular printer lens to which reference is made has been calculated to compensate only for the distortion of a particular Topogon lens of 100 millimeter focal length. It is to be expected that it will operate well with other camera lenses of the same focal length and type produced by the same manufacturer, particularly as a special effort has been made to maintain the typical distortion curve for lenses of this type. However, a printer lens of this kind will not compensate properly when used with an aerial camera lens having a different degree of distortion. The indiscriminate use of camera lenses of different kinds and types will require the calculation of special printer lenses in each case so it is important to keep the type of aerial camera lenses employed at a minimum.

For purposes of comparison the major changes made in the reduction printer are stated below for the benefit of anyone who may be interested in contrasting the two designs.

1. The need for correction charts has been eliminated by the employment of a printer lens that compensates for distortions of certain aerial camera lenses.

2. The diapositive is supported in the printer in a manner identical with that used in the vertical projector.
3. The printer light source can be adjusted to aid in shading the negatives during printing.
4. The printer can be set readily to other slightly different reduction ratios.
5. The printer lamp employs 110 volts, thus eliminating the need for a special transformer.
6. The negative is adjusted to proper position by microscopes of considerable power.
7. Collimation adjustments of the printer can be made more readily due to the movable negative and diapositive supporting planes
8. The printer can be operated more easily and better control of printing is possible.

The Tracing Table

The small tracing table is the only part of the multiplex equipment having any freedom of motion. It carries the small platen on which the stereoscopic image is observed, the luminous point used in following the contour lines and other detail throughout the model, and finally the drawing pencil by means of which the path of the tracing table is recorded on the drawing paper These motions are all simple and one would not think that they would cause any great degree of wear in the various parts of such an instrument. However, the continual up and down motion of the platen does cause wear on the vertical standards and this affects the functioning of the platen with its luminous mark. The illumination of the tracing table is controlled by a small toggle switch which must function many times each day. These small switches often fail and require replacement as a result of this heavy use. Nor were the Zeiss tracing tables provided with verniers to facilitate the reading of the vertical positions of the platen which was a distinct drawback until these were added by the Geological Survey. It was found also that the height of the vertical standards might be advantageously increased and thus reduce the need for the extension that must be attached to the platen when excessive relief of the terrain is encountered.

The new tracing tables should overcome many of these disadvantages. The range of motion of the platen has been increased from 70 millimeters to 100 millimeters and an optical reading device with magnification has been provided to facilitate the reading of the vernier against the *Z* scale, which avoids the awkward position that must otherwise be assumed by the operator.

The platen can be inclined 45° around an axis passing through the luminous point. This feature will add to the convenience with which the luminous point can be set on points on the far side of the models, which because of their distant position, are the most difficult ones to read.

It is a prerequisite to accurate work that the luminous point move in a vertical line as the platen is traversed throughout its range of motion. To assure this action great care has been given to the design of the ball bearings which will control the motion of the platen, as it did not seem possible to achieve the necessary accuracy of motion or maintain the adjustment of this part with the old design. It is equally important that the drawing pencil be at all times in a vertical line passing through the luminous point. This position of the pencil can be readily accomplished by convenient adjustment features that have been provided for this purpose. The cam for lowering the pencil to the drawing paper surface has been re-designed in such manner that this action can be accomplished most gently.

The pitch of the elevating screw and the size of the knurled nut by which the

screw is rotated have been so changed that the platen can be easily and rapidly traversed throughout its range. The new construction permits the platen to be "rocked" up and down by a very slight motion of the operator's finger as all back lash has been removed.

It is not anticipated that the loss of time occasioned by breakage of the toggle switches controlling the table lights will occur in the future as micro-switches will be used for this duty. These switches operate with a light touch of the operator's finger and are guaranteed against failure for at least one million operations.

The increased height of the standards and the addition of the tilting platen and reading device fixtures have materially increased the weight of the new tracing tables This is not believed to be a serious disadvantage as the tracing table tested was found to glide over the paper with all necessary freedom. It is important that the table move freely under the operators hands but not so freely as to slide of its own accord. The necessary smooth operation will be attained with these tables notwithstanding the fact that they are about 50 per cent heavier than tables previously used.

The following features are listed as being of greatest importance among the improvements made in the design of the tracing tables·

1. Positive alignment of the vertical motion of the luminous mark and reduction of wear in the parts involved. Ease of adjustment for any wear that may occur is possible without affecting the alignment of the vertical motion.
2. The design of the actuating screw as well as the nut of the elevating device is such as to eliminate all back lash and permit the small rocking motion of the platen which is so essential to accurate measurements of elevation.
3. The construction of the pencil point centering mechanism and the device by which the pencil point is lowered to the drawing paper have been greatly improved.
4. The delays due to frequent breakage of the light switch have been eliminated by the use of a micro-switch having extremely long life and ease of operation.
5. The platen has a type of light hole which was designed and successfully used by the Geological Survey.

The changes that have been incorporated in the multiplex equipment by the Geological Survey have been tried out in service and it can now be stated that each modification has brought about a real improvement in the multiplex instrument.

BROCK PROCESS OF TOPOGRAPHIC MAPPING

Photogrammetric Staff—Aero Service Corporation

THE Brock Process of mapping bears the distinction of being the greatest contribution to Photogrammetry this country has made. Like so many other things that are produced far in advance of general recognition of their importance, it requires much educational work to convince the public of its value. Those difficulties would not be so troublesome were the method being introduced today, as the importance of photogrammetric methods is now being generally recognized. The Brock Process was devised in this country without the slightest regard for developments abroad, which accounts for the unique design of the instruments. This method is remarkable for the simplicity and ruggedness of construction of the various instruments and the fact that the method lends itself well to the production of maps in quantity.[1]

From 1918 to 1921 many experiments were made related to methods of making topographic maps from aerial photographs. The knowledge gained from these experiments led to the following conclusions:

1. In making topographic maps from aerial photographs, some operations can be performed without the use of a machine; those operations performed on machines do not require the same amount of time. Therefore, a single machine performing all operations would not be economically successful, as its output would be limited by the operation requiring the most time.
2. Because of liability of relatively large errors introduced by angular displacement of optics, due to lost motion and deflection of supporting members, non-movable optical systems should be used where possible.
3. That until film free of non uniform shrinkage or expansion is available, aerial photographs should be made on glass plate, for accurate determination of the contours.

It is desired to emphasize here that it is due to accuracy requirements for determining contours that plates were adopted. The disadvantages of plates are weight and bulk. Breakage is a negligible factor, as no work has been affected due to loss of a plate, but plates are more accurate and are more easily handled and developed.[2]

Many aerial photographers would undoubtedly object to the use of this camera due to the time required in loading its magazine, but a Photogrammetrist who has worked on models obtained from glass plates as well as film negatives, will appreciate the greater value of the glass plates. Until such time as film manufacturers can produce a flexible base which will hold its shape with the permanence of a glass plate, this condition will remain unchanged. The convenience of film is paid for now in reduced map accuracy.[3]

The camera is of the full automatic type and is provided with daylight loading magazines which hold 48 glass plates, 6.5″ by 8 5″ of 1/16″ thickness. These plates are placed on individual shelves in the magazine and moved into the focal plane and back into the magazine by septums which only act as guides and cannot cause any deviation from the optical axis.

The cone with the lens mounted and doweled therein constitutes a unit inter-

[1] From *Photogrammetric Engineer*, Volume 6, No 2, p. 87, April, 1940, by T. P. Pendleton, U. S Geological Survey.

[2] From *Journal of Optical Society of America*, March, 1932, by Major Edward H. Cahill.

[3] From *Photogrammetric Engineer*, Volume 6, No. 2, p. 88, April, 1940, by T. P. Pendleton, U. S. Geological Survey.

changeable in the camera proper When the glass plate is in position for exposure, it is pressed firmly against the machined surface of the cone and held in place by four springs located near each corner of the cone. The camera is hand operated. Turning of a crank charges the shutter at the same time the plate moves into position When the plate is in position, the mechanism automatically locks and is released only after the plate is exposed. After the plate is exposed and returned to the original compartment from which it came, the magazine rises and places the next plate in position to resume the operation. As the magazine rises, the light door on the magazine closes by this same amount. This operation continues until the last plate is exposed. The camera door cannot be opened unless the magazine door is closed, which prevents exposure of plates to daylight.

The next magazine is merely placed in the camera which is then ready to operate. The time required for an experienced cameraman to change magazines is about ten seconds. The number of plates that can be carried is limited only by the hours of photographic light available.

The camera is suspended in the plane by a gimbal mount, the oscillation being damped by hydraulic dash-pots. An easy means is also provided to compensate for the unequal weights of the loaded magazine.

The plates are exposed with the longer axis of the plates in the direction of the flight.

The plates must be absolutely flat glass, as any plate that is not such will only cause incorrectable trouble later on in the operations. Plates which deviate from a flatness by more than 2/1,000 inch are rejected and cannot be used. The best type of glass for this operation is of a Belgian make.

Ground Control

As the horizontalization of plates is accomplished by "parallax" differences, the only horizontal control points needed are those from which the radial triangulation is extended. These points, if properly located, can be minimized to five points per quadrangle, one located in each of the corners and the other point somewhere near the center.

Vertical control points are required near the center of each plate and at right angles to the line of flight on each side of the center, in the overlap area of the adjoining flights A lock point is also located somewhere near the center of the model. With the control so located, we have a net gain of four vertical control points per model. These points with the exception of the set point and lock point are so selected to place them in the overlap area of successive plates or the adjoining flights.

At this time, it might be well to mention that good maps are very essential to produce the type of flying required for stereoscopic mapping. Although the flying is executed in strips it practically amounts to spot photography in order to have the flight lines and photographs lined up in such a manner that the ground control can be held to a minimum.

In many cases where good maps are not available, it is cheaper to fly the area for coverage to compile a photographic map for the flying of precision photography than it is to try to fly an area with poor maps.

In recent years, we have probably all heard many people say, "What a shame the country is almost completely covered by aerial photographs which are of little use for topographic mapping." This is not the case. It is merely the first step of the topographic program, since the material covering these areas is available for the compilation of a good reconnaissance map by which the flying for stereoscopic mapping and ground control programs can be carried out.

The cost of topographic mapping by any method varies in direct proportion to the number of models required and therefore the least amount of overlap both in line of flight and on adjoining strips that can be secured without gaps is the most practical. An overlap of 53% in line of flight and a side overlap of 10% are ideal for the Brock process.

Any method of securing the elevations of the selected control points may be used as long as the error in the elevation of the points does not exceed one-tenth of the contour interval.

The points selected for vertical control should always be objects on the ground such as center lines of cross roads, fence corners, field corners, railroad crossings, elevations beside prominent features such as trees, houses, etc. Poor points are such objects as house gables, mountain peaks or any other object that is so small and pointed as to be obliterated when it is etched on the glass plate.

Enlarged Glass Positives

After the plates to be used in the mapping have been selected, the overlap and quality checked and the plates indexed, a glass positive is made on the enlarging projector to confrom in size to a photograph taken with a camera having a focal distance of approximately 13.4″. These positives are made on plate glass 14″×17″ having a thickness of $\frac{1}{8}$″ and a variation from flatness throughout of less than 3/1,000 inch.

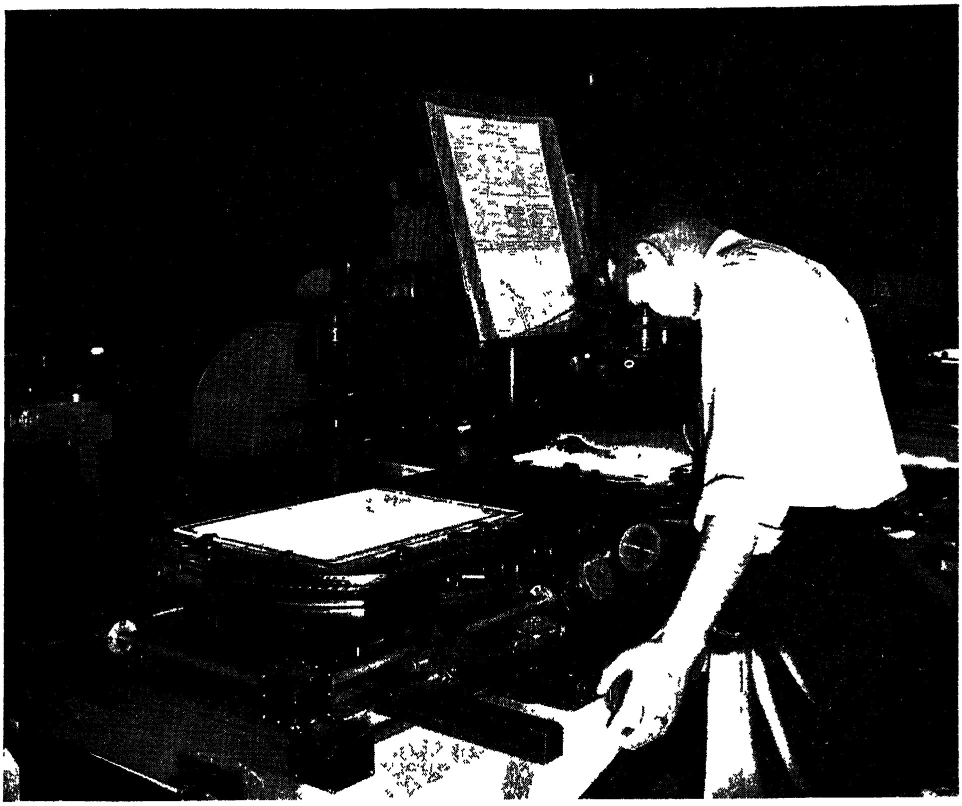
Extra fine grain emulsions and developers are used in each photographic step throughout the process. The enlarging projector is constructed with the lens and receiving plane carriers located in their 1:1 and 1:2 positions by dowel pins. A slow motion is provided to move these carriers from their positions to any other desired location. Jeweled dials reading to one-thousandth inch are provided to record such movement. The illumination is provided by Cooper-Hewitt lights.

Point Marking

After the glass positives have been made the "C" point or center of the glass plate is recovered by etching a cross on the glass, using the indices which were photographed on the plate by the taking camera for its recovery

The plates are now placed on the large stereometer which is a precision measuring instrument and is used later in the process for the contouring (see Fig. 1). This stereometer is provided with two plate holders, each rotating about centers on the same horizontal axis, a means being provided to optically correct any deviation of the center of rotation of either of the plates from this axis. The plates can be moved together as a unit on both the horizontal and vertical axis or the left plate can be moved independently in the horitzontal axis. A dial indicator is provided for the measuring of this movement The dial is calibrated to 1/1,000 inch but .0003 inch can be read easily. All the optics of the stereometer are stationary. The measurements of the desired images are made by moving the plates under the optical system until the images appear under the cross-hairs of the eyepieces. The cross-hairs are etched on optically flat glass and can be independently moved up and down until they appear to be in focus on the same plane as that of the objective without any apparent measurable parallax. The power of the lens in the eyepiece is approximately $2\frac{1}{2}$ diameters.

In order that the plates may be centered as fast as possible, a rule is provided which has a long etched line and center cross on glass. This rule is detachable and fits over the rotating plate holders, the cross being fixed in the axis of the rotation of the plate holder.



Aero Service Corporation, Philadelphia

FIG. 1 Detail view of Brock stereometer.

The left plate is now moved under the corresponding eyepiece until the cross on the rule is directly under the cross-hair of the left eyepiece The rule having served its purpose is then removed. The plate is now moved by two thumb screws on independent X and Y axes (which are built inside of the circular plate holder), until the center of the etched cross is directly under the cross-hairs of the eyepiece. The plate is then locked. The same operation is repeated for the right-hand plate. A sheet of low shrink transparent plastic overlay placed over one of the glass plates will later serve as a templet in the extension of the radial triangulation.

The markers are now swung into position under the eyepiece and the etchers removed, leaving a hole of about an inch to peer through and examine the vertical and horizontal control points which are to be reconstructed on the plates. As the plates are now on the center of rotation of the horizontal axis, one or both of the plate holders is rotated the amount necessary to bring the images of the selected point into stereoscopic fusion without throwing the plates out of their proper alignment. When the conjugate images selected are in such agreement the etchers are placed back in their holders and twisted. This operation breaks through the vellum templet and the emulsion of the plate and records the control point by leaving a tiny clear hole.

At the time the selected point was in stereoscopic fusion and before the marking was made, the dial indicator was read. Now the cross-hairs are independently checked by "blinking," that is, one eye is closed and the plates are independ-

ently examined to see if the etched marking is directly under the cross-hair. If either of the points is not under the cross-hairs, they are moved in coincidence and the dials reread. These differences which have the effect of shortening or lengthening the base are recorded as high or low by the amount of the discrepancy. As the stereoscopic image of the point is sought, these errors in the markings are applied to all further calculations or readings throughout the process.

After all the points have been marked, the plates are rotated back to the original lineup so that the plate centers and conjugate centers all lie on the same horizontal axis.

One of the elevation points near the center of either plate is selected as the set point This set point is assumed to have no parallax and is used as the base distance for the comparison of all bases measured on the model The plates are now separated until the etched markings of the set points lie under the cross-hairs of the corresponding eyepieces and the dial gauge is set on zero and locked.

The plates are now moved under the eyepieces until another control point is visible in the right eyepiece. This etched marking is made to coincide with the vertical cross-hair of the right eyepiece Holding the right plate in this position, the left plate holder is now moved independently on the horizontal axis until the image of the control point on this plate comes into coincidence with the left cross-hair.

The gauge now records the distance of spread or separation from the base distance of the set point. This difference, which is called the original parallax, is recorded and will be used later in the preliminary tilt analysis. While this difference is called the original parallax, it must be remembered that it is a combination of three factors, error due to tilt, scale differences due to variation of altitude in the exposure stations of the two plates, and parallax.

The original parallaxes are measured in the same manner for all vertical control points on the model.

Throughout the Brock process, the difference is distance between two pairs of conjugate image points measured parallel to the horizontal axis, due to the difference in elevation of the two corresponding ground points, is called "parallax." The distances from the horizontal axis of the two conjugate images, measured parallel to the vertical axis are referred to as "ordinates " The differences in the ordinates of the four vertical control points which are located in the corners of the model are also measured and recorded. These ordinate differences are also used in the tilt analysis as well as the parallaxes.

Photogrammetric Control

The low shrink transparent plastic overlays which were over the glass positives now have the control points marked thereon by a very fine needle-like point. The templets are placed over a light table and very fine radials are constructed from the "C" point through all the vertical and horizontal control points. These points are sufficient to extend the radial triangulation without the addition of other points.

The extension of radial triangulation by the Brock method has been used by many mapping organizations and is probably well enough known that it will not be necessary to go into this phase of the process.

Lens Altitude Computations

The lens height for each vertical control point is computed thus: measure the distance from the "C" point to the control point on both the plate and its

corresponding position on the radial plot; obtain the ratio of these bases by dividing the distance on the plate into the distance on the plot; obtain the theoretical lens height by multiplying the scale of the plot in feet per inch by the focal length of the taking camera in inches and multiply this product by the enlargement factor of the plate being used. Then multiply the ratio of the bases by the theoretical lens height to secure the preliminary lens altitude for this particular point. To the preliminary lens altitude the elevation for the vertical control point is added. If the plates were exposed while the camera was truly vertical each of these preliminary lens altitudes would be the correct lens altitude for the particular point, but as the ratios from which we started the computations were calculated using distances scaled from a tilted plate, each of the computations is in error.

Since the control points are so distributed that they lie approximately opposite or 180° from each other, the tilted bases are compensated and a mean of the preliminary lens altitudes is taken as the lens altitude for the plate.

From this lens altitude, the elevation of the set point is subtracted to secure the lens height above the set point.

Computations of Parallax for Tilt Analysis

In order to simplify the computations for parallax, tables have been compiled giving the parallax value resulting from any elevation difference from the set point up to 1,000′ These tables are computed for a lens height of 10,000′ and a base distance between picture centers of 10″. In the preliminary analysis, the distance between the center of the plate and the conjugate center of the adjoining plate is used as the base distance.

To find the parallax of any elevation change of ground for any lens height above the "set point" or datum, divide the table lens height by the lens height in question and the result will be a constant in reference to the pair of plates involved. Multiply the elevation change by this factor and in the tables opposite the apparent ground change will be found the correct parallax. Since the table base is 10″, the final result must be the product of the table parallax and the real base.

Now that the parallax for the vertical control points has been computed and the original parallax for the same points has been read on the stereometer, the difference between these results is the amount of error caused by tilt in the plates and the difference in scale caused by the variation in altitude of the two exposure stations.

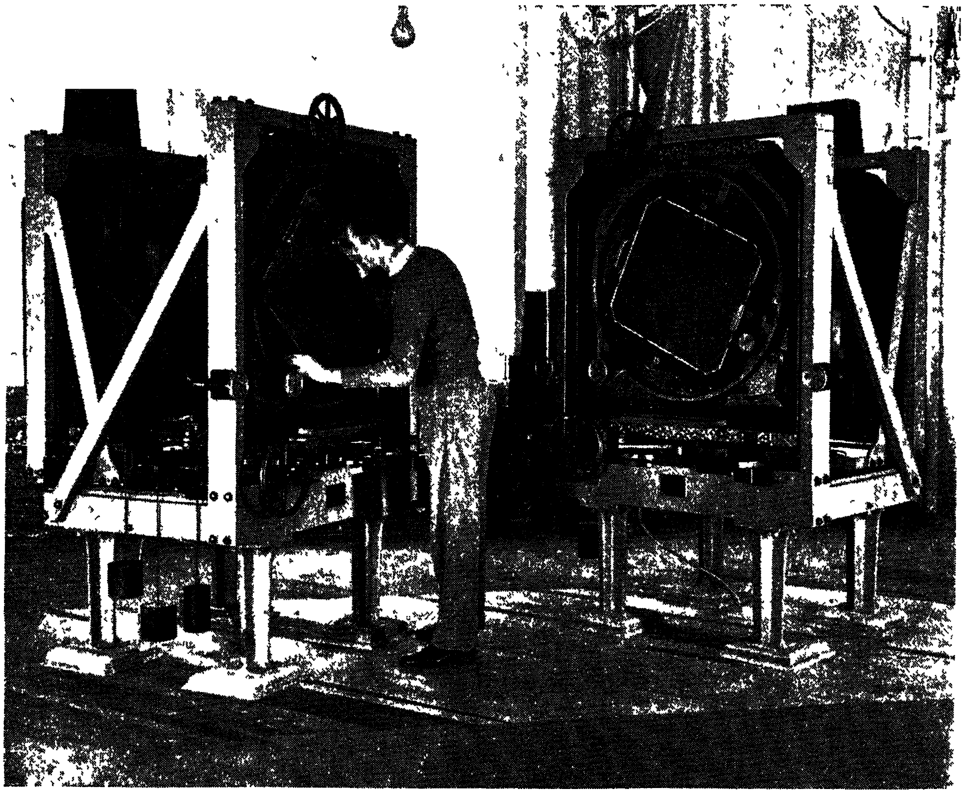
Tilt Analysis

A set of transparent celluloid tilt charts, each having a range of ten minutes and running up to $2\frac{1}{2}°$, is available. These charts show the movement of any point in the area in both the parallax and ordinate directions if this amount of tilt is applied. A vellum templet is traced from one of the plates showing the location of vertical control points in reference to a line projected from the center of the plate to the conjugate center of the adjoining plate.

Since all of the error in the plates is not caused by tilt, a study is made of the lens heights of the two plates and an enlargement factor applied to bring both plates of the model to a uniform scale.

The result of this scale change as it affects the individual control points is deducted from the difference between the original parallax readings and the computed parallax. The difference is assumed to be the error caused entirely by tilt.

The tilt charts are laid over the vellum overlay, orienting the charts on the



Aero Service Corporation, Philadelphia

FIG 2. Correcting projectors used to horizontalize plates.

X and Y axes of the plate in reference to the center and conjugate center and the movements of the various points are tabulated for the top tilts and side tilts which are required to bring the errors to an apparent zero. When the computations show all the points to be within 2/1,000 inch of their computed base distance, the pair is ready to place in the correcting projectors (see Fig. 2).

Now that the top tilts and side tilts for each plate have been computed, these results are referred to a set of resultant tilt tables which show the azimuth of the axis of tilt and degree of tilt for such a condition.

It will be noted that each model is handled separately for the tilt analysis and that the entire plate cannot be considered as corrected. This condition results from the tilts of each model being referenced to its own axis which is the base between the centers of the plate on the model, and also that a scale change has in most cases been applied to bring the exposure stations to a uniform datum.

CORRECTING PROJECTOR

The plates are now placed in a pair of duplicate projectors. The plate holder of the projector is mounted in a rotator having a horizontal axis and means are provided to bring the center of the plate into this axis. Behind each plate is a lamp house fitted with Cooper-Hewitt lights by which the image of the plate is projected its exact size onto a matte finished precision grid with $\frac{1}{2}''$ spacings in both directions intersecting at right angles.

The intersection of the two main grid lines is adjusted to lie in the horizontal axis of the image plane rotator and in the axis of the lens when the focal planes are not tilted. Means are provided to move the grid screen on both the X and Y axes a sufficient distance to measure the distance from any image point to the closest grid line The grid is provided with dial indicators to record these measurements. The plates are placed in the projectors with the center point of the plate and the conjugate center of the adjoining plate both lying on the horizontal axis of the grid.

Each grid is rotated to its azimuth of tilt and clamped. The plate holder is rotated until the center and conjugate center coincide with the horizontal axis of the central grid line. By turning a hand wheel both plates of the projector are tilted through equal angles and the rotators of the plate holder and the grid screen are both equally displaced towards the intersection of the two prolonged tilted planes. This displacement is accomplished by a linkage which is based on the function of a taking camera having a focal distance of 13.4″ and the $9\frac{1}{2}''$ focal distance of the projector lens. Therefore, as the planes are tilted, the true scale line moves into the tilt axis of the focal plane.

As the machines are tilted, the intersections of the main grid lines simultaneously moves onto the points where the plumb lines through the lens pierce the pictures. The rotators of each projector are rotated until the main grid line is in coincidence with the conjugate images of the respective "V" points or nadir points of each plate. While in this position, the bases to all the control points are measured and compared with the base of the set point. If the results are not the same as the preliminary analysis showed, a touch up is made from these readings and the same procedure carried out until the results are satisfactory.

The markers of the machines are set on the main X and Y grid lines of the machine, which will photograph on the plate glass negative which is about to be made. The grids are removed from the machines, plate holders inserted and the plates exposed.

Final Glass Positives

From these corrected negatives, the final glass positives are made, the scale change, if any is required, being taken care of at this time.

Computations of Contour Parallax

The computation for the contour parallax is essentially the same as that for the preliminary parallax except the new base distance between the "V" points of the model is substituted for the distance between the center of the plate and the conjugate center of the adjoining plate. The difference of elevation from the set point and contour datum is used in each case

Contouring

After the plates have been horizontalized and corrected for scale, they are placed on the stereometer with the "V" points being used for the alignment of the plates. The plates are set at a separation with the images of the set point under the cross-hairs of the eyepieces. The parallax slide is fastened and the dial set to read zero.

A transparent sheet is superimposed on the right-hand plate on which the contours and detail are to be drawn. The "V" point and control points are marked on the transparent sheet. The plates are separated and set at a proper separation corresponding to the parallax of the particular contour. By means of

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FIG. 3. Brock stereometers used to contour plates.

hand wheels all parts of the plate are brought into the field of the optics (see Fig. 3).

Now the reticule lines which are perpendicular to the horizontal axis of the plates have blended into one pair of intersecting lines. These lines appear to float in space over any terrain that is below the elevation of the parallax setting and when the elevation of the ground is higher than the parallax setting, the reticules appear to split. Where the point in the line appears to touch the ground, the contour is drawn on the transparent paper. This operation is repeated until the whole meandering contour is drawn (see Fig. 4).

The parallax setting for the next contour is made and the operation continued until the entire plate is contoured. The culture which is to appear on the final map is delineated while being viewed stereoscopically. This detail is also drawn on the transparent paper which is over the right-hand plate.

SCALE EQUALIZATION

The sheet containing the contours, cultural detail and drainage is now placed over the radial control plot and oriented by radials originating from the "V" point. When all the radials pass through the control points on the plot, the "V" point is transferred to the radial plot. The final lens altitude computations are now made in the same manner as those of the preliminary lens altitude computations except the distances for the ratios are measured from the "V" point to the control points instead of from the old picture centers.

These new results from the lens altitudes usually check the preliminary

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FIG. 4 Drawing contours directly over an image on Brock stereometer.

results by 10′ or 15′. This average lens altitude which we have now determined, minus the theoretical lens altitude, gives us the ground elevation at which the sheet is the same scale as the plot. Since the contoured sheet is now in a conic projection, each contour, as represented, has a different lens altitude equal to the amount of the contour interval. So, in order to bring this sheet to an equal scale drawing on an orthographic projection it is necessary to change the scale setting for each of the contours before they are retraced.

This is accomplished by drawing a ten-inch line on the contoured sheet and computing the length to which this line will be required to be reduced or enlarged to bring each contour individually to the scale of the plot The lens height above the contour divided by the theoretical lens height and multiplied by ten will give us the result. The contoured sheet is now placed between two glass plates in the frame at the rear of another projector called the T.I. machine.

The image of this sheet is projected to another transparent paper placed over the image plane of the machine. This machine is so constructed that the scale of the drawing can be changed by turning the hand wheel to the left which also operates the automatic focus.

By means of vertical horizontal controls near the rear of the glass top table, the "V" point on the drawing is brought into register with an etched intersection on the glass top of the table which marks the optical center of the machine.

By means of the scale change control wheel, the scale of the drawing is changed so that the 10″ line on the drawing measures the required length for the highest contour (see Fig. 5). This highest contour and all adjoining detail are

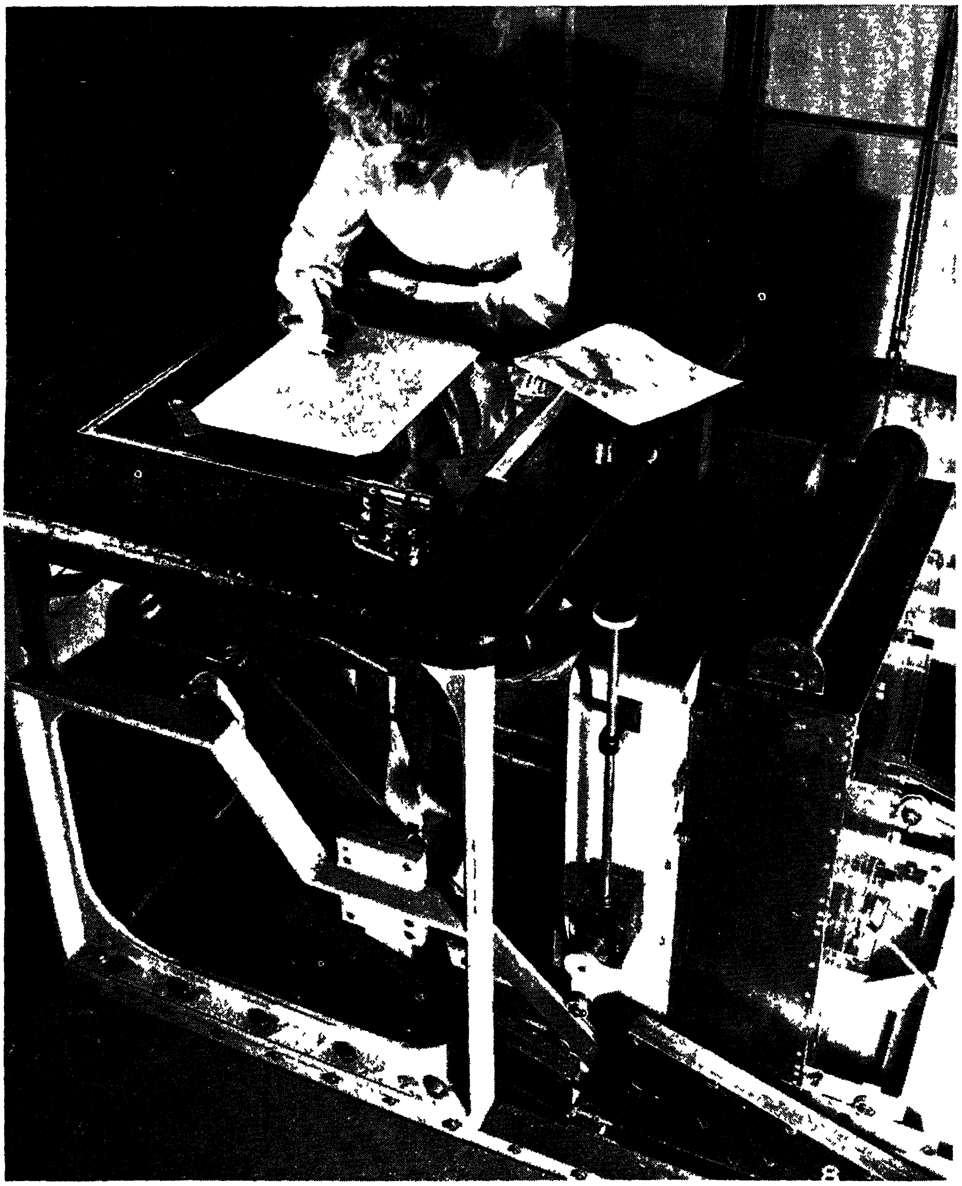

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FIG. 5. Tracing instrument used to convert conic contour drawing to orthographic projection.

now traced on the new sheet. The machine is set so that the original 10″ line now measures the new required length for the next lowest contour and this contour and adjoining detail are traced. This operation is continued until the entire conic projection has been reconstructed as a true scale drawing to the scale of the final map compilation.[4]

[4] From *Photogrammetric Engineering,* Volume 6, No. 2, April, 1940, by E. A. Schuch, Aero Service Corporation.

To prove the accuracy of the Brock Process many test maps were made during the early twenties. Various areas in Italy, France, Canada and the United States were selected for this work using contour intervals from 1 foot to forty feet. The results showed that any interval contour map could be made by this process with equal accuracy. The controlling factor for producing different contour intervals is the altitude of the airplane above the terrain, thereby producing original glass plate exposures of larger scale when a 2 foot contour interval is to be shown than if a 10 foot contour interval is desired.

Terrain conditions and the accuracy required, affect the figure used to express the relationship of altitude and contour intervals.

Normally, with the Brock Process, a 1,500 multiple of the contour interval is used to secure the altitude at which the airplane is to be above the mean datum plane of the area. Example: for a 5 foot contour (5×1500) the altitude equals 7,500 feet. To produce a contour map to the usual accuracy specifications in that 90% of the contour elevations shown on the map will not be more than one half a contour interval from the correct elevations as determined by third order level lines, and the remaining 10% of the contour elevations checked will not be in error by more than a contour interval If the terrain to be mapped is rolling in nature, heavily wooded, or lacking in well defined image detail, a lower factor is used to determine the contour altitude relationships. On the other hand, decrease in the accuracy specifications would tend to increase the factor used to secure the altitude of the airplanes.

A tabulation of recent projects completed by the Brock Process are as follows:

Area	*Contour Interval*	*Horizontal Scale*
61,912 acres	2′	1″=100′
117,210 acres	5′	1″=200′
93,565 acres	10′	1″=200′&1″=400′
4,365 sq. miles	20′	1″=1666′
1,600 sq miles	20′	1″=3333′

CHAPTER XII

ANALYTICAL COMPUTATIONS

ANALYTICAL COMPUTATIONS IN AERIAL PHOTOGRAMMETRY

Earl Church

MATHEMATICAL analysis in aerial photogrammetry or the computation of survey data from measurements on aerial photographs, serves several important purposes It provides an effective means of teaching the fundamental theory of photogrammetry in a thorough, coherent, and systematic manner. It provides a powerful method of attack for research problems in photogrammetry It exposes completely all possibilities of applying photogrammetry, not exclusively to topographic mapping, but to many other types of surveying. It can be combined effectively with some of the simpler and more practical applications of photogrammetry to planimetric and topographic mapping.

Although analytical methods are susceptible to wide variations, there are in reality four fundamental problems, namely: (1) Space resection; (2) Orientation of the photograph in space; (3) Space intersection; and (4) Photogrammetric extension of surveys without ground control. This paper presents brief explanations of these four problems in order, together with a brief discussion of an additional practical method for computing approximate areas, distances, and horizontal positions of ground points

Space Resection

In surveying work it is convenient to define the positions of ground points by three rectangular coordinates in space, referred usually to a north-and-south y-axis, an east-and-west x-axis, and a vertical z-axis, with the horizontal xy-plane taken at the zero elevation for the system of levels used, usually sea level. The problem of space resection consists of determining the three rectangular coordinates of the point in the air from which a photograph was exposed, with these coordinates referred to the same axes as those used in the ground surveying. This point where a photograph was taken, generally called the "exposure station" of the photograph and designated by L in all of the accompanying geometric figures, is technically the space position of the incident nodal point of the camera lens at the instant of exposure of the photographic film. The required space coordinates of L are called X, Y, Z.

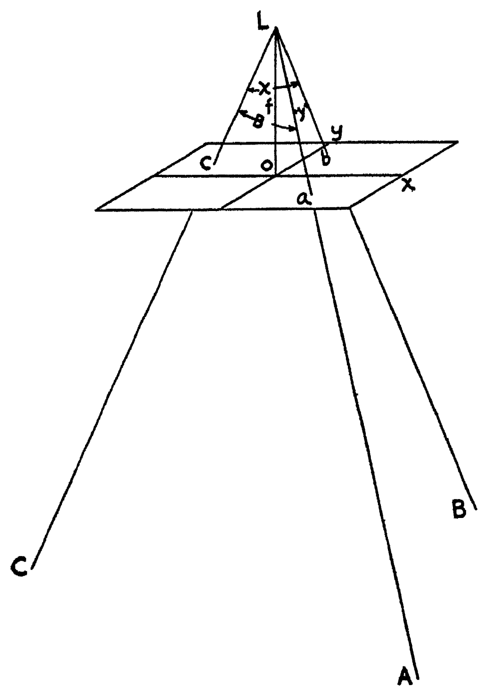


Fig. 1

The data for the problem consist of: (1) The survey space coordinates of three ground control points whose images are sharply defined in the photograph being used; and (2) the plane coordinates on the photograph itself, of the images of these three points, re-

ferred to the two rectangular axes of the photograph indicated by the camera marks. The survey coordinates of the ground points must have been previously determined upon the ground by usual geodetic procedure, such as triangulation, traverse, and levelling. The photographic coordinates of the images may be measured by any ordinary means such as an engineer's scale, but the accuracy of the results of the problem can be increased considerably if they are measured with a comparator, a precise photogrammetric instrument for this purpose.

The three ground control points are designated by A, B, and C, and their photographic images by a, b, and c, respectively. The survey coordinates of A, B, and C, are designated by (X_A, Y_A, Z_A), (X_B, Y_B, Z_B), and (X_C, Y_C, Z_C), and the photographic coordinates of a, b, and c, by (x_a, y_a), (x_b, y_b), and (x_c, y_c)

In Figure 1, abc represents the plane of the film; L represents the emergent nodal point of the camera lens; a, b, and c are the images designated in the previous paragraph; f is the effective focal length of the camera; o is the principal point of the photograph; and ox and oy are the rectangular geometric axes of the photograph defined by the camera marks. The angle aLb is designated by γ, bLc by α, and cLa by β.

It is easily shown that these three angles can be found from the expressions

$$\cos\gamma = \frac{x_a x_b + y_a y_b + f^2}{(La)(Lb)}$$
$$\cos\alpha = \frac{x_b x_c + y_b y_c + f^2}{(Lb)(Lc)} \qquad [2]$$
$$\cos\beta = \frac{x_c x_a + y_c y_a + f^2}{(Lc)(La)}$$

where

$$La = \sqrt{x_a^2 + y_a^2 + f^2}$$
$$Lb = \sqrt{x_b^2 + y_b^2 + f^2} \qquad [1]$$
$$Lc = \sqrt{x_c^2 + y_c^2 + f^2}$$

These three angles are identical with those subtended at the exposure station by the three ground points A, B, and C. The problem of space resection now consists of finding the survey coordinates (X, Y, Z) of a point L in space, such that the three angles subtended at L by A, B, and C, or the angles ALB, BLC, and CLA, will be exactly equal to γ, α, and β, respectively.

This is accomplished by stating the cosines of the angles ALB, BLC, and CLA in terms of the unknown coordinates X, Y, and Z, equating these to the known numerical values for $\cos\gamma$, $\cos\alpha$, and $\cos\beta$, respectively, and solving these equations for the three unknown quantities X, Y, and Z.

It can be proved that

$$\cos ALB = \frac{(X - X_A)(X - X_B) + (Y - Y_A)(Y - Y_B) + (Z - Z_A)(Z - Z_B)}{(LA)(LB)}$$
$$\cos BLC = \frac{(X - X_B)(X - X_C) + (Y - Y_B)(Y - Y_C) + (Z - Z_B)(Z - Z_C)}{(LB)(LC)} \qquad [3]$$
$$\cos CLA = \frac{(X - X_C)(X - X_A) + (Y - Y_C)(Y - Y_A) + (Z - Z_C)(Z - Z_A)}{(LC)(LA)}$$

where

$$
\begin{aligned}
LA &= \sqrt{(X - X_A)^2 + (Y - Y_A)^2 + (Z - Z_A)^2} \\
LB &= \sqrt{(X - X_B)^2 + (Y - Y_B)^2 + (Z - Z_B)^2} \\
LC &= \sqrt{(X - X_C)^2 + (Y - Y_C)^2 + (Z - Z_C)^2}
\end{aligned}
\qquad [4]
$$

The equations, therefore, which are to be solved for the desired values X, Y, and Z, are

$$
\begin{aligned}
&(X-X_A)(X-X_B)+(Y-Y_A)(Y-Y_B)+(Z-Z_A)(Z-Z_B)-(LA)(LB)\cos\gamma=0 \\
&(X-X_B)(X-X_C)+(Y-Y_B)(Y-Y_C)+(Z-Z_B)(Z-Z_C)-(LB)(LC)\cos\alpha=0 \\
&(X-X_C)(X-X_A)+(Y-Y_C)(Y-Y_A)+(Z-Z_C)(Z-Z_A)-(LC)(LA)\cos\beta=0
\end{aligned}
\qquad [5]
$$

in which $\cos\gamma$, $\cos\alpha$, and $\cos\beta$, as well as the survey coordinates of the control points, are known numerical values, but in which LA, LB, and LC are radical expressions involving the unknown quantities. The solution of these equations in this form is virtually impossible.

To effect a solution of these equations there are first found approximate values for the coordinates X, Y and Z, designated by (X), (Y), and (Z) These are found by making a quick graphical solution of the three-point problem to find (X) and (Y), and by a quick approximate determination of (Z) by the fundamental scale and altitude relation for aerial photographs, assuming for the moment true verticality of the camera axis Then these quantities are substituted in [5] to find the numerical values of the left-hand members, called v_1, v_2, and v_3, respectively These quantities of course show the amounts by which conditions [5] fail to be satisfied by the approximate values (X), (Y), and (Z). In order to find the correct values of X Y, and Z, it is necessary to determine the corrections ΔX, ΔY, and ΔZ to be applied to the approximate values such that the changes produced by these corrections in the left members of [5] will be respectively equal to $-v_1$, $-v_2$, and $-v_3$. The changes produced in the left members of [5] by small changes ΔX, ΔY, and ΔZ in the three variables, are found by partial differentiation; and by equating these total increments to $-v_1$, $-v_2$, and $-v_3$ there are obtained three very simple linear equations which can be easily solved for X, Y, and Z. These equations are

$$
\begin{aligned}
U\,\Delta X + V\,\Delta Y + W\,\Delta Z + v_1 &= 0 \\
U'\,\Delta X + V'\,\Delta Y + W'\,\Delta Z + v_2 &= 0 \\
U''\Delta X + V''\Delta Y + W''\Delta Z + v_3 &= 0
\end{aligned}
\qquad [6]
$$

where the coefficients are given by the following expressions

$$
\begin{aligned}
U &= [1 - (LA/LB)\cos\gamma](X - X_B) + [1 - (LB/LA)\cos\gamma](X - X_A) \\
V &= [1 - (LA/LB)\cos\gamma](Y - Y_B) + [1 - (LB/LA)\cos\gamma](Y - Y_A) \\
W &= [1 - (LA/LB)\cos\gamma](Z - Z_B) + [1 - (LB/LA)\cos\gamma](Z - Z_A) \\
U' &= [1 - (LB/LC)\cos\alpha](X - X_C) + [1 - (LC/LB)\cos\alpha](X - X_B) \\
V' &= [1 - (LB/LC)\cos\alpha](Y - Y_C) + [1 - (LC/LB)\cos\alpha](Y - Y_B) \\
W' &= [1 - (LB/LC)\cos\alpha](Z - Z_C) + [1 - (LC/LB)\cos\alpha](Z - Z_B) \\
U'' &= [1 - (LC/LA)\cos\beta](X - X_A) + [1 - (LA/LC)\cos\beta](X - X_C) \\
V'' &= [1 - (LC/LA)\cos\beta](Y - Y_A) + [1 - (LA/LC)\cos\beta](Y - Y_C) \\
W'' &= [1 - (LC/LA)\cos\beta](Z - Z_A) + [1 - (LA/LC)\cos\beta](Z - Z_C)
\end{aligned}
\qquad [7]
$$

The coefficients of ΔX, ΔY, and ΔZ, are computed in [7] using the approximate values of the coordinates of the exposure station.

Space Resection

Photograph No. f	a x_a y_a b x_b y_b c x_c y_c			La [1] Lb Lc	cosine γ [2] cosine α cosine β	
L (X) (Y) (Z)						
A X_A Y_A Z_A	$(X)-X_A$	$(Y)-Y_A$	$(Z)-Z_A$	(LA)[3]	$\frac{LB}{LA}\cos\gamma$	$\frac{LC}{LA}\cos\beta$
B X_B Y_B Z_B	$(X)-X_B$	$(Y)-Y_B$	$(Z)-Z_B$	(LB)	$\frac{LA}{LB}\cos\gamma$ $\frac{LC}{LB}\cos\alpha$	
C X_C Y_C Z_C	$(X)-X_C$	$(Y)-Y_C$	$(Z)-Z_C$	(LC)	$\frac{LB}{LC}\cos\alpha$	$\frac{LA}{LC}\cos\beta$

1st 3 terms 4th term [4]	1st terms of U, V, W 2nd terms of U, V, W [5]	U X V Y W Z v_1 U' X V' Y W' Z v_2	
v_1 1st 3 terms	1st terms of U', V', W' 2nd terms of U', V', W' [5]	U'' X V'' Y W'' Z v_3	
4th term [4]	1st terms of U'', V'', W''		
v_2	2nd terms of U'', V'', W'' [5]	Solutions of equations	
1st 3 terms	(X) (Y) (Z)		
4th term [4]	ΔX ΔY ΔZ X Y Z		
v_3			

	Second Solution		

	(X) (Y) (Z) ΔX ΔY ΔZ		
	X Y Z Exposure station		

$X-X_A$ $Y-Y_A$ $Z-Z_A$ $X-X_B$ $Y-Y_B$ $Z-Z_B$ $X-X_C$ $Y-Y_C$ $Z-Z_C$	LA [3] LB LC	1st 3 terms 4th term [4] v_1	1st 3 terms 4th term [4] v_2	1st 3 terms 4th term [4] v_3

[1] See formulas [1].
[2] See formulas [2].
[3] See formulas [4].
[4] From equations [5].
[5] See formulas [7].

After the linear equations [6] are solved for ΔX, ΔY, and ΔZ, the application of these corrections to the approximate values (X), (Y), and (Z) gives the corrected coordinates X, Y, and Z of the exposure station The corrected values for

Space Resection

Photograph I				*q*	3.68	−71 56		166 236			.907084
f 150 00 mm				*b*	82 29	−74 88		186 758			648273
				a	83 56	83 56		190 957			.530115
L	4600	34500	19785								
Q	5000	25000	400	− 400	9500	19385		21591 39	986		.591
B	15000	25000	1000	−10400	9500	18785		23479 49	834	664	
A	15000	45000	800	−10400	−10500	18985		24059 10		633	476

458557225	− 6	133	271	−7311	− 662	13279	−1163209	
−459850611	−1726	1577	3118	−5614	−1616	17876	−2944302	
− 1293386	−3494	3192	6312	−1732	1710	3389	−1293386	
365043225	−3817	−3854	6967	−1866	− 169	3389	− 296868	
−366206434				−1064	− 306	3389	− 558192	
	− 164	3886	7928					
− 1163209	−5450	−5502	9948	134	1879		− 996518	
272434225				802	− 137		− 261324	
−275378527	4600	34500	19785					
	411	501	339	10	137		− 72657	
− 2944302								
	5011	35001	20124	812			− 333981	

L	5011	35001	20124	Second Solution						
Q	5000	25000	400							
B	15000	25000	1000	11	10001	19724	22114 62	975		574
A	15000	45000	800	− 9989	10001	19124	23780 82	844	653	
				− 9989	− 9999	19324	23941 12		644	490

477111898	0	250	493	−7022	− 90	13515	244895	
−477038920	−1558	1560	2983	−5089	− 839	18257	367963	
72978	−3466	3470	6636	−1558	1810	3476	72978	
369332298	−3556	−3560	6879	−1806	− 23	3476	62986	
−369087403				− 969	− 160	3476	70057	
	5	4260	8402					
244895	−5094	−5099	9855	248	1833		9992	
281036698				837	− 137		7071	
−280668735	5011	35001	20124					
	− 9	− 4	− 23	19	137		747	
367963								
	5002	34997	20101	856			7818	
	Exposure station							

2	9997	19701	22092 29	476228814	368628414	280229014
−9998	9997	19101	23764 43	−476228787	−368630745	−280231562
−9998	−10003	19301	23927 99			
				27	− 2331	− 2548

these coordinates should be substituted again in [5] to find whether residuals still remain; and if these residuals are still not negligible, a second solution must be made. In such cases it is seldom necessary to re-compute the coefficients of ΔX, ΔY, and ΔZ in equations [6].

Although the symbols used, together with the mathematics underlying this explanation, make this exact method appear somewhat involved, there is after all little difficulty in the computation itself. In fact the form for the computation covers but half a sheet of paper, and two complete solutions require only about an hour with a calculating machine. This method will not fail under extreme conditions of large tilts and extensive topographic relief.

Photograph Orientation

The orientation of a tilted aerial photograph in space can be resolved into three component elements in several slightly different ways. A convenient resolution is into the "tilt," the "swing," and the "azimuth." The last is often called the "azimuth of the principal plane." These are shown in Figure 2, and are defined, together with some other terms, as follows: The vertical plane vLo containing the camera axis is called the principal plane; the line ov in which the principal plane intersects the plane of the photograph is called the principal line; the point o where the camera axis meets the plane of the photograph is called the principal point; the point v where a vertical line through the emergent node of the camera lens intersects the plane of the photograph is called the nadir point, the angle between the plane of the photograph and a horizontal plane, shown by the equivalent angle vLo since Lv is vertical and Lo is perpendicular to the plane of the photograph, is called the "tilt," and is designated by t; the direction on the photograph of the principal line ov, referred to the geometric axes of the photograph and measured conventionally like an azimuth clockwise from the positive y-axis to ov, is called the "swing," and is designated by s; the usual survey azimuth of the principal plane VLO, measured clockwise from north to the direction VO, is called the "azimuth" or the "azimuth of the principal plane." This last element is actually the azimuth of the camera axis at the instant of exposure, in accordance with the usual definition of azimuth in ordinary surveying It is designated by α_{VO}, the subscript showing that the symbol designates the azimuth of VO, but not of OV. It should be noted by the reader that these three elements, the tilt, the swing, and the azimuth of the principal plane, actually *orient* the photograph in space, and that these three angles together with the survey coordinates of the exposure station, actually fix the photograph in space.

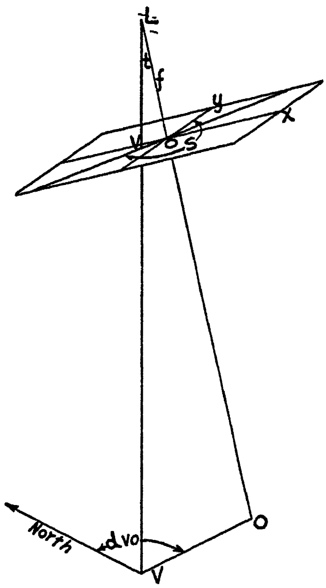


FIG. 2

The problem in hand is to determine t, s, and α_{VO}, after the coordinates of the exposure station have been found.

In Figure 3, L represents the exposure station; A, B, and C represent the three ground control points at different elevations; a, b, and c represent the images of these three points on a tilted aerial photograph; o represents the principal point and v the nadir point of the photograph. The three vertical angles VLA, VLB, and VLC, called m_A, m_B, and m_C, respectively, measured outward from the vertical line LV to the lines LA, LB, and LC, respectively, are obviously given by

$$\begin{aligned} \cos m_A &= (Z - Z_A)/LA \\ \cos m_B &= (Z - Z_B)/LB \\ \cos m_C &= (Z - Z_C)/LC \end{aligned} \qquad [8]$$

in which all numerators and denominators are values which have already been calculated.

It now becomes necessary to locate the nadir point v on the photograph in

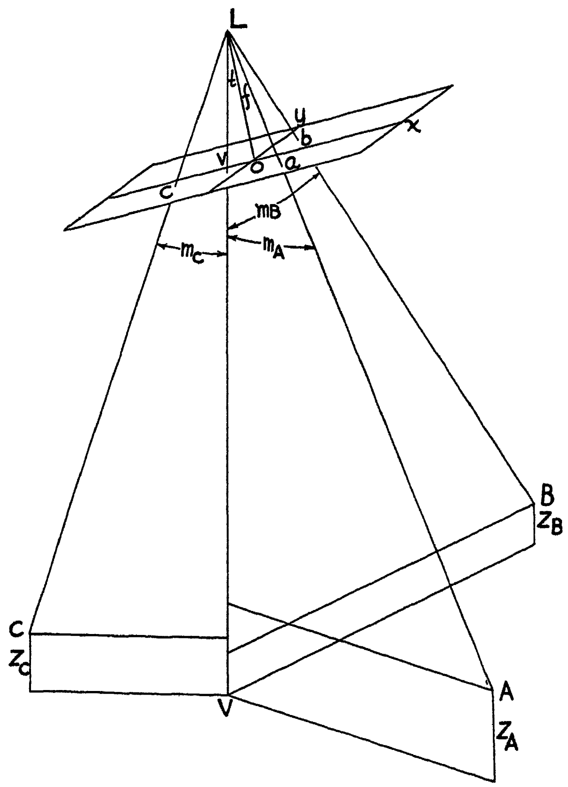


FIG. 3

such a manner that angle vLa will be equal to m_A, angle vLb to m_B, and angle vLc to m_C. If the unknown photographic coordinates of the nadir point v are designated by x, y, it can easily be proved that

$$\begin{aligned} \cos vLa &= (x_a x + y_a y + f^2)/(La)(\sqrt{x^2 + y^2 + f^2}) \\ \cos vLb &= (x_b x + y_b y + f^2)/(Lb)(\sqrt{x^2 + y^2 + f^2}) \\ \cos vLc &= (x_c x + y_c y + f^2)/(Lc)(\sqrt{x^2 + y^2 + f^2}) \end{aligned} \qquad [10]$$

in which

$$\begin{aligned} La &= \sqrt{x_a^2 + y_a^2 + f^2} \\ Lb &= \sqrt{x_b^2 + y_b^2 + f^2} \\ Lc &= \sqrt{x_c^2 + y_c^2 + f^2} \end{aligned} \qquad [9]$$

If the right-hand members of equations [10] are set equal respectively to the known numerical values $\cos m_A$, $\cos m_B$, and $\cos m_C$ found by [8], we have three equations with but two unknown quantities x and y. Then if the radicals are eliminated from these three equations, simple algebraic transformation leads to the two simple linear equations

$$\begin{aligned} ux + vy + w &= 0 \\ u'x + v'y + w' &= 0 \end{aligned} \qquad [11]$$

in which the coefficients are given by

$$
\begin{aligned}
u &= x_a/(La \cos m_A) - x_b/(Lb \cos m_B) \\
v &= y_a/(La \cos m_A) - y_b/(Lb \cos m_B) \\
w &= f^2/(La \cos m_A) - f^2/(Lb \cos m_B) \\
u' &= x_b/(Lb \cos m_B) - x_c/(Lc \cos m_C) \\
v' &= y_b/(Lb \cos m_B) - y_c/(Lc \cos m_C) \\
w' &= f^2/(Lb \cos m_B) - f^2/(Lc \cos m_C)
\end{aligned} \qquad [12]
$$

These coefficients of x and y are known definite numerical values. The solution of the simple equations [11] gives the coordinates x and y of the nadir point v on the photograph.

Then the following relations follow at once

$$ov = \sqrt{x^2 + y^2} \qquad [13]$$

$$\tan t = ov/f \qquad [14]$$

$$\tan s = x/y \qquad [15]$$

with the swing s placed in the proper quadrant in accordance with the algebraic signs of x and y. These give two of the three elements of the space orientation of the photograph.

To continue with finding the azimuth of the principal plane, it is easiest to

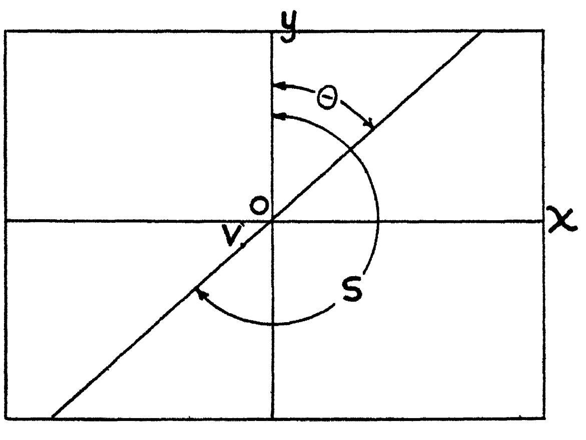


FIG 4

transform first the photographic coordinates of the images a, b, and c, by rotating the rectangular axes through the angle θ which is $180°-s$, and then by translating the axes by the algebraic addition of the value of ov to all of the new ordinates. It can be seen from Figure 4 that these transformations place the origin at the nadir point v, and the positive y-axis along the principal line in the direction vo. The analytical geometry formulas for this rotation and translation are

$$
\begin{aligned}
\text{New } x &= x \cos\theta + y \sin\theta \\
\text{New } y &= -x \sin\theta + y \cos\theta + ov
\end{aligned} \qquad [16]
$$

To avoid confusion, the photographic coordinates of the images a, b, and c after these transformations, will henceforth be designated by the same symbols as those heretofore used for the original coordinates, namely, (x_a, y_a), (x_b, y_b), and (x_c, y_c).

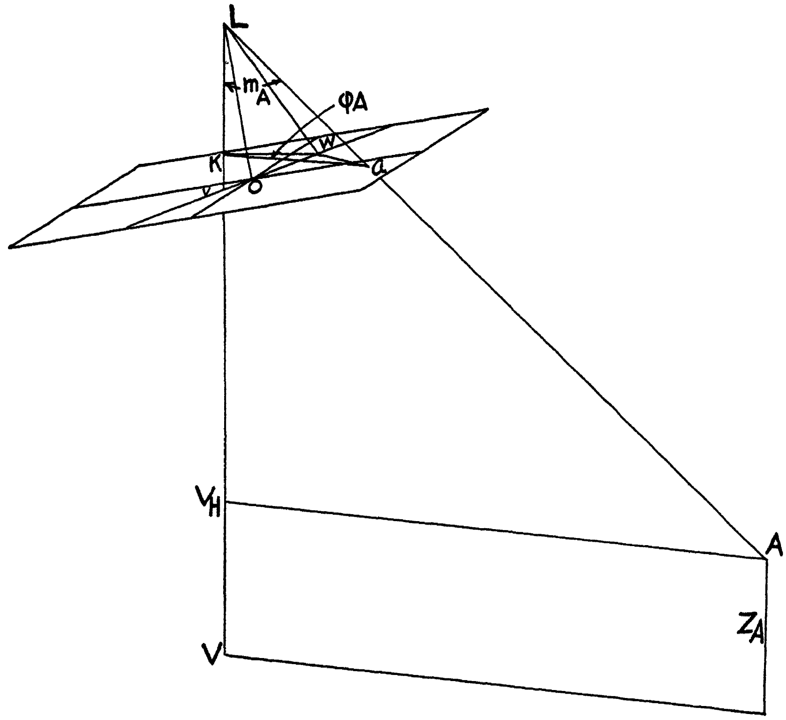


FIG. 5

Figure 5 shows a tilted photograph with the image a of one of the control points A. If aw is drawn perpendicular to the principal line, it lies wholly within the plane of the photograph and is equal to the new x_a. Also vw equals y_a. If wk is drawn perpendicular to Lv and k is joined to a, then kw equals $y_a \cos t$. The angle wka, designated by ϕ_A, is a horizontal angle, and is actually the survey angle between the principal plane and the vertical plane VLA. From the figure it is seen that

$$\tan \phi_A = x_a/(y_a \cos t)$$

and similarly

$$\tan \phi_B = x_b/(y_b \cos t) \qquad [17]$$
$$\tan \phi_C = x_c/(y_c \cos t)$$

Moreover, the survey azimuths of the vertical planes VLA, VLB, and VLC, or the survey azimuths of LA, LB, and LC, or of VA, VB, and VC, according to the usual plane surveying practice, are given by

$$\tan \alpha_A = (X - X_A)/(Y - Y_A)$$
$$\tan \alpha_B = (X - X_B)/(Y - Y_B) \qquad [18]$$
$$\tan \alpha_C = (X - X_C)/(Y - Y_C)$$

with these azimuths placed in the proper quadrants in accordance with the relative positions of L and the points A, B, and C, respectively. Then the azimuth of the principal plane is given by direct subtraction as follows:

$$\alpha_{VO} = \alpha_A - \phi_A$$
$$\alpha_{VO} = \alpha_B - \phi_B \qquad [19]$$
$$\alpha_{VO} = \alpha_C - \phi_C$$

The agreement between these three values furnishes an excellent check on the computations.

This completes the determination of the three elements of space orientation of the aerial photograph. This entire computation requires about a half-hour with a calculating machine.

SPACE ORIENTATION

a	$\cos m_A$ [1]	$\tan m_A$ [2]	$La \cos m_A$ [3]	1st terms of u, v, and w [4]
b	$\cos m_B$	$\tan m_B$	$Lb \cos m_B$	2nd terms of u, v, w, 1st of u', v', w'
c	$\cos m_C$	$\tan m_C$	$Lc \cos m_C$	2nd terms of u', v', and w'

$\tan t$ [6]	$\tan s$ [7]	u	v	w [4]
t	s	u'	v'	w'
$\sin t$	θ [8]	Solution of equations [11]		
$\cos t$	$\sin \theta$			
$f \sec t$	$\cos \theta$			
		x	y	ov [5]

$x_a \cos \theta$	$y_a \sin \theta$	$-x_a \sin \theta$	$y_a \cos \theta$ [9]	New x_a	New y_a [9]	$y_a \cos t$	$\tan \phi_A$ [10]
$x_b \cos \theta$	$y_b \sin \theta$	$-x_b \sin \theta$	$y_b \cos \theta$	New x_b	New y_b	$y_b \cos t$	$\tan \phi_B$
$x_c \cos \theta$	$y_c \sin \theta$	$-x_c \sin \theta$	$y_c \cos \theta$	New x_c	New y_c	$y_c \cos t$	$\tan \phi_C$

ϕ_A	$\tan \alpha_A$ [11]	α_A	αvo [12]	$y_a \sin t$	13	16	19
ϕ_B	$\tan \alpha_B$	α_B	αvo	$y_b \sin t$	14	17	20
ϕ_C	$\tan \alpha_C$	α_C	αvo	$y_c \sin t$	15	18	21
			αvo [22]				

[1] By formula [8].
[2] Found from cosines.
[3] Use La, Lb, Lc, from resection computation
[4] See formulas [12]
[5] By formula [13].
[6] By formula [14].
[7] By formula [15].
[8] $\theta = 180° - s$.
[9] Four of the terms for formulas [16]. New x and new y found by formulas [16].
[10] By formulas [17].
[11] By formulas [18]
[12] By formulas [19].
[13] $f \sec t - y_a \sin t$.
[14] $f \sec t - y_b \sin t$.
[15] $f \sec t - y_c \sin t$.
[16] $\sqrt{x_a^2 + y_a^2 \cos^2 t}$.
[17] $\sqrt{x_b^2 + y_b^2 \cos^2 t}$
[18] $\sqrt{x_c^2 + y_c^2 \cos^2 t}$.
[19] Use formula [23] of Space Intersection, inverted, to find $\tan m_A$.
[20] Same for finding $\tan m_B$.
[21] Same for finding $\tan m_C$.

Checks:

[22] The three values for αvo should be identical In case of small discrepancies in the three values, enter in this space the mean

[19,20,21] These tangents of the m's should be identical with their values found at the top of the computation in the second column

SPACE INTERSECTION

Every surveyor understands the problem of intersection in plane surveying. If from two known points, horizontal directions and vertical angles are measured to some undetermined point, the horizontal position and the elevation of the new point can be calculated. In fact, after the horizontal position is determined, the identity of the elevations computed with each of the two vertical angles furnishes a complete check on the whole intersection problem.

The space intersection problem in aerial photogrammetry is exactly analogous to this. If an undetermined point has its image appearing in each of two photographs for which the foregoing calculations of exposure station coordinates

SPACE ORIENTATION — Photograph I

q	89176	.50744	148 243	+ 0248	− 4827	+151 7778
b	80376	74023	150 109	+ 5482	− 4988	+149 8911
a	80663	73274	154 032	+ 5425	+ 5425	+146 0735

$\tan t =$ 03491	$\tan s =$ 1 0084	− 5234	+ 161	+ 18867
$t =$ 2°00 0′	$s =$ 45°14 3′	+ 57	−10413	+ 38176
$\sin t =$ .03490	$\theta =$ 134°45 7			
$\cos t =$ .99939	$\sin \theta =$ + 71004	+ 1	− 161	+ 590
f $\sec t =$ 150 092	$\cos \theta =$ − 70416			
		− 5233		+ 19457
		$x =$ +3 718	$y =$ +3 687	$ov =$ 5 236

q	− 2 591	−50 810	− 2 613	+50 390	− 53 401	+ 53 013	+ 52 981	1.0079
b	−57.945	−53 168	−58 429	+52 728	−111 113	− 0 465	− 0 465	238 95
a	−58 840	+59 331	−59 331	−58 840	+ 0 491	−112 935	−112.866	00435

314°46 5′	00020	180°00 7′	225°14 2′	+1 850	148 242	75 224	50744
269 45 6	1 0001	134 59 7	225 14 1	−0 016	150 108	111 114	74023
179 45 0	99950	44 59 1	225 14 1	−3 941	154 033	112 867	73275
			$\alpha_{VO} =$ 225 14 1				

and orientation elements have been completed, then by means of horizontal directions and vertical angles measured by the photographs from their respective exposure stations, the horizontal position and the elevation of the new point can be computed

In this explanation of photogrammetric intersection in space, the same symbolism as that used heretofore will be preserved, except that primes will be added to designate data for the second one of the two photographs.

First the photographic coordinates are measured (with the comparator if possible) on each of the two photographs, of the image p or p' of the point P whose survey position is to be determined.

Then the coordinate axes of each photograph are rotated and translated exactly as described in the discussion of space orientation, giving coordinates of the image of the undetermined point on each photograph referred to the nadir point for the origin and the principal line ov or $o'v'$ for the positive y-axis. These transformed coordinates on one photograph will be designated by x_p, y_p and on the other by x_p', y_p'.

At this point the reader should refer to Figure 5 and imagine the control point A replaced by the undetermined point P and the image a by the image p. In accordance with the explanation of the space orientation, the horizontal angles taken clockwise from the principal planes are given by

$$\tan \phi = x_p/(y_p \cos t) \qquad \tan \phi' = x_p'/(y_p' \cos t')$$

and the survey azimuths from the exposure stations L and L' respectively to the undetermined point are

$$\alpha_P = \alpha_{VO} + \phi \qquad \alpha_P' = \alpha_{VO}' + \phi' \qquad [20]$$

These two survey azimuths from two known points L and L' to P are sufficient for determining the horizontal position of P. Although plane trigonometry suffices for the calculation, it is perhaps preferable to use the analytics method; namely, that of finding the equations of the horizontal projections of LP and $L'P$ and solving them simultaneously for X_P and Y_P. Following this method, if

we call s and s' the analytical geometry slopes (positive upward to right, negative upward to left) of the horizontal projections of LP and $L'P$, respectively, then

$$s = \cot \alpha_P \quad \text{and} \quad s' = \cot \alpha_P' \qquad [21]$$

with the proper signs assigned. The equations of these horizontal projections of LP and $L'P$ are

$$\begin{aligned} Y_P - Y_L &= s(X_P - X_L) \\ Y_P - Y_L' &= s'(X_P - X_L') \end{aligned} \qquad [22]$$

with X_P and Y_P the only unknown quantities. These equations are quickly solved simultaneously for the survey coordinates X_P and Y_P, giving the horizontal position of the new point P.

Again reference to Figure 5 in which control point A has been replaced by the undetermined point P and image a by image p, shows that

$$\begin{aligned} kp &= \sqrt{kw^2 + wp^2} = \sqrt{x_p^2 + y_p^2 \cos^2 t} \\ kv &= y_p \sin t \qquad Lv = f \sec t \\ Lk &= f \sec t - y_p \sin t \end{aligned}$$

SPACE INTERSECTION

Point	Photo	Point	Photo
x_p	y_p	x_p'	y_p'
$x_p \cos \theta$ $y_p \sin \theta$	$-x_p \sin \theta$[1] $y_p \cos \theta$	$x_p' \cos \theta'$ $y_p' \sin \theta'$	$-x_p' \sin \theta'$ $y_p' \cos \theta'$
New x_p	New y_p [2] $y_p \cos t$	New x_p'	New y_p' $y_p' \cos t'$
$\tan \phi$ [3] α_P [4]	ϕ Slope [5]	$\tan \phi'$ [3] α_P' [4]	ϕ' Slope [5]
Equations in X and Y[6]			
X		Y	
$y_p \sin t$ [9]	[7] $\cot m_P$ [11]	$y_p' \sin t'$ [10]	[8] $\cot m_P'$ [11]
$V_H P$ [12]	Z_L $(Z_L - Z_P)$ [13]	$V_H' P$ [12]	Z_L' $(Z_L' - Z_P)$ [13]
	Z_P		Z_P
Point	X_P	Y_P	Z_P

[1] First four terms of formulas [16].
[2] By formulas [16].
[3] By formulas [17].
[4] By formulas [20]
[5] By formulas [21].
[6] Equations [22], solved for X and Y.
[7] $f \sec t - y_p \sin t$.
[8] $f \sec t' - y_p' \sin t'$
[9] $\sqrt{x_p^2 + y_p^2 \cos^2 t}$.
[10] $\sqrt{x_p'^2 + y_p'^2 \cos^2 t'}$.
[11] By formulas [23].
[12] By formulas [24].
[13] By formulas [25].

Check:
The two values for Z_P should be identical. In case of a small discrepancy attributable to small errors in measurements, the mean is entered at the bottom of the form.

Space Intersection

Point $B3$ Computed from Measurements on Photographs II′ & II

Resection and Orientation Data for Photographs II′ & II

Photo II′		Photo II	
Exposure Station	Orientation	Exposure Station	Orientation
$X=14997$ ft.	$t=0°59\ 7'$	$X=15003$ ft	$t=\ 1°29\ 3'$
$Y=15002$	$s=180°29\ 0'$	$Y=34995$	$s=\ 0°20\ 3'$
$Z=20201$	$\alpha_{VO}=\ 0°29\ 0'$	$Z=20000$	$\alpha_{VO}=180°20\ 3'$
	$\sin t=\ 01737$		$\sin t=\ 02599$
	$\cos t=\ 99985$		$\cos t=\ .99966$
	$f \sec t=150\ 022$		$f \sec t=150.051$
	$ov=\ 2\ 606$		$ov=\ 3\ 899$

Space Intersection

$b3$	Photo II′	$b3$	Photo II
+80 77	+78.16	+81 28	+77 38
+80 767 − 0 660	+ 0 682 +78.157	−81 278 − 0 457	− 0 480 +77 378
+80 107	+81 445 +81.433	−81 735	+80 797 +80 770
98372 45°00 8′	44°31.8′ + 99954	1 0119 134°59 9′	314°39 6′ − 99994
$Y-15002=+\ 99954(X-14997)$ $Y-34995=-\ 99994(X-15003)$			
$X=24999$ ft.		$Y=25000$ ft.	
+ 1 415 114 230	148 607 1.300945	+ 2.100 114 910	147.951 1.287538
14142 1	20201 18398	14135 8	20000 18200
	1803		1800
$B3$ $X=24999$		$Y=25000$	$Z=1801$

and that

$$\cot m_P = (f \sec t - y_p \sin t)/\sqrt{x_p^2 + y_p^2 \cos^2 t}$$
$$\cot m_P' = (f \sec t' - y_p' \sin t')/\sqrt{x_p'^2 + y_p'^2 \cos^2 t'} \qquad [23]$$

These give the two vertical angles needed to find the elevation of P. Following the same figure with the same changes,

$$V_HP = \sqrt{(X_P - X_L)^2 + (Y_P - Y_L)^2}$$
$$V_H'P = \sqrt{(X_P - X_L')^2 + (Y_P - Y_L')^2} \qquad [24]$$

Then

$$Z_L - Z_P = V_HP \cot m_P \qquad [25]$$
$$Z_L' - Z_P = V_H'P \cot m_P'$$

Subtracting these from the known values Z_L and Z_L', respectively, gives two values of Z_P. Their equality furnishes a complete check on the computations.

The survey coordinates can therefore be found for any point whose image appears on each of any two overlapping aerial photographs whose exposure stations and space orientations have been calculated. With a calculating machine, this space intersection computation requires about twenty minutes for each point.

Analytical Bridging or Photogrammetric Extension of Surveys without Ground Control

The ultimate problem in the analytical geometry of aerial photogrammetry consists of extending the exposure station and orientation computations from one photograph to a succeeding one without additional ground control data This is theoretically possible. If an initial photograph of a flight strip has its exposure station and its space orientation determined from three ground control points appearing in this photograph, then, even though the second photograph which overlaps the first may not contain any of the original control points, its exposure station and space orientation can theoretically be determined analytically if the photographic rectangular coordinates are measured for the images of *five* common points on the two photographs. The theory underlying this five-point procedure is not difficult, but the equations encountered are extremely difficult to solve.

However, a slight variation of this five-point procedure produces a much easier process which has been called the "four-point method" For example, suppose in Figure 6 that the two initial photographs contain three ground con-

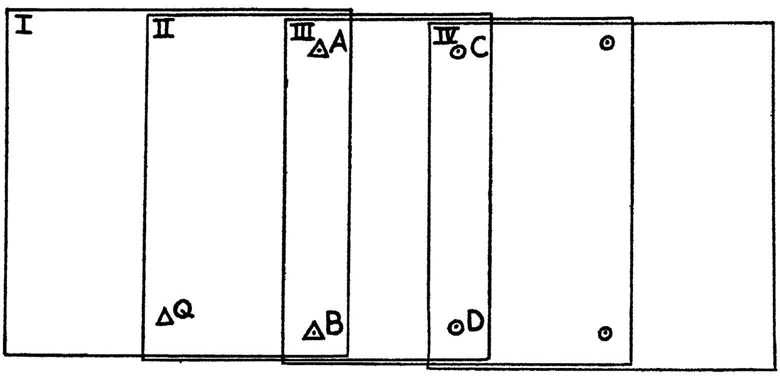


FIG. 6

trol points A, B, and Q, disposed somewhat as shown, with A and B approximately opposite the center of photograph II. It is always possible to have the three indispensable ground control points control the first two photographs as they do in this figure There are to be used no additional control points. Suppose C and D are merely two points chosen approximately opposite the center of photograph III. Now, the four-point method consists of calculating the exposure station and the complete space orientation for photograph III, from the known survey coordinates of A and B, together with the measurements of the photographic coordinates of the images of C and D on photograph II and of the images of A, B, C, and D on photograph III. Incidentally this calculation yields as a by-product the complete space coordinates of C and D on the survey datum.

Then when the extension to photograph IV is undertaken, points C and D, whose survey coordinates will then be fully known, become regular control points. That is, they become the "A" and "B" points for computing photograph IV. Two new points are then chosen approximately opposite the center of photograph IV for the "C" and "D" points and the photographic plane coordinates of their images are measured on photographs III and IV. Then these are the complete data for carrying out the four-point computation for photograph IV, and the position of its exposure station and its space orientation can be completely computed. Again positions become available for the new C and D points, and the computations can be extended in a similar manner throughout the flight strip.

All operators of stereoscopic instruments such as the multiplex projector, acquire confidence in stereoscopic orientation and stereoscopic bridging, because they actually *see* the orientations perfected by the elimination of so-called y-parallaxes in the field of overlap of successive photographs. Similarly, the ultimate test of the precision of the analytical determination of the exposure stations and orientation data for two overlapping photographs is the subsequent determination of positions of ground points by intersections. The check in these intersections, which consists of the identity of the two values of each elevation found by equations [25], as many readers have doubtless already discovered, is directly analogous to the instrumental elimination of the y-parallaxes. This check establishes the same confidence in analytical computations as that which stereoscopic operators feel in the instrumental orientation of aerial photographs and in stereoscopic bridging.

Now the four-point method to be explained at this time, as the introductory discussion has already suggested, actually removes y-parallaxes analytically at the A, B, C and D points, or at the four corners of the field of overlap of the two photographs. This feature adds considerable interest to this analytical process.

Furthermore, if difficulty is encountered in obtaining satisfactory checks in intersection computations using two photographs which are fixed in space independently by individual groups of three control points, the difficulty is attributable to inconsistency between the geodetic determinations of the control data, or to errors in image identification, or to inaccuracies in the photographic measurements. The first of these difficulties does not enter into computations of intersections using two photographs where one has been computed from the preceding one by the four-point method. Although the four-point calculations will always give satisfactory results in subsequent intersection computations, there is considerable danger in extending bridging calculations through too large a number of photographs without additional ground control data.

These remarks of the last three paragraphs might well be placed at the end of this discussion, but they are inserted here with the hope of adding interest to the explanation to follow of the process of analytical bridging.

In this discussion, the method will be explained for photograph III shown in Figure 6. The explanation given above of the use of the A, B, C, and D points shows that the four-point method for any other photograph is identical with that for photograph III, except that the A and B points for any subsequent photograph will have their survey coordinates determined photogrammetrically from the preceding bridging calculations, whereas for photograph III the A and B points are control points. The four-point method itself will not be affected by this difference.

It is assumed that the survey coordinates of the exposure station for photograph II, called X_{II}, Y_{II}, Z_{II}, have already been computed, together with the space orientation elements, the tilt called t_{II}, the swing called s_{II}, and the azimuth of the principal plane called α_{II}. The survey coordinates for the A and B points are known, as they will always be in the bridging method, as explained before. These coordinates will be called (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B). The photographic rectangular coordinates of the images a, b, c, and d on photograph III which is being computed, are called (x_a, y_a), (x_b, y_b), (x_c, y_c), and (x_d, y_d), and those of c and d on photograph II just preceding are called (u_c, v_c) and (u_d, v_d).

The space intersection method already presented shows how the survey azimuths from the exposure station L_{II} of photograph II to the ground points C and D can be computed. These azimuths are called α_C and α_D respectively. The formulas used for obtaining them are [16], [17], and [20] The space intersection method likewise shows how to find the vertical angles at L_{II} measured from a vertical line down from L_{II} outward to the lines $L_{II}C$ and $L_{II}D$ in space. These angles are called m_C and m_D The formula for obtaining them is [23].

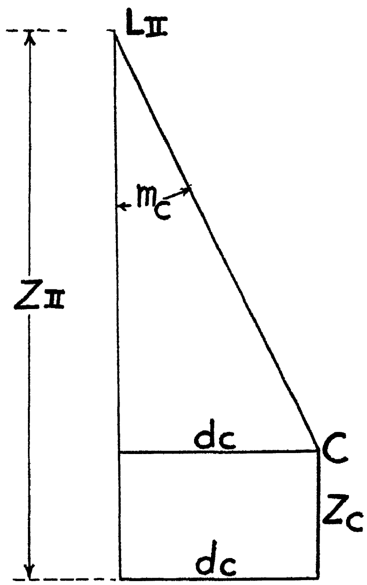


FIG. 7

Formulas [16], [17], [20], and [23] may be restated for this case as follows For transforming the coordinate system on photograph II to the nadir point for the origin and to the principal line for the y-axis,

$$\theta = 180^\circ - s_{II} \qquad [26]$$

$$\begin{aligned} \text{new } u_c &= \quad u_c \cos\theta + v_c \sin\theta \\ \text{new } v_c &= -u_c \sin\theta + v_c \cos\theta + ov \\ \text{new } u_d &= \quad u_d \cos\theta + v_d \sin\theta \\ \text{new } v_d &= -u_d \sin\theta + v_d \cos\theta + ov \end{aligned} \qquad [27]$$

(New coordinates are henceforth designated by the same symbols as before without ambiguity)

$$\begin{aligned} \phi_C &= \tan^{-1} (u_c/v_c \cos t_{II}) \\ \phi_D &= \tan^{-1} (u_d/v_d \cos t_{II}) \end{aligned} \qquad [28]$$

$$\begin{aligned} \alpha_C &= \alpha_{II} + \phi_C \\ \alpha_D &= \alpha_{II} + \phi_D \end{aligned} \qquad [29]$$

$$\begin{aligned} \tan m_C &= \sqrt{u_c^2 + v_c^2 \cos^2 t_{II}}/(f \sec t_{II} - v_c \sin t_{II}) \\ \tan m_D &= \sqrt{u_d^2 + v_d^2 \cos^2 t_{II}}/(f \sec t_{II} - v_d \sin t_{II}) \end{aligned} \qquad [30]$$

It might be noted that, if the elevations of the ground points C and D were known, it would be possible to find the horizontal survey coordinates of these two ground points. For, calling d_C the horizontal projection V_HC of $L_{II}C$ and

d_D that of $L_{II}D$, it is obvious that

$$\begin{aligned} d_C &= (Z_{II} - Z_C) \tan m_C \\ d_D &= (Z_{II} - Z_D) \tan m_D \end{aligned} \qquad [31]$$

and then from the usual plane surveying method, that the differences in X and Y between L_{II} and the ground points C and D, called ΔX_C, ΔY_C and ΔX_D, ΔY_D, respectively, are

$$\begin{aligned} \Delta X_C &= d_C \sin \alpha_C & \Delta X_D &= d_D \sin \alpha_D \\ \Delta Y_C &= d_C \cos \alpha_C & \Delta Y_D &= d_D \cos \alpha_D \end{aligned} \qquad [32]$$

and

$$\begin{aligned} X_C &= X_{II} + \Delta X_C & X_D &= X_{II} + \Delta X_D \\ Y_C &= Y_{II} + \Delta Y_C & Y_D &= Y_{II} + \Delta Y_D \end{aligned} \qquad [33]$$

But of course from the very nature of the problem, the ground elevations of C and D are not known.

However at this point in the computation it becomes necessary to find roughly approximate values for d_C and d_D The best way to find them is to make a very rough estimate of the elevations of C and D. In practice it is in most cases satisfactory to take an average of the two known elevations of A and B, and use this as the approximate value for each of the elevations of C and D It might be preferred to place photographs II and III under a viewing stereoscope and to estimate the elevations of C and D from the known elevations of A and B; the rough approximation desired does not warrant taking any further trouble in estimating these elevations. With parentheses used throughout this discussion to designate approximate values, the assumed approximate elevations of C and D are (Z_C) and (Z_D), approximate values of d_C and d_D are

$$\begin{aligned} (d_C) &= [Z_{II} - (Z_C)] \tan m_C \\ (d_D) &= [Z_{II} - (Z_D)] \tan m_D \end{aligned} \qquad [34]$$

and the approximate coordinates (X_C), (Y_C), (X_D), and (Y_D) can be found from formulas [32] and [33]. Although the Z's, the d's, and the coordinates of the C and D points are all approximations, it is to be noted that they are all rigidly consistent between themselves and with the known fixed values of the α's and the m's.

If the solution of the four-point method gave as part of the results the corrections necessary to be applied to (Z_C) and (Z_D) to obtain the true elevations, then the complete ground positions of C and D would be determined. However it simplifies the method enormously to determine instead of these, the corrections necessary to be applied to (d_C) and (d_D) to obtain their true values. In this way the correct ground survey coordinates of C and D will likewise be determined. Using Δ's to signify the corrections to all approximate values to obtain the correct ones,

$$\begin{aligned} d_C &= (d_C) + \Delta d_C \\ d_D &= (d_D) + \Delta d_D \end{aligned} \qquad [35]$$

with Δd_C and Δd_D to be determined by the solution of the problem, and

$$\left.\begin{aligned} Z_C &= (Z_C) + \Delta Z_C \quad \text{where} \quad \Delta Z_C = -\Delta d_C \cot m_C \\ Z_D &= (Z_D) + \Delta Z_D \quad \text{where} \quad \Delta Z_D = -\Delta d_D \cot m_D \end{aligned}\right\} \qquad [36]$$

$$\left.\begin{aligned} X_C &= (X_C) + \Delta X_C \quad \text{where} \quad \Delta X_C = + \Delta d_C \sin \alpha_C \\ Y_C &= (Y_C) + \Delta Y_C \quad \text{where} \quad \Delta Y_C = + \Delta d_C \cos \alpha_C \\ \\ X_D &= (X_D) + \Delta X_D \quad \text{where} \quad \Delta X_D = + \Delta d_D \sin \alpha_D \\ Y_D &= (Y_D) + \Delta Y_D \quad \text{where} \quad \Delta Y_D = + \Delta d_D \cos \alpha_D \end{aligned}\right\} \qquad [37]$$

It is likewise necessary at the beginning of this four-point problem to find approximate values for the unknown survey coordinates of the exposure station of photograph III, as in the space resection method already explained. These coordinates of the exposure station are called (X, Y, Z) and their approximate values are designated by $[(X), (Y), (Z)]$. The (X) and (Y) can be found by a rough graphical solution of the three-point problem, using the A and B points and the approximate position of either C or D. The (Z) can be found roughly from the ground length of AB and the photographic length of ab, using the

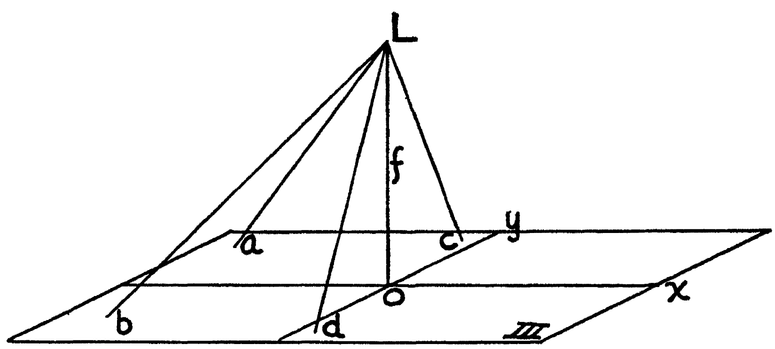


FIG. 8

ordinary scale relationship for a truly vertical photograph, assuming verticality for this approximation.

Now, to summarize to this point, we have the following symbols for coordinates of which we have actual numerical values:

X_A, Y_A, Z_A, space coordinates of A on the survey datum,

X_B, Y_B, Z_B, space coordinates of B on the survey datum,

(X_C), (Y_C), (Z_C), approximate survey coordinates of C, consistent between themselves and the angles α_C and m_C from photograph II,

(X_D), (Y_D), (Z_D), approximate survey coordinates of D, consistent between themselves and the angles α_D and m_D from photograph II,

(x_a, y_a), (x_b, y_b), (x_c, y_c), (x_d, y_d), rectangular coordinates for the four images a, b, c, and d on photograph III;

and the following symbols for unknown coordinates:

$\left.\begin{array}{l} X_{II}+d_C \sin \alpha_C \\ Y_{II}+d_C \cos \alpha_C \\ Z_{II}-d_C \cot m_C \end{array}\right\}$, survey coordinates of C, expressed in terms of a single unknown quantity d_C, which is to be determined,

$\left.\begin{array}{l} X_{II}+d_D \sin \alpha_D \\ Y_{II}+d_D \cos \alpha_D \\ Z_{II}-d_D \cot m_D \end{array}\right\}$, survey coordinates of D, expressed in terms of a single unknown quantity d_D, which is to be determined,

X, Y, Z, unknown survey coordinates of the exposure station of photograph III, to be determined.

As in the space resection problem, the angles subtended at the emergent node of the camera lens by pairs of the images a, b, c, and d on photograph III,

are given by

$$\begin{aligned}
\cos aLb &= (x_a x_b + y_a y_b + f^2)/(La)(Lb) \\
\cos aLc &= (x_a x_c + y_a y_c + f^2)/(La)(Lc) \\
\cos aLd &= (x_a x_d + y_a y_d + f^2)/(La)(Ld) \\
\cos bLc &= (x_b x_c + y_b y_c + f^2)/(Lb)(Lc) \\
\cos bLd &= (x_b x_d + y_b y_d + f^2)/(Lb)(Ld)
\end{aligned} \qquad [38]$$

where $La = \sqrt{x_a^2 + y_a^2 + f_2}$, $Lb = \sqrt{x_b^2 + y_b^2 + f^2}$, etc. There are six of these angles, but only the above five are used in this problem.

Inasmuch as these angles are identical with the angles ALB, ALC, ALD, BLC, and BLD, equation [5] in Space Resection gives rise to six condition equations, five of which are stated for this problem as follows:

$$\begin{aligned}
&(X - X_A)(X - X_B) + (Y - Y_A)(Y - Y_B) + (Z - Z_A)(Z - Z_B) \\
&\qquad - (LA)(LB)\cos aLb = 0 \\
&[X - X_A][X \overset{*}{-} (X_{II} + d_C \sin \alpha_C)] + [Y - Y_A][Y - (Y_{II} + d_C \cos \alpha_C)] \\
&\qquad + [Z - Z_A][Z - (Z_{II} - d_C \cot m_C)] - (LA)(LC)\cos aLc = 0 \\
&[X - X_A][X - (X_{II} + d_D \sin \alpha_D)] + [Y - Y_A][Y - (Y_{II} + d_D \cos \alpha_D)] \\
&\qquad + [Z - Z_A][Z - (Z_{II} - d_D \cot m_D)] - (LA)(LD)\cos aLd = 0 \\
&[X - X_B][X - (X_{II} + d_C \sin \alpha_C)] + [Y - Y_B][Y - (Y_{II} + d_C \cos \alpha_C)] \\
&\qquad + [Z - Z_B][Z - (Z_{II} - d_C \cot m_C)] - (LB)(LC)\cos bLc = 0 \\
&[X - X_B][X - (X_{II} + d_D \sin \alpha_D)] + [Y - Y_B][Y - (Y_{II} + d_D \cos \alpha_D)] \\
&\qquad + [Z - Z_B][Z - (Z_{II} - d_D \cot m_D)] - (LB)(LD)\cos bLd = 0,
\end{aligned} \qquad [39]$$

in which

$$\begin{aligned}
LA &= \sqrt{(X - X_A)^2 + (Y - Y_A)^2 + (Z - Z_A)^2} \\
LB &= \sqrt{(X - X_B)^2 + (Y - Y_B)^2 + (Z - Z_B)^2} \\
LC &= \sqrt{[X - (X_{II} + d_C \sin \alpha_C)]^2 + [Y - (Y_{II} + d_C \cos \alpha_C)]^2 + [Z - (Z_{II} - d_C \cot m_C)]^2} \\
LD &= \sqrt{[X - (X_{II} + d_D \sin \alpha_D)]^2 + [Y - (Y_{II} + d_D \cos \alpha_D)]^2 + [Z - (Z_{II} - d_D \cot m_D)]^2}
\end{aligned} \qquad [40]$$

It is to be noted the five equations [39] contain but five unknown quantities, namely, X, Y, Z, d_C, and d_D. Theoretically these equations suffice to determine these quantities, giving the space position of the exposure station and indirectly the survey coordinates of C and D. The solution of these equations in this form is of course impossible.

The actual method of solving them is analogous to that used in space resection. First, approximate values are found for LA, LB, LC, and LD, by substituting in equations [40] the approximate values already found for X, Y, Z, d_C, and d_D Then all of the approximate values are substituted in [39] to find the amounts by which these equations fail to be satisfied. Let us call the numerical values of the left members of equations [39], when the approximate values are substituted therein, v_1, v_2, v_3, v_4, and v_5, respectively

Now the amounts by which the left-hand members of equations [39] change with small changes ΔX, ΔY, ΔZ, Δd_C, and Δd_D, in the five unknowns, can be found by partial differentiation of the left-hand members of [39]. If these total increments are equated to $-v_1$, $-v_2$, $-v_3$, $-v_4$, and $-v_5$, respectively, we have five linear equations in ΔX, ΔY, ΔZ, Δd_C, and Δd_D, the necessary corrections to the assumed approximate values to cause equations [39] to be satisfied. These equations are

$$
\begin{aligned}
&U'\ \Delta X + V'\ \Delta Y + W'\ \Delta Z && + v_1 = 0 \\
&U''\,\Delta X + V''\,\Delta Y + W''\,\Delta Z + M''\,\Delta d_C && + v_2 = 0 \\
&U'''\Delta X + V'''\Delta Y + W'''\Delta Z \qquad + N'''\Delta d_D && + v_3 = 0 \\
&U^{\mathrm{IV}}\,\Delta X + V^{\mathrm{IV}}\,\Delta Y + W^{\mathrm{IV}}\,\Delta Z + M^{\mathrm{IV}}\,\Delta d_C && + v_4 = 0 \\
&U^{\mathrm{V}}\ \Delta X + V^{\mathrm{V}}\ \Delta Y + W^{\mathrm{V}}\ \Delta Z \qquad + N^{\mathrm{V}}\ \Delta d_D && + v_5 = 0
\end{aligned}
\qquad [41]
$$

in which

$$
\begin{aligned}
U' &= [1 - (LA/LB)\cos aLb](X - X_B) + [1 - (LB/LA)\cos aLb](X - X_A) \\
V' &= [1 - (LA/LB)\cos aLb](Y - Y_B) + [1 - (LB/LA)\cos aLb](Y - Y_A) \\
W' &= [1 - (LA/LB)\cos aLb](Z - Z_B) + [1 - (LB/LA)\cos aLb](Z - Z_A) \\
U'' &= [1 - (LA/LC)\cos aLc](X - X_C) + [1 - (LC/LA)\cos aLc](X - X_A) \\
V'' &= [1 - (LA/LC)\cos aLc](Y - Y_C) + [1 - (LC/LA)\cos aLc](Y - Y_A) \\
W'' &= [1 - (LA/LC)\cos aLc](Z - Z_C) + [1 - (LC/LA)\cos aLc](Z - Z_A) \\
M'' &= -(X - X_A)\sin\alpha_C - (Y - Y_A)\cos\alpha_C + (Z - Z_A)\cot m_C \\
&\quad + [(LA/LC)\cos aLc][(X - X_C)\sin\alpha_C + (Y - Y_C)\cos\alpha_C \\
&\quad \pm (Z - Z_C)\cot m_C] \\
U''' &= [1 - (LA/LD)\cos aLd](X - X_D) + [1 - (LD/LA)\cos aLd](X - X_A) \\
V''' &= [1 - (LA/LD)\cos aLd](Y - Y_D) + [1 - (LD/LA)\cos aLd](Y - Y_A) \\
W''' &= [1 - (LA/LD)\cos aLd](Z - Z_D) + [1 - (LD/LA)\cos aLd](Z - Z_A) \quad [42] \\
N''' &= -(X - X_A)\sin\alpha_D - (Y - Y_A)\cos\alpha_D + (Z - Z_A)\cot m_D \\
&\quad + [(LA/LD)\cos aLd][(X - X_D)\sin\alpha_D + (Y - Y_D)\cos\alpha_D \\
&\quad - (Z - Z_D)\cot m_D] \\
U^{\mathrm{IV}} &= [1 - (LB/LC)\cos bLc](X - X_C) + [1 - (LC/LB)\cos bLc](X - X_B) \\
V^{\mathrm{IV}} &= [1 - (LB/LC)\cos bLc](Y - Y_C) + [1 - (LC/LB)\cos bLc](Y - Y_B) \\
W^{\mathrm{IV}} &= [1 - (LB/LC)\cos bLc](Z - Z_C) + [1 - (LC/LB)\cos bLc](Z - Z_B) \\
M^{\mathrm{IV}} &= \pm (X - X_B)\sin\alpha_C - (Y - Y_B)\cos\alpha_C + (Z - Z_B)\cot m_C \\
&\quad + [(LB/LC)\cos bLc][(X - X_C)\sin\alpha_C + (Y - Y_C)\cos\alpha_C \\
&\quad - (Z - Z_C)\cot m_C] \\
U^{\mathrm{V}} &= [1 - (LB/LD)\cos bLd](X - X_D) + [1 - (LD/LB)\cos bLd](X - X_B) \\
V^{\mathrm{V}} &= [1 - (LB/LD)\cos bLd](Y - Y_D) + [1 - (LD/LB)\cos bLd](Y - Y_B) \\
W^{\mathrm{V}} &= [1 - (LB/LD)\cos bLd](Z - Z_D) + [1 - (LD/LB)\cos bLd](Z - Z_B) \\
N^{\mathrm{V}} &= \pm (X - X_B)\sin\alpha_D - (Y - Y_B)\cos\alpha_D + (Z - Z_B)\cot m_D \\
&\quad + [(LB/LD)\cos bLd][(X - X_D)\sin\alpha_D + (Y - Y_D)\cos\alpha_D \\
&\quad - (Z - Z_D)\cot m_D]
\end{aligned}
$$

The computation of the coefficients in [41] is made by substituting in [42] the approximate values of all of the quantities, the work following a cyclical routine and being much easier than it first appears. Equations [41] are very easily solved simultaneously for ΔX, ΔY, ΔZ, Δd_C, and Δd_D, by simple algebraic elimination. Then formulas [36] and [37] give the desired corrected survey coordinates of the exposure station and of the C and D points

A substitution of the corrected values in equations [39] may show that the residuals still remaining are not negligible. In this case a second solution may be necessary.

The form for the entire computation by the four-point method covers but one sheet of paper. The work with a calculating machine resolves itself into

mere routine operations, and the entire procedure requires only about two hours.

After the four-point computation of the coordinates of an exposure station, the elements of space orientation of the photograph, namely, the tilt, the swing, and the azimuth of the principal plane, are calculated by exactly the same procedure as that previously explained. There are, however, additional checks with four points now available instead of three as before.

FOUR POINT COMPUTATION

PRELIMINARY COMPUTATION FROM PRECEDING PHOTOGRAPH

Photograph No.			Points:			$C=$		$D=$
c d	u_c u_d	v_c v_d	$u_c \cos\theta$ $u_d \cos\theta$	$v_c \sin\theta$ $v_d \sin\theta$	$-u_c \sin\theta$ $-u_d \sin\theta$	$v_c \cos\theta$ [1] $v_d \cos\theta$	New u_c New u_d	New v_c [2] New v_d
$v_c \cos t$ $v_d \cos t$	$\tan\phi_C$ [3] $\tan\phi_D$	ϕ_C ϕ_D	α_C [4] α_D	$v_c \sin t$ $v_d \sin t$	[5] [6]	[7] [8]		
$\tan m_C$ [9] $\tan m_D$	(Z_C)[10] (Z_D)	$Z_L-(Z_C)$ $Z_L-(Z_D)$	(d_C) [11] (d_D)	$(d_C)\sin\alpha_C$ $(d_D)\sin\alpha_D$	$(d_C)\cos\alpha_C$ $(d_D)\cos\alpha_D$	(X_C) [12] (X_D)	(Y_C) [13] (Y_D)	(Z_C) [14] (Z_D)
$\sin\alpha_C$ $\sin\alpha_D$	$\cos\alpha_C$ $\cos\alpha_D$	$\cot m_C$ [15] $\cot m_D$	To be used for Photograph No.					

[1] See formulas [26] and [27]
[2] By formulas [27]
[3] By formulas [28]
[4] By formulas [29].
[5] f sec $t-v_c \sin t$.
[6] f sec $t-v_d \sin t$.
[7] $\dfrac{\sqrt{u_c^2+v_c^2 \cos^2 t}.}{\sqrt{u_d^2+v_d^2 \cos^2 t}.}$
[9] By formulas [30].
[10] Assumed values.
[11] By formulas [31]
[12] See formulas [32] and [33].
[13] See formulas [32] and [33].
[14] Same assume values as before.
[15] By formulas [30]

COMPUTATION OF LENGTHS, AREAS, AND HORIZONTAL POSITIONS BY SIMPLE PARALLAX METHODS

The analytical method to be explained at this time provides a simple means for calculating lengths between any desired ground points and for calculating farm areas. An absolute minimum of control is used, consisting of only the length of one line on the overlap of the first pair of photographs, without even requiring the elevations of the extremities of this line. From this single line the computation of lengths throughout a strip of photographs follows very easily, without any additional control data, and without requiring or even determining any elevations of any ground points whatever, yet giving results for both lengths and areas which are practically corrected for topographic relief. This is all accomplished by using the photographs in pairs instead of singly

The method utilizes the well known parallax formulas which are first derived for the ideal case of two overlapping vertical phtographs exposed from exactly equal altitudes. In Figure 9, I and II represent two vertical overlapping photographs; H is the common elevation of the exposure stations above the ground datum plane; P is a ground point with elevation h, having its images at p and p' respectively, the point P being taken, for convenience in drawing the figure, in the vertical plane containing the exposure stations; x and x' are the two abscissas of the images measured with respect to the rectangular axes in the photographs having their x-axes in the vertical plane of the line of flight; X is the ground abscissa of the point P, measured with respect to ground rectangular

FOUR POINT METHOD

Photograph No.								
a	x_a	y_a	La [16]	cos aLb [17]	L	(X)	(Y)	(Z)
b	x_b	y_b	Lb	cos aLc	A	X_A	Y_A	Z_A
c	x_c	y_c	Lc	cos aLd	B	X_B	Y_B	Z_B
d	x_d	y_d	Ld	cos bLc	C	(X_C)	(Y_C)	(Z_C) [18]
				cos bLd	D	(X_D)	(Y_D)	(Z_D)

$(X)-X_A$	$(Y)-Y_A$	$(Z)-Z_A$	(LA) [19]	[20]	[22]	[24]		
$(X)-X_B$	$(Y)-Y_B$	$(Z)-Z_B$	(LB)	[21]			[26]	[28]
$(X)-(X_C)$	$(Y)-(Y_C)$	$(Z)-(Z_C)$	(LC)		[23]		[27]	
$(X)-(X_D)$	$(Y)-(Y_D)$	$(Z)-(Z_D)$	(LD)			[25]		[29]

1st 3 terms [30]	1st 3 terms	1st 3 terms	1st 3 terms	1st 3 terms
4th term [30]	4th term	4th term	4th term	4th term
v_1	v_2	v_3	v_4	v_5

1st terms of U', V', W' [31] 2nd terms of U', V', W'	$\sin \alpha_C$ $\sin \alpha_D$		$\cos \alpha_C$ $\cos \alpha_D$		$\cot m_C$ $\cot m_D$
1st terms of U'', V'', W'' 2nd terms of U'', V'', W''	U'' U^{IV}	V'' V^{IV}	W'' W^{IV}	M'' M^{IV}	v_2 [32] v_4
1st terms of U''', V''', W''' 2nd terms of U''', V''', W'''	U''' U^{V}	V''' V^{V}	W''' W^{V}	N''' N^{V}	v_3 v_5
1st terms of U^{IV}, V^{IV}, W^{IV} 2nd terms of U^{IV}, V^{IV}, W^{IV}	Elimination of Δd_C				
1st terms of U^{V}, V^{V}, W^{V} 2nd terms of U^{V}, V^{V}, W^{V}	Elimination of Δd_D				

1st three terms	2nd three [31] terms	M''	U'	V'	W'	v_1 [32]	$\Delta X=$ $\Delta Y=$ $\Delta Z=$ $\Delta d_C=$ $\Delta d_D=$		ft.
			Solution of equations [41]						
1st three terms	2nd three terms	N'''					(X_C) ΔX_C [33]	(Y_C) ΔY_C [33]	(Z_C) [34] ΔZ_C
1st three terms	2nd three terms	M^{IV}					X_C	Y_C	Z_C
1st three terms	2nd three terms	N^{V}					(X_D) ΔX_D [33]	(Y_D) ΔY_D [33]	(Z_D) ΔZ_D [34]
			(X) ΔX	(Y) ΔY	(Z) ΔZ		X_D	Y_D	Z_D
			X	Y	Z				

[16] By formulas [1].
[17] By formulas [38].
[18] From preliminary computation above.
[19] By formulas [40].
[20] (LB/LA) cos aLb.
[21] (LA/LB) cos aLb.
[22] (LC/LA) cos aLc.
[23] (LA/LC) cos aLc.
[24] (LD/LA) cos aLd
[25] (LA/LD) cos aLd.
[26] (LC/LB) cos bLc
[27] (LB/LC) cos bLc
[28] (LD/LB) cos bLd.
[29] (LB/LD) cos bLd.
[30] From equations [39].
[31] See formulas [42].
[32] Equations [41].
[33] See formulas [37].
[34] See formulas [36]

axes in the datum plane with the X-axis in the direction of the line of flight and the origin at the ground plumb point for the left photograph; and B is the air-base or the distance LL' between exposure stations.

From similar triangles we have $X/x=(H-h)/f$. If the triangle $L'o'p'$ is placed adjacent to the triangle Lop, with $L'o'$ coinciding with Lo, the side pp' is then equal to the algebraic difference between the abscissas x and x'. This difference is called the parallax for the point P and is denoted by p. Then by similar triangles it is seen that $B/p=(H-h)/f$. If the ground point P is not situated in the vertical plane containing the exposure stations and therefore has both X and Y ground coordinates, the first equation above of course still holds; and furthermore, by similar reasoning, $Y/y=(H-h)/f$, where y is the ordinate of the image p on photograph I. Hence the well known parallax formulas follow at once:

$$X = (B/p)x \qquad Y = (B/p)y \qquad H - h = (B/p)f \tag{43}$$

The method to be explained also utilizes the principle that if (x, y) and (x', y') are respectively the rectangular coordinates in photographs I and II of the images p and p' of the ground point P outside the vertical plane containing the line of flight, if the x-axes of the photographs are in the vertical plane containing the line of flight, then the ordinates y and y' are exactly equal. This can be proved very easily by similar triangles.

Photograph I contains the images a and b and photograph II contains the images a' and b', of the extremities A and B of a ground line whose horizontal

FOUR POINT COMPUTATION
PHOTOGRAPH III′

Data used from preceding computation of Photograph II′.

Exposure Station	Photo II′ Orientation	Constants
$X=14997$ ft	$t=$ 0°59 7′	$\sin t=$ .01737
$Y=15002$	$s=180°29$ 0′	$\cos t=$ 99985
$Z=20201$	$\alpha_{vo}=$ 0°29 0′	$f \sec t=150$ 022 mm.
		$ov=$ 2 606 mm.

Survey Coordinates of A and B Points

	X	Y	Z
$A=C1$	15000 ft	5000 ft.	800 ft.
$B=C5$	15000	25000	1000

PRELIMINARY COMPUTATION FROM PRECEDING PHOTOGRAPH

Photo II′			$C=E1$				$D=E5$	
$e1$	+78 85	−81 48	+78.847	+0 688	+0 665	−81 477	+79 535	−78.206
$e5$	+80 77	+78 16	+80.767	−0 660	+0 682	+78 157	+80.107	+81 445
−78.194	1 0171	134°30.8′	134°59.8′	−1 358	151.380	111.535		
+81 433	.98372	44 31.8	45 00 8	+1 415	148.607	114 230		
.73679	900	19301	14220 8	+10056	−10055	25054	4952	900
.76867	900	19301	14836 1	+10493	+10488	25491	25495	900
+ 70715	−.70707	1.357242	To be used for					
+ 70727	+ 70694	1 300945	Photograph III′					

Four Point Method

Photograph III′								
				645877	*L*	26000	14200	19550
				.907815	*C*1	15000	5000	800
*c*1	−85.22	−74 23	187 810	.503747	*C*5	15000	25000	1000
*c*5	+88 78	+86 82	194 730	513208	*E*1	25054	4952	900
*e*1	− 6 39	−73 35	167 096	906355	*E*5	25491	25495	900
*e*5	− 6 49	+88 46	174 262					

*C*1	+11000	+ 9200	+18750	23605.14	.660	801	465		
*C*5	+11000	−10800	+18550	24119 34	632			443	.820
*E*1	+ 946	+ 9248	+18650	20838 50		1 028		594	
*E*5	+ 509	−11295	+18650	21809 60			.545		1.002

+369452500	+445175100	+251372500	+256485100	+473542500
−367723868	−446550304	−259338356	−257943918	−476772783
+ 1728632	− 1375204	− 7965856	− 1458818	− 3230283

+ 3740	+ 3128	+ 6375		+ 70715		− 70707	1 357242	
+ 4048	− 3974	+ 6826		+ 70727		+ 70694	1 300945	
+ 2189	+ 1831	+ 3731	+2163	+1572	+ 3209	−7882		−1375204
− 26	− 259	− 522	+6511	−2261	+17904	−8761		−1458818
+ 5885	+ 4922	+10031	+6117	− 217	+18517		−7270	−7965856
+ 232	− 5139	+ 8486	+1979	−1921	+ 3302		−7983	−3230283
+ 6127	− 6016	+10332	+2163	+1572	+ 3209	−7882		−1375204
+ 384	+ 3755	+ 7572	+5858	−2034	+16108	−7882		−1312453
+ 1980	− 1944	+ 3339	+6117	− 217	+18517		−7270	−7965856
− 1	+ 23	− 37	+1802	−1749	+ 3007		−7270	−2941771

− 7779	+ 687		+7788	− 846	+13201	+1728632	$\Delta X = -1017$ ft.		
+ 6505	− 6722	− 7882	+3695	−3606	+12899	+ 62751	$\Delta Y = +847$		
+25448	−26021		+4315	+1532	+15510	−5024085	$\Delta Z = +523$		
							$\Delta d_C = -$ 71 59		
− 7780	+ 196		+7610	− 827	+12899	+1689086	$\Delta d_D = -$ 644 05		
− 6504	− 4352	− 7270	+3695	−3606	+12899	+ 62751			
+24393	−13223		+3589	+1274	+12899	−4178315	25054	4952	900
							− 51	+ 51	+ 97
− 7779	+ 397		+3915	+2779		+1626335			
− 7636	− 3884	− 8761	+ 106	−4880		+4241066	25003	5003	997
+25177	−15036								
			+ 60	−2779		+2415148			
− 7780	+ 361		+3975			+4041483	25491	25495	900
+ 7635	− 8001	− 7983					−456	−455	+838
+24133	−24311		26000		14200	19550			
			−1017		+847	+523	25035	25040	1738
			24983		15047	20073			

FOUR POINT METHOD

Photograph III′ (2nd solution)				.645877	L	24983	15047	20073	
				907815	$C1$	15000	5000	800	
				503747	$C5$	15000	25000	1000	
				513208	$E1$	25003	5003	997	
				906355	$E5$	25035	25040	1738	
$C1$	+9983	+10047	+19273	23917 59	.640	818	440		
$C5$	+9983	− 9953	+19073	23717 12	651			467	.798
$E1$	− 20	+10044	+19076	21558 67		1 007		565	
$E5$	− 52	− 9993	+18335	20881 45			577		1 029

+367256427	+468364156	+252451668	+263668956	+448644668
−366377831	−468097947	−251588359	−262408158	−448870370
+ 878596	+ 266209	+ 863309	+ 1260798	− 225702

+ 3594	+ 3617	+ 6938		+ 70715		−.70707	1.357242		
+ 3484	− 3474	+ 6656		+ 70727		+ 70694	1 300945		
+ 1817	+ 1829	+ 3508	+1817	+1759	+ 3374	−7035		+ 266209	
0	− 70	− 134	+5312	− 936	+18464	−6858		+1260798	
.+ 5590	+ 5626	+10793	+5568	+1399	+18549		−6951	+ 863309	
− 22	− 4227	+ 7756	+2019	−1721	+ 3321		−7064	− 225702	
+ 5321	− 5305	+10166	+1771	+1715	+ 3289	−6858		+ 259511	
− 9	+ 4369	+ 8298	+5312	− 936	+18464	−6858		+1260798	
+ 2017	− 2011	+ 3853	+5568	+1399	+18549		−6951	+ 863309	
+ 2	+ 290	− 532	+1987	−1693	+ 3268		−6951	− 222092	
− 7059	− 14		+7078	+ 143	+13594	+ 878596	$\Delta X = +13$ ft.		
+ 7104	− 7151	− 7035	+3541	−2651	+15175	+1001287	$\Delta Y = -13$		
+26158	−26073		+3581	+3092	+15281	+1085401	$\Delta Z = -71$		
							$\Delta d_C = +3\ 67$		
− 7061	− 21		+7078	+ 143	+13594	+ 878596	$\Delta d_D = -58\ 45$		
− 7103	− 4076	− 6951	+3172	−2375	+13594	+ 896968			
+25073	−13763		+3186	+2751	+13594	+ 965574	25003	5003	997
							+ 3	− 3	− 5
− 7059	− 8		+3906	+2518		− 18372			
− 7037	− 4013	− 6858	+ 14	+5126		+ 68606	25006	5000	992
+25887	−14628							$C=E1$	
			+ 7	+2518		+ 33701	25035	25040	1738
							− 41	− 41	+ 76
− 7061	− 38		+3899			− 52073			
+ 7036	− 7269	− 7064					24994	24999	1814
+24813	−24545		24983		15047	20073		$D=E5$	
			+ 13		− 13	− 71			
Exposure Station			24996		15034	20002			

FOUR POINT METHOD

				645877	*L*	24996	15034	20002
				907815	*C*1	15000	5000	800
				503747	*C*5	15000	25000	1000
				513208	*E*1	25006	5000	992
				906355	*E*5	24994	24999	1814
*C*1	+9996	+10034	+19202	23860 39				
*C*5	+9996	− 9966	+19002	23671 02				
*E*1	− 10	+10034	+19010	21495 61				
*E*5	+ 2	− 9965	+18188	20738 96				

+364797576	+465611216	+249277158	+261129216	+444939558
−364791180	−465612538	−249274001	−261132041	−444940851
+ 6396	− 1322	+ 3157	− 2825	− 1293

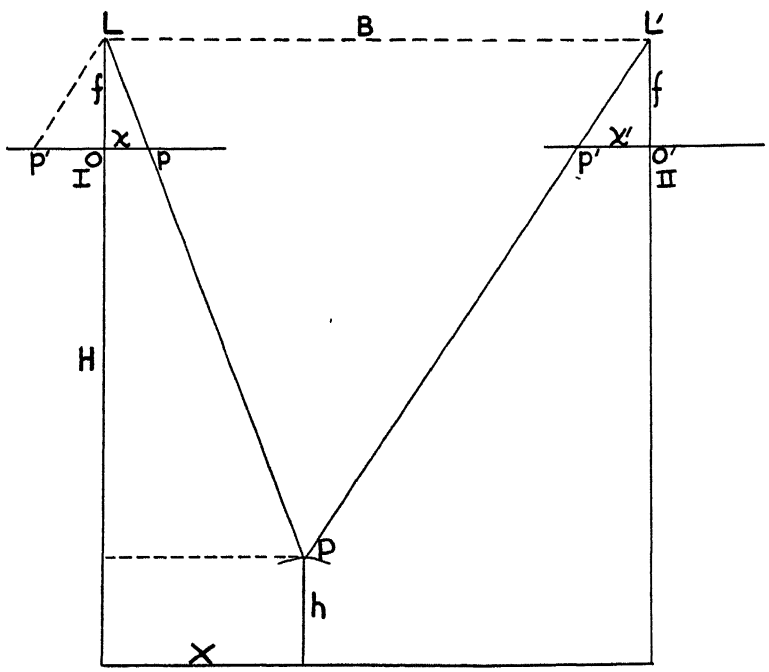


FIG. 9

length has been determined by ground surveying. It has been mentioned that this single distance constitutes all the ground control work needed except for subsequent checks, and that even the elevations of the points *A* and *B* are not required.

Sharply defined pass points are chosen somewhat as shown in Figure 10. That is, *c* and *d* are chosen about opposite the center of photograph II, near the lateral margins of the photographs, *e* and *f* are chosen opposite the center of photograph III, etc. Photographic rectangular coordinates are then measured for all of these points upon all of the photographs in which they appear, these coordinates being referred to the geometric axes of the photographs as indicated

by the camera marks, and the measurements being made with a comparator if possible. On photograph I the coordinates are measured for the images a, b, c, and d; on photograph II for a, b, c, d, e, and f, on photograph III for c, d, e, f, g, and h; etc At the same time, the coordinates are measured of the images of the extremities of any line whose length is desired, or of the corners of any property whose area is to be determined, on both of the two photographs in which they appear

At the beginning it is necessary to determine for each pair of overlapping photographs the direction on each one of the principal point ray toward the principal point of the other. To do this, for the first pair, for example, there is found the angle θ through which the axes of photograph I must be rotated in order that the positive x-axis may contain the principal point of photograph II, and also the angle θ' through which the axes of photograph II must be rotated

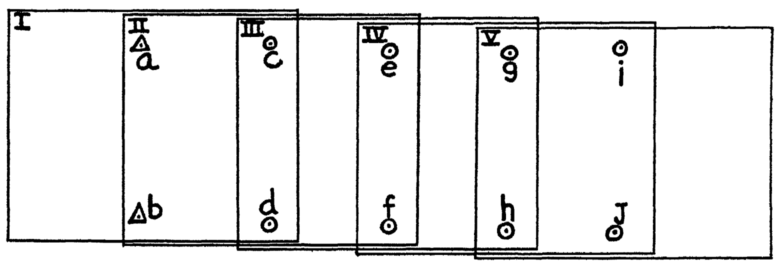


FIG 10

in order that the negative x-axis may contain the principal point of photograph I.

In each pair of photographs there have been measured the coordinates of four common points, those in the first pair being a, b, c, and d, those in the second pair being c, d, e, and f, etc. For the four points common to the first pair, let us call the coordinates on photograph I (x_a, y_a), (x_b, y_b), (x_c, y_c), and (x_d, y_d), and on photograph II the same symbols with primes. Since the rotation of the axes through the as yet undetermined angles θ and θ' places the x-axes into the vertical plane of the line of flight, for the ideal case of two vertical photographs from equal altitudes the ordinates of corresponding points become identical, as stated above That is, we would have

$$\begin{aligned} -x_a \sin\theta + y_a \cos\theta &= -x_a' \sin\theta' + y_a' \cos\theta' \\ -x_b \sin\theta + y_b \cos\theta &= -x_b' \sin\theta' + y_b' \cos\theta' \\ -x_c \sin\theta + y_c \cos\theta &= -x_c' \sin\theta' + y_c' \cos\theta' \\ -x_d \sin\theta + y_d \cos\theta &= -x_d' \sin\theta' + y_d' \cos\theta' \end{aligned} \tag{44}$$

Theoretically any two of these equations would be sufficient to solve for θ and θ'. However, owing to the fact that the two photographs are probably taken from slightly different altitudes, and to the fact that either or both may be slightly tilted, the four equations are never quite consistent. In practice the best results will be obtained by weighting the first two equations inversely proportionally to the ordinates of a and b respectively on photograph I, combining the resulting equations; by weighting the second pair of equations inversely proportionally to the ordinates of c and d respectively on photograph II, combining the resulting equations; and then solving simultaneously for θ and θ' the two combined equations. This is equivalent to solving for θ and θ' the two equations

obtained by eliminating cos θ between the first two equations, and by eliminating cos θ' between the last two. Thus the weighting merely simplifies the solution of the equations. Since the ordinates of a and b on photograph I and those of c and d on photograph II will ordinarily change very little with the rotation of the axes, the values of θ and θ' thus determined will so place the axes that the two pairs of residuals in the four inconsistent equations will be approximately proportional to the ordinates of the respective points after the rotation of the axes. This places the axes in the proper positions to pass through the conjugate principal points regardless of any difference in altitude between the exposure stations

The two trigonometric equations resulting from the combination just described may be solved very easily for θ and θ'. Since θ and θ' are always small angles, both cosines can first be taken as unity and an approximate solution made for sin θ and sin θ'. Then another solution can be made with better values for cos θ and cos θ'. This is continued until values of θ and θ' are found which satisfy the equations. However two solutions usually suffice and require but a few minutes

Corresponding angles are determined for the pair of photographs II and III in a similar manner, using the four common points c, d, e, and f. This process is carried out for every pair of photographs in the strip.

The next step in the computation is to rotate the axes of the photographs through these calculated angles. That is, for the first pair of photographs we find the coordinates of the four common points a, b, c, and d on photograph I with the axes rotated through the angle θ, and on photograph II with the axes rotated through the angle θ'; for the second pair of photographs II and III, we find the coordinates of the four common points c, d, e, and f on photograph II with the axes rotated through the angle θ and on photograph III with the axes rotated through θ'; and so on throughout the strip. These coordinates after the rotation of the axes are found very easily by means of a calculating machine, using the well known analytical geometry formulas

$$\begin{aligned} \text{New } x &= x \cos\theta + y \sin\theta \\ \text{New } y &= -x \sin\theta + y \cos\theta \end{aligned} \qquad [45]$$

To avoid complication no new symbols are introduced for the coordinates of the various points after the rotation of the axes. There will be no ambiguity in using the same symbols as before to designate now the coordinates after rotation.

The next step for each pair of photographs is to make the ordinates of each of the four common points on one photograph *exactly* equal to the corresponding ordinates on the other photograph. This is brought about by a proportional numerical increase or decrease in both the abscissa and ordinate of each point on one of the photographs of the pair. For instance, in the case of photographs I and II, we find the factor by which y_a' after the rotation of the axes must be multiplied to give y_a, and we multiply both x_a' and y_a' by this factor; the same operation is carried out for the other three of the common points.

Which one of the two photographs of the pair is used to transform the coordinates of the four common points is of no consequence as far as the results are concerned. Usually the coordinates of the four common points on photograph II are transformed to make the ordinates equal to the corresponding ordinates on photograph I; similarly in the second pair of photographs the coordinates of the four common points on photograph III are transformed to make the ordinates equal to the corresponding values on photograph II; etc.

Again no new symbols will be introduced in this step for the corrected values

PARALLAX METHOD

Photo No a x_a y_a b x_b y_b c x_c y_c d x_d y_d	Photo No. x_a' y_a' x_b' y_b' x_c' y_c' x_d' y_d'	$\sin\theta$ $\cos\theta$ $\sin\theta'$ $\cos\theta'$ Coefficients for Equations [44]
Photo No. $x_a\cos\theta$ $y_a\sin\theta$ $-x_a\sin\theta$ $y_a\cos\theta$ $x_b\cos\theta$ $y_b\sin\theta$ $-x_b\sin\theta$ $y_b\cos\theta$ $x_c\cos\theta$ $y_c\sin\theta$ $-x_c\sin\theta$ $y_c\cos\theta$ $x_d\cos\theta$ $y_d\sin\theta$ $-x_d\sin\theta$ $y_d\cos\theta$		Elimination of $\cos\theta$ from 1st pair Elimination of $\cos\theta'$ from 2nd pair
Coordinates after rotation by formulas [45]	p_a Formulas p_b [46] p_c p_d	Elimination of $\sin\theta'$ from new pair
Photo No $x_a'\cos\theta'$ $y_a'\sin\theta'$ $-x_a'\sin\theta'$ $y_a'\cos\theta'$ $x_b'\cos\theta'$ $y_b'\sin\theta'$ $-x_b'\sin\theta'$ $y_b'\cos\theta'$ $x_c'\cos\theta'$ $y_c'\sin\theta'$ $-x_c'\sin\theta'$ $y_c'\cos\theta'$ $x_d'\cos\theta'$ $y_d'\sin\theta'$ $-x_d'\sin\theta'$ $y_d'\cos\theta'$		Elimination of $\sin\theta$ from new pair $\cos\theta$ $\cos\theta'$ $\sin\theta =$ In terms of the cosines of θ and θ' $\sin\theta' =$
Coordinates after rotation by formulas [45]	Coordinates after equalization of ordinates	$\theta =$ $\theta' =$ $\sin\theta =$ $\sin\theta' =$ $\cos\theta =$ $\cos\theta' =$

$10000/p_a =$ $10000/p_b =$ See formula [43]	(X_A) (X_B) Diff	(Y_A) (Y_B) Diff.	$(D) =$ Square root of sum of squares of differences
$B = [D/(D)]10000$			See formula [47]
$B/p_a =$ $B/p_b =$ See formula [43]	X_A X_B Diff.	Y_A Y_B Diff.	$D =$ Should check given value
$B/p_c =$ $B/p_d =$	X_C X_D Diff	Y_C Y_D Diff.	$CD =$

of the coordinates on the one photograph in each pair throughout the strip. The original symbols will now indicate the coordinates after rotation of the axes and after the correction for scale difference in the last step.

Next there are found the values of the parallaxes mentioned above in connection with the parallax formulas. In the first pair of photographs we find the parallaxes for the four points a, b, c, and d from the expressions

$$p_a = x_a - x_a' \qquad p_c = x_c - x_c' \qquad [46]$$
$$p_b = x_b - x_b' \qquad p_d = x_d - x_d'$$

using the abscissas obtained after the three preceding steps. Then we continue

PARALLAX METHOD

	Photograph IV		V	
e	+ 2 36	+34.67	−62 90	+52 38
f	−10 86	−40 20	−76 72	−21 74
g	+83 31	+23 07	+12 56	+40 18
h	+65 47	−54 73	− 4 32	−39 24

	Photograph IV			
e	+ 2.30	− 7 90	+ 0 54	+33 76
f	−10.57	+ 9 16	− 2 47	−39 14
g	+81 12	− 5 26	+18 98	+22 46
h	+63 75	+12 47	+14 91	−53 29

e	− 5 60	+34 30	p=69 68
f	− 1 41	−41 61	67 54
g	+75 86	+41.44	74 37
h	+76 22	−38 38	70 13

	Photograph V			
e	−60 67	−13 81	−16 59	+50 53
f	−74 00	+ 5 73	−20 23	−20 97
g	+12 12	−10 60	+ 3 31	+38 76
h	− 4 17	+10 35	− 1 14	−37 85

e	−74 48	+33.94	−75 28	+34 30
f	−68 27	−41 20	−68.96	−41 61
g	+ 1.52	+42 07	+ 1 50	+41 44
h	+ 6 18	−38 99	+ 6 08	−38 38

sin θ	cos θ	sin θ′	cos θ′
− 2 36	+34 67=	+62.90	+52 38
+10 86	−40 20=	+76 72	−21 74
−83 31	+23 07=	−12 56	+40 18
−65 47	−54 73=	+ 4 32	−39 24
− 2 74	+40 20=	+ 72 93	+60 74
+10 86	−40 20=	+ 76 72	−21.74
+ 8 12	=	+149 65	+39 00
−83 31	+23 07=	− 12 56	+40 18
−67 04	−56.04=	+ 4 42	−40 18
−150 35	−32 97=	− 8 14	
+ 0 44	=	+ 8 14	+ 2 12
−150 35	−32 97=	− 8 14	
−149 91	−32 97=		+ 2 12
+ 8 12	=	+149 65	+39 00
− 8 12	− 1 78=	− 0 44	
	− 1 78=	+149 21	+39 00

	cos θ	cos θ′
sin θ =	− 21994	− 01414
sin θ′=	−.01194	− 26134

θ =−13°10 1′	θ′=−15°17 5′
sin θ =− 22780	sin θ′=− 26372
cos θ =+.97370	cos θ′=+ 96459

10000/69 68=143 5235	− 803 73	+4922 28	
10000/67 54=148 0604	− 209 65	−6161 83	(EF)
	594 08	11084 11	11100 02
B (10031/11100)10000	9037 ft		
9037/69 68=129 7005	− 726 32	+4448 21	
9037/67 54=133 8004	− 189 46	−5568 37	EF
	536 86	10016.58	10031 ft.
9037/74.37=121 5157	+9218 67	+5035 73	
9037/70 13=128 8571	+9820 85	−4945.15	GH
	602 18	9980.88	9999 ft.

by finding the parallaxes for c, d, e, and f in the second pair of photographs, and so on throughout the strip.

The next operation is to use the known ground length of AB, the control line appearing in the first pair of photographs, to determine the air base B for this pair of photographs, together with the horizontal ground distance CD; then from the known length of CD to determine the air base for the second pair of photographs, together with the ground distance EF; etc. Thus the air bases will be determined for all the pairs of photographs in the entire strip.

DETERMINATION OF THE AREA OF A FOUR-SIDED FIGURE

Measured coordinates of vertices

	Photo IV		Photo V		
g4	+ 2 36	+34 67	−62.90	+52.38	
h4	+40 .67	+28 48	−26 38	+45 70	B = 9037 ft.
i5	+85 19	+59 65	+20 39	+77 05	
h5	+45 56	+64 17	−17 55	+81 48	

Photo IV. $\sin\theta = -.22780$ $\cos\theta = +\ 97370$

$x\cos\theta$	$y\sin\theta$	$-x\sin\theta$	$y\cos\theta$	New x	New y	Parallax
+ 2 30	+ 7.90	+ 0.54	+33 76	− 5 60	+34 30	69 68
+39 60	− 6 49	+ 9 27	+27.73	+33 11	+37.00	70.48
+82 95	−13 59	+19.41	+58 08	+69 36	+77.49	70.00
+44 36	−14 62	+10.38	+62 48	+29.74	+72 86	67.59

Photo V. $\sin\theta' = -\ 26372$ $\cos\theta' = +\ 96459$

$x\cos\theta'$	$y\sin\theta'$	$-x\sin\theta'$	$y\cos\theta'$	New x	New y	Coordinates with ordinates equalized	
−60 67	−13.81	−16.59	+50 53	−74 48	+33 94	−75 28	+34 30
−25.45	−12.05	− 6 96	+44 08	−37 50	+37 12	−37.37	+37 00
+19 67	−20 32	+ 5 38	+74 32	− 0 65	+79 70	− 0 63	+77 49
−16 93	−21 49	− 4 63	+78 60	−38 42	+73 97	−37.84	+72 86

		X	Y	Area:
9037/69.68=129.7005	G4	− 726 3	+ 4448 2	143957807.12
9037/70 48=128 2191	H4	+4245.6	+ 4743 6	94074509 32
9037/70 00=129 1057	I5	+8955.0	+10004 0	
9037/67 59=133 7074	H5	+3977.0	+ 9742 0	2) 49883297 80
	G4	− 726 3	+ 4448 2	
				24941649 sq ft
				572 581 acres

For the first pair let us designate by D the known ground length AB. First assume temporarily for the air base B some arbitrary value such as unity, or as in the illustrative computations to follow, 10,000 feet. Corresponding ground coordinates are then found for A and B by means of the parallax formulas [43],

$$X_A = (10000/p_a)x_a \qquad X_B = (10000/p_b)x_b$$
$$Y_A = (10000/p_a)y_a \qquad Y_B = (10000/p_b)y_b$$

Then we find the ground distance (D) between these points corresponding to these coordinates, by taking the square root of the sum of the squares of the differences in X and Y. Then the correct air base will be equal to $D/(D)$ multiplied by the arbitrary value taken for the air base; that is, in this case,

$$B = [D/(D)]10000 \tag{47}$$

Then with air base B known, correct ground coordinates for the points C and D can be found from the parallax formulas [43],

$$X_C = (B/p_c)x_c \qquad X_D = (B/p_d)x_d$$
$$Y_C = (B/p_c)y_c \qquad Y_D = (B/p_d)y_d$$

Then the ground length CD is found by taking the square root of the sum of the squares of the differences in X and Y.

The length of CD is then used as the known length in the second pair of

photographs, from which the second air base is computed together with the ground distance EF. This process is continued throughout the entire flight strip.

Finally, with the air base and the angles θ and θ' determined for every overlapping pair of photographs, it becomes a simple matter to find the ground lengths of any desired lines whatever On a pair of photographs where there appears a line ST, for example, whose length is desired, the coordinates are measured as before for the extremities of the line on each photograph The values of these coordinates after the rotation of the photographic axes through the angles θ and θ' and after the equalization of the corresponding ordinates, are found as described above Then parallaxes are found for the points S and T, as shown in formula [46]. The parallax formulas [43] then give ground coordinates of the points S and T, from which the length can be found at once

If a property survey is desired, it is simple to find lengths of all the property boundaries and also the angles at the property corners, from the ground coordinates of the corners found in this photogrammetric process.

The area of a piece of property is found very easily from the ground coordinates of the corners, using a calculating machine, by the usual analytical geometry method for finding the area of a polygon from the coordinates of the vertices The process requires no more time than finding the area by planimeter, and it gives far more accurate results than the planimeter. Furthermore, the planimeter method requires information regarding the elevation of the tract or the elevations of the corners, whereas this parallax method requires no information whatever regarding the elevations of the property corners.

Test of the Parallax Method on a Strip of Five Photographs

Computations by the parallax method, which has been described, were carried through a strip of five photographs as illustrated in Figure 10. The five photographs used were from a set of synthetic photographs calculated by Co. B, 30th Engineers, at Fort Belvoir, for test purposes.

The photographs in this strip had tilts ranging from 0° to 3°. The topographic relief for the extremities A and B of the control line, for the pass points used as shown in Figure 10, and for the vertices of the area determined in the preceding illustrative computation, comprised the following widely different elevations:

	Point	Elevation		Point	Elevation
Control line	A	150 feet	Vertices of area	$G4$	500 feet
	B	1800	determined	$H4$	900
				$I5$	1050
Pass points	C	2200		$H5$	300
	D	1000			
	E	2000			
	F	750			
	G	500			
	H	0			
	I	2000			
	J	800			

Although neither the tilts nor these elevations are used at all in the computations of the ground distances or areas by the parallax method, nevertheless the wide variation in the elevations, coupled with the large tilts, severely tests the method, which is supposed to correct the distances and areas determined for topographic relief without actually using the elevations. How effectively the method does this can be seen from the tabulation of the results of the computation.

RESULTS OF THE COMPUTATION OF THE STRIP OF FIVE PHOTOGRAPHS

Photo pair	θ	θ'	Starting length	B	Length determined	Correct value†	Error
I– II	2°02′	2°00′	AB=10000*	10035	CD=10029	10000	0.29%
II–III	– 5 03	– 4 59	CD=10029	10892	EF=10062	10000	0.62%
III–IV	6 51	– 3 32	EF=10062	9747	GH=10031	10000	0.31%
‡IV– V	–13 10	–15 18	GH=10031	9037	IJ= 9999	10000	0 01%

* Ground control
† Correct lengths are known in this case since the photographs are synthetic ones.
‡ The complete computation of this pair is previously shown as the illustration.

Area determined in the foregoing illustrative computation,	572.581 acres
Correct value of this area	573.921 acres
Error	1.340 acres or 0 23%

RADIAL TRIANGULATION

Several variations of radial plotting have been used in photogrammetry for the graphical determination of horizontal positions of ground points. The differences between them lie principally in the different ray centers used, in the method of transferring the ray centers to adjacent photographs, and in the manner of making and using templates.

The various ray centers which have been used are the principal points, the nadir points, the isocenters, and substitute center points. The geometric principles governing the choice between the first three can be briefly stated as follows: (1) With truly vertical photographs, image displacements caused by topographic relief radiate from the principal points, and angles between the principal point rays are exactly equal to the corresponding horizontal angles on the ground; (2) with tilted photographs, image displacements caused by topographic relief radiate from the nadir points, but angles between nadir point rays are not exactly equal to the corresponding angles on the ground; (3) with tilted photographs taken of absolutely flat terrain, regardless of the magnitudes of the tilts, angles measured on the photographs between rays from the isocenters are *exactly* equal to the corresponding ground angles. Consequently, with truly vertical photographs, radial plotting using principal point rays will give horizontal positions of ground points which are absolutely free from errors, regardless of the magnitude of topographic variations. If the photographs have tilts, no matter how large the tilts may be, and if the ground is perfectly flat, radial plotting using isocenter rays will give horizontal positions of ground points absolutely free from errors. If the problem involves both tilts and topographic relief, no ray center will give results free from errors. If the tilts are as small as those usually encountered, and if the ground has considerable topographic relief, radial plotting with nadir point rays will give by far the best results, and will ordinarily give horizontal positions in which the errors are negligible.

Obviously neither the nadir points nor the isocenters can be used unless the individual tilts of the photographs are known approximately. Also none of the three points, the principal point, the nadir point, or the isocenter, can be used without some scheme for marking the conjugate points on adjacent photographs. The stereoscopic method for transferring these points to adjacent photographs has proved very satisfactory in practice. The substitute center point has a distinct advantage in that conjugate points can be marked on adjacent photo-

graphs by mere identification of the images of the points chosen. Of course small errors in the horizontal positions determined arise from the use of the substitute center points, but these errors are frequently negligible in practical work.

At the outset of the previous discussion of the calculation of distances and areas by the parallax method, a very simple and satisfactory method was shown for calculating on each photograph the directions of the principal point rays toward the principal points of the adjacent overlapping photographs. If the directions of the principal point rays on each photograph to the various images arranged approximately as shown in Figure 10, together with the directions found from the θ's to the conjugate principal points, are considered as horizontal directions read from a direction theodolite at each exposure station, it becomes possible to compute a net work of triangulation consisting of the two control points, the pass points of Figure 10, and the principal points, as vertices. In this manner the X and Y survey coordinates can be calculated for all of the points in this net work and for any other points desired. Thus complete planimetric survey data become available for the flight strip as well as data for computing any desired lengths or areas. Inasmuch as the methods of computing this triangulation are identical with those universally used for any triangulation in either plane or geodetic surveying, the explanation of the procedure is scarcely required here

Of course it is obvious that this calculation of radial triangulation is not confined to principal point triangulation. If the approximate tilts are known so that approximate nadir points and isocenters become available, then the entire scheme of triangulation can likewise be calculated using nadir points or isocenters in place of the principal points.

In fact the problem of radial triangulation, like all other problems in aerial photogrammetry, is susceptible to many variations.

Scale Determination

The complete determination of the scale of a single *vertical* photograph consists of only the determination of the altitude of the exposure station. This can be done by using two ground points A and B whose images a and b appear in the photograph, with the distance AB measured on the ground and with the elevations of A and B determined on the ground. Let the photographic coordinates of the images a and b be called (x_a, y_a) and (x_b, y_b), and let the horizontal ground length AB be called D and the elevations of A and B be called Z_A and Z_B. A very rough value for the altitude H of the exposure station can be found from the scale relation

$$(H - h)/f = D/d$$

in which

$$d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

and in which h is taken as the mean of Z_A and Z_B. Then consider that the images are displaced inward radially toward the center of the photograph to compensate for the elevations Z_A and Z_B, giving the positions of the images for corresponding points at zero elevation. To do this, multiply x_a and y_a by $(H-Z_A)/H$ and x_b and y_b by $(H-Z_B)/H$, using the approximate value of H. Now find d again from these revised photographic coordinates, and then find a better value of H from the sea-level scale relation.

$$H/f = D/d,$$

using in this the new d. With this revised H, displace the images again, and

repeat this until the value of H will no longer affect the photographic coordinates used to find d Two approximations almost invariably suffice to find the correct H.

Then if a length is desired for a line joining any two ground points of unequal elevations, the usual expressions for scale relationship

$$X = [(H - h)/f]x \qquad Y = [(H - h)/f]y \tag{48}$$

in which H is the exposure station altitude and h is the elevation of one of the ground points, give ground coordinates X, Y directly from the photographic coordinates x, y, for each of the two points. Then the true ground length is at once obtained by finding the square root of the sum of the squares of the differences in X and Y between the two ground points.

The complete scale determination for a *tilted* photograph consists of finding the elevation of the exposure station, and also the tilt and the swing. The space resection problem and the space orientation problem for a single photograph supply these data completely, together with additional data, not a part of the scale determination, for uniquely fixing the photograph in space.

In this case, if the correct length is desired for a line joining any two ground points having different elevations, the exact expressions for the scale relationship for tilted photographs

$$\begin{aligned} X &= [(H - h)/(f \sec t - y \sin t)]x \\ Y &= [(H - h)/(f \sec t - y \sin t)]y \cos t \end{aligned} \tag{49}$$

in which t is the tilt, and x, y, are the photographic coordinates after the transformations by formulas [16], will give the *exact* X and Y ground coordinates for each point. Then the correct length can be found as before from the horizontal ground coordinates.

Absolutely correct values can be found for areas in either of the above two cases, if the elevations of the vertices on the ground are known. The photographic coordinates substituted in [48] for a vertical photograph, or in [16] and then [49] for a tilted photograph, give the correct X and Y ground coordinates of the vertices, and the area can then be found using the usual analytical geometry method for finding the area of a polygon from the coordinates of the vertices.

Of course the ground X and Y coordinates given by [48] from a vertical photograph, are referred to the ground plumb point as the origin and to vertical coordinate planes including the rectangular axes of the photograph, and likewise the ground X and Y coordinates given by [49] from a tilted photograph, are referred to the ground plumb point as the origin and to vertical coordinate planes perpendicular to each other with the YZ-plane coinciding with the principal plane of the photograph. But this has no bearing on the lengths of lines or the areas of polygons found from the scale determination of the photograph.

However, if it should be desired to find the exact space coordinates on the survey datum for a ground point whose elevation is known, using measurements on a tilted photograph for which the resection and space orientation problems have been solved, the method is shown in the discussion of the Four-Point Method, and consists of applying formulas [16], [17], [20], [23], [31], [32], and [33]. But the horizontal coordinates X, Y, of the exposure station, and α_{Vo}, the azimuth of the principal plane, required for this work, are not strictly a part of the scale determination problem.

Some analytical solutions of the complete scale determination problem other than space resection and space orientation previously described, furnishing as

they do only the Z coordinate of the exposure station together with t and s, but not αv_O, will merely give lengths and areas, but not actual ground survey coordinates, from measurements on the photographs. Some of these analytical solutions of the scale determination problem may be found extremely useful in certain practical work. None of these has been explained here on account of the fact that the analytical solutions explained for space resection and space orientation of the photograph include the complete scale determination

Scale Determination

Case 1, assuming that the photograph is truly vertical.

Point	Measured photo coordinates x	y	Given ground elevations	
$d2$	$+39.25$	-41.87	$D2$ 1000 ft.	
$d4$	$+42$ 90	$+40.28$	$D4$ 2800	
	3 65	82.15		

$f=150.00$ mm

Given distance $D2$ to $D4=10000$ ft

$\sqrt{3.65^2+82.15}=82$ 23

$(H-h)/150.00=10000/82$ 23 $\qquad H-h=18242$ ft.

Approx. $(H)=18242+1900=20142$ ft

Point		Coordinates corrected for topographic relief	
$d2$	$19142/21042=.950$	$+37$ 29	-39.78
$d4$	$17342/20142=.861$	$+36$ 94	$+34.68$
		0 35	74 46

$\sqrt{0.35^2+74\ 46}=74.46$

$H/150$ 00$=10000/74$ 46 $\qquad H=20145$ Final

To find the distance from $B2$ to $B4$, given the following data:

Point	Measured photo coordinates x	y	Ground elevations
$b2$	-42 35	-44.97	$B2$ 2400 ft.
$b4$	-38 49	$+35$ 88	$B4$ 800

Point	$H-h$	$(H-h)/f$	Ground Coordinates X	Y
$B2$	17745	118 300	-5008	-5320
$B4$	19345	128 967	-4964	4627
			44	9947

$\sqrt{44^2+9947^2}=9947$ ft Desired distance.

Case 2, using the photograph as tilted.

The space resection and the space orientation for this photograph have already been calculated. In fact this is the same photograph for which these computations have already been shown under those topics. The part of the results which constitute the scale determination are as follows:

$$H \text{ or } Z_L=20{,}201 \text{ ft.} \qquad t=0°59.7' \qquad s=180°29.0'$$

To find the distance from $B2$ to $B4$, given the same data as before, namely,

Point	Measured photo coordinates x	y	Ground elevations
$b2$	-42.35	-44 97	$B2$ 2400 ft.
$b4$	-38.49	$+35.88$	$B4$ 800

Using the following constants

$$\theta = 180° - s = -0°29.0' \quad \sin\theta = -\ 00844 \quad \cos\theta = +.99996$$
$$\sin t = .01737 \quad \cos t = .99985 \quad f \sec t = 150.02 \quad ov = 2.61$$

from the space orientation data, we first substitute the given coordinates in formulas [16] thus

	$x \cos\theta$	$y \sin\theta$	$-x \sin\theta$	$y \cos\theta$	New x	New y
$b2$	-42.35	$+0.38$	-0.36	-44.97	-41.97	-42.72
$b4$	-38.49	-0.30	-0.32	$+35.88$	-38.79	$+38.17$

Then we find

	$y \cos t$	$y \sin t$	$(f \sec t - y \sin t)$	$H-h$	$\frac{(H-h)}{(f \sec t - y \sin t)}$
$b2$	-42.71	-0.74	150.76	17801	118.075
$b4$	$+38.16$	$+0.66$	149.36	19401	129.894

and by formulas [49] we have the ground coordinates

	X	Y
$B2$	-4957	$-\ 5043$
$B4$	-5039	$+\ 4957$
	82	10000

Required distance $B2$ to $B4 = \sqrt{82^2 + 10{,}000^2} = 10{,}000$ ft.

The method for Case 1 is correct for truly vertical photographs. The results for the exposure station altitude and for the distance from $B2$ to $B4$ are in error, however, because the photograph, instead of being a truly vertical one, actually has a tilt of 0°59.7′. The method for Case 2 is rigorously correct.

The photograph used is one of the Fort Belvoir synthetic photographs, and the correct length of $B2$–$B4$ is therefore known. It is 10,000 ft. When the tilt is neglected the length found for this line has an error of 53 ft. or 0.53%. When the tilt is considered, the length determination is exactly correct

A Simple Method for the Photogrammetric Extension of Surveys without Ground Control Using Two Parallel Overlapping Strips of Photographs

After some experience with analytical computations it will be found that the space resection, the space orientation, and the space intersection problems can be solved easily and rapidly. The four-point method, however, for extending a photogrammetric survey through a strip of photographs without using ground control after the initial triangle, does require considerable time even for an experienced computer. To avoid the four-point calculations, therefore, a simple means has been devised for extending the photogrammetric survey through two overlapping parallel strips of photographs, using only the resection, orientation, and intersection methods.

Figure 11 shows two overlapping parallel strips of photographs. Let us suppose that there are four initial control points A, B, C, and Q situated as shown in the figure, points whose ground coordinates on the survey datum have been determined by the usual geodetic methods. Then let sharply identifiable pass points be chosen situated as shown by the figure, with a_3, b_3, and c_3 approximately opposite the centers of photographs III and III′, a_4, b_4, and c_4 approximately opposite the centers of photographs IV and IV′, etc. Measurements are first made with the comparator on all of the photographs of the rectangular

coordinates of the images of all these points, with these coordinates referred to the geometric axes of the respective photographs as usual. That is, on photograph I, x and y coordinates are measured for the images a, b, and q; on photograph I′ for b, c, and q; on photograph II for a, b, q, a_3, and b_3; on photograph II′ for b, c, q, b_3, and c_3; on photograph III for a, b, a_3, b_3, a_4, and b_4; on photograph III′ for b, c, b_3, c_3, b_4, and c_4; etc

Then the space coordinates on the survey datum for the exposure stations, and the elements of space orientation, can be computed for photographs *I* and *II*, using the control points A, B, and Q; and the same calculations for photographs I′ and II′ can be made using control points B, C, and Q, by the space resection and space orientation methods previously explained.

Then the ground coordinates on the survey datum for the point B_3 can be computed by the space intersection method previously explained, using measurements on photographs II and II′.

Next, the exposure station and elements of space orientation are computed

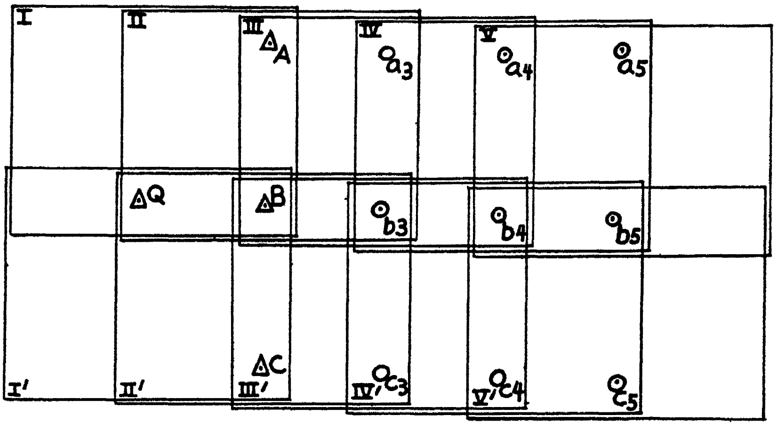


FIG. 11

for photograph III using as control the three points A, B, and B_3, and the same calculations are made for photograph III′ using the points B, C, and B_3.

Then by the space intersection method again, the survey space coordinates are computed for A_3 using measurements on photographs II and III, those for C_3 using measurements on photographs II′ and III′, and those for B_4 using measurements on photographs III and III′. After this, points A_3, B_3, and B_4 are used for control for exposure station and orientation calculations for photograph IV, and the points B_3, C_3, and B_4 are used for photograph IV′. Then in turn photographs III and IV are used for space intersection to compute the survey coordinates of A_4, III′ and IV′ are used to determine C_4, and IV and IV′ to determine B_5.

By the continuation of this procedure both of the entire strips of photographs can be computed and the space positions of all of the pass points can be determined, all with no additional control after the four initial points, and all by using only the simple space resection, orientation, and intersection methods.

Subsequently of course the space intersection method will determine the space coordinates on the survey datum for any desired points situated anywhere within the area covered by the two strips of photographs.

It might be thought at first that this method would fail unless the photo-

RESULTS OF COMPUTATIONS FOR THE TWO PARALLEL STRIPS OF PHOTOGRAPHS SHOWN IN FIGURE 11

‖	Photo	Point	Control points used	Photos used	Results			Correct values* X, Y, Z'
					Exp. sta. X, Y, Z	Orientation $t, s, \alpha vo$	Ground coordinates X, Y, Z	
R, O‡	I		A† B† Q†		5002 34997 20101	2°00′ 45 14 225 14		
R, O	I′		B C† Q		5002 15003 20001	0°01′ 32 28 212 28		
R, O	II		A B Q		15003 34995 20000	1°29′ 0 20 180 20		
R, O	II′		B C Q		14997 15002 20201	1°00′ 180 29 0 29		
I§		$B3$		II II′			24999 25000 1801	25000 25000 1800
R, O	III		A B $B3$		25000 34999 20399	3°00′ 305 00 125 00		
R, O	III′		B C $B3$		24997 15005 20001	3°01′ 305 05 125 05		
I		$A3$		II III			25000 44998 801	25000 45000 800
I		$C3$		II′ III′			25000 5000 1001	25000 5000 1000
I		$B4$		III III′			34997 25001 253	35000 25000 250
R, O	IV		$A3$ $B3$ $B4$		35002 34994 19998	0°01′ 133 58 313 58		
R, O	IV′		$B3$ $C3$ $B4$		35001 15014 20001	1°59′ 235 26 45 25		

RESULTS OF COMPUTATIONS FOR THE TWO PARALLEL STRIPS OF PHOTOGRAPHS SHOWN IN FIGURE 11—*Continued*

‖	Photo	Point	Control points used	Photos used	Results: Exp. sta X, Y, Z	Results: Orientation t, s, avo	Results: Ground coordinates X, Y, Z	Correct values* X, Y, Z
I		*A*4		III IV			35000 44998 205	35000 45000 200
I		*C*4		III′ IV′			35004 4998 145	35000 5000 150
I		*B*5		IV IV′			44991 25003 1058	45000 25000 1050
R, *O*	V		*A*4 *B*4 *B*5		45021 34965 19788	1°56′ 268 04 88 06		
R, *O*	V′		*B*4 *C*4 *B*5		45029 15043 19887	1°35′ 99 28 267 25		
I		*A*5		IV V			45003 45001 681	45000 45000 700
I		*C*5		IV′ V′			45013 4998 22	45000 5000 50
I		*B*6		V V′			54965 25006 611	55000 25000 600

* These correct values are known because these photographs are synthetic ones.
† *A*, *B*, *C*, and *Q* are the only ground control points used for the entire group of photographs.
‡ See illustrative computations for Space Resection and Space Orientation.
§ See illustrative computation for Space Intersection.
‖ Resection and Orientation, or Intersection.

graphs in the two strips were situated exactly opposite each other laterally, as they are in Figure 11. This is not true however. If the photographs in each strip have the customary sixty percent overlap, even if the photographs of the two strips are staggered in any possible manner, the procedure described will always apply. The slight variations in the procedure which may be necessary in any particular case are so obvious when a sketch is made to show the overlap, that no further explanation is required here.

It should be noted that if two parallel strips of photographs are available, this method really offers a much easier means than the four-point method for extending photogrammetric surveys into inaccessible territory.

Test of the Method for the Photogrammetric Extension of Surveys through Two Parallel Overlapping Strips of Photographs

Computations by this resection and intersection method of extending photogrammetric surveys without ground control, using two parallel overlapping strips of photographs, were made for the group of photographs shown in Figure 11. These are a part of the series of synthetic photographs computed by Co. B, 30th Engineers, at Fort Belvoir, for test purposes.

It will be noticed that, although the tilts were large enough and the variations in topographic relief great enough to test the method rather severely, the results are very satisfactory.

The computations involve only the resection, orientation, and intersection methods, for all of which illustrative examples have already been shown. Therefore there is given here only a tabulation of the results of these computations for this group of photographs. However the illustrative computations for resection and for space orientation are actually those for photograph I in this group shown in Figure 11; and the illustrative computation for space intersection is actually the computation of the survey coordinates of $B3$ using photographs II and II′, as it was done in this series of computations.

Conclusion

In conclusion two points should be mentioned regarding this paper. First, the demands of recent research work have brought analytical methods in photogrammetry to the attention of several very eminent mathematicians, and it is entirely possible that their findings may partially supersede some of the methods presented here.

Secondly, it should be emphasized that the methods presented here can be extended to countless surveying problems other than those specifically shown, such as the determination of the bearings and lengths of property boundaries, earth-work calculations, determination of tree heights and timber estimating in forestry surveys, etc. It is of course impossible to present anything more than the most fundamental problems in this brief discussion of analytical methods in aerial photogrammetry.

CHAPTER XIII

MAPPING FROM OBLIQUE PHOTOGRAPHS

THE USE OF OBLIQUE AERIAL PHOTOGRAPHS FOR MAPPING

Theory of Plotting from Oblique Aerial Photographs

AN OBLIQUE aerial mapping photograph as used by the Topographical Survey is one taken by a previously calibrated fixed-focus aerial survey camera with the apparent horizon included in its field of view. The inclusion of the apparent horizon line serves with the known camera constants obtained by calibration, to determine the orientation of the photograph relative to the horizontal plane containing the lens. If, in addition, two suitably situated ground control features are included in the field of view, the altitude of the lens at the time of exposure above such ground control features, together with the direction of camera pointing, can also be determined.

With the photograph thus fixed in space, the location in the ground plane of any object is determined by joining the image point to the perspective centre of the photograph and producing this ray to intersect the ground plane. This assumption indicates that the method is applicable only to those areas where for the purposes of the map the ground may be considered a level plane.

The Use of Perspective Grids.—The determination of the camera air position and of the drawing of the rays to intersection as above referred to, are in practice mechanically affected by the use of perspective grids in the construction of which the particular orientation and altitude of air position of

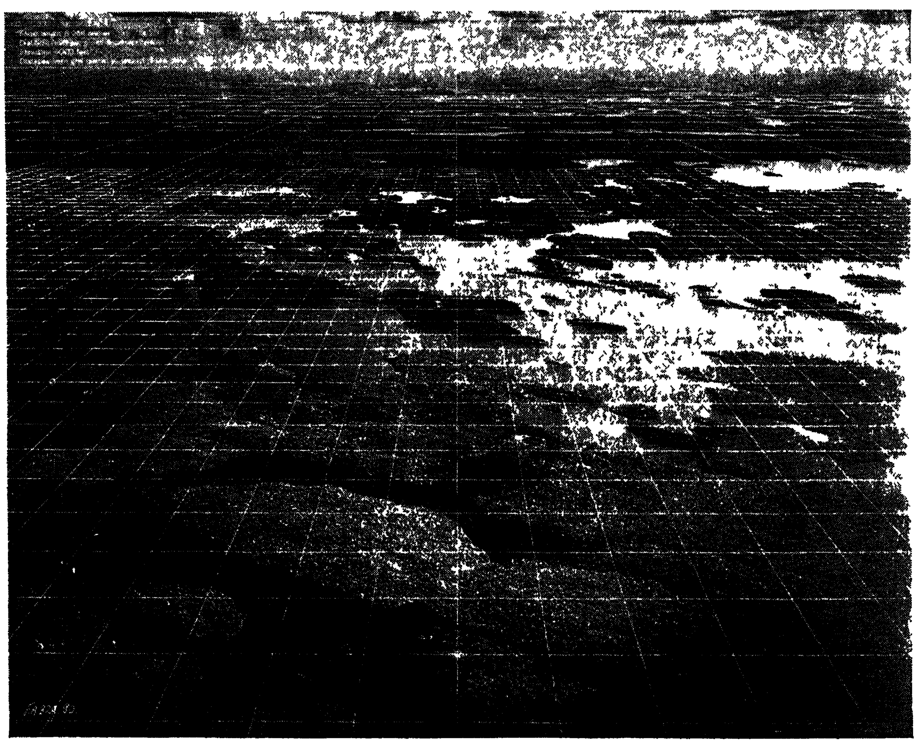


FIG. 1. Oblique aerial mapping photograph with plotting grid superimposed. For plot developed from this photograph see Fig. 2.

the photograph have been considered These perspective grids representing a system of squares which are made photographically on transparent glass plates, are mechanically superimposed in correct position on the photograph to be plotted. The objects are then plotted by noting their location in the particular square and marking this position in the corresponding square of the compilation sheet (See Figs 1 and 2). These operations are described in more detail in a publication of the Topographical Survey entitled "A Graphical Method of Plotting Oblique Aerial Photographs"

The way in which these grids have been evolved will be further considered.

In sextant observations at sea, made by the determination of altitudes of celestial objects, the apparent horizon serves a definite purpose. This purpose relates to the locating of the vertical plane in which the altitude angle must be measured containing the observer's eye and the celestial object as well as to the position in this plane, of a horizontal line through the point of observation The latter may be obtained when the height above sea level of the observer's eye is known. The dip of the apparent horizon below the true horizon for use in such observations is determined from the expression:

$$\text{Dip} = 59\sqrt{h}^{*} \tag{1}$$

where Dip = the dip angle expressed in seconds of arc, and h = the distance above sea level of the observer's eye expressed in feet.

Perspective Elements of the Oblique Photograph.—In taking oblique aerial mapping photographs, the apparent land or sea horizon is included in the field of view for a similar purpose. This purpose is to fix the relation of the photograph to the vertical plane containing the camera axis at the time of exposure, as well as to the horizontal plane passing through the rear node of the camera lens or perspective centre of the photograph, so that the directions from such perspective centre to image points in the photograph will be in agreement with the true directions from such point to the objects themselves. In other words, this determines the orientation of the photograph in space.

By the use of a special camera sight, the apparent horizon line is made to occupy a position parallel to and as close as is practical to the upper limit of the photograph In this way the amount of dead ground—sky and unplottable

* The derivation of this formula is given in Fig 1 of the bulletin of the Topographical Survey "A Graphical Method of Plotting Oblique Aerial Photographs" Briefly it is as follows.—

$$\text{Tan Dip} = \frac{SH}{OH} = \frac{\sqrt{SO^2 - OH^2}}{OH} = \sqrt{\frac{2h}{R} + \frac{h^2}{R^2}}$$

As $\frac{h^2}{R^2}$ is very small, it may be neglected without appreciable error

$$\therefore \text{ Tan Dip} = \sqrt{\frac{2h}{R}} = \sqrt{\frac{h}{10444314}}$$

Also, Dip being a small angle, we may write

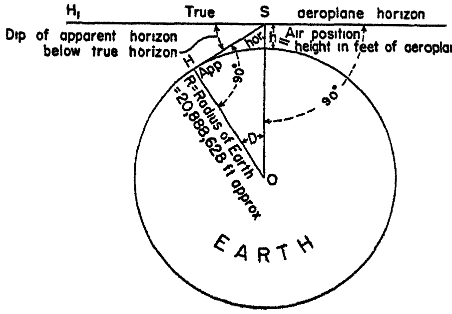
$$\text{Tan Dip} = \text{Dip}'' \tan 1''$$

$$\therefore \text{ Dip}'' \tan 1'' = \sqrt{\frac{h}{10444314}}$$

$$\therefore \text{ Dip} = 206265\sqrt{\frac{h}{10444314}} = 63''.82\sqrt{h}$$

To correct for refraction multiply value of Dip above by 0.9216

$$\text{Dip in seconds} = 58\ 82\sqrt{h} = 59\sqrt{h}.$$



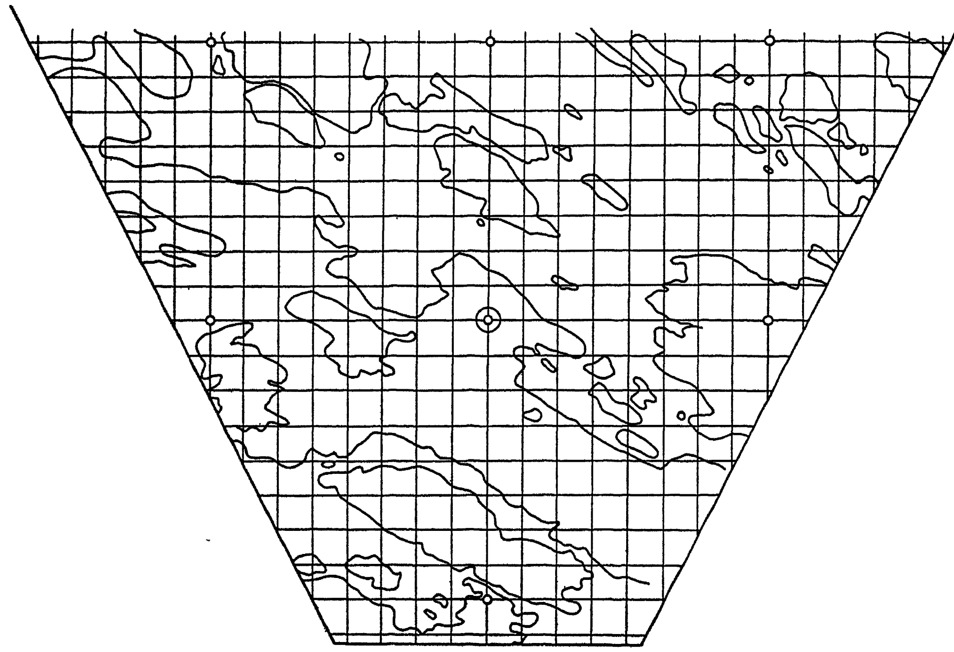


FIG 2 Plot developed from oblique aerial mapping photograph shown in Fig. 1.

area near the apparent horizon—is kept as small as possible The record of the apparent horizon on a mapping oblique photograph taken over a level area departs slightly from a straight line. The wider the angle of the camera the greater becomes the departure. In the case of a $7'' \times 9''$ oblique mapping view taken with a lens of 8″ focal length, this curvature amounts to a few one-hundredths of an inch.

The apparent horizon line itself consists of a hyperbolic curve obtained as a section, by the plane of the photograph, of the right cone formed by the apparent horizon with its apex at the lens centre, the depression of its generating line below the true horizon being obtained from the dip expression given by equation (1) The chord of the portion of the apparent horizon line intercepted by the limits of the photograph may, without sensible error, be assumed to be parallel to the true horizon line

As indicated in the previous section, the position of the principal point of the photograph, the foot of the normal from the rear node of the camera lens, or perspective centre, to the photo plane, is shown on each view marked by a small cross. This cross is etched on the focal plane glass plate at the principal point, at the time when the camera is calibrated and thus appears on each negative and the prints therefrom. A normal drawn on the photograph from the principal point to the chord of the apparent horizon will thus define the intersection with the photo plane of the vertical plane containing the camera axis, and this normal is usually called the principal line of the photograph. Thus, in Fig. 3, which represents an oblique aerial mapping photograph,

P is the principal point.

ACB is the apparent horizon line.

A normal from P to AB meets the apparent horizon line in C. PC represents

the principal line of the photograph, and the tangent to the apparent horizon at C is parallel to the chord AB and also parallel to the true horizon line formed by the intersection of the photo plane with the horizontal plane containing the perspective centre, which cuts PC produced in H. The distance CH from the apparent horizon must be determined, and the manner in which this is done is as follows:—

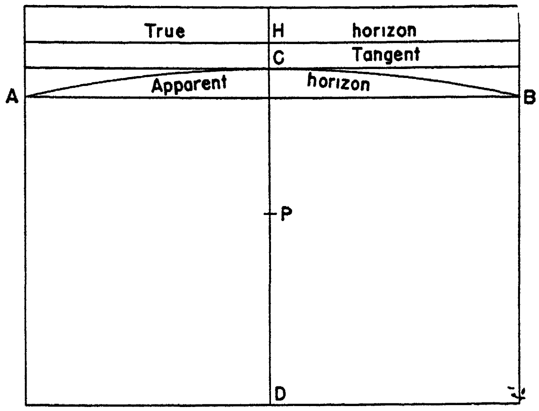


FIG. 3

Fig. 4 represents a section by the principal plane of the photograph.

O represents the perspective centre or camera lens rear node.

OP represents the normal from the perspective centre meeting the plane of the photo in the principal point P and therefore equals the principal distance or focal length of the camera lens.

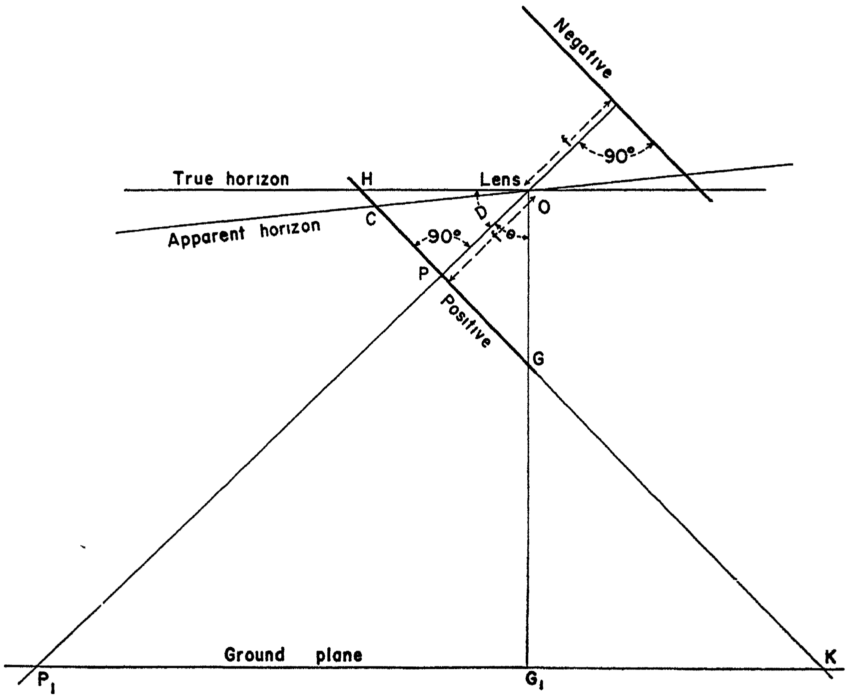


FIG. 4

PC represents the principal line of the photo and meets the apparent horizon line in C and the horizontal plane containing the camera lens rear node in H

HOC represents the angular depression of the apparent horizon below the

true horizon and is obtained by substituting in equation (1) for h the altitude of the aeroplane above the ground when the photograph was taken, as given by the record of the altimeter readings obtained in the flight from which is subtracted the altimeter reading when leaving the ground. At the ordinary photographic altitude of 5,000 feet at which oblique mapping views are taken, an error of 1,000 feet in altitude introduces an error in the apparent horizon dip angle of about seven minutes which corresponds to an error of 0 017 inches in the distance CH when using a lens of 8.2 inches focal length, so that for substitution in equation (1) an approximate value of h only is necessary.

Referring to Fig. 4, to determine CH the distance CP is measured in the photograph. The distance $PO=f$ is known from the camera calibration. As the angle CPO is a right angle, the remaining sides and angles of triangle POC are readily determined, and as the dip angle HOC is given from equation (1), triangle CHO is readily solved either graphically or by calculation from the known base CO and the previously determined angles HCO and COH. The principal line of the photograph is thus produced from C above the apparent horizon to H and a normal drawn through H to the principal line represents the true horizon line of the photograph. On this line on the photograph are located the vanishing points of all systems of parallel horizontal lines on the ground. Thus, horizontal lines on the ground parallel to the principal plane of the photograph, that is, the vertical plane containing the camera principal line, vanish at the central vanishing point H; and horizontal lines on the ground making an angle of 45° with this plane vanish at a point distant from H an amount equal to f sec Dip where Dip = camera depression angle (HOP of Fig. 4) Horizontal lines perpendicular to this plane vanish at an infinite distance from H, that is, they are parallel to the horizon line.

Perspective Grids.—The information appearing upon an oblique aerial photograph exhibits a large variation of scale. For plotting it to a uniform scale, the method most practical is that of "gridding" the photograph, that is, of applying to it a grid or perspective representation of a system of squares, rectangles or triangles. The particular form of grid to be decided upon should ordinarily be determined by its ease of construction and development both on the plot and on the photograph. In this connection, it might be noted that if a transversal making an angle of 45° therewith be applied to a system of equally spaced parallel lines and through the points of intersection normals to the parallel lines be drawn a system of squares is formed. This is illustrated in Fig. 5.

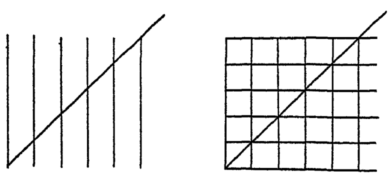


FIG. 5

If the direction of such parallel lines be made to coincide with the known principal line of the photograph, the corresponding perspective diagram can readily be constructed. This is done by: (1st) drawing the lines vanishing at the central vanishing point representing the equally spaced parallel lines, (2nd) drawing each of the two transversals thereto to its respective vanishing point for diagonals which, as stated previously, is located on the true horizon line of the photograph, and (3rd) drawing the lines parallel to this horizon line, through the points of intersection, thus completing the system of squares.

Thus, in Fig. 6a, which represents the photograph,

P is the principal point,

V_1HV is the true horizon line,

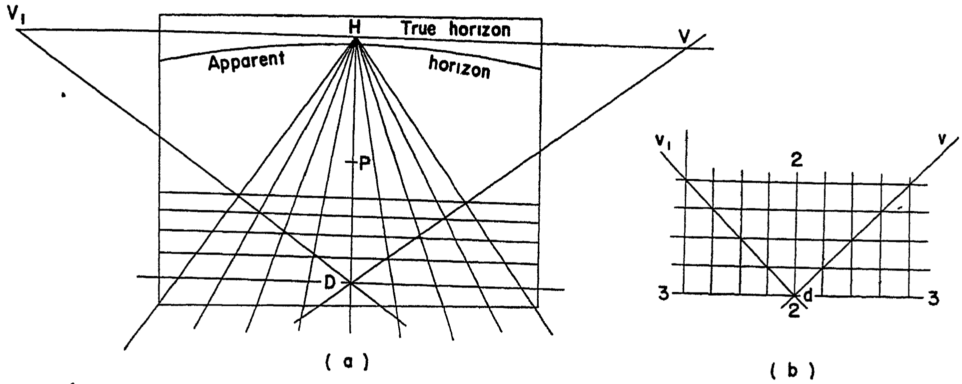


FIG. 6

V_1 and V are the vanishing points for lines making angles of 45° with the direction of camera pointing

To construct a grid on the photograph and the corresponding system of squares on the plot (see Fig. 6b) proceed in the following manner· A number of equal divisions should be ticked off on a straight line drawn on the photograph parallel to its horizon line through any point D on the principal line. Straight lines should then be drawn through such divisions to converge on the central vanishing point H of the photograph. A corresponding system of equally spaced parallel lines such as Fig. 6b, should be drawn for the plot.

On one of the vertical lines of the plot which may be taken to represent the principal line of the photograph, select a point d to represent point D of the photograph. Any point on any of the vertical lines of the plot would effect the same result, but for reasons that will appear later, it is better to make the selection of the point d as indicated. From this point d diagonal lines dv and dv_1 should be drawn and also lines such as 3–3 perpendicular to lines 2–2 on the compilation through the intersection points, thus forming the corresponding plotting squares. On the perspective (Fig. 6a) the diagonals are drawn from D to V and V_1 intersecting the central converging lines. Through the intersection of the diagonals with the converging lines are drawn lines parallel to the true horizon. This completes the perspective grid of the plotting squares.

The squares can readily be made as small as required and the plotting of the area shown in the photograph resolves itself into a square by square transferring by sketching of the topographic detail from photograph to plot. In plotting according to this procedure, no information of the plotting scale is available until the location of two features of a known distance apart have actually been plotted and measured upon the plot.

Uniformity of Grid Construction an Advantage.—If the plotting of a number of individual photographs were to be combined it would be necessary to reduce or enlarge each compilation to a common even scale. This would involve considerable labour. To offset this, the converging lines are drawn on the photograph so that they represent lines spaced 10 chains or 660 feet apart on the ground and the plotting squares are all drawn to the common scale desired. Referring to Fig. 4, the principal line of the photograph when extended to the ground meets it at K at a distance measured in the photo-plane equal to the altitude of the air position multiplied by the secant of the angle of camera depression below the true horizon. Thus, in constructing our grid for a known

altitude of air position we can (see Fig. 7) select the point D on the principal line so that when a distance of one inch is ticked off on the transversal, cutting it at right angles, the line joining such ticked point to the central vanishing point H will when produced meet the ground on the photo-plane at a point 660 feet from the principal plane at K.

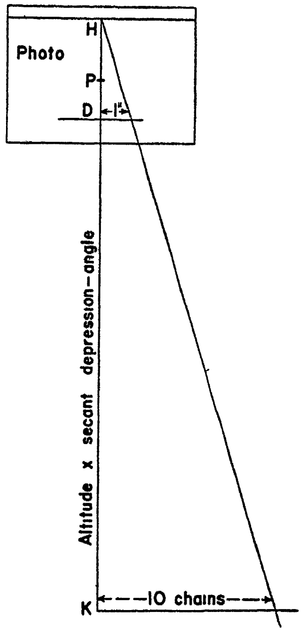


FIG. 7

By proportion then

$$\frac{HD \text{ (inches)}}{1 \text{ inch}} = \frac{HK \text{ (feet)}}{660 \text{ feet}}.$$

Thus, if the altitude of the air position is 5,000 feet and the depression angle of the camera is 20° below the true horizon,

$$HD = \frac{1 \times 5{,}000 \text{ sec. } 20^\circ}{660}.$$

The one-inch ticks on the line through D can then be joined to the central vanishing point H as before.

It is preferable to have the principal point mark the corner of a grid square and its corresponding point on the plotting sheet will serve as an origin for the plotting. This is readily accomplished if the diagonal lines are drawn through the principal point P instead of any other point on the converging lines. As the plotting squares will then be identical for all photographs, one only need be drawn and duplicates thereof may be made preferably on transparent paper by photolithography.

It might be noted that a knowledge of the focal length of the camera lens and the approximate altitude of the air position is necessary for the determination of the camera depression angle where the location of the principal point and the apparent horizon are shown on the photograph. The location of the principal point, as stated before, is etched in its true position on the focal plane plate of the camera in use. The focal length or principal distance is determined by the calibration which is usually done in the laboratory, but the contraction of film from its predevelopment dimensions necessitates, for use in the construction of the grid, a corrected focal length or principal distance, the correction amounting to about one-half of one per cent. Where many photographs are to be plotted it would be a large undertaking to construct on each photograph a grid as described. It has, therefore, been found advisable to construct for each focal length of lens employed a number of grids for depression angles differing by regular intervals and also for altitudes differing by regular intervals, both through a limited range.

In the actual preparation of these grids for regular use, they are drawn to a scale four times their natural size, reduced copies of them being obtained by photographing to transparent glass plates. An endeavor is then made to have all photographs for mapping purposes taken within the range for which the grids are available, a special camera sight and altimeter being used for the purpose. By marking the position of the apparent horizon line on the glass grid, the true horizon line need not be drawn on the photograph as the former will serve equally well as the latter for the correct super-imposition of the grid

on the photograph. Furthermore, in the aerial camera in regular use, a rectangle of dimensions 9.125 by 6.966 inches is etched on the focal plane glass plate marking the limits of the photograph. Instead of determining the camera depression angle as previously outlined, this may equally well be expressed by the distance in tenths of an inch that the apparent horizon line appears to be located below the upper marginal line of the photograph when no swing has occurred. The grid is thus constructed for the depression angle corresponding to a particular position of the apparent horizon on the photograph.

In deciding what intervals of depression angle and of altitude should be adopted so as to keep small the number of grids necessary for plotting a group of photographs, it should be borne in mind that the procedure in taking and plotting these oblique aerial photographs is as follows· Sets consisting of three oblique views, slightly overlapping each other, are usually taken at intervals of not more than two miles along the line of flight. The plotting is carried out on a scale of one mile to an inch, the scale of final publication being four miles to an inch. In the central strip of views an area is plotted in each photograph extending from its principal point, a distance on the ground of one mile towards the foreground and one mile towards the background. If a mapping oblique aerial photograph taken from an altitude of 5,000 feet with a lens of 8.2 inches focal length and at a camera depression angle below the true horizon of 19°, were plotted by means of a grid constructed for this altitude but for a depression angle of 19°20′ instead of 19°, the two-mile distance extending along the principal line one mile from the principal point would measure 10,603 feet instead of 10,560 feet. There would, thus, be a plotting error of 43 feet incurred along the line of flight which on the working scale, one mile to an inch, would amount to 0.008 inches, an error equal to that caused by using a grid constructed for an altitude differing by 24 feet from that which should have been employed in the correct grid of the proper depression angle. A tenth of an inch variation in the marginal distance of these oblique aerial mapping views taken from an altitude of 5,000 feet with a depression of from 18° to 19° corresponds to a variation in depression angle of approximately 36 minutes.

Grids Used by the Topographical Survey.—The Topographical Survey have constructed grids through the range of depressions encountered in taking oblique mapping views of this kind for intervals represented by a tenth of an inch of difference in marginal distance. If the available grid nearest to the marginal distance be employed, the error resulting from this cause will, when the scale of compilation is considered, always be so small that it is practically negligible. The altitude interval adopted in the construction of the grids is either 25 feet or 50 feet. The former is considered to be of slightly smaller interval than the adopted interval of marginal distance would indicate to be required for the same order of accuracy. Each grid is drawn on paper to four times its natural size (as stated before) and then reduced photographically, one or more contact prints as required being made on glass plates from the resulting negative. Before being photographed, the following information entering into the construction of the grid is marked thereon:—

1. The focal length of the lens for which the grid is constructed.
2. The marginal distance of the apparent horizon (instead of the angle of depression referred to the principal horizon).
3. The altitude for which the grid has been constructed.
4. The distance to the ground plumb point from the point where the plate normal meets the ground plane.

The latter, as seen by inspection of Fig. 4, is equal to the altitude of the air

position multiplied by the cotangent of the angle of camera depression below the true horizon. This information is used when it becomes necessary to employ more than one grid in the plotting of different parts of the same photograph, that is, when it is desired to extend the method to plotting areas which depart from a horizontal plane. In this case, the location of the ground plumb point is fixed on each plot and as this point is common to both plots it enables them to be combined. From Fig. 4, it can readily be shown that the photo plumb point is located on the principal line at a distance equal to $f(\tan \text{Dip}+\cot \text{Dip})$ below the true horizon at H where $\text{Dip} = <HOP$ or angle of depression of camera axis. The photo plumb point is the vanishing point for vertical lines.

In photographing obliquely, as stated before, the practice is to take at intervals of two miles along the lines of flight a set of three oblique views, in the central one of which the camera is directed in the line of travel over the ground, due allowance being made for the drift of the plane. The principal line of the first central view in a straight flight in which all central views are taken exactly in the direction of ground travel, when extended, would pass through the image points which lie on the principal lines of all succeeding central views. By having the plotted principal lines collinear, the plot of each central view could readily be joined to the next in correct bearing to make a continuous strip. Any relief of the area would not affect the accuracy of the direction obtained by joining image points along this line.

Conditions such as these, however, cannot be fulfilled. Accordingly, for joining plots of successive central views it is the practice to select two image points lying near the principal line of the first central view and, upon the line joining them (and the line produced, if necessary), to choose image points which can be clearly identified on the succeeding central views of each set. In this manner the line may be extended from the start to the end of the flight, that is, from ground control to ground control. These direction points are plotted to establish a straight line common to the successive plots and to ascertain if the length of strip from ground control to ground control agrees with that plotted on the projection or assembly sheet or, if not, the extent of the discrepancy. If, through drifting or other errors of flying, a selected direction line cannot be found that, when produced, will pass through the central views, advantage may be taken of the common vanishing point in the true horizon line of the photograph to establish on each succeeding photograph as required a parallel direction. Or since it is preferable to keep the direction line as near as possible to the principal line of the photograph, the former may be deflected at a point on it by noting where the direction line as deflected intersects the true horizon line of the photograph. By referring to Fig. 4, it can easily be shown that the distance along the horizon line from the principal line to the vanishing point for a horizontal line making an angle ϕ with the principal plane is equal to $f \sec \text{Dip} \tan \phi$ where Dip represents the angle of camera depression. The angle between the original and the deflected direction lines is thus readily obtained by referring each of them to the principal line. The direction line as deflected can still be used for the purpose of controlling the azimuth of the strip plot. When the plot of the central views has been adjusted throughout its length, the plots of the photographs, joining it at either side, are tied to it by the features shown in the area of overlap.

Mapping From Oblique Aerial Photographs

Photographing the Area to be Mapped.—Previous to going into the area which is to be photographed, the navigator, who accompanies the aeroplane

on its photographic flights, is furnished with the best existing map of that area, on which the various projected photographic flight lines have been laid down in their respective positions. For economic mapping the photographic lines of flight must be laid down in such a manner as to allow continuous photography during the operation of the plane.

In this connection, it is to be noted that while it is possible to obtain good photographs with the plane flying in a northerly direction, obviously a return flight with the plane facing the sun cannot give the best results photographically. On this account the photographic lines of flight are usually laid down in parallel lines running in an east and west direction.

For a flying altitude of 5,000 feet above the ground surface, these lines are spaced six miles apart. The areas when the lines of the Dominion lands system of survey have been laid down in whole or in part, can thus be made to readily fit in with base lines, with east and west township outlines, or with other subdivision lines, which may serve as ground control. In other areas, control must be otherwise provided. A very suitable condition exists when the flight lines commence and terminate at points controlled by ground survey and pass over intermediate controlled points at intervals of about 20 miles. However, this condition seldom obtains and recourse must often be had to other methods of providing the necessary control (see Fig. 8).

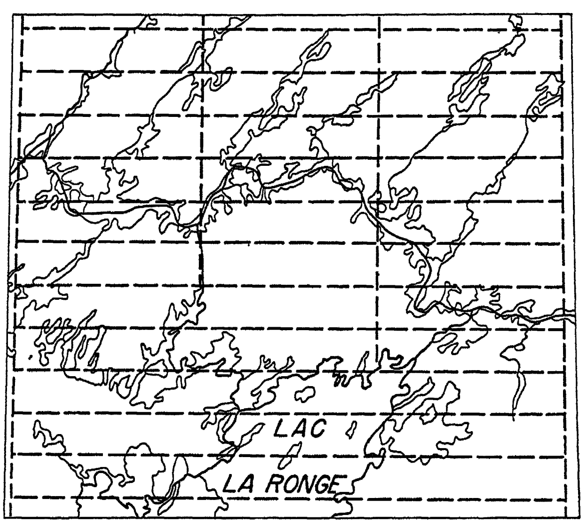


FIG 8. Projected flight lines to cover four-mile map sheet.

In this sheet, covering two degrees of longitude and one degree of latitude, in territory previously almost entirely unmapped, the heavy full lines indicate lines of control (monumented traverse surveys) and the broken lines the projected lines of flight for the taking of mapping oblique aerial photographs.

By experience in mapping to the four-mile scale from oblique photographs it has been found quite practicable on flight lines to "carry a good azimuth" for considerable distances between and beyond controlled points. Therefore, lacking ground control at any twenty-mile interval, if a northerly flight line—itself controlled by passing over two or more surveyed points—is flown to intersect the main east and west flight lines, the points of intersection may be made to serve as subsidiary control.

Along flight lines the oblique aerial views are taken in fan-shaped sets of three, the central photograph of the fan looking ahead along the line of flight and the two-side photographs being taken at angles of 45° to the line of flight, the overlap in each case amounting to about 25 per cent.

The sets are taken at intervals of about two miles so that the greater por-

tion of the photographic detail appearing on one set is duplicated in the next succeeding set.

Flying at an altitude of 5,000 feet with flight lines spaced about six miles apart this arrangement will allow the side pictures of adjacent flight lines to overlap at a distance of about two miles beyond the centres of the prints, thus allowing the whole area to be completely covered. The time of taking each set of pictures is recorded, as well as a reading upon a sensitive aneroid barometer, the latter giving the barometric elevation of the plane at the instant of exposure.

Since each photograph represents a different perspective and since the photographs vary according to the altitude of the plane and also according to the tilt of the camera, they must be treated separately. They are first reduced to their respective plans at a uniform scale, generally that of one mile to one inch, which is the scale of the projection sheet. This treatment is termed "plotting the photographs" and is accomplished by means of a perspective grid on a transparent base, described in the previous section of this article (see Figs. 1 and 2).

These separate plans or "plots," as they are called, must then be placed in their correct relationship on a projection sheet upon which the ground control has been plotted. This is the real problem which confronts the cartographer. Towards its solution uniformly successful methods have been adopted which may be briefly described as follows.

The various flight lines are first oriented and adjusted with the ground control. They are then correlated and adjusted with each other. The projection sheet is thus covered by a network made up of a series of east and west lines representing the principal flight lines crossed by lines representing ground surveys and northerly flight lines.

Then commencing at the flight line possessing the strongest ground control throughout the greatest portion of its length, the individual pictures are correlated, oriented and adjusted to each other. The adjacent flight lines are dealt with in like manner until the whole map has been built up.

Azimuth Lines.—In order to orient the flight lines between the control points a simple method of alignment is adopted known as an "Azimuth Line" method. Its essential feature consists merely of a straight line drawn on the central pictures of the various sets in the flight line, in the direction of the line of flight and passing from one control to another, thereby governing the flight in azimuth.

Generally speaking, in the type of country to which mapping by oblique aerial photography is adapted, the maximum change in ground relief is not over 200 feet. This marks the change in elevation from water areas to the hilltops. The elevation at which the water areas themselves lie is more or less constant. Hence the intersections of a line drawn on the photograph with the boundaries of these water areas may be considered to lie all in the same plane on the ground. An exception to this assumption would occur when there are rapids or falls indicating a rugged country with various water areas lying at entirely different elevations.

Any straight line on the ground will appear as a straight line on the photograph when the ground is level. Over sloping ground, however, the only straight lines which will appear straight on a photograph will be those which happen to pass through the ground plumb point.

The photo plumb point for the regular oblique mapping photograph falls beyond the photo limits. The straight line which can most readily be drawn to pass through it is the principal line of the photograph, that is, the line passing

through the principal point or centre of the photograph and perpendicular to the apparent horizon. For this reason, and in order that the azimuth line may not deviate too far from the central portion of the field of view and disappear to one side when produced through successive photographs, it is necessary to keep it as near as possible to the principal line of the photographs.

Drawing an Azimuth Line.—To draw an azimuth line on the central set of pictures of a flight it is best first of all to lay them out in the order in which they were taken so as to readily determine the tendency of the plane to drift or waver. Drift is the inclination of the plane to depart from the flight line projected on the map due to wind; it generally results in a curved line. Waver is the tendency of the plane to depart from the flight line in a more or less zigzag manner due generally to the pilot being unable to pick up a sufficient number of prominent ground features en route for orienting his flight.

If the water features in the extreme background of one picture disappear off at the left-hand side of some of the later photographs then the plane is swinging to the right. A projected azimuth line drawn to these features would also pass off the pictures at the left-hand side, so that, in order to lay down an azimuth line throughout the entire series, it must be commenced to the right of the principal line. Similarly, if the plane is seen to be swinging to the left, the azimuth line must be commenced on the left-hand side of the principal line. A brief inspection will reveal the extent of deviation, if any, and in practice it has been found better on long flights to start drawing the azimuth line on the pictures covering the middle portion of the flight.

Having made this preliminary study, the azimuth line is then drawn. On the first photograph selected two well-defined objects are connected up in laying it out. These may consist each of the point of an island, a point of land on the shore of some lake, or some other equally prominent point, one being in the extreme foreground and the other in the extreme background of the picture. As succeeding pictures are treated, the latter point will, of course, approach to the foreground of the photographs. It should be noted that in the type of country usually mapped by this method the water features are the most prominent general features recognizable.

Several well-located points on this line, such as for instance those on lake shores or in marshes, which can be readily identified on the adjoining central picture are now chosen and their locations are carefully pin-pricked upon it. A straight line is drawn through these points and on the succeeding picture the procedure is repeated, other prominent points being selected when required. In this way the common line is carried through all the central pictures of the flight until it is produced from one control to another.

Generally speaking, when the pictures are taken at an elevation of 5,000 feet if the pilot has maintained a reasonably straight course, a line may thus be successfully carried throughout the entire flight without departing more than three-quarters of a mile from the principal line. When they are taken at 4,000 feet this amount would be about one-half mile.

When there is a greater deviation, a new line should be tried out, preferably one parallel to the original azimuth line. This latter is chosen in preference to another to facilitate in later adjustments which may have to be made along these lines. Since parallel lines meet at a point on the true horizon and as the position of the true horizon is given on the plotting grid, this can be transferred to the picture, and the new line then established parallel to the old azimuth line by drawing it to the point of intersection of the original azimuth line with the true horizon.

Scale Control Points—Having completed the azimuth line, or series of lines as the case may be, we must now decide upon the scale of the pictures. In other words, we must determine upon the proper grids which are to be used in plotting the pictures.

To do this choose two points on each central picture, one of which is common to the succeeding picture and the other common to the preceding picture. If, by means in each case of the proper grid, these points were plotted, as well as the portion of the azimuth line nearby, and if their separation distances in the direction of the azimuth line or as projected thereon, were scaled off, the sum total of these distances will give a result representing the length of the flight from control to control. It should agree with the true distance between controls as shown on the projection sheet.

In practice, in choosing these two points in each picture, one is selected in the extreme foreground which will appear at about the middle of the preceding picture and the other far enough ahead to appear in the foreground of the following picture. They should, whenever possible, be at water level at points where the view of the shore line is unobstructed and easily identified on the neighboring pictures. Where it is not possible to have them at water level they may be upon marshes or low-lying land—never upon hilltops or undulating country. The reason for this is to keep at a minimum any possible errors which might occur due to changes of elevation. They should be placed on the outlines of water features as far as possible from the azimuth line, since the distance from each of these points to the azimuth line is used to calculate the altitudes at which the pictures are taken, thus checking the barometer readings.

On the initial and final pictures these points should coincide with points established by the ground control systems. If no barometer readings have been furnished, two sets of scale control points on opposite sides of the azimuth line should be made use of on each picture and as, by these alone, the scale must be carried from control to control, they must be as far apart as possible in order to detect by means of a grid any change in altitude of 50 feet or more. (Flying at 5,000 feet, a change of 50 feet in altitude between pictures causes a change of 0.8 of a chain in two points spaced 80 chains apart or 1/100 of an inch at the scale of the projection.) The points should also be placed as low down in the foreground of the picture as possible in order to take advantage of the larger scale of the grid there. It should be here stated that the accuracy and entire progress of compiling a map by the method described is seriously affected by the lack of barometer readings. The latter actually should be considered essential to the work.

Determining the Grid Elevations for the Initial Photograph.—In determining the altitudes at which the photographs were taken, recourse is therefore had, where at all possible, to the barometer readings recorded as each set of pictures is taken. These readings are put down in a form as shown in Fig. 9. The time and barometer readings are noted when the plane leaves the base, when each set of pictures is exposed throughout the flight, and again when the plane returns to the base.

The temperature is also recorded upon leaving the base, upon returning thereto, and several times throughout the flight, so that when conditions warrant, a correction for it may be applied. This correction may be taken as two feet per thousand feet of elevation for each degree Fahrenheit by which the average air column temperature differs from fifty degrees Fahrenheit. It is to be added to the height obtained for temperatures above fifty degrees Fahrenheit and subtracted when the temperature is below fifty degrees Fahrenheit.

AERIAL PHOTOGRAPHY

Operation Order TS 95 Flight No Aeroplane V B Date June 26 1929

Leave Thicket at 10 15 Aneroid Bar TS, 1382 Reading 1600 Time 10 15 Temp 66°

Arrive Thicket at 14 15 Aneroid Bar T.S. 1382 Reading 1760 Time 14.15 Temp 62°

Total length of flight 240 miles Photographic mileage 140

FILM #A. I. M.51.

Camera K.3 14 Cone K3 14 8 Filter Aero I. Exposure 1/50

Location From South end, Northward up line 18 to Birch Tree Lake

Weather Very bright, horizon clear, a few cloud shadows toward end of roll very bumpy

Time	Aneroid	Altimeter	Pict Sets	Temp.	Calc Mean Elev	Remarks	Time	Aneroid	Altimeter	Pict Sets	Temp.	Calc Mean Elev	Remarks
111800	6530		1	40°	4850	Set of 3	114630	6700		61		5010	Sets of 3
111930	6840	5300	4		5150		114800	6300	4700	64		4610	
112100	6640		7		4950	1 extra East	114930	6190		67		4500	
112230	6730		11		5050	of baseline	115100	6010	4500	70		4310	
112400	6540	5000	14		4850		115230	6120		74		4420	Sets of 4 1 extra left
112530	6460		17		4770		115400	6700	5100	78		5000	
112700	6770		20		5080		115530	6520		82		4820	
112830	6570		23		4900		115700	6360		86		4660	
113000	6340		26		4650		115830	6220		90		4520	
113130	6340	4800	29		4650		120000	6150		94		4450	
113300	6400		32		4710		120130	6000	4500	98		4300	
113430	6950	5400	35		5260		120300	6120		102	40°	4420	
113600	6870		38		5160								
113730	6860	5400	41		5150	1 extra R							
113900	6940		45		5250	1 extra East							
114030	6940	5500	49		5250	on baseline							
114200	7060	5600	52		5370								
114330	6640	5100	55		4950								
114500	6720		58		5030								

FIG. 9

Example: Average air column temperature = 35°F.
Height of plane from aneroid readings = 5,000 feet.

$$\text{Correction} = (50-35)\times 2\times\frac{5{,}000}{1{,}000} = 150 \text{ feet.}$$

Hence true height of plane = 4,850 feet.

The difference between the barometer reading at the base and that of the first set taken above the ground control, properly corrected for temperature

when necessary, gives the elevation of the first set of photographs above the base. Correcting this for any difference in ground elevation between the base and the ground control the altitude of the plane above the ground control is obtained. This furnishes the approximate elevation of the first set of photographs and must be checked by applying to the photograph containing the ground control a grid of that elevation.

How to Use the Grid.—(1) As before noted, the principal point is usually etched as a cross on the glass plate in the focal register of the camera lens, and is thus recorded on each negative and on prints therefrom. However, if it is not so marked, locate it by joining the opposite collimating marks.

(2) Measure the marginal distance, that is, the distance from the apparent horizon to the marginal edge of the photograph along the line passing through the centre of the picture and perpendicular to the apparent horizon.

(3) Note the serial number of the lens used during the operation and ascertain its focal length from the camera records.

(4) Ascertain from the barometer records the approximate altitude at which the picture was taken.

(5) Having selected a grid which will most nearly conform to the above requirements superimpose it on the photograph so that the principal line of the grid passes through the principal point of the photograph and the apparent horizon on the grid lies along the apparent horizon on the photograph.

The apparent horizon line on the grid is a straight line and the apparent horizon on the photograph is slightly curved. When the grid is correctly placed on the photograph, the horizon line on the grid should coincide with the horizon line of the picture at the intersection of the principal line with the apparent horizon and will be slightly above the horizon line on the photograph at the edges.

(6) Now upon the squared plotting paper draw in the required information square for square as shown by the intersections of the grid lines. The plotting paper used is thin transparent paper on which are printed large squares whose linear dimensions measure one mile at the scale at which the map is being compiled. These large squares are subdivided into smaller squares representing ten chains on the map and corresponding to the small subdivisions on the grid

Obtaining the Grid Elevations of the Sets.—The initial picture is now taken containing the two "scale control points," that is, the two points which have been identified in the ground control system and whose distance apart on the ground is therefore known. The grid is now selected which most nearly conforms to this picture in focal length of lens, marginal distance, and altitude as approximately obtained. This is superimposed on the picture and a graphical determination of the grid distance between the two scale control points on the photograph is made by transferring them to the plotting paper and scaling the distance between

If this distance does not agree with the actual distance as measured on the ground, then the wrong grid elevation has been used.

The correct grid elevation is then obtained by the formula:

$$\frac{\text{Correct grid elevation}}{\text{Grid elevation used}} = \frac{\text{Correct ground distance}}{\text{Distance as obtained by measuring on grid}}.$$

Having thus obtained the elevation above the ground control at which the picture was taken this elevation is now compared with the barometric reading recorded for this photograph and a correction factor thus obtained to apply to

barometric readings of the succeeding sets. Going now to the picture taken over the next ground control, in like manner is obtained the elevation above that ground control at which the picture was taken and, by a comparison with the recorded barometer reading for that set, a second correction factor.

These two correction factors generally agree very well. When, however, a slight difference occurs due to a change in atmospheric pressure or ground elevation, it may be distributed proportionately among the intervening sets unless for some specific reason a different treatment is indicated. The latter case might occur for instance when the first few sets are taken over a lake and the remainder over a stretch of country higher than this lake. The first correction factor would then naturally apply to those pictures covering the lake and the remainder could be corrected proportionally.

We thus obtain a corrected elevation for every set of pictures. These elevations should be marked on the centre picture of each set for the guidance of the plotter. They will serve for the initial plotting of the azimuth line and scale control points; and when a uniform altitude has been maintained by the photographic plane the resulting scale will be such that very little extra adjustment is required.

However, the reading of the barometer readings is done under rather severe conditions so that errors in recording may occur, even to the inadvertent skipping of a set; again the plane may change in altitude so rapidly that the barometer lags. Thus, for several reasons, it has been found necessary to check the elevations derived from the barometer readings by means of the scale control points.

Plotting the Azimuth Line and Scale Control Points.—With the elevations for each set thus determined, the azimuth line and scale control points are now plotted from each central photograph on sheets of plotting paper, one sheet being used for each photograph. They are indexed with the roll number of the flight and the individual number of the respective print.

This procedure is carried on throughout the series of pictures, making such changes in grid elevations as are necessary to have the plots agree in scale between the scale control points common to each picture and the azimuth line. This finally takes the form of a sort of trial line from one ground control to another.

When there are no barometer readings, the elevations from each picture must be calculated by measuring the distance between the two scale control points on opposite sides of the azimuth line on each picture and comparing it with the distance between the same points on the adjacent picture. In actual practice it will be found better to work from both ground controls to the middle of the strip rather than to carry the elevations throughout the entire series of pictures. In this way the elevations are carried the shortest possible distance from that one established directly from the ground control, thus keeping at a minimum errors from changes in ground elevation.

Assembling the Centre Plots.—To assemble the plots, a strip of transparent paper of sufficient length to stretch from one ground control to the next is now taken and a straight line is drawn down its length corresponding to the initial azimuth line. The first plot is now placed beneath and oriented so that the azimuth line on the plot is coincident with the line on the strip of tracing paper. The ground control and scale control points are now traced through and lettered as in the plot.

The next successive plot is now placed beneath the strip, oriented as to

azimuth line and adjusted so that the corresponding scale control points coincide. The scale control points on this plot are then traced and lettered.

This procedure is followed from plot to plot until the plot containing the next ground control is reached.

We now have a trial line extending from one ground control to another which has to be made to fit between the corresponding ground controls on the projection.

Adjusting Points.—This strip of tracing paper is now placed upon the projection sheet and oriented so that the azimuth line passes through corresponding points on the ground control systems.

If, when superimposing the initial ground control points on the tracing over the corresponding ground control points on the projection sheet, it is found that the final ground control at the other end of the tracing does not coincide with that on the projection sheet, then the elevations of the grids used in plotting the pictures must be increased or diminished according as the plotted distance between the controls is smaller or greater than the true distance.

The adjustment of this lack of correspondence must depend on the nature of the ground elevations. If the intervening country is assumed to have a constant elevation, then it may be distributed uniformly by a uniform change in the successive grid elevations. But if, for any reason, a more pronounced change in ground elevation is assumed in any particular portion of the flight then, of course, the plots covering this particular portion will be more greatly affected and the grid elevations should be changed accordingly. If the error is less than one per cent the points can be moved proportionally without changing the grid elevations.

Having decided upon the probable source of the lack of agreement the photographs involved are then replotted and the strip made to fit between the ground controls.

The strip is now laid down on the projection sheet with the plotted ground control positions superimposed on the corresponding ground control positions thereon. The azimuth line and the scale control points are then transferred by means of carbon paper, the points being lettered as on the photograph for identification purposes.

In practice, it is better to plot and transfer the azimuth lines of each flight first of all before transferring any of the topography. With all the flight lines laid down in this way, it is at once seen how successful the pilot has been in adhering to the six-mile interval between flights. In some cases the distance between the flight lines will be found much greater than six miles. In this event the side pictures will have to be plotted farther into the background. Furthermore, with all the flight lines laid down in this manner it is possible to check one with another by plotting some feature common to both, further adjustment at these points being made when necessary. If this is done at this stage considerable trouble in fitting topography later on may be avoided.

Preparing the Photo-prints for Plotting.—In preparing the photo-prints for plotting, it is well to remember that the foreground of the picture more nearly resembles a vertical view and, of course, gives the more accurate representation of ground conditions.

Examination and intense study of photographs is most trying upon the eyesight, the effect being even more pronounced when such examination is carried on underneath a glass grid. To reduce the eye strain and to insure against unnecessary duplication in the actual plotting, the information required to be plotted from the photographs is first of all traced over on the prints with

chinese white or aero white ink. This offers a good contrast to the grey and black background of the photograph; it is quite easily put on with a fine pen and quite as easily erased, by merely moistening the finger and rubbing it out when, for any reason, it is desired to remove it.

It is found in practice much easier to work from the end of the flight line towards the beginning since one may always have before him the previously prepared set, indicating at a glance how far into the background it is necessary to study and outline the topography on the succeeding photographs. In this manner a great deal of duplication in inking in the pictures and in subsequent plotting is avoided and the full value is obtained from the foreground of each set.

When outlining the information to be plotted it is important to remember that the photograph has been taken at an oblique angle and consequently near sides of features such as lakes and rivers may be partially obscured by the banks, trees, etc. This must be allowed for and the line drawn back sufficiently to give a correct representation of the true positions of such features.

Plotting the Photographs.—The photographs are now plotted, usually employing for the side pictures grids constructed for the same elevations as were used for the centre one of the corresponding set. In order to tie the plots of the side pictures to those of the corresponding centre pictures, two common points are chosen on each picture known as "tie points."

As the positions of the side plots depend on these points great care should be taken in their selection. They should, whenever possible, be at water level and sufficiently far apart to insure against errors creeping in, due to improper orientation

The plots of the central pictures are now superimposed on the projection sheet along the azimuth line, they are fitted to the adjusted positions of the scale control points, and the plotted information transferred by means of transfer paper. The plots of the side pictures are then superimposed and adjusted so that the tie points common to both centre and side pictures coincide, and the plotted information upon them also transferred. The principal point and the direction of the principal line are also transferred and the number and roll of the picture marked thereon, thus forming a complete index of the flight.

Occasions may arise when it is necessary to plot features having different known ground elevations appearing on the same picture. Such a case would occur, for instance, where a small lake or lakes would lie in a plateau bordering upon a large water area. The difference in the water elevations in such cases might amount to as much as 100 feet or more.

To plot the topography here two different grids are used for the two different ground elevations. In each case the ground plumb point and principal line are carefully marked on the plot. The ground plumb point is constant for the two plots and by superimposing one plot on the other with the ground plumb points and principal lines coincident the true relationship between the features is obtained.

If the elevation of the larger water area is known and that of the smaller lakes upon the plateau is not known, the true positions of the latter can always be obtained by drawing on the plots radial lines to the plotted positions of points on their shore-lines from the ground plumb points of different pictures. The intersections of such lines as derived from successive plots will give the true position of any points on the shore-lines and thus locate the features.

Summary of Oblique Plotting Method.—Summarizing the above method, first of all draw the azimuth lines on the central prints of the flight and mark

the control points necessary to carry on the picture traverse. Then plot the azimuth lines and control points. Fit the strip and adjust between the ground control surveys. Transfer the azimuth lines and control points to the projection sheet and record the corrected elevations at which the various sets are to be plotted. Whiten in the detail which is required from the photographs and the side or tie points to connect the side pictures with the central ones of the corresponding set. Then plot the pictures and transfer the plotted information to the projection sheet.

After all the pictures involved in the sheet are thus dealt with, they are all carefully gone over and inspected for omissions which sometimes occur, chiefly in that portion of the map which lies between flights in the background of the side pictures. One of the chief difficulties in oblique mapping is in identifying creeks, and all drainage should be carefully checked up at this stage.

In order to achieve the best results, the horizons must be clearly defined, and the pictures should be clear and free from cloud shadows. The flight lines should be spaced as correctly as possible, thus avoiding the necessity of plotting far into the background of the side pictures. As the difficulty in plotting the scale control points is increased with the distances between sets, these should not exceed two miles. The ground control should also be readily recognizable on the photographs.

With these conditions maintained it is possible to produce, at much less cost, a map which will for the scales used compare very favorably with one compiled from vertical photographs.

OBLIQUE PHOTOGRAPHS FOR THE SURVEYOR

R. M. Wilson

THE principles described in this paper will interest the surveyor or engineer who wishes to make practical use of the wealth of information displayed in oblique photographs. Whether the camera used be one of the precision instruments of formal photogrammetry, or an ordinary inexpensive hand camera, the principles are the same in application. The purpose may be to extend reconnaissance mapping, to develop auxiliary control for detailed topographic mapping, or to make a close-up dimensional study of details in a construction project from "photographic memoranda." In practice the graphical method requires only the ordinary drafting equipment already available to any engineer.

Oblique photographs, as referred to here, are those taken from either air or ground stations with the camera pointed up or down at any angle of elevation or depression preferably not greater than about 45° from the horizon. Such photographs are too often regarded merely as pretty pictures of the landscape, interesting only because they present a general view in the kind of perspective most familiar to the eye. But the information they contain may be interpreted accurately and with but little effort in terms of true horizontal and vertical angles, useful in determining distances and differences of elevation.

In previous articles[1,2] the writer has described the photoalidade, which provides an adjustable holder for the photograph and a telescopic sighting device similar to that of a transit. When the photograph has been properly placed with respect to the axes of this instrument, sights may be taken readily to details shown upon it just as they would be taken to actual features in the real landscape. But how many surveyors can have one of these instruments conveniently at hand? The need for a fully graphical method was pointed out by Col. Thomas North, and he has published[3] a description of a simple procedure that should find wide application. His method uses angles transferred from the plane of the photograph through the isocenter to the map plane. These angles are useful for rapid radial-line compilations, although they are measured at a point that is not near the nadir. The writer has been tempted to use the first part of the method described by Colonel North, but to develop it according to the somewhat different principles involved in operating the photoalidade, employing true horizontal angles measured at the nadir. The result contains ideas that are closely related to those already developed[4] by Capt. D. R. Crone, of the Survey of India, in a solution of the problem which has been referred to as the "Indian Method." However, the following procedure seems so simple and direct as to deserve presentation in its own right, and it is believed that an entirely new technique is used in adjusting the observations to obtain the most probable results.

THE PROBLEM

The object is to determine the true horizontal angles at the camera station as subtended between points or features on the ground that are shown on the

[1] Wilson, R. M., A new Photoalidade: *The Military Engineer*, November–December 1937.

[2] Wilson, R. M., Oblique Photographs and the Photoalidade: PHOTOGRAMMETRIC ENGINEERING, issue of April–May–June, 1938

[3] North, Col. Thomas, Locating Points from Oblique Airphotos—. *Field Artillery Journal*, September–October 1940

[4] Jenney, Capt. R.C N., R. E., The Indian Method of Compiling High Oblique Air Photographs and its Routine Use in Determining Spot Heights for Contouring Vertical Air Photographs: *Photogrammetria*, 1939, Vol. II No. 3.

photographs and, also, to determine the true vertical angles of depression or elevation to these points from the camera station. These are the angles that might be measured with a transit—on the limb and on the vertical arc—if the station could be occupied with a transit instead of a camera.

Just as in using a transit, it is necessary to know or to find the position of the station from which the observations are taken, and to orient the observations with respect to known directions before they can be used to determine new directions, positions, or elevations. The simple special case, when the camera has been leveled and its axis pointed horizontally in a known direction from a known position, is described in many text books on surveying. The representation of the true horizon line then passes through the center of the photograph and through the fiducial marks at the middle of the right and left edges of the photograph if the camera is equipped and adjusted to show these correctly.

Generally, however, oblique photographs are not taken with a leveled camera. Therefore the true horizon line may be neither through the center of the photograph, nor parallel to its edges or its geometric axes. At a ground station, sights might be taken with a leveling instrument to find points in the field of view which lie in the plane of the true horizon; these points, identified on the photograph, would provide the means to draw the horizon line there. But auxiliary observations of this kind cannot be made from camera stations in the air.

The general problem, therefore, involves finding the horizon line on the photograph, and then determining the position, elevation and orientation of the camera, entirely through information that appears in the photograph itself. Usually the solution is possible if at least three ground points of known position and elevation can be identified in the photograph. These control points on the ground compose the base upon which the solution rests, just as a surveyor's tripod rests firmly upon the sharp points of its three legs. Obviously they should form a triangle as large and wide as is feasible within the field of view, in order to attain maximum stability in the resulting solution. If the three points are in line the solution will fail, just as the tripod would not stand alone with the points of its three legs set all in line.

Focal Length

When taking the photograph the rear nodal point of the camera lens is the station point of perspective for the view as projected upon the sensitized film or plate held in the focal plane. If the camera is focussed at infinity, the length of the perpendicular dropped from the nodal point to the focal plane is the focal length of the lens It is assumed that this perpendicular coincides with the axis of the lens. The foot of the perpendicular determines the principal point of the photograph. But in the present problem, when using a paper print, it is necessary to know the effective distance, f', from the plane of that print to its own perspective station point. This is not likely to be exactly the same as the focal length of the lens, unless the photograph is a contact print, made without expansion or contraction in the paper or the negative, and from an exposure made with the camera focussed at infinity. A finished print may be compared with its negative and, if it has changed in size or scale, a proportionate correction may be applied to the known focal length of the lens to obtain the distance required. Prints made glossy by "squeegee" drying or by "tinning" are often so badly distorted as to be useless for the purposes described here. If the lens was drawn out a little to focus on near objects, the amount of that movement should be added as a further correction. Although such corrections suffice for ordinary purposes, a more formal means of calibrating the camera, prints, or diapositives should be employed if the extreme precision of this method is to be attained.

General Relationships

Referring to the perspective diagram, Figure 1, suppose that the photograph is held with its perspective station coinciding with the camera station at C and is also adjusted to the particular angles of tilt, swing and azimuth that represent the circumstances at the instant of exposure. The objects G_1, G_2, G_3, shown as the corner of a building, a hilltop triangulation station, and a street intersection, respectively, are the three ground control points required for the solution; it is assumed that accurate position and elevation are known for each of these points. From the corners near G_1 and G_3 the photographed area spreads wider and wider with distance, and in this example its length stretches out to the horizon.

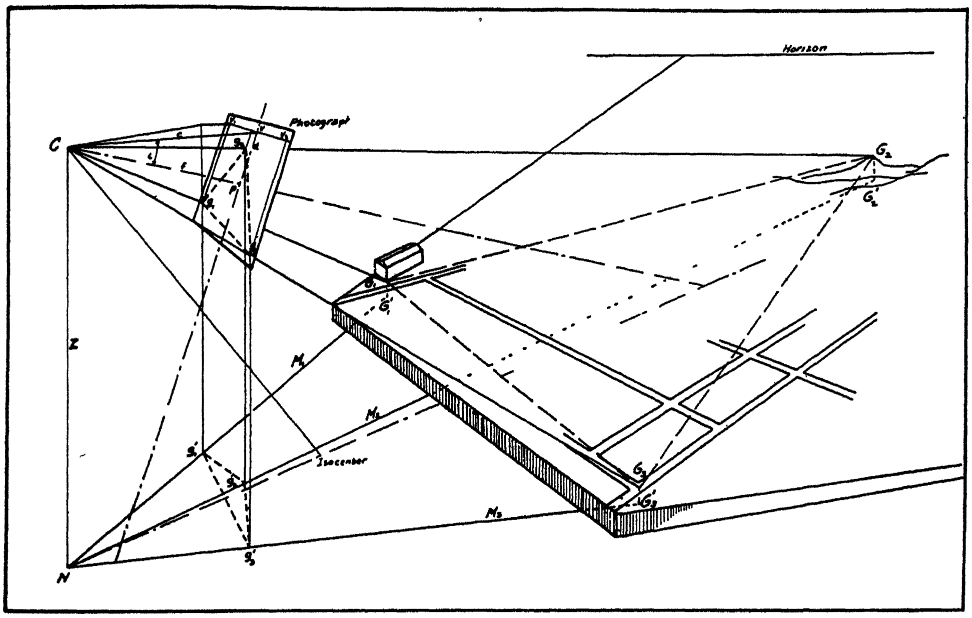


Fig. 1

The plumb line dropped down from the camera station reaches the horizontal datum plane at the nadir, N, and the distance, Z, is the altitude of the camera.

The rays from C to G_1, G_2, G_3 will pass through g_1, g_2, g_3 exactly where the images of those objects appear on the photograph. The line v_1v_3 is the trace of the horizontal plane through C upon the plane of the photograph; it is the line to be determined on the photograph that will represent the true horizon of the camera station.

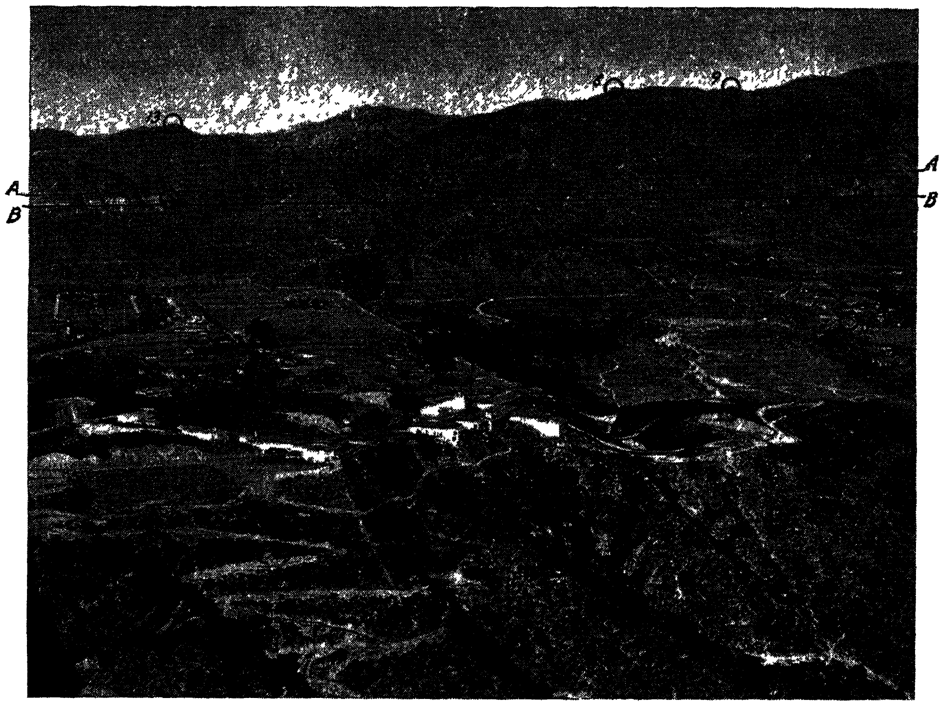
Part of the problem is to measure the horizontal angles between directions observed from C to features in the field of view. Plumb lines dropped to the datum plane from the ground features G_1, G_2, G_3, and others, would determine the projections of those points at G_1', G_2', G_3', etc. Then the lines from N through the projected points indicate the true directions in the datum plane, and angles between them are equal to the corresponding horizontal angles that are to be measured, by means of the photograph, at C.

Similarly, the points g_1, g_2, g_3 and others projected down from the sloping photograph to the datum plane at g_1', g_2', g_3' fall, respectively, upon the hori-

zontal lines of direction already referred to that radiate from N to G_1', G_2', G_3', etc., and which are the projections of the sight rays from C. These same points might be considered also as projected up to the horizontal plane containing C. It is apparent, therefore, that the horizontal angles sought can be found by projecting upon a horizontal plane the information contained in the photograph while the photograph is held in its natural sloping position. The horizon line v_1v_3 is the record on the photograph defining the swing and tilt of that position.

Preparation for Graphical Construction

The first step in the procedure of interpretation is to find the horizon line on the photograph. This may be done correctly and at once in the special cases



Fairchild Aerial Surveys, Inc., L. A.

Fig. 2

already mentioned, but in general it can be drawn only tentatively at first, by estimation. The apparent horizon often appears in the photograph and is an excellent guide to the estimation. The true horizon is approximately $58.82''\sqrt{Z}$ above the sea horizon in seconds of vertical angle;[5,6] or, in inches on the photograph, $(f'\sqrt{Z} \div 3507 \cos^2 i)$ in which Z is elevation of camera in feet, f' is in inches, and i is the angle of inclination of the camera axis as determined below. Subsequent calculations will indicate the amount of error in the estimation, and provide the corrections to determine the true horizon line. But in order to demon-

[5] Canada Topographical Survey, A Graphical Method of Plotting Oblique Aerial Photographs: *Bulletin of the Topographical Survey* 1928, Department of the Interior, Dominion of Canada.

[6] Canada Topographical Survey. The use of Aerial Photographs for Mapping: *Topographical Survey Bulletin* No. 62, 1932, Department of the Interior, Dominion of Canada.

strate the geometry of the problem it will be assumed that the true horizon line is already drawn correctly instead of only tentatively as shown by A–A, Figure 2. Carefully measure the perpendicular distance from p to v, which will be called d, and divide it by f'; the result is tan i. If d is down from the horizon it is negative and then tan i, and i, are also negative. Find i and cos i, to determine the hypotenuse, from C to v, which will be called c, in the triangle Cvp, Figure 1.

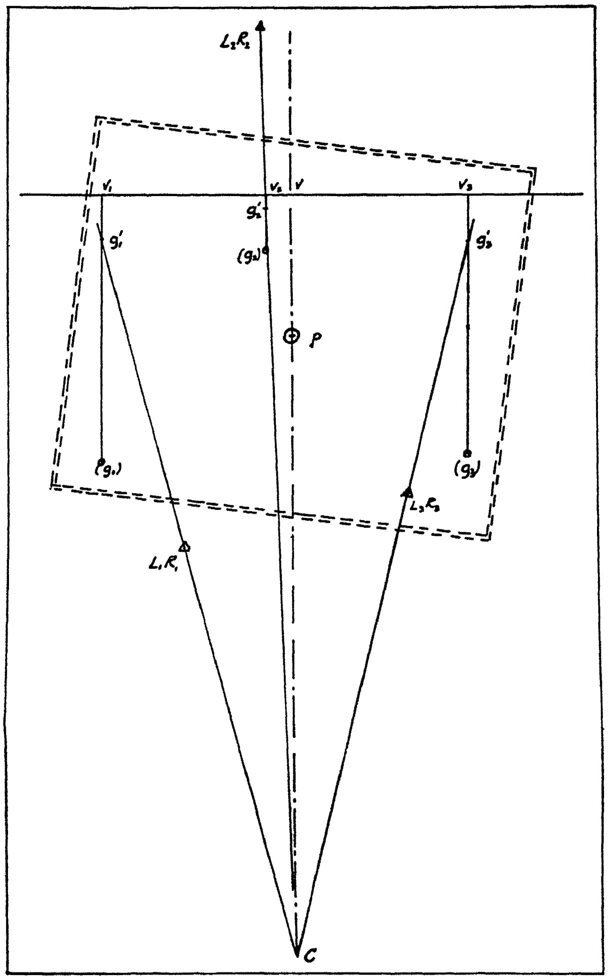


FIG. 3

The same units, generally either inches or centimeters, should be used for all these lengths.

Now lay a piece of tracing paper over the photograph, as shown in Figure 3, making it large enough to provide, later, for plotting the distance c below the horizon line. Also it may be found desirable for later operations to have the tracing paper extend far enough above the horizon line to accommodate distances from C to points of interest in the field of view as reduced to the scale of the map or work sheet upon which the compiling of information is to be done.

The tracing paper should be fastened over the photograph in a way to prevent relative movement, although provision may be made for lifting it at one side for clear inspection of the photograph.

Horizontal Angles

With these arrangements made, trace the line v_1v_3 and construct the perpendicular to it that passes through the traced center of the photograph, (p). Extend this perpendicular, as indicated, to C at the computed distance c from the horizon line (Fig. 3). The points on the photograph, representing features

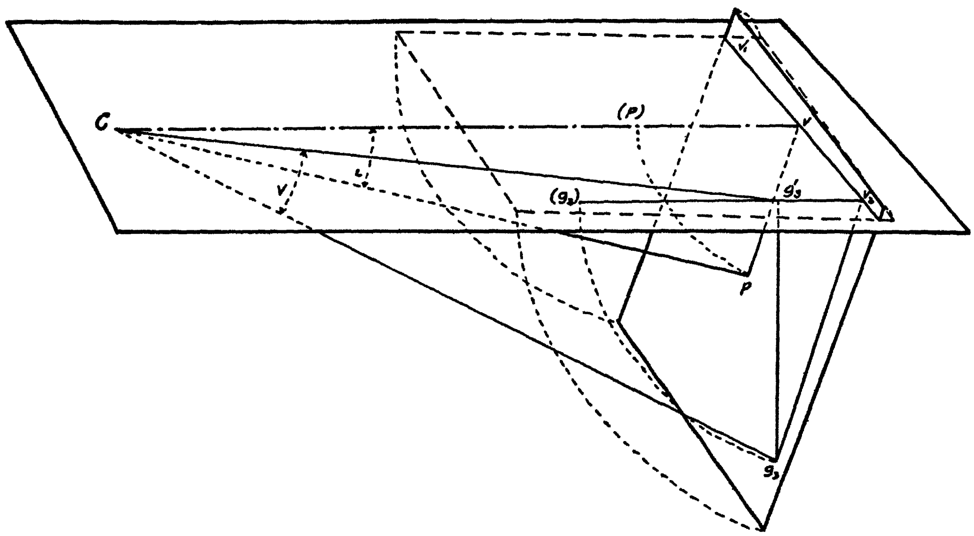


Fig. 4

upon which observations are desired, are traced through on the paper as (g_1), (g_2), (g_3), etc. From each of these points draw a perpendicular to the horizon line. Set a pair of proportional dividers (or use a scale and a slide rule) to the ratio d:c and plot g_1', g_2', g_3', etc., so that each of the perpendiculars will be divided into parts in the same ratio as c is divided by point (p). Now draw the lines of direction from C through g_1', g_2', g_3', etc., which define the required horizontal angles. The tracing paper may be used to transfer these angles to the map or compilation sheet.

The geometric proof of this procedure may be seen in Figure 1, or perhaps more clearly in the locally enlarged view, Figure 4. The horizontal plane through C, and the natural plane of the photograph, are shown intersecting in the horizon line v_1v_3. The photograph as actually used, however, lies on the drafting table with a sheet of tracing paper fastened over it, which is to be considered as a horizontal plane. Its center at (p), and points upon it such as (g_1), (g_2), (g_3) are marked through upon the tracing paper. Now it may be imagined that the photograph is rotated downward about the horizon line v_1v_3 as axis, to its natural position, swinging its center and other points on it down (or up) along the indicated arcs. Then point g_3' is the point on the tracing paper directly above the natural position of g_3. Now the triangle $g_3'v_3g_3$ lies in a plane that is parallel to the principal vertical plane which contains the triangle Cvp. Both are right triangles, and the angle at v in one is equal to the angle at v_3 in the other; therefore

the triangles are similar. The hypotenuse of each of these triangles is represented individually on the tracing paper by a line perpendicular to the line v_1v_3. The dividers were set to the ratio $d:c$, because the distance from v to $(p)=d$. Therefore:

$$\begin{aligned} d:c &= \text{short leg}:\text{hypothenuse} && \text{(in triangle } Cvp\text{)} \\ &= v_3g_3':v_3g_3 && \text{(in similar triangle } g_3'v_3g_3\text{)} \\ &= v_3g_3':v_3(g_3) \end{aligned}$$

Thus the distance v_3g_3' can readily be plotted to determine g_3' which is the projection of g_3 upon the horizontal plane. Any points so determined upon the tracing paper are in fact parts of the orthometric projection of the slanting photograph upon a horizontal plane. Therefore the angles between them, about C as a center, are the true horizontal angles sought. It will be convenient to use the letter y for the distance v_3g_3 measured on the photograph, making its value negative when measured down from the horizon line

Vertical Angles

To measure the vertical angle at C, a somewhat similar proportional method might be employed to determine graphically the projection of v_3g_3, or y, into a vertical plane. It is simpler, and more precise, however, to scale y directly from the photograph and multiply it by cos i to obtain the vertical distance g_3g_3'. Then scale the horizontal distance Cg_3' from the tracing paper, and designate it by the letter m, to use with g_3g_3' in finding the tangent of the vertical angle, V, thus:

$$\tan V = \frac{y \cos i}{m} \cdot$$

The use of this formula and others to follow is illustrated later in a numerical example Both y and m must be measured in the same units, and both might be measured from the tracing paper. But to avoid the inaccuracies of tracing, it is best to measure y directly from the photograph where the image can be seen clearly; accuracy is required in this measurement because it is the critical one in determining the difference of elevation. The angle V, from C to g_3, is the vertical angle to the ground point G_3 because the sight ray through g_3 continues through to G_3. Generally it will not be necessary to look up the angle itself, because only tan V, already available, is needed in solving for the difference of elevation. Tan V (and V) should be negative in sign if V is down from the horizon.

The Photoalidade

The operations just described show how the horizontal and vertical angles can be measured graphically from an oblique photograph. The photoalidade (Fig. 5), introduced in 1937, is an instrument designed to measure the horizontal and vertical angles directly, without computation or graphical construction. Results are obtained more rapidly with the photoalidade than is possible either graphically or by the computing method to be described later in this paper. The results obtained with the photoalidade are more nearly accurate than results obtained by the graphical method

The well-known and much used plane table, with a telescopic alidade, is a device for plotting lines of direction to sighted objects directly upon a map or work sheet, and for measuring vertical angles to those objects. The photoalidade serves the same purpose, the principal difference being that the plane table is used on ground stations in the field, sighting to objects or features in the actual

landscape, whereas the photoalidade is designed for use in the office to sight features shown in photographs of the landscape. Each instrument has a telescope with cross wires through which the selected features may be sighted accurately, with an arc and vernier for measuring vertical angles to them, and each has also a straightedge to guide the drawing of lines of direction corresponding to the pointings of its telescope.

The photoalidade may be adjusted to use oblique photographs taken with the camera axis at any angle of inclination from 10° above to 50° below the horizon. Contact prints may be used if the photographs were made with a camera having a lens with a focal length of more than 4 inches and less than 14

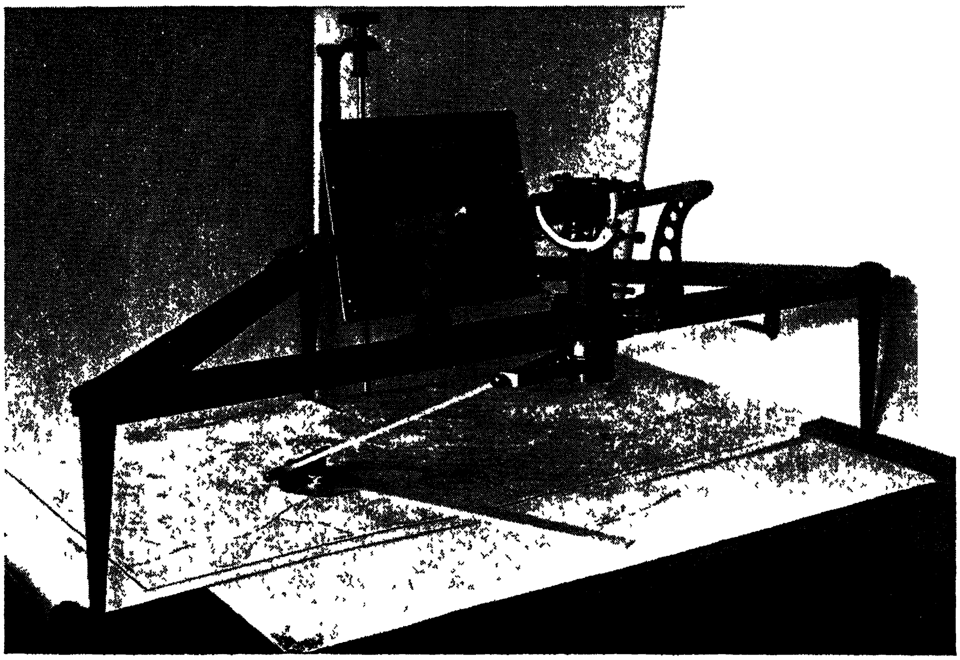


FIG. 5. Geological Survey Photoalidade

inches. Photographs that show the horizon near and about parallel to their upper edges are preferred, as it is possible to place them in the instrument immediately with approximate settings of the adjustments. Final settings for any photograph are obtained by a trial and correction method.

When the photograph is correctly adjusted in the instrument, observations are taken in the same way as with an ordinary plane table. The illustration of the photoalidade shows how easy it is to imagine that the view displayed is a real scene, framed in a small open window, rather than a mere photograph. In fact, manipulation of the instrument will seem a surprisingly familiar operation to any one accustomed to the use of a plane table.

The instrument is mounted for convenience on a triangular frame supported by three 9-inch legs. This brings the sighting telescope up to the level of the eye of the operator who sits at a table of normal height. The spread of the legs allows ample room on the table for maps or work sheets, which can be conveniently shifted into position because of the free space between table and

instrument. The straightedge of the photoalidade is on a double-hinged arm, which can be folded so that the straightedge will be out of the way when not in use or lifted from time to time for inspection of the map beneath it. Thus it is possible to sketch freely on the map without disturbing its position in relation to the instrument. A centering microscope fitted with cross wires is provided, so that the station point on the map can be placed exactly in the vertical axis of the instrument. A clamp with slow-motion screw allows the telescope to be turned and set at any desired angle in relation to the straightedge, so that final orientation can be effected without rotating the map on the table. The simplicity of operation and the comparatively low cost of construction are attractive features of this instrument.

The precision attained in position and elevation appears consistent with what would be expected with a plane table and telescopic alidade under like conditions of map scale and with the same kind of control. The time necessary to obtain the final instrumental adjustments for a photograph is about half a day for the first time that the photograph is used, but if a record of the settings is made, it can be replaced in the instrument in five or ten minutes.

Mathematical principles fundamental to the photoalidade, and to the graphical and computational methods also, have been outlined in a previous paper.[2] In that outline, the coordinate system used to define the image-point positions on the photograph was based on the principal point of the photograph as origin. In the graphical method, and in the solution by computation, both described in this paper, the origin for the reference system in the plane of the photograph has been chosen at v, the intersection of the horizon line and the principal line which were used as the axes of x and y, respectively. Furthermore, in the graphical method the positive direction of y is up, but in the computation method it is down. The purpose is to simplify the practical solutions, although y is given a different meaning in each of the three systems used.

Angles by Computation

A still more precise method is available if the coordinates of images on the photograph can be measured closely with a comparator. From these coordinates the horizontal and vertical angles can be computed accurately, without any graphical operation to weaken the result. The horizontal angles obtained may still be laid off on tracing paper and used graphically if so desired, but better advantage may be taken of their inherent precision by using them to compute triangulation instead. Customary triangulation methods should be used for computing "three-point fixes" and for the positions intersected from camera stations so established.[7]

If work of high quality is to be undertaken, the measurements should be made directly from the negative, or at least from a glass diapositive, in order to avoid the distortion of a paper print.

The formulas for obtaining the angles from coordinates measured on the photograph are given on the specimen computing form, entitled "Oblique Photographs." In general, the same symbols are used on this form that have been employed in the discussion of the graphical method. The horizon line and the line perpendicular thereto through the center of the photograph, are the axes of the system in which the coordinates x and y are measured on the photograph. The greater proportion of sights from airplane camera stations are down from the

[7] Wilson, R. M., Triangulation Adjustments in Plane Coordinate Systems: *Engineering News-Record*, October 3, 1935.

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

State______________

Quadrangle____________

OBLIQUE PHOTOGRAPHS

Computation of Horizontal and Vertical Angles

$c^2=(f')^2+d^2$; $\tan O=\frac{c\,x}{c^2-d\,y}$; $\tan V=\frac{f'y\cos O}{c^2-d\,y}$, $Z'=M\tan V+E-(\text{Curv \& Refr})$

Photo No_______ f'_____ d____ c²________ c_____ Horizon______

1 Point						
2 Photo x						
3 Photo y						
4 d y						
5 $c^2-d\,y$						
6 $\tan O=\frac{c\,x}{[5]}$						
7 sin O						
8 cos O						
9 $\tan V=\frac{f'y}{[5]}[8]$						
10 Ground M						
11 Ground E						
12 M tan V						
13 Curv & Refr						
14 $Z'=[11]+[12]-[13]$						
1 Point						
2 Photo x						
3 Photo y						
4 d y						
5 $c^2-d\,y$						
6 $\tan O=\frac{c\,x}{[5]}$						
7 sin O						
8 cos O						
9 $\tan V=\frac{f'y}{[5]}[8]$						
10 Ground M						
11 Ground E						
12 M tan V						
13 Curv & Refr						
14 $Z'=[11]+[12]-[13]$						

f' = focal length of camera reduced to scale of photograph, in inches
d = distance from center of photograph to assumed horizon, in inches,
x = distance to right of axis of photograph, in inches
y = distance below assumed horizon, in inches
O = horizontal angle at camera
V = vertical angle at camera
M = ground distance from camera station, in feet.
E = ground elevation, in feet
Z' = elevation of projection of point in plane of camera and assumed horizon, in feet

Photo measured by______________
Date________________
Map measured by______________
Date________________
Computed by______________
Date________________

horizon; this form was arranged, therefore, to treat y, V and tan V as positive when measured down, in order to avoid too frequent negative signs. These quantities have thus been reversed in sign for convenient use in this computation, but otherwise the symbols are consistent with those used in the graphical method. Each form provides space to compute 12 sights.

The three methods described here are essentially the same in principle. In using any one of them an estimated horizon line must be drawn as a beginning, and subsequently corrected to the true horizon. It may be convenient to use the graphical method as a preliminary to either of the other two, so that a horizon line that is very nearly correct may be drawn before the photograph is placed in either the photoalidade or the comparator.

Resection for Camera Position

Having established the method of measuring horizontal and vertical angles from the photograph, the next step in a practical problem is to find the position, elevation, and orientation of the camera with respect to the ground control.

The positions of the control points being known, they may be plotted accurately on the map or compilation sheet. The horizontal angles between them, as viewed from the camera, are available on the tracing paper referred to in the preceding paragraphs. Therefore, the tracing paper may be laid over the compilation sheet and shifted about until each of the lines of direction radiating from C passes through the plotted position of its own control point. This is the usual and well known tracing paper resection.[8] With the tracing paper so adjusted, the position of C is pricked through with a needle to the compilation sheet to mark the nadir of the camera station; also, the positions of the control points are marked on the tracing paper, to show them there at proper distances from C, on the scale of the compilation sheet. Now the horizontal distances, M, from the camera station to the control points can be scaled either from the compilation sheet or from the tracing paper. These distances, with the tangents of the corresponding vertical angles already available, are used in solving right triangles to find differences of elevation.

Datum for Elevations

The true datum for elevations is the curved sea-level surface of the earth. In this problem, however, it will be less confusing to refer elevations to a datum plane which is exactly horizontal at the nadir of the camera and tangent to the curved sea-level surface there. At all other places this datum plane will be above the sea level surface. Refraction, which is the curving of lines of sight through the atmosphere, really is not concerned in this relationship; it is customary to assume sight lines to be straight, and to compensate for that assumption by using a smaller curvature correction. Accordingly, the combined effect of curvature and refraction should be applied at once to the elevations, E, of all ground points to refer them to the simple datum plane. Such translated elevations, E', will always be less than the actual elevations above sea level; occasionally they may be so much less as to be negative in sign. The relation is expressed by the formula: $E' = E - (\text{curv. \& refr.})$.

The usual expression for curvature and refraction, .574 (miles)^2 may be transformed to .00002059 M^2, using thousand-foot units for M and for the resulting correction also. The curvature and refraction table given here has been computed accordingly.

[8] Wilson, R. M Strength of Three-Point Locations: *Field Engineers Bulletin* No. 11, U. S. Coast and Geodetic Survey, December 1937.

Certain approximations have been admitted into this step of the procedure. There may be a figure slightly different from the constant .574 that would represent more closely the curvature and refraction for high and long observation lines from an airplane. The direction of a computed difference in elevation is perpendicular to the reference plane; this is not exactly the same as the true vertical direction through the control point along which the difference of elevation should properly be measured. These approximations do not introduce appreciable errors in the results obtained.

Differences of Elevation

The formula for the elevation, Z', above the datum plane, for a point on the reference plane just over an observed ground point is:

$$Z' = E' - M \tan V$$
$$= E' - \frac{My \cos i}{m}.$$

Curvature and Refraction

Distance and Correction both in thousand-foot units

$C \,\&\, R = 00002059\ M^2$

M	$C \& R$	M	$C \& R$	M	$C \& R$	M	$C \& R$
0	.0000	30	.0185	60	.0741	90	.1668
1	.0000	31	.0198	61	.0766	95	1858
2	.0001	32	.0211	62	.0791	100	.2059
3	.0002	33	0224	63	0817	105	2270
4	.0003	34	.0238	64	.0843	110	.2491
5	.0005	35	0252	65	.0870	115	.2723
6	.0007	36	.0267	66	.0897	120	.2965
7	.0010	37	.0282	67	.0924	125	.3217
8	.0013	38	.0297	68	.0952	130	3480
9	.0017	39	.0313	69	.0980	135	.3753
10	.0021	40	.0329	70	.1009	140	.4036
11	.0025	41	.0346	71	.1038	145	.4329
12	.0030	42	.0363	72	.1067	150	.4633
13	.0035	43	.0381	73	.1097	155	.4947
14	.0040	44	.0399	74	.1128	160	.5271
15	.0046	45	.0417	75	.1158	165	.5606
16	.0053	46	.0436	76	.1189	170	.5951
17	.0060	47	.0455	77	.1221	175	.6306
18	.0067	48	.0474	78	.1253	180	6671
19	.0074	49	.0494	79	.1285	185	7047
20	.0082	50	.0515	80	.1318	190	.7433
21	.0091	51	.0536	81	.1351	195	.7829
22	.0100	52	.0557	82	.1384	200	.8236
23	.0109	53	.0578	83	.1418	205	.8653
24	.0119	54	.0600	84	.1453	210	.9080
25	.0129	55	.0623	85	.1488	215	.9517
26	.0139	56	0646	86	.1523	220	.9966
27	.0150	57	0669	87	.1558	225	1.0424
28	.0161	58	.0693	88	.1594	230	1.0892
29	.0173	59	.0717	89	.1631	235	1.1371

The distance, y, and the vertical angle, V, usually will be negative, or down from the horizon. The horizontal distance to the control point, M, is always positive. It will be convenient to express Z', E', and M all in thousand-foot units; y and m in inches or centimeters.

The Sloping Reference Plane

The difference of elevation ($M \tan V$) computed from each control point may be regarded as the length of the line extending up or down from that point to the reference plane used in measuring the vertical angle. It must be remembered that this reference plane is approximately horizontal and passes through the camera station; it should not be confused with the datum plane that is exactly horizontal at N. If the reference plane were exactly horizontal the ends of such vertical lines from the several control points would all be just at the same elevation—the elevation of the plane and of the camera station. Then all values of Z' would be equal to Z.

But unfortunately the plane probably is not horizontal. It will be recalled that, at the beginning of a practical problem, only an estimated or tentative horizon line can be drawn on the photograph to represent the reference plane. Observations on the photograph referred to the tentative horizon line result in angles measured in, and downward (or possibly upward) from, the corresponding approximately horizontal plane. Such angles miss being true horizontal and vertical angles, respectively, because of the tilted reference system. The effect of this tilt upon horizontal angles is relatively small, and the position determined for the camera station, using the tentative reference plane, is not likely to be greatly in error. But vertical angles may contain the full amount of that tilt and be so seriously affected as to yield widely different figures of elevation for points on the reference plane.

Care has been taken to explain that the elevation, Z', determined from a control point through the scaled distance and preliminary vertical angle is not the elevation at the camera station; it is the elevation of a point on the tentative reference plane just over the control point Therefore, widely different figures of elevation for different points on the tentative plane are to be expected, because it is known to be only approximately horizontal.

However, after the elevations and positions have been found for three points in the plane, if they are not in line, the slope of the plane can be determined and the error existing in the initial estimation of the horizon line will be disclosed. This information can be used to level up the reference system, and to draw the true horizon line correctly on the photograph.

Correcting the Reference Plane

While the tracing paper was still adjusted over the compilation sheet, the positions of the plotted control points were traced. Now, on the tracing paper at each of these points, write the elevation of the tentative plane as determined there. The slope of the tentative plane may be represented graphically by drawing its contours of elevation on the tracing paper, just as is done on a contour map to represent a sloping ground surface. The contours for the plane must be parallel, equally-spaced straight lines, fitted to the three spotted elevations. Then it is easy to scale off the components of slope in the direction of the principal vertical plane in which the camera was pointed and in the (transverse) direction at right angles thereto. This method has been fully described already,[2] and it will not be expanded again here. Instead, a more recently devised analytical method will now be described.

It is convenient to use a system of rectangular coordinates with its origin at N, measuring Z upward, L horizontally forward and R to the right with respect to the pointing of the camera. The equation for a plane in this system may be written thus:

$$Z' = Z + aL + bR$$

in which a is the component of slope of the plane in the direction forward from the camera, b, the component toward the right, and Z is the particular value of Z' where the vertical axis of the system intersects the plane, that is, the elevation at the camera station in the present problem.

An observation upon a control point from the camera station provides the value of Z' for the spot on the plane which has the same horizontal coordinates as the control point, L and R, which may be scaled from the tracing paper. These three known coordinates are then substituted into the general equation of the plane. The same thing is done for the other control points, so that three observation equations are available to solve simultaneously for the three unknown constants Z, a and b. Using values thus found for a and b, which define the slope of the tentative plane, small distances above or below the ends of the tentative horizon line can be calculated to find points through which the true horizon line should be drawn. These distances are found by the formula:

$$e = \frac{ac + bw}{\cos i}$$

The distance along the horizon line from v to the place where e is to be plotted—in the side margin of the photograph for example—is represented by w, which should be in the same units used for other measurements on the photograph, and should be positive to the right of v and negative to the left. Where the sign of e is positive, the tentative horizon is above the true horizon line.

All of this procedure, which may seem complicated in the description, may be clarified by a numerical example. The photograph (Fig. 2) has been selected for this purpose because some of the identified points on it are above the horizon, and the use of the algebraic signs in the vertical measurements can so be illustrated. The horizon line $A-A$ was chosen simply by estimation. The small distances $+.122$ inches and $+.268$ inches can be plotted in the margins, -4.5 inches to the left and $+4.5$ inches to the right of v, respectively. In this example they are both positive, therefore the tentative horizon line is above the true horizon at both ends. The new horizon line $B-B$ is then drawn below the tentative line, as indicated.

The position and pointing of the camera represented by the position of C, the rays from that point through the image points on the photograph, and the ground points to which the rays go, compose a rigidly fixed system. A tentative reference plane was passed through that fixed system; this should be replaced now by the new plane that is more nearly horizontal. This leveled-up reference plane does not make any change in the actual directions and slopes of the rays observed from C, it is simply in a better position than the old one to measure those directions and slopes as true horizontal azimuths or bearings and as vertical angles from the true horizon, respectively.

When the new horizon line has been drawn, improved determination of horizontal and vertical angles should be made. Resection for the position of the nadir should be repeated, and new differences of elevation calculated from the control points to check the elevation of the camera station.

Theoretically, the new measurements to the three control points should result

REFERRED TO ESTIMATED HORIZON LINE $A-A$

$f'=11.583$ $\quad d=1\ 705$ $\quad c=11\ 708$
$\tan i=-.14720$ $\quad i=-8°22'25''$ $\quad \cos i = 98934$

Point	Feet ÷ 1,000 Scaled from Tracing			Inches Scaled from		Feet ÷ 1,000			Ft. r
	L	R	M	Photo y	Tracing m	$\frac{My \cos i}{m}$	$E'=E-$Curv. & Ref	Z'	
1	25 21	−3 79	25 49	+0 325	11 89	+0.689	3 114− 013	2 412	
2	9 51	−3 08	10 00	−1 192	12 13	−0 972	1 194− 002	2 164	
3	17 55	+5 86	18 50	−0 215	12 32	−0 319	2 128− 007	2.440	

Simultaneous observation equations

$$
\begin{array}{lrrrl}
Z & +25\ 21a & -\ 3\ 79b & -2\ 412 & =0 \\
Z & +\ 9\ 51a & -\ 3\ 08b & -2\ 164 & =0 \\
Z & +17\ 55a & +\ 5\ 86b & -2\ 440 & =0 \\
\hline
 & +15\ 70a & -\ 71b & -\ 248 & =0 \\
 & -\ 8\ 04a & -\ 8\ 94b & +\ 276 & =0 \\
 & -15\ 70a & -17\ 46b & +\ 539 & =0 \\
\hline
 & & -18\ 17b & +\ 291 & =0
\end{array}
$$

$Z=\ 2\ 056,\ a=+\ 0165,\ b=+\ 0160$

Computation of e, in left margin $w=-4\ 5$, right, $w=+4\ 5$

$$e=\frac{+.0165(11\ 708)+\ 0160(\mp 4\ 5)}{98934}=\ 195\mp\ 073$$

$$=+\ 122 \text{ in left margin} \qquad +\ 268 \text{ in right margin}$$

in three exactly equal figures of elevation for the reference plane above the datum plane. But, practically, due to the unavoidable inaccuracies in scaling and drafting, perfect agreement will seldom be achieved Usually it is fruitless to draw a third horizon line hoping to eliminate any very small residual slope that may be found in the adjusted reference plane. There is a method, to be described later, whereby the effect of such a residual slope can easily be eliminated from the results of observations made upon the photograph

ADJUSTMENT OF REFERENCE PLANE TO MANY OBSERVATIONS

Three ground control points, suitably situated, are theoretically sufficient to locate and orient the camera station. But in using only three points there is no check against errors in identifying them on the photograph, or in the subsequent graphical construction and computation. It is desirable, therefore, to use more than three control points. But absolutely perfect observations cannot be expected, and if different three-point combinations are used in several different solutions, their several results will all be slightly different. The average of these might give a reasonably accurate general result, but to carry through the several similar solutions required would be tedious.

An adjustment by the method of least squares, to include impartially the influence of more than three control points in a simultaneous solution, is not difficult. It has been shown already how each observation upon a control point provides a set of coordinates, Z', L, and R, that can be substituted numerically into the general equation of a plane. This gives an equation with three quantities,

a, b, and Z, still unknown, but with numerical coefficients which represent the observation made upon that control point. When a, b, and Z have been found by solving three such equations, they are written back into the general equation, thereby defining the reference plane. But if there are more than three such observation equations, they cannot all be satisfied exactly by any single set of values for a, b, and Z. Each observation equation, to be mathematically complete, should therefore include a term, r, to represent the amount by which that equation is not satisfied. Thus.

$$Z + aL_1 + bR_1 - Z_1' = r_1$$

$$Z + aL_2 + bR_2 - Z_2' = r_2$$

etc.

The residuals, r_1, r_2, etc , are analogous to the "deviations from the mean" that appear when each one of a number of separate measurements of any single quantity is individually compared with the mean of them all. If the basic data, the computations, and the observations were all absolutely perfect, the residuals would all be zero; but of course this is never the case in practice The least squares solution obtains the set of values for a, b, and Z that makes the sum of the squares of the residuals as small as possible; in other words, it defines the plane that most nearly fits all the observations.

The numerical example of an algebraic solution, using only three control points, provided corrections applicable to the tentative reference plane. A new horizon line, $B-B$, Figure 2, was drawn on the photograph to conform as nearly as possible to the indicated corrections The following table contains the data for observations made upon the original 3 control points and 10 more, making

REFERRED TO CORRECTED HORIZON LINE $B-B$

$f'=11\ 583$ $\quad$ $d=1.505$ $\quad$ $c=11\ 680$

$\tan i = -.12993$ $\quad$ $i=-7°24'10''$ $\quad$ $\cos i = .99166$

Point	Feet ÷ 1,000 Scaled from Tracing			Inches Scaled from		Feet ÷ 1,000			Ft.
	L	R	M	Photo y	Tracing m	$\frac{My \cos i}{m}$	$E'=$ $E-$Curv. & Ref.	Z'	r
1	25.18	−3.92	25 43	+0 483	11 875	+1.026	3 114− 013	2.075	−19
2	9 56	−3 07	10 02	−1 065	12 140	− .872	1 194− 002	2 064	− 6
3	17.62	+5 92	18 55	+0.047	12 325	+ 070	2.128− 007	2 051	− 1
4	19.89	−1.66	19.92	+0.354	11 755	+ .595	2 652− 008	2 049	+ 6
5	20 00	−5 16	20.62	+0 080	12 065	+ .136	2.185− 009	2 040	+17
6	34.29	−2 66	34 32	+0 728	11 805	+2 099	4.197− 024	2 074	−21
7	8 33	−2 86	8 79	−1.260	12 200	− .900	1.159−.002	2 057	+ 1
8	25 04	+3.06	25 18	+1.121	11 910	+2.350	4.405−.013	2.042	+ 9
9	24.98	+5 50	25 53	+1.112	12 095	+2 328	4.380− 013	2 039	+10
10	18.29	+0 11	18 25	+0.090	11.690	+ .139	2.200− 010	2.051	+ 3
11	8.68	+2 82	9.11	−1.206	12.115	− .899	1.164−.002	2.061	− 7
12	28.85	−2.90	28.94	+0.472	11.795	+1.148	3.223−.017	2.058	− 4
13	33.83	−8.45	34.80	+0.818	12.135	+2.326	4.395−.025	2.044	+14

By least squares solution:

$a=-.00016$ $\quad$ $b=-.00071$ $\quad$ $Z=2.0569$

13 in all, which are referred to the corrected horizon line. In this table the values of $\sqrt{L^2+R^2}$ are slightly larger than M; the scaled values of M were reduced by proportional corrections because the scale of the map from which they were measured was not quite correct. The residuals in the right hand column are obtained after the equations are solved.

Solution by Least Squares

To apply least squares to these data, the 13 observation equations might be written out in full, as was done for the previous algebraic solution. But there are contained in the table all essential figures for each observation equation, so that the usual normal equations can be formed directly from the table. The coefficient of each Z term is equal to 1; the coefficients of a and b are listed under the headings L and R, respectively. The absolute term is found in the Z' column, but it must be given a negative sign to transpose it to the same side of the equation with the other terms. Let "Sum" mean the sum of the indicated products taken successively from the lines 1 to 13 inclusive. Using such sums, the terms of the three normal equations (omitting the terms to the left of the diagonal terms as is customary) are represented thus:

Z	a	b	Z' (absolute)
$+\text{Sum}(1^2)$	$+\text{Sum}(1\times L)$	$+\text{Sum}(1\times R)$	$-\text{Sum}(1\times Z')$
	$+\text{Sum}(L^2)$	$+\text{Sum}(L\times R)$	$-\text{Sum}(L\times Z')$
		$+\text{Sum}(R^2)$	$-\text{Sum}(R\times Z')$

The detailed theory for forming and solving such equations[7,9] is given in textbooks on least squares, but brief practical directions to continue the solution are given here in the form of a diagrammatic "Key to Routine Solution." To avoid large figures it is permissible to subtract arbitrarily some convenient amount from all values of Z'. In the present example they have been diminished by 2.000 (two thousand feet), which has the effect of referring the solution to a plane at that distance above datum; when the solution is finished this amount can be replaced. Meanwhile, there will be only two significant figures for Z', instead of four, which will simplify the solution. With this modification, the numerical substitution in this example produces these terms:

Z	a	b	$Z'-2.000$ (absolute)
+13.	+ 274.54	− 13.27	− 0.705
	+6714.36	−404.02	−14.836
		+231.86	+ 0.856

Still another modification may be employed to make the solution easier. The coefficients for a and b in these normal equations are large; to make them more consistent with the magnitude of the Z coefficient it is better to solve for $10a$ and $10b$, which justifies shifting the decimal point 2 places to the left in (L^2), $(L\times R)$, and (R^2), and one place to the left in $(1\times L)$, $(1\times R)$, $(L\times Z')$, and $(R\times Z')$.

Solution of Normal Equations

The modified normal equations appear in the table entitled "Solution of Normal Equations." The column, S, to the right of the "Absolute" column, con-

[9] Norris, Clarence and Speert, Julius L., An Improved Method for Adjusting Level and Traverse Surveys: *Am. Soc. C. E. Transactions* 1940, page 1376, 1940 Paper No. 2087, see appendix II.

tains the check terms usually carried through such computations to guard against errors.

Perhaps the simplest way to show the systematic routine of the solution is by a diagrammatic "key." Each line in the solution in this example has been given a designating number to use, with the letters in the column headings, for reference purposes Thus, $b2$ means the figure appearing in column b, line 2. The figures found for the coefficients of the terms in the normal equations are written as indicated in lines 1, 3, and 7 in both of the following tables.

Referring now to the "Key to Routine Solution," the first step is to add selected given figures, as indicated (remembering to change the sign of the sum) to obtain the check figures for the S column, lines 1, 3, and 7. When that has been done, the solution progresses as indicated, line by line, to the bottom of the form. The figures found in each line marked by an asterisk should add nearly to zero as a check on the computation. Lines 2, 6, and 11 will each begin with -1; this should not be forgotten when adding along these lines to obtain the check. The "key" provides a guide that will not be needed after the first ex-

KEY TO ROUTINE SOLUTION

Ref. No.	Z	$10\times a$	$10\times b$	$Z'-2$ 000 (Absolute)	S (Sum)
1	+13 0000	+27 4540	−1.3270	− 7050	(−Sum· Z to Z' line 1)
2		(Z to S inclusive, line 1, divided by $-Z1$)*			
3		+67 1436	−4.0402	−1.4836	(−Sum· $a1$, $a3$, $b3$, $Z'3$)
4		(a to S, incl , line 1, multiplied by $a2$)			
5		(Sum, lines 3 and 4)*			
6		(a to S, incl., line 5, divided by $-a5$)*			
7			+2.3186	+0.0856	(−Sum: $b1$, $b3$, $b7$, $Z'7$)
8			(b to S, incl., line 1, multiplied by $b2$)		
9			(b to S, incl., line 5, multiplied by $b6$)		
10			(Sum, lines 7, 8, and 9)*		
11			(b to S, incl., line 10, divided by $-b10$)*		
12			(Copy $Z'11$)	$=10\times b$	
13		$b12\times b6$, $+Z'6$		$=10\times a$	
14	($a13\times a2$ $+b12\times b2$ $+Z'2$)			$=Z-2.000$	

perience, when the computer should discover and remember the systematic sequence in the line-by-line progress of such solutions. With a computing machine or a slide rule the solution of the three normal equations may be done in less than an hour.

The immediate results of this solution gives figures for $10\times a$, and $10\times b$, and $Z-2.000$ because of the modified normal equations used. Therefore the first two must be divided by 10, and 2.000 must be added to the third to obtain the results sought In this example, then, $a=-.00016$, $b=-.00071$, $Z=2.057$; the latter is the elevation of the camera station in thousand-foot units, and agrees well with the figure of elevation found in the algebraic solution using three control points.

The values of a and b indicate that a little too much correction was applied to the first estimation, and the reference plane now has a small slope in the

SOLUTION OF NORMAL EQUATIONS

Ref. No.	Z	$10\times a$	$10\times b$	$Z'-2\ 000$ (Absolute)	S (Sum)
1	+13 0000	+27.4540	−1.3270	− .7050	−38.4220
2	− 1	− 2 1118	+ .1021	+ .0542	+ 2 9555
3		+67.1436	−4.0402	−1.4836	−89.0738
4		−57.9774	+2.8024	+1.4888	+81.1396
5		+ 9.1662	−1.2378	+ .0052	− 7.9342
6		− 1.	+ .1350	− .0006	+ .8656
7			+2.3186	+ .0856	+ 2.9630
8			− .1355	− .0720	− 3.9229
9			− .1671	+ .0007	− 1.0711
10			+2.0160	+ .0143	− 2 0310
11			−1.	− .0071	+ 1 0074
12			− .0071	$=10\times b$	
13		− .0016		$=10\times a$	
14	+ .0569			$=Z-2\ 000$	

opposite direction. But, instead of trying to draw another horizon line, it is better to preserve the new reference plane, and with the accurate knowledge of its slight slope, make appropriate corrections to any observations referred to it.

The elevation of a point on the adopted reference plane, situated just above a ground position whose horizontal coordinates are L and R, is equal to $Z+aL+bR$, as referred to the datum plane tangent to the earth's surface at N. In the example being discussed, this elevation is:

$$2.057 - .00016\,L - .00071\,R$$

The figure so found should be used with the vertical angle observation to transfer an accurate elevation to a ground point in that position. In ordinary ground surveying the analogous procedure uses the elevation of the station itself, because the vertical angle is then controlled by a spirit level, and so it is taken for granted that the reference plane is horizontal.

RESIDUALS

The value of Z and the slope of the reference plane were determined by combining or averaging the observations upon 13 different ground points. To see how nearly each individual observation is satisfied by the adopted reference plane, the elevation on the plane may be compared with the elevation indicated by that particular observation. It must be remembered here that elevations are being referred to the datum plane, not to the curved surface of the earth beneath it. In the present example, the elevation of the reference plane over point No. 9 is:

$$2.057 + (-\ .00016)(+\ 24\,98) + (-\ \ 00071)(+\ 5.50) = 2\,049.$$

The value for Z' indicated by the observation is given in the table as 2.039, so this particular observation deviates from the combined result of all thirteen by 10 feet. However, if the slope of the reference plane were not considered, the deviation obtained by comparing this observation directly with the elevation

of the camera station would be 18 feet. What has just been done is the substitution of known figures into the observation equation:

$$Z + aL + bR - Z' = r.$$

For point No. 9, therefore, $r = +10$. By similar substitutions made into the observation equations for other points, always using $Z = 2.057$, $a = -.00016$, and $b = -.00071$ to represent the adopted plane, the other residuals are obtained. They are all given in the right hand column of the table of observations

Magnitude of Residuals as a Test of Map Accuracy

The example given is based on the graphical method alone. The 13 points used to control the solution were not established particularly for that purpose. They were merely identifiable features in the field of view, selected from a topographic map published on the scale of 1:24,000. The elevations of some of them were estimated by interpolating between 10-foot contours.

Residuals are caused by a combination of various accidental errors, such as the adoption of inaccurate position or elevation for the ground points or errors in the map, identifying the ground points imperfectly on the photograph, distortion of the print or in the camera lens, inaccurate graphical construction on the tracing paper.

The average magnitude of the 13 residuals just listed is 9 feet, and is a numerical index of the precision of the data used as well as of the graphical method itself.

Accurately plotted positions and indication of correct elevation are qualities that make a good topographic map. If details have been correctly mapped, there will appear only very small relative discrepancies when observations to them are compared simultaneously through a least squares solution. Even if the points are not formal control points, the photograph will still provide information as to the accuracy of their relative elevations. If equivalent precision in method and similar photographs are used in different areas covered by maps of various degrees of accuracy, it is reasonable to expect that the magnitude of the residuals will vary according to map accuracy. The adoption of one of the more nearly accurate methods as standard procedure would soon show what magnitude to expect in the residuals corresponding to each designated grade of map. Considered thus, one of the methods described may so be used to test the accuracy of elevations on any map, regarding the average residual in each test as the numerical index of the quality of the map tested. The scheme is at the same time effective in testing relative accuracy of plotted horizontal positions, because errors in position will produce discrepancies of elevation as observed by the vertical angles and scaled distances. Also, if appreciable errors in horizontal position exist in the map, they will be disclosed when resecting for the camera station.

Interpolation of Additional Control

New points, required for the objective purposes of the survey, may now be selected and observed upon. Lines of direction to them, obtained by means of the tracing paper, may be plotted on the compilation sheet. Similar operations with a photograph taken from another camera station will provide intersecting lines to locate the new points in position. The camera stations should be situated so as to avoid sharp intersections. Vertical angles from one or both of the camera stations with scaled horizontal distances, provide the means to determine the

Fairchild Aerial Surveys, Inc., L. A.

FIG. 6. Glendale, California

U S Navy Photo

FIG 7 Twin Glaciers, Alaska

elevations of the new intersected points This is the same method that is employed with a plane table.

Oblique photographs include large fields of view as compared to the usual vertical photographs. They show near and distant control points in proper relation to foreground detail; this quality makes them adaptable to the interpolation of supplementary control, both horizontal and vertical, to aid many of the established procedures of surveying and photogrammetry. Slotted templets[10] may be made using horizontal directions extracted from oblique photographs; these may be included in an adjustment of vertical photograph templets to tie in the whole group with distant control

In conclusion the author wishes to offer his opinion that many promising potential uses for oblique photographs are now suffering undeserved neglect It is hoped that the principles[11,12] described here may help to promote their use.

Figures 6 and 7 show a pair of typical Oblique Aerial Photographs

[10] Kelsh, Harry T., The Slotted-Templet Method for Controlling Maps made from Aerial Photographs: U S. Department of Agriculture, *Miscellaneous Publication No 404*, November, 1940.

[11] Wang, Dr. Chih-cho, China Institute of Geography, Height Determinations with High Oblique Photographs: *Photogrammetric Engineering*, Vol. VII, No 3, July-August-September 1941.

[12] See also references listed in PHOTOGRAMMETRIC ENGINEERING, April-May-June issue, 1938, page 74, under the article (2) above.

TOPOGRAPHIC MAPPING FROM HIGH OBLIQUE AIR PHOTOGRAPHS

WITH SPECIAL REFERENCE TO THE TECHNIQUES DEVELOPED AT THE AMERICAN GEOGRAPHICAL SOCIETY

O. M. Miller

INTRODUCTION

HIGH oblique air photographs are those taken with the camera axis pointing more nearly horizontal than vertical.

Topographical mapping from high obliques, as described in this article, is an offspring of ground photographic surveying of a type first practiced to any great extent by the Canadian Government under the leadership of its famous Surveyor, General E. Deville. Though writers such as B. M. Jones and J. C. Griffiths[1] had discussed the principles involved and outlined possible procedures, little was done in a practical way prior to 1930 to realize the possibilities of high oblique air photographs except for the making of planimetric maps of flat country. Since then, however, various techniques have been proposed and developed and considerable topographical mapping has been accomplished with them.

The techniques and instruments described in this article have been, for the most part, developed at the American Geographical Society and have been in use since 1931. Projects already completed at this institution are maps of Northernmost Labrador[2] and the Sierra Nevada de Santa Marta in Colombia.[3] Work is proceeding on extensive surveys made in the Yukon Territory. Other completed projects include a map of the Queen Maud Mountains in the Antarctic from photographs taken on the First Byrd Expedition[4] and sketch maps made from Leica photographs taken by Lincoln Ellsworth on his flight across the Antarctic Continent.[5]

The use of high obliques is particularly suitable for the mapping of mountainous country on scales smaller than 1:50,000. This is so for a variety of reasons. The flying height at which the photographs are taken need not be nearly so great as in vertical photography in order to cover the ground efficiently for small scale mapping. The taking of high obliques does not necessarily depend on perfect weather conditions. Furthermore straight line flying and a constant flight altitude are not at all necessary. Accurate height control may be obtained from points many miles distant. Finally when topographical features are to be

[1] B. M Jones and J. C Griffiths· Aerial Surveying by Rapid Methods, Cambridge, 1925, Chapter VI and Appendix IV

[2] Alexander Forbes. Northernmost Labrador Mapped from the Air, *American Geographical Society Special Publication No. 22*, New York, 1938 Map of Northernmost Labrador, 6 sheets, in accompanying slip case.

[3] W. A. Wood: Mapping the Sierra Nevada de Santa Marta: The Work of the Cabot Colombian Expedition, *Geographical Review*, Vol. 31, October, 1941, pp. 639–643, accompanied by map, 1:100,000.

[4] Laurence M. Gould: Some Geographical Results of the Byrd Antarctic Expedition, *Geographical Review*, Vol. 21, April, 1931, pp. 177–200, accompanied by map, 1.500,000.

[5] W. L. G. Joerg: The Cartographical Results of Ellsworth's Trans-Antarctic Flight of 1935, *Geographical Review*, Vol.27, July, 1937, pp. 430–444, accompanied by maps.

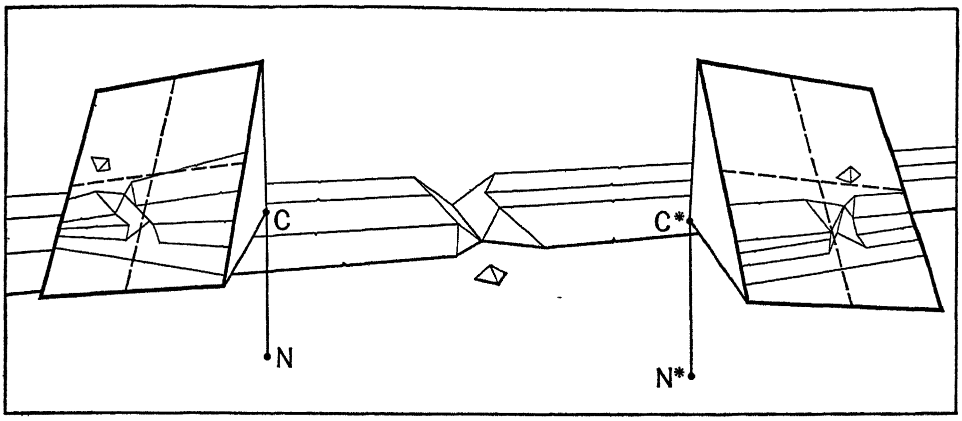


FIG. 1

delineated on small scales by means of sketched contours, oblique views are generally more helpful for this purpose than verticals. One superficial objection to the use of high obliques in mountainous country which has often been made in the past is that there must necessarily be much dead ground not imaged on the photographs. In the vast majority of cases, however, provided the scheme of photography has been carefully planned and executed, what is dead ground from one pair of photographs will not be dead ground from another

The chief technical problem involved in utilizing high obliques is that of determining the exterior orientations of the camera that took the photographs. When this has been done, it is possible to measure horizontal directions and differences of height from the photographs and the procedure becomes essentially the same as the well-established ground photographic method. The method, therefore, is essentially one of radial line intersections. Contours are sketched as in plane-table surveying, but the photographs, instead of field sketch notes are used as guides. Thus each topographical feature can be studied in the comfort of an office from several different aspects. This aids in the truthfulness of the sketching. Stereoscopic examination of the terrain from pairs of photographs is generally possible and is a valuable additional aid.

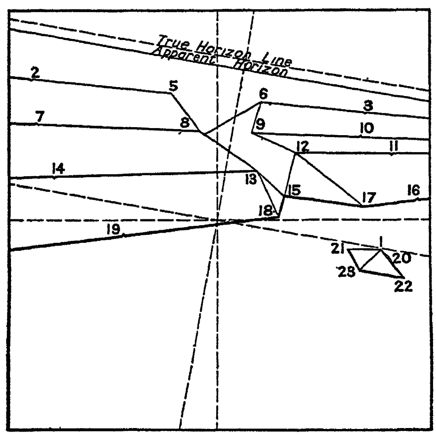


FIG. 2

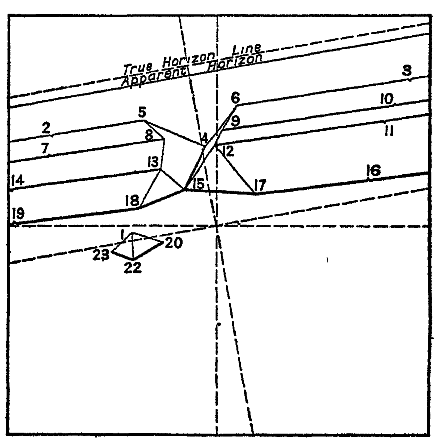


FIG. 3

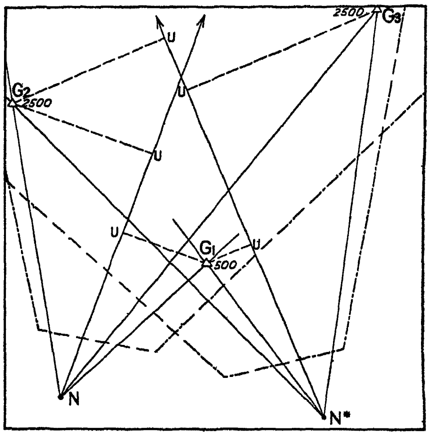


FIG. 4

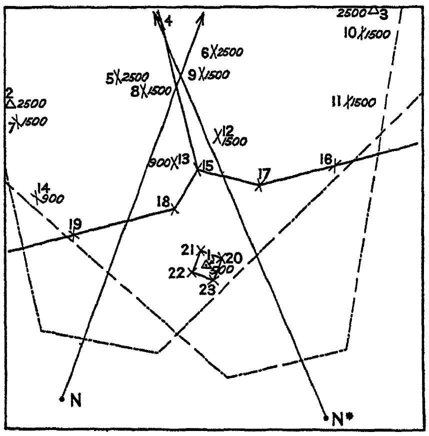


FIG. 5

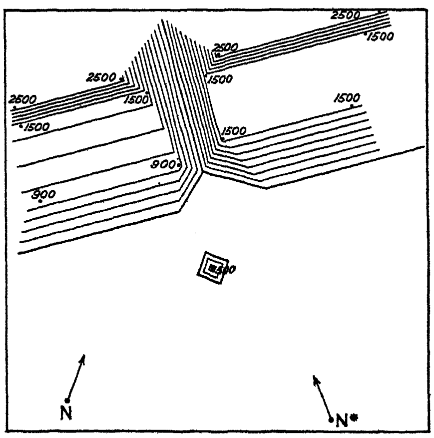


FIG. 6

An outline of the general process of mapping from high oblique photographs is shown schematically in Figures 1 to 6. Figure 1 is a perspective view of two camera stations and the area which is to be mapped from them. Figures 2 and 3 are the photographs. In the area, and imaged on the photographs, are three points, 1, 2, and 3, whose positions and elevations are known. From these three control points the exterior orientations of the two photographs are obtained The stage in the construction of the map now reached is shown in Figure 4. Shorelines and streams are then plotted and the position and elevations of as many points in the terrain as are considered necessary for constructing the map are determined. The map has now reached the stage shown in Figure 5 From the spot elevations obtained and with the aid of the photographs as guides the contour lines are sketched and thus the final map as shown in Figure 6 is produced.

Most of the work involved in mapping from high obliques is either graphical or instrumental. However, for the analytical processes which required a high order of accuracy such as exterior orientation, computational methods are used advisedly, not only because they are more accurate, but because they provide checks and records of the working and avoid cluttering the plotting sheet with construction lines, and also because in the long run they save time and fatigue. Furthermore, although, in the development of the techniques employed for orientation use is made of spherical trigonometry and elementary differential calculus, in actual practice the computational methods require only the ability to solve linear algebraic equations numerically.

DEFINITIONS

The meaning of the symbols used in the article can be ascertained by reference to the figures. A few special definitions are necessary, however, for a ready understanding of the text and these are given below in alphabetical order.

Air Station (denoted by C) (Fig. 1). The point in space from which a photograph is exposed or the position in space of the *perspective center.*

Apparent Horizon (Figs. 2, 3 and 7). The image on a photograph of the terrestrial horizon.

Exterior Orientation. In this article it denotes the combined process of determining the position of an *air station* relative to the ground and the angular orientation of the photograph.

Horizon Line (*True*) (Figs. 2, 3 and 7). The imaginary trace on the photograph of the *horizon plane.*

Horizon Plane (Figs. 8, 10 and 11). The horizontal plane containing the *perspective center C.*

Horizontal Angle (Fig. 9). In this article it means specifically the horizontal angle between the *principal plane* and the vertical plane containing the *perspective center* and a ground point G. It is denoted by the symbol O. If G is to the right of the *principal plane* O is positive. If to the left, negative.

Isometric Parallel (Fig. 7). The line on the photograph perpendicular to the *principal line* at an equal angular distance from the *true horizon line* and the *principal parallel.* If j is the intersection of this line with the *principal line* then $pj = f \tan T/2$. (Fig. 8.)

Perspective Center (denoted by C). A single point assumed for mapping purposes where the angular relationships between *perspective rays* to points in the photograph are the same as the angular relationships between rays to corresponding points in the object photographed. Actually the *perspective center* of a photograph corresponds almost exactly to the position, relative to the photograph, of the back nodal point of the camera lens (see also *air station*).

Perspective Ray. The straight line from a *perspective center* to any point object photographed.

Plate Perpendicular (Fig. 8). The perpendicular from the *perspective center* to the photograph. Its distance, commonly called the calibrated focal length or principal distance, is denoted by f.

Principal Line (Fig. 7). The trace on the photograph of the *principal plane.*

Principal Parallel (Fig. 7). The line on the photograph perpendicular to the *principal line* passing through the *principal point.*

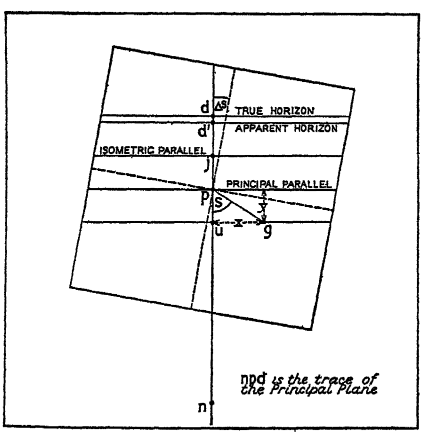


FIG. 7. The paper represents the plane of the photograph.

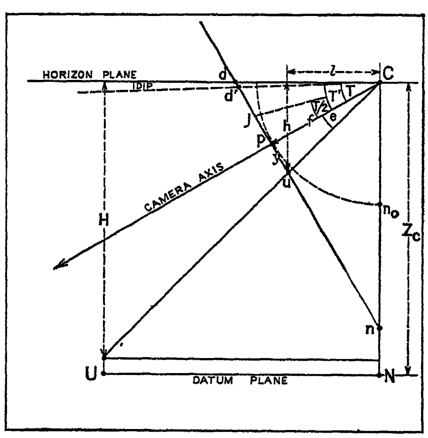


FIG. 8. The paper represents the the principal plane.

Principal Plane. The vertical plane containing the *plate perpendicular.*

Principal Point (Fig. 7). The foot of the *plate perpendicular* on the plane of the photograph. It is denoted by *p*.

Radial Angle (Fig. 7). The angle subtended at the *principal point p* by the *principal line* and any point image *g*. It is denoted by *S* and is measured clockwise on a positive photograph from the *principal line.*

Swing (Fig. 7). The angle of rotation of a photograph in its own plane about its *plate perpendicular.* In this article the term is used to mean specifically the angle at *p* made by the *principal line* and a line passing through *p* between the top and bottom collimating marks of the photograph. It is denoted by ΔS and is measured clockwise from the *principal line* on a positive photograph. As it is obvious that a small change in *swing* will cause an equal or corresponding change in any *radial angle, dS* denotes this change in both cases.

Tilt (denoted by *T*) (Fig. 8). In this article it refers specifically to the angle of depression below the *horizon plane* of the *plate perpendicular* of a positive photograph. Note that in all figures, except 1, 24 and 25, which show negative positions of the photographs, the positive position of the photograph is shown. The positive position of a photograph is assumed to lie in a tilted plane interposed between the *perspective center* and the object space at a perpendicular distance from the *perspective center* equal to *f* (See *plate perpendicular*).

Tilt Axis. The horizontal line containing the *perspective center* which is perpendicular to the *principal plane.*

Vertical Angle (denoted by *V*) (Fig. 10). In this article it refers specifically to the angle of depression below the *horizon plane* of a *perspective ray.*

Horizontal and Vertical Angles from Rectangular Coordinate Measurements on a Photograph

In actual practice it would be a laborious procedure to determine the positions of points in the terrain from horizontal and vertical angles derived from rect-

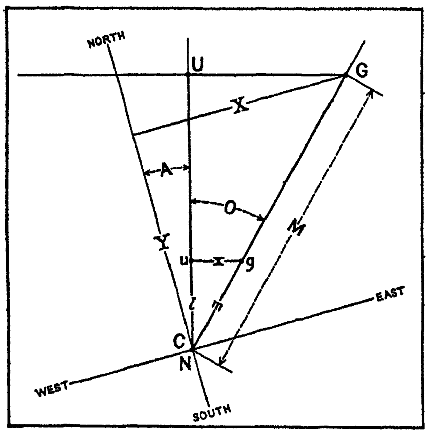


FIG. 9. Horizontal relationships.

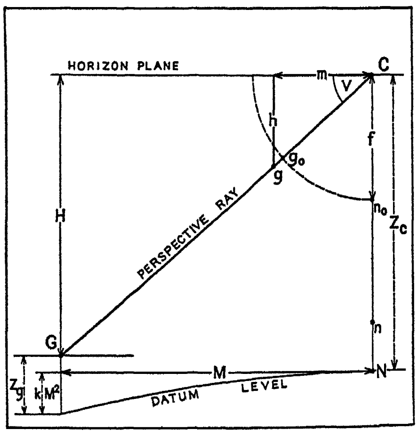


FIG. 10. The paper represents the vertical plane containing a perspective ray.

angular coordinate measurements. Nevertheless, the formulae below constitute the necessary background for the development of the simple graphical methods described later.

From Figure 8

$$\tan e = y/f \tag{1}$$

$$l = \mathrm{Cu} \cos (T - e)$$

$$\mathrm{Cu} = f/\cos e\,.$$

From figure 9

$$\tan O = x/l$$

Therefore by substitution

$$\tan O = x \cos e/f \cos (T - e). \tag{2}$$

From figure 10

$$\tan V = h/m.$$

From figure 8

$$h = l \tan (T - e).$$

From figure 9

$$m = l/\cos O.$$

Therefore

$$\tan V = \tan (T - e) \cos O. \tag{3}$$

Differences of Height Formulae

From Figure 10 the difference of height H between the horizon plane of an air station and a ground point G may be obtained by the usual surveyors' formula

$$H = M \tan V. \tag{4}$$

Substituting for $\tan V$ in (3)

$$H = M \tan (T - e) \cos O. \tag{5}$$

Again from Figure 10 the height Z_c of an air station above datum level is

$$Z_c = M \tan V + Z_g - kM^2 \tag{6}$$

where k is a constant of curvature and refraction.

Differentiating Z_c in respect to V

$$dZ_c = M \sec^2 V \cdot dV \text{ when } dV \text{ is in radians}$$

$$= M \sec^2 V \operatorname{arc} 1' \cdot dV \text{ when } dV \text{ is in minutes of arc}$$

but

$$\sec^2 V = 1 + \tan^2 V$$

and

$$\tan^2 V = H^2/M^2$$

so

$$dZ_c = M(1 + H^2/M^2) \text{ arc } 1' \, dV. \tag{7}$$

Formula (7) is of importance in dealing with the problem of exterior orientation.

Differential Relationships Between Angles Measured from a Photograph and Angular Elements of Exterior Orientation

Certain of the fundamental relationships between directions from a perspective center can be determined by means of spherical trigonometry in much the same way as the surveyor or navigator uses the astronomical triangle.

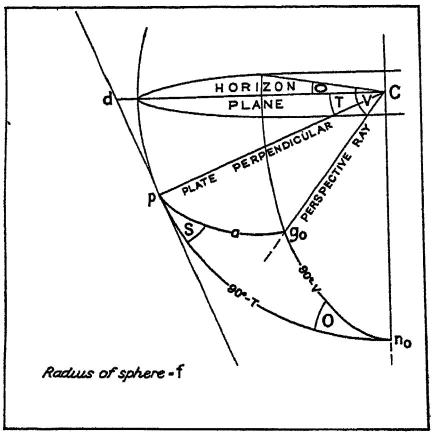


Fig. 11

Assume the perspective center of a photograph as the center of a sphere of radius f. The plane of the photograph will then be tangent to the sphere and the principal point p will lie on the surface of the sphere (Fig. 11).

Assume that the horizon plane contains the equator of the sphere and that the direction of the nadir point n is the polar axis. Then in the spherical triangle n_0pg_0 the side n_0p will be the complement of the tilt T, the side n_0g_0 will be the complement of the vertical angle V of any point G and the side pg_0 will be the angle a whose tangent is equal to pg/f. The angle at p will be the radial angle of the point G and the angle at n_0 will be the horizontal angle O of the point G.

Using the conventional formulae for solving an oblique spherical triangle

$$\cos(90° - V) = \cos(90° - T)\cos a + \sin(90° - T)\sin a \cos S$$

and

$$\cot O = [\cot a \sin(90° - T) - \cos(90° - T)\cos S]/\sin S$$

or

$$\sin V = \sin T \cos a + \cos T \sin a \cos S \tag{8}$$

and

$$\cot O = [\cot a \cos T - \sin T \cos S]/\sin S. \tag{9}$$

Differentiating V and O in respect to T and S

$$dV = \cos O \, dT + \sin O \cos T dS \tag{10}$$

$$dO = \sin O \tan V dT + (\sin T - \tan V \cos O \cos T)dS. \tag{11}$$

These differential equations are important as they enable the effects on the measurement of vertical and horizontal angles due to small errors in the as-

sumptions of tilt and swing to be readily determined. They will be referred to later in dealing with the problem of exterior orientation.

Differential Variations in the Ground Coordinate System

When great accuracy is required in the determination of position, differential methods are efficient tools for this purpose.

Table of Intersection Errors in Feet
Arguments—Angle of intersection in degrees and half the sum of the distances in miles.

	10°	20°	30°	40°	50°	60°	70°	80°	90°
50	910	447	301	235	196	173	159	150	146
45	819	402	270	212	176	156	144	135	131
40	728	358	240	188	157	138	128	120	117
35	637	313	210	165	137	121	112	105	102
30	546	268	180	141	118	104	96	90	88
25	455	224	150	118	98	87	80	75	73
20	364	179	120	94	78	69	64	60	58
15	273	134	90	71	59	52	48	45	44
10	182	89	60	47	39	35	32	30	29
5	91	45	30	24	20	17	16	15	15
	170°	160°	150°	140°	130°	120°	110°	100°	90°

Fig. 12

The azimuth of any point G from an air station is the angle $(A+O)$. Therefore from figure 9

$$X_{cg} = Y_{cg} \tan (A + O) \tag{12}$$

$$Y_{cg} = X_{cg} \cot (A + O). \tag{13}$$

Differentiating X and Y in these equations and neglecting the subscripts

$$dX = \tan (A + O)dY + Y \sec^2 (A + O) \text{ arc } 1'd(A + O)$$

$$dY = \cot (A + O)dX - X \operatorname{cosec}^2 (A + O) \text{ arc } 1'd(A + O).$$

Substituting (Fig. 9) linear ratios for the trigonometrical functions and as $d\ (A+O)$ will be the same as dA

$$dX = (X/Y)dY + (M^2/Y) \text{ arc } 1'dA \tag{14}$$

$$dY = (Y/X)dX - (M^2/X) \text{ arc } 1'dA. \tag{15}$$

Standards of Accuracy

A question that is often asked is, "How accurate is mapping from high oblique photographs?" If the question is phrased, "Given a camera and photographic materials which will insure a given standard of accuracy in measurement at what maximum distance from the camera stations can a specified map accuracy be maintained?" then answers for varying conditions may be given. In Figure 12 is given the maximum error in feet to be expected in the horizontal position of a point intersected from two air stations whose positions are known when the order of accuracy in determining direction is 1 minute of arc. The table has been computed assuming the NG distance to be very much longer than the N^*G distance in the ground triangle NN^*G of Figure 4. This is the most unfavorable condition. In this connection, however, it is of interest to

note that under the most favorable condition (which is when the ground triangle is isoceles) the maximum errors are still very nearly the same. The biggest improvement (as might be expected) is when the n gle of intersection is 90°. Even in this case, however, the maximum error from that given in the table is reduced only by about 25 per cent. The table in Figure 12 can be put to

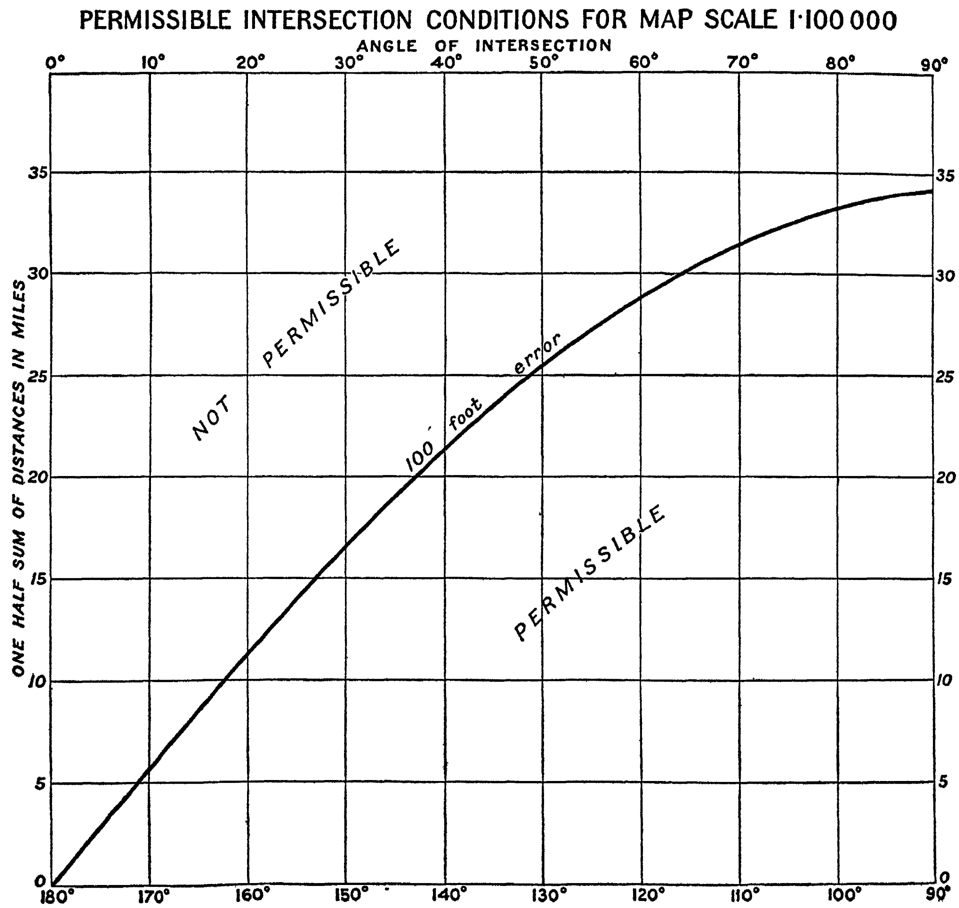


FIG. 13

various practical uses. For instance, if a map is being constructed which will eventually be printed to a scale of 1:100,000 errors of position of less than 100 feet will be unappreciable. The thick line in the body of the table, therefore, is the dividing line for this scale between permissible and non-permissible types of intersections. For any specific scale a graph constructed from the table as in Figure 13 is more convenient to use. Such a graph is also useful in choosing suitable control points for the resection of air stations and in planning the necessary intervals between consecutive exposures and adjacent flight lines.

When it is assumed that the horizontal distances between the air stations and ground points are accurately known, Figure 14 shows the possible errors (dZ_v) in height determinations introduced by a one minute error in the measured vertical angle when the air station is 10,000 feet above the ground points. For

POSSIBLE ERRORS IN MEASURING HEIGHTS
from high oblique photographs
Height of Air Station—10,000 feet

Distance in miles	05	10	15	20	25	30	35	40	45	50
dZ_v	09	15	23	31	38	46	54	61	69	76
dZ_k	00	01	03	06	09	12	16	21	27	34

FIG. 14

any other set of conditions values of dZ_v may easily be computed from (7).

An independent source of error in height determinations from high obliques is introduced by the fact that in applying a correction to a measured height for curvature and refraction, the precise value of the coefficient of refraction (0.070 is the value normally adopted) is not known. dZ_k in Figure 14 is the error in height introduced by assuming the coefficient of refraction to be 0 010 in error.

The plotting of detail such as shore lines is often undertaken from single photographs when the difference of height between the air station and the ground level is known Figure 15 shows for various values of V and M and a difference of height of 10,000 feet the horizontal displacements in feet of a plotted point due to one minute errors in the measurement of the V and O angles, dM being in the horizontal direction of the plotted line and the dR being perpendicular to it.

GRAPHICAL METHODS OF MEASUREMENT

In the graphical methods to be described, it is assumed that a positive paper print is used on which to make measurements from the photograph.

TWO METHODS OF EXTRACTING HORIZONTAL ANGLES[6]

Method 1

All points lying in the same vertical plane containing the perspective center of a photograph must obviously have the same horizontal angle O (Figs 9 and 10). The trace of this vertical plane on the plane of the photograph is a straight line as the intersection of two planes is a straight line. Furthermore, as the vertical plane contains C it must also contain n, the nadir point of C in the plane of the photograph (Fig. 7). It follows that lines of equal horizontal direction on an oblique photograph are straight lines radiating from n. However, the angle formed at n in the plane of the photograph by two lines of equal horizontal direction is not the correct difference in horizontal direction (Fig. 17) except in the case of the truly vertical photograph when T is 90° and n coincides with p (Fig. 16). This statement can be more readily appreciated if one considers the other extreme case when T is zero. Then the nadir line from C will never intersect the plane of the photograph. In other words, in the latter case n will be at an infinite distance from p, and the lines of equal horizontal direction will be parallel to each other and to the principal plane (Fig. 18).

[6] It is believed that the first method is here described in print for the first time. Though the author has been aware of the geometrical theory of the method for a number of years it was first suggested by him as a practical procedure in October, 1939, in a letter to Captain Thomas North of the Engineer Board, Fort Belvoir. The second method is the Survey of India method devised by Captain D. R. Crone. See Appendix I. Report of the Air Survey Committee No. 2, 1935. London. His Majesty's Stationery Office.

HORIZONTAL DISPLACEMENT OF POINTS PLOTTED FROM SINGLE PHOTOGRAPHS

due to one minute errors in the measurement of the horizontal and vertical angles from the perspective center

HEIGHT OF AIR STATION:—10,000 feet

V	80°	70°	60°	50°	40°	30°	20°	10°	05°
M	1763	3640	5774	8391	11917	17320	27475	56712	114300
dM	3.00	3.25	4.00	5 00	7.00	11.50	25 00	97.00	393 00
dR	0 50	1.00	1.75	2 50	3.50	5 00	8 00	16 50	33.00

FIG. 15

The formal proofs of these statements may easily be obtained from formula 2, for when T is 90°

$$\tan O = x \cos e/f \cos (90° - e)$$
$$= x/f \tan e$$
$$= x/y$$

and when T is zero

$$\tan O = x/f.$$

In all other cases when T has some value between 0° and 90°

$\tan O = x/f$ only when e equals $T/2$.

This is demonstrated by inserting this value of e in formula 2.

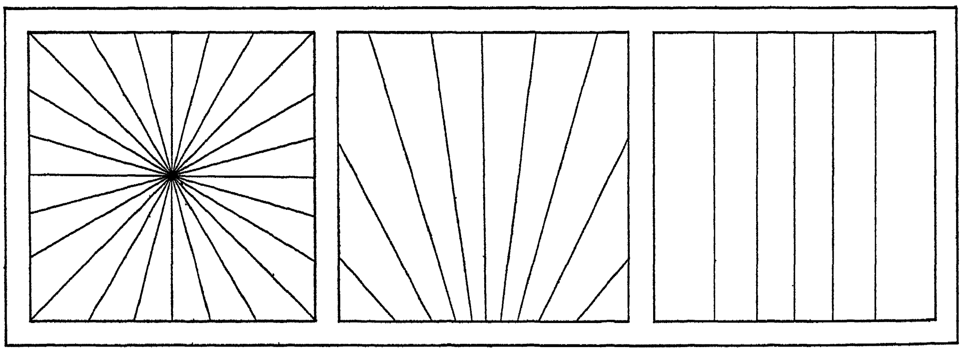


FIG 16. Lines of equal horizontal direction on a vertical photograph. Angular difference 15°.

FIG. 17. Lines of equal horizontal direction on a high-oblique photograph ($T = 30°$). Angular difference 15°.

FIG. 18. Lines of equal horizontal direction on a photograph when T is zero. Angular difference 15°.

Stated in other words

$\tan O = x/f$ when $y = f \cdot \tan T/2$

or

$\tan O = x/f$ when a point image g lies on the isometric parallel.

From the above argument a very simple construction for extracting the horizontal angle O of an image point g may be developed provided the tilt and swing of the photograph are known.

With reference to Figure 19, lay off a straight line on a piece of paper. Mark a point n at one end of it. Measure a distance np equal to f cot T (Fig. 8)

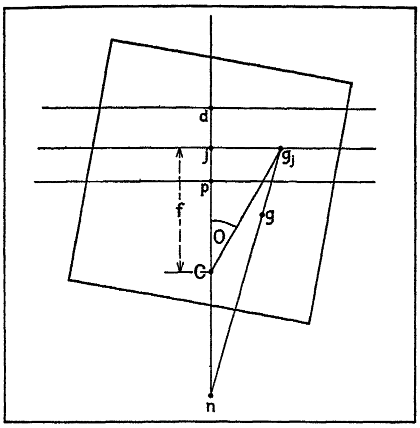


FIG. 19

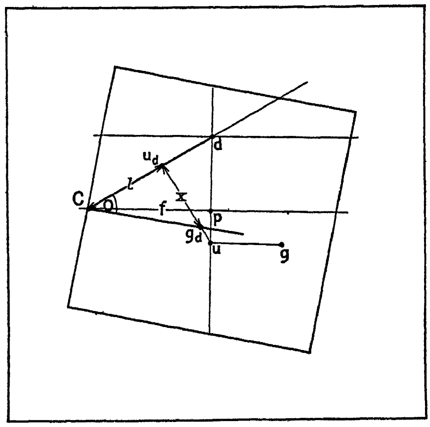


FIG. 20

From p on the same straight line, but in the opposite direction from n, lay off a distance pj equal to f tan $T/2$. Draw lines perpendicular to np through p and j. These lines correspond to the principal and isometric parallels respectively (Fig. 7).

Make marks on the edges of the photograph indicating the principal line and principal parallel and lay the photograph on the paper so that the principal line coincides with the line np and the principal parallel coincides with the perpendicular line through p.

Lay off on a piece of cellulose acetate (or other transparent drawing material) considerably larger than the photograph a straight line. Mark a point C at one end of it. Measure a distance Cj equal to f. Draw a line perpendicular to Cj passing through j. Lay the cellulose acetate over the photograph and drawing paper so that the line Cj coincides with np and the perpendicular at j coincides with the isometric parallel.

The angle O of any image point g on the photograph is then obtained on the cellulose acetate at C in the following manner: Join ng by a straight line and produce this line to cut the isometric parallel at g_j. Join C and g_j. Now because by construction g_j lies on the line of equal horizontal direction passing through g and also on the isometric parallel, O equals the angle jCg_j.

When a large number of O angles are to be measured a simple device will greatly expedite this process. Set a thumb tack at n and tie a piece of strong thread to it. Holding the thread taut, line it up with a g point and mark the corresponding g_j point on the cellulose acetate.

This method is not very satisfactory when T approaches zero as then the distance np becomes long. For example, when $f=10$ and T equals 5° np is 114.3. When $f=10$ and T equals 10° np is 56.7. Under ordinary conditions, however, T is usually considerably greater than 10°.

Method 2

When np is too long for convenient handling on a table the following method may be used, though it is not as rapid as Method 1.

With reference to Figure 20, on a piece of cellulose acetate lay off the right triangle Cpd in Figure 8. Where Cp equals f, Cd equals f sec T, and pd equals f tan T. Extend Cp and Cd as necessary. Make marks on the sides of the photograph indicating the principal line and principal parallel, and lay the cellulose

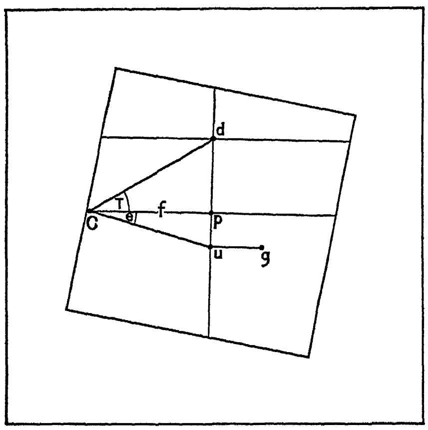


FIG. 21

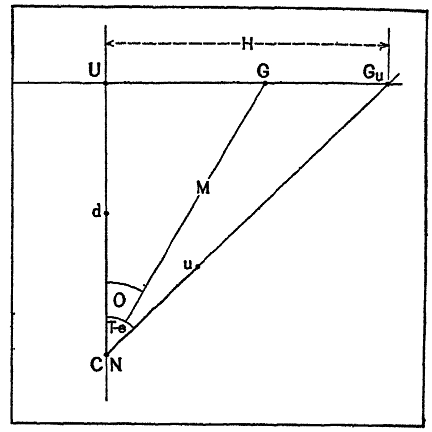


FIG. 22

acetate over the photograph so that Cp coincides with the principal parallel and pd with the principal line.

Drop a perpendicular from g to pd cutting at u. Drop a perpendicular from u to Cd cutting at u_d From u_d measure a distance $u_d g_d$ equal to ug. Join Cg_d. By comparing Figures 9 and 20 it will be evident that the angle dCg_a is the required O angle.

A METHOD OF OBTAINING DIFFERENCES OF HEIGHT[7]

Before differences of height can be measured between an air station and a ground point, it is necessary to know the horizontal distance between them.

In this graphical method it is assumed that the horizontal positions are plotted on the map being constructed together with the line NU which is the trace of the principal plane in the datum plane, or in other words the horizontal direction of the axis of the camera taking the photograph.

Referring to Figures 21 and 22, make marks on the edges of the photograph indicating the principal line. On a piece of cellulose acetate lay off a straight line. Mark a point C at one end of it. Measure a distance Cp equal to f. Draw a line perpendicular to Cp through p and measure the distance pd from p along it (pd equals f tan T). Join Cd.

Place the cellulose acetate over the photograph so that the points p and d lie over the corresponding points on the photograph. Drop a perpendicular from g to pd cutting at u. Mark the u point on the tracing. Join C and u.

Now transfer the tracing to the plotting sheet so that C lies on N and the line Cd lies along the line NU. Draw the perpendicular from the point G to

[7] This is essentially the same as the Survey of India method. See reference in footnote 6.

U on the plot cutting the line Cu (produced if necessary) at G_u. Then the length UG_u is the required difference of height H on the scale of plotting.

That this is so is because by construction the angle UNG_u is equal to $(T-e)$ and the length NU is equal to $M \cos O$. Then from (5)

$$\begin{aligned} UG_u &= M \cos O \tan (T - e) \\ &= H. \end{aligned}$$

When a number of elevations have to be determined from one photograph at the same time the method is very fast, especially if set squares or transparent right triangles are used for setting up the perpendiculars from g and G. However, to maintain accuracy great care is required in making the necessary construction.

Horizontal Position of a Point from One Station

If formula 6 is used in order to determine M when Z_c and Z_g are known, it is necessary to solve a quadratic equation Rather than attempt to do this directly a first approximate value of M is obtained assuming no curvature and refraction In other words, assuming H to equal $Z_c - Z_g$ determine a value of $'M$ on the plot which will give this value of H. With $'M$ as argument extract the curvature and refraction element from tables and now assume H to equal $Z_c - Z_g + k('M)^2$ and obtain a second value of M, namely $''M$. Repeat the process until the difference between two successive values of M is insignificant

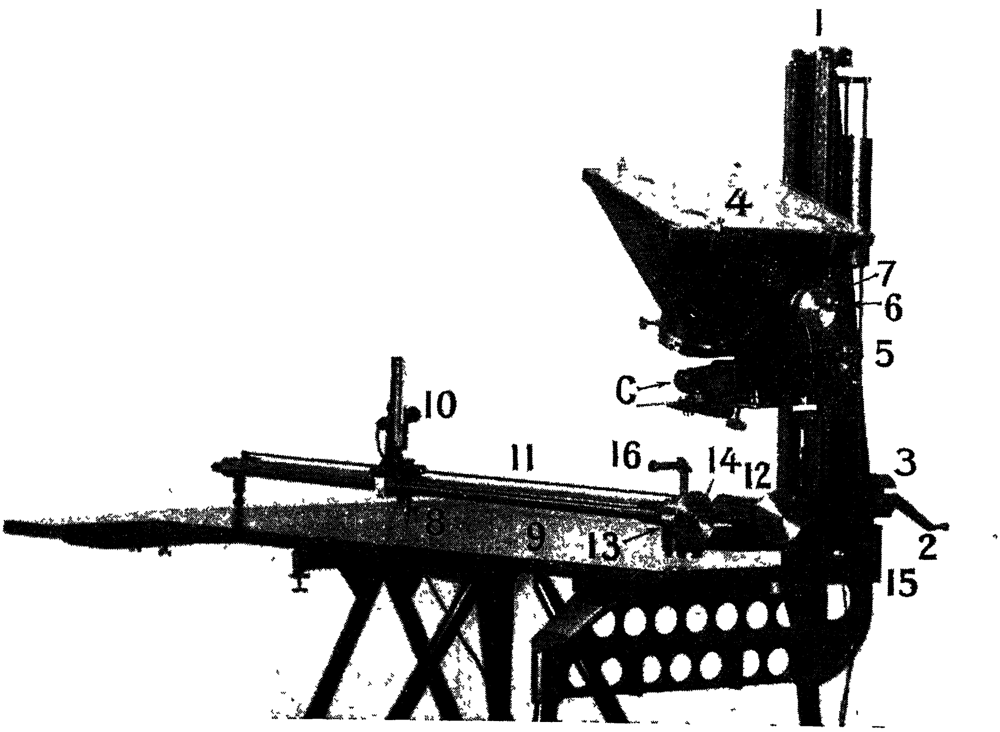


Fig. 23

From consideration of Figure 15, it is obviously poor practice to attempt to fix the horizontal position of a point in this way when M is very large in comparison to H and, consequently, when it is permissible to use the method, never more than two or three approximations will be necessary, as under these conditions the correction kM^2 will always be small

Instrumental Methods of Measurement

The Single Eyepiece Plotter

The single eyepiece plotter, developed in 1935 by the American Geographical Society, is an instrument with the aid of which a considerable number of things can be done Its operation may first be described in simple terms. A photograph is placed in the holder shown near the top of the accompanying illustration in Figure 23 and viewed through the eyepiece, which appears just below and to the right of the uppermost of the three drums. The observer sees a point of light superimposed on the photograph and, by turning the handles on the drums, he can make this point appear to move up or down or to the right or left on the photograph. When the light is so maneuvered, a pencil attached to it is moved on the drawing board and, if the photograph is properly set, shore lines may be plotted, points intersected, and heights determined.

Figure 24 is a diagrammatic outline of the first model of the oblique plotter. A horizontal axis t–t is mounted on the vertical standard (1) so that it can be moved up and down the latter by turning the crank (2); the height of the axis being read on a drum (3). t–t is the tilt axis about which a "pinhole mirror" C, a photograph holder (4) in which a negative is placed, and a viewing apparatus (5) can be rotated together by turning the crank (6); the tilt being read on the drum (7). The mirror consists of a fully reflecting plane surface in the center of which is a pinhole aperture about 0.4 mm. in diameter. The mirror is adjusted so that the tilt axis lies in its plane and the pinhole is centered on the tilt axis The photograph holder is adjusted so that the plate perpendicular corresponding to the optical axis of the camera cuts the tilt axis normally at the pinhole. Slow motion movements are provided for swinging the photograph in its own plane about the plate perpendicular and for setting the photograph at its calibrated focal length from the pinhole, and vernier scales are provided for reading the amounts of these movements. The viewing apparatus (5) consists of a telescope the optical axis of whose objective always points to the pinhole.

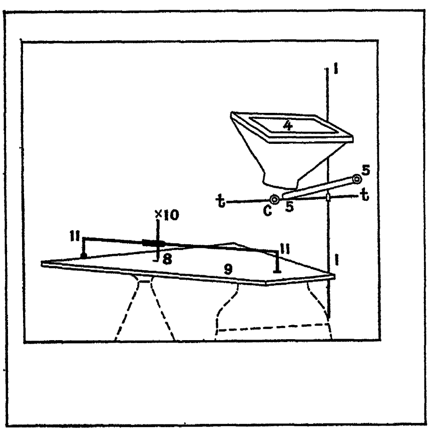


Fig. 24

Attached to and vertically above a plotting pencil (8) which, when operating, touches the surface of the plotting table (9), is an index mark (10) consisting of a small aperture illuminated from behind through a condenser lens. The index mark and pencil are attached to a carriage which is free to move along the horizontal bar (11). The bar (11) is free to swing around the vertical axis passing through the pinhole. By turning the crank (12) the bar is made to swing

and a drum is attached which reads horizontal (O) angles. By turning the crank (13) the pencil and index mark carriage is made to move along the bar and a drum is attached which reads M distances out from the center of rotation. The pencil can be lifted off the plotting table by means of a bar lever operated at (14). The intensity of the illumination of the index mark is controlled by a rheostat situated at (15) and it has been found convenient to make the color of the illumination green or red. The angular scales on the instrument read to the nearest minute and the linear scales all read to the nearest 0.1 of a mm.

It will be apparent from the above description that the pinhole is the perspective center C and that a very thin pencil of rays (virtually a perspective ray) from the index mark on passing through the pinhole is enveloped by a corresponding pencil of rays from a point image in the photograph. The effect to the observer looking through the telescope is to see the image of the index mark superimposed on the photograph. The telescope functions solely as a viewing apparatus whereby the eye is placed virtually at the pinhole.

With the telescope focussed on the photograph the index mark can be seen in sharp definition over a wide range, which on the instrument built is from less than 5 cms. to a meter out from the pinhole.

The operation of the instrument is simple. To plot planimetry once the photograph has been correctly oriented, the operator merely moves the index mark so that its image follows the outline of the feature on the photograph. The scale of the drawing is determined by the ratio between the height of the pinhole above the map and the height of the camera station above the ground. To intersect a point above the map datum level it is necessary to plot first the relative horizontal positions of the two photographs on a plotting sheet and the azimuth lines of their principal planes. The first photograph is taken and correctly oriented, and the sheet is then oriented on the board so that the azimuth line of the photograph's principal plane points at right angles to the tilt axis and the position of the camera station lies directly under the pinhole To facilitate the latter a centering device (16) has been provided. The image of the index mark is then made to superimpose the image of the point to be intersected, and the pencil is placed in operation and a line is drawn on the paper by moving the index mark in towards the operator The same process is repeated with the second photograph and where the two lines intersect on the paper is the position of the point on the map. Then by placing the pencil at this intersection and lowering the tilt axis until the image of the index mark again covers the photographic image of the point the difference of height (H) (Fig. 10) can be read directly on the height scale, the distance (M) (Fig. 9) can also be read on the distance scale and the necessary correction to the height for curvature and refraction can be applied at once from tables.

Recently a second model of this instrument has been designed and may possibly be constructed in the near future. It will function in exactly the same way as the first but will incorporate many improvements of a mechanical and operational character. Chief amongst the latter are fast and slow motions for the M, O, and H movements; optical magnification of the image as seen by the operator; an optical linkage system which will enable the whole of a 90° field of view to be scanned over without moving the eyepiece. Ample space (not true of the first model) on the plotting board for orienting a large sheet of paper without interference from supporting structures and more easily read scales are also provided. Mechanically the new design is more rigid, can be more easily adjusted and can be taken apart for shipment and reassembled without difficulty. It will rest on any strong table rather than having a special stand, as in the first model.

Exterior Orientation

The Method in General

This is the process of determining from measurements on a photograph the position of the air station, the tilt and azimuth of the optical axis of the camera taking the photograph and the swing of the camera at the moment of exposure. The process requires that a minimum of three points whose positions on the ground have already been fixed be imaged on the photograph.

In general the tilt (T), (Fig. 8), of the camera axis and the swing (Δs), (Fig. 7) of the horizon in respect to the sides of the photograph are assumed. With these assumptions horizontal angles to the control points are measured and a horizontal position of the station is determined by the familiar tracing paper method. The height of the air station is then determined independently through each control point and if these heights do not agree the differences are used in solving differential equations which relate the effect of small changes in the tilt and swing to small changes in the height of the air station. Thus, as will later be explained, a consistent solution for the tilt, swing, and height of the air station may be found As will be gathered from examination of formula 11, small errors in the assumed tilt and swing affect the horizontal angles very little so that only when the assumed values of tilt and swing have large errors or when maximum accuracy is desired for control purposes will the horizontal position have to be redetermined.

Choice of Control Points

If only three control points are available the strongest condition for orientation purposes is when two of the control points are imaged in the background of the photograph on either side with consequent large O angles and small V angles, and the third point is imaged in the center of the photograph near the principal point with a small O angle and a V angle approximately equal to the angle of tilt. There are three reasons for this. In the first place, this arrangement insures against the condition when the horizontal position of the air station lies near the circle containing the horizontal positions of the three control points, for when this happens, as every planetabler knows, no accurate determination of the position of the station is possible. Secondly, errors in the assumed tilt and swing will have the least effect on the resected horizontal position. Again this can be readily appreciated by examination of formula 11. Thirdly, if the expression for dV in (10) is inserted in (7) there results the expression

$$dZ_c = M(1 + H^2/M^2) \cos O \text{ arc } 1'dT + M(1 + H^2/M^2) \sin O \cos T \text{ arc } 1'dS \quad (16)$$

which relates the effects of small errors in tilt and swing on the measured height of the air station. Now in the suggested distribution of the control points it will be observed that the differences between corresponding coefficients in this expression between any two control points will be large. The importance of this will become apparent when the actual process of determining the true tilt and swing from the assumed values is described.

In practice it will very often happen that the control points available are not distributed in the best manner for the resection process. Strong determinations of tilt and swing are always possible, however, provided the horizontal position of the air station does not lie near the circle containing those of the control points and that there is a wide variation in the measured O angles and M distances to the control points.

Assumption of Tilt and Swing

In exposing the camera the endeavor is made to tilt it so that the apparent horizon will appear near the top edge of the photograph. If a clear cut apparent horizon is imaged then a good estimate of tilt and swing may be obtained by determining the so-called dip angle (Fig. 8). This angle in minutes equals

$$0.9878\sqrt{Z_c} \text{ (in feet)}$$

or for purposes of a first approximation

$$\sqrt{Z_c} \text{ (in feet)}$$

Therefore, from Figure 8

$$pd = \tan\,(T' + dip)/j$$

where

$$\tan T' = pd'/f.$$

Often in mountainous country or when there is haze the apparent horizon may be only poorly defined or not defined at all. Under these circumstances its position on the photograph must be estimated. It is surprising, however, how accurately this can be done with a little experience. In the Survey of Northernmost Labrador, for instance, the average error in estimating the tilt of a large number of photographs was only 14′ and the greatest single error 1°25′. Only 2 per cent were over 1° in error. In the same test the average error in estimating the swing was 18′ and the greatest single error was 2°56′. Only 5 per cent were over 1° in error.

Having determined the assumed tilt and swing and the tentative position for the point d, it is necessary to indicate these elements on the photograph before measurements can be made. When an instrument is used together with negatives or transparent positives, all that is required to set up the assumed tilt and swing in the instrument is to indicate the position of the points p and d. This should be done very carefully by scratching fine crosses on the emulsion side of the photograph with a needle point. Later when the adjustment is complete small corrections to the assumed tilt and swing may be set on directly by means of the mechanical movements. When graphical methods are used plot the position of p with a fine ink cross and the assumed positions of d and j with similar crosses made with a very soft pencil. Indicate the assumed true horizon line, isometric parallel, and principal parallel with marginal marks on the sides of the photograph also made with a soft pencil.

Tilt and Swing Determinations

With the assumed tilts and swing draw the horizontal direction lines from the perspective center to the three control points and obtain the approximate position of N on the plotting sheet by slipping the drawing under it and orienting them together until the three lines pass through the plotted positions of the control points. Then determine the height of the air station through each control point by the methods already outlined. Now if $'Z_c$ is the height of the air station as determined in this way, then $dZ_c = Z_c - 'Z_c$ Substituting in equation (16)

$$M \cos O(1+H^2/M^2) \text{ arc } 1\, dT + M \sin O \cos T(1+H^2/M^2) \text{ arc } 1\, dS = Z_c - 'Z_c. \quad (17)$$

$M \cos O$ and $M \sin O$ are obviously the distances NU and UG of Figure 4 and are therefore easily derived by direct measurement. Also arc 1, if dT and dS are in minutes, may be given the approximate value 0.0003.

Compute the coefficients of dT and dS for each control point Then by subtracting one equation from another, Z_c is eliminated and a pair of simultaneous equations will result which enable values of dT and dS to be determined. Z_c can then be determined by solving each of the initial equations and the results should agree. Check the adjustment further by setting the corrected tilt and swing or indicating them on the photograph and then remeasuring the height of the air station through each of the control points If the results do not check within reasonable limits the whole process should be repeated, using, however, the same coefficients in the equations. A worked out example taken from actual practice is shown in Figure 25

If graphical methods of measurement are being used, indicate the final positions of the true horizon, isometric and principal parallels with ink lines on the sides of the photograph.

G	1	2	3
M	19,790	31,610	29,980
UG	8,202	−1,535	−8,623
UN	18,011	31,573	28,713
Z_g	828	1,236	1,082
H	501	110	297
kM^2	−26	−68	−61
$'Z_c$	1,303	1,278	1,318
dZ_t	+79	+140	+128
dZ_s	+31	− 5	− 32
Z_c	1,413	1,413	1,414

Equation 17 for $\begin{cases} G_1 \\ G_2 \\ G_3 \end{cases}$

$+5.2\,dT-2.4\,dS=(1,303)$
$+9.2\,dT+0.4\,dS=(1,278)$
$+8.4\,dT+2.5\,dS=(1,318)$

(2−1) $+4.0\,dT+2.8\,dS=-25$
(3−1) $+3.2\,dT+4.9\,dS=+15$

$dT=+15'$

$dS=-13'$

FIG. 25

Only slide rule accuracy is necessary for computing the coefficients and solving the equations.

Refining the Horizontal Position and the Process of Grouping

If only three control points are used in the orientation process a consistent result will always be obtained, but this may not be the true result because of the impossibility of measuring angles closer than one minute or because of poor distribution of control or personal errors of measurement and identification. Therefore, it is always desirable to use more than three control points if these are available. Let us suppose that horizontal directional rays have been drawn to half a dozen or more control points from a station. It is a familiar experience in making a graphical resection to find that some of the rays give an apparently good result at one point and another selection of rays gives a similarly good result at another point. How is a decision to be made as to which is the best of the points or which the best compromise position?

One obviously practicable way of attacking the problem is to use the most distant control in making the tilt and swing adjustment and in determining the

bearing of the optical axis and to make the determination of horizontal position from the near-by control. Another method is the process of grouping. Suppose, for instance, that on one photograph several control points imaged are peaks of a distant mountain range appearing on the right. Others are peaks of a much nearer mountain range appearing on the left, and in the foreground there is a series of fixed points on a river or lake. A graphical resection is made to determine approximately the position of the station and the azimuth of the optical axis Assuming one of the coordinates of the position—say the X coordinate—and the azimuth of the principal plane, the Y coordinate position is computed by means of formula 13 through each of the control points. If the computed Y coordinates do not agree with each other within prescribed limits compute the coefficients in equation (15) for each point and put $dY = Y_c - 'Y_c$ where $'Y_c$ is the computed Y coordinate of the air station. Then solve for dX and dA in the same way as described for determining dT and dS. Y_c can then be deduced from the initial equations.

As only three sets of initial equations are necessary to get a consistent solution, the control points are placed in three groups according to their locations and the coefficients and values of $'Y_c$ of each point within a group are meaned. Thus the finally computed horizontal position is an average position in which the errors within any one group are balanced. If Y is assumed and X computed, formulae (12) and (14) may be used. Y is assumed when the control is more northerly or southerly than easterly or westerly and X when the opposite is the case. The same process of grouping may be employed in the determination of tilt and swing As only slide rule accuracy is needed for the computations, this process of grouping takes very little longer than if only three points are used, but the resulting increase in accuracy is very marked.

Intersections and the Plotting of Detail

Identification of Points

In making intersections from high oblique photographs, the hardest task and the one that consumes the greatest amount of time is the identification of points on the photographs. The strongest intersections geometrically are, of course, obtained if the optical axes of the two cameras are at right angles to each other. Identification, on the other hand, is difficult because the pictures show features from such entirely different aspects. If the optical axes of the camera taking the photographs were parallel at the moments of exposures, the intersections will be acute but the identification of points will be easy, especially if a hand stereoscope is available so that the terrain imaged can be viewed three-dimensionally. In taking the photographs, therefore, the scheme should be such that both types of pairs of photographs are available, the first for obtaining strong intersections and the second for easy identification of points.

In choosing points experience shows that it is best to choose small surface indications such as edges of rock outcrops, abrupt changes in vegetation, or snow patches, appearing near the topographical point that has been chosen for mapping. Culture, such as roads, houses, etc., are always easy to identify and the same applies to streams. Shorelines, on the other hand, if the pair of photographs have been taken at widely different times of the day should be treated with suspicion as far as identification of points is concerned because if the tidal range in the vicinity is at all large, the shoreline may present completely different outlines from one photograph to another. Similarly, it is always desirable to be extremely skeptical in the identification of features near the surface of the water, such as very small islands, shoals, or sand bars.

AUXILIARY PLOTTING SHEETS

When a large number of intersections have to be made from a pair of photographs, it is wise to indicate the points to be intersected on extra prints of the photographs. Auxiliary plotting sheets on which to make the intersections and height determinations are also desirable. If both the auxiliary plotting sheets and the master plot are made of transparent material such as cellulose acetate, it is easy to transfer points and lines from one to the other, and this procedure has the advantage of keeping the master plot clean and free of cluttering construction lines.

NUMBER OF INTERSECTING RAYS

For purposes of checking the accuracy of the work, it is desirable to make three ray intersections, but when the triangle of error by one three ray intersection is found to be negligible, in order to save time in working over a small area, all other points in the immediate vicinity need thereafter only be intersected from the two stations giving the strongest intersections. However, if a point being intersected is to be used later for extending the control it should be intersected from as many stations as possible. If in this case the intersecting rays do not plot precisely to a point the differential methods already outlined in the horizontal resection processes can be used with advantage and the intersecting lines may be grouped as before in order to get a balanced position. Occasionally points on a shoreline at sea level or on a lake, whose height above sea level has already been determined by elevation determinations to other points on it, may be fixed in horizontal position when only imaged on one photograph by the method of successive approximations already described. This method, however, should only be used when the points are not too far away from the air station (see Fig. 15).

ORDER OF PLOTTING

In plotting points and planimetrical features it is advisable to do this in two stages. Only when the culture and the drainage have been plotted over a section should the points whose spot elevations are to be used for contour sketching be chosen. The reason for this, of course, is to break down the terrain into comparatively small areas, for by so doing the process of choosing and identifying the points necessary for adequate contour sketching is much simplified.

TANGENT RAYS

Shore outlines and streams of low gradient can be plotted directly on the single eyepiece plotter or, if an instrument is not available, by the Canadian grid method described on pages 578–586. Such plottings will usually be distorted to a certain extent because they are made from one photograph (Fig. 15). Consequently a sound practice before drawing such outlines on the master plot is to draw tangent rays from other photographs to headlands and embayments or prominent curves on streams and then to fit the detail to these guiding lines. Streams of high gradient should always have well distributed spot elevations determined along their courses.

SKETCHING TOPOGRAPHICAL FEATURES

A knowledge of planetabling and physiography is of great help in the selection of points to be used as control. As in all other mapping processes, experience improves the quality of sketching from photographs. Before sketching a topo-

graphical feature by means of contour lines it should be studied from as many photographs as are available, stereoscopically if possible. Again it is convenient to do the sketching on the auxiliary plotting sheets and only to draw the finished lines on the master compilation.

FIELD WORK

TYPE OF PLANE AND CHOICE OF CAMERA

In mapping projects in which high obliques are going to be used circumstances beyond the surveyor's control generally dictate the type of plane and camera that he will use. This, incidentally, brings out one of the advantages of high oblique surveying, which is that this process of mapping is not completely dependent on special types of planes or precise mountings for cameras. The cameras themselves, therefore, though they should be precise survey cameras, may be hand cameras. When this is so it is obvious that lightness is advantageous, so that cameras having short focal lengths are to be preferred. The airplane used should, if possible, have either a clear range of view on both sides of the line of flight or there should be clear views both forward and to the rear. If neither of these conditions exists and the fields of view from the plane are very limited, the amount of flying necessary to cover the ground adequately with photographs is, of course, increased. If the airplane is sufficiently well adapted it would be an efficient plan to use two cameras and two operators, one working on either side or at either end of the aircraft. The ideal camera equipment would be a multiple lens camera so mounted in the plane that the coverage at one exposure extends around the whole horizon.

PROGRAM OF PHOTOGRAPHY

Experience shows that the height of flight should, if possible, be at least as high above the highest point in the terrain as the latter is above the lowest point. On the other hand, it is impossible to give any general rule as to the frequency of exposures or the separation of the flight lines in high oblique surveying. The tables in Figures 12 to 15 will be of aid, but nevertheless each mapping project has its own individual problems and in planning flight lines consideration must be given to the intended scale on which the map is to be published, to the type of country being photographed, its ruggedness, etc , the meteorological conditions in the area, the time of day when the photographs should or must be taken, and the types of airplane and camera available. Even then, when all the contributing factors have been studied, it is largely a matter of individual judgment as to what will be the best scheme.

As has already been remarked, in order to obtain strong intersections but easy identification of points the scheme of photography should be planned as far as possible in such a way that 50 per cent of the photographs are taken at right angles to each other and 50 per cent with parallel axes. One special point should be emphasized. The photographer taking the photographs should know and understand how they are to be used later. Furthermore, he should be encouraged to use his own initiative and take additional photographs beyond those planned in order to get especially good views or when he sees that if they are not taken certain features will be hidden.

INITIAL GROUND CONTROL

Under ideal conditions where secondary or tertiary triangulation systems cover the terrain being mapped, it is still necessary to put in additional ground

control. This is because small survey beacons are generally not identifiable from high oblique photographs. This is clearly brought out in Figures 26 and 27.

The additional control would consist of points intersected from the triangulation stations which can be readily identified in the photographs. It is highly desirable, therefore, to have this field work done after the photographs have been examined. Unfortunately, this will rarely be possible in practice.

Local Triangulation System

The more usual circumstances are that no triangulation system exists and that a ground field party must put in whatever control they can within the limited period during which the reconnaissance and photographic flights are being undertaken. Under these circumstances the most efficient ground survey work will consist of small local triangulation nets extended from small measured base lines fixed geographically by astronomical methods. From the extended local triangulation stations prominent features in the landscape will then be intersected.

Auxiliary Ground Photographs

As it is very difficult for the surveyor on the ground to know what points are likely to be conspicuous from the air and to safeguard against his intersecting nothing but useless points, it is highly desirable that he be provided with a surveying camera Then at the local triangulation stations, instead of making sketches for identification purposes and taking rounds of angles with a transit, he will cover the whole circumference of the horizon with photographs. This procedure saves a great deal of time in the field, especially as the necessary and desirable intersection of points for control purposes may be made later from the ground photographs in the home office using similar techniques to those already described.

Extending Control

In reconnaissance surveying from high obliques it sometimes happens that there is no time to put in even these local triangulations When this is the case the photographs can still be used to make a map by extending the control through the photographs by alternately intersecting and resecting, but the scale and orientation of the map will then only be approximate and will depend to a large extent for this on previous work in the area by others, such as independently determined positions in latitude and longitude and small local surveys.

General Office Procedure

If a plotting instrument is available one set of paper prints in addition to the negatives or transparent positives should be made. If graphical methods are used two sets of prints should be made, one for measurement purposes and one on which to mark the identification of points.

Before the field work has been commenced a map from all available sources should have been compiled for planning purposes. As soon as the photography is completed and the photographs have been received in the home office together with navigation and flight data, the approximate courses of the flight lines and the positions of the exposure stations should be plotted on this preliminary map. When this has been done the individuals who are actually going to construct the map should then learn the country from the photographs. It saves time in the long run to spend plenty of time on this. In fact, those who are undertaking the work should not attempt to start mapping until they feel that they know

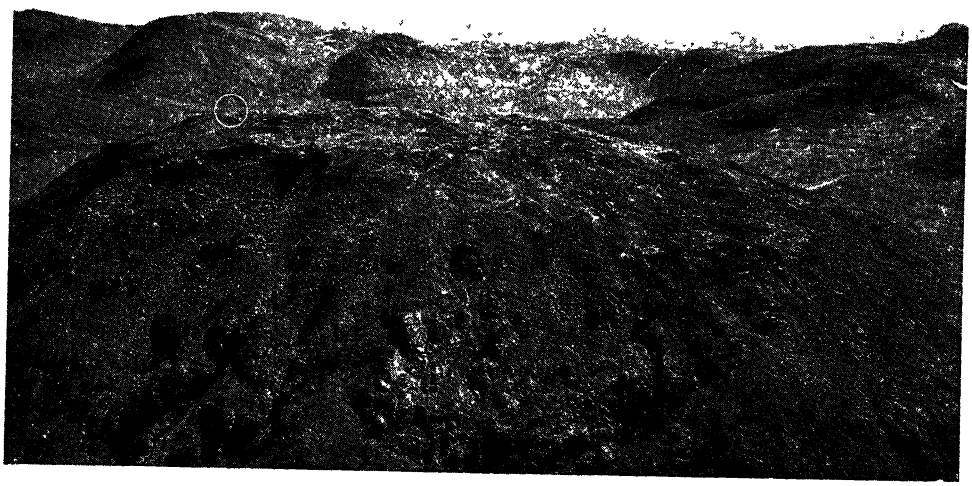


FIG 26 (top) and FIG 27 (bottom) are much reduced reproductions of two air photographs, the first taken at a low altitude and the second from about 7500 feet. They both show the same hill on the top of which was a survey flag 7 feet high. In the original photograph of FIG 26 the flag is just discernible within the white circle

ILLUSTRATIONS

SECTION I

GENERAL INTRODUCTION

A. *Purpose.* This paper describes, for purposes of reference, a method of compiling cartographic data from trimetrogon photography for the rapid production of reconnaissance maps or charts.

B. Scope. This paper includes a description of the method of taking trimetrogen aerial photography; a brief treatment of the theory of planimetric mapping from high obliques; a detailed description of the steps involved in the preparation of planimetric maps from trimetrogen photography; and a description of the material and equipment used in this method.

C. Trimetrogon Photography. 1. GENERAL. In the United States the tri-lens system of map compilation is now usually designated as the "Trimetrogon" method. The word "Metrogon" is the commercial name of the kind of lens generally used, but the method of compilation is not affected by the kind of lens nor its focal length. This type of photography is accomplished by an assembly of three cameras, one pointing vertically downward in the conventional manner and two cameras mounted obliquely, pointed in directions perpendicular to the line of flight. The two obliquely mounted cameras are so placed that they will photograph both the horizon and a small part of the area covered by the vertical camera. All three cameras are exposed simultaneously and thus photograph a strip of ground extending from horizon to horizon in a direction perpendicular to the flight line. (See Fig. 1 for an illustration of a trimetrogon camera installation.)

For purposes of small scale reconnaissance mapping, trimetrogon photography, as compared to standard vertical photography, has the following advantages.

a. A single flight covers a wider strip of terrain, thus permitting the distance between flight lines to be approximately six times as great.
b. The highly precise flying required for vertical photography is not essential, permitting photography to be obtained under adverse conditions.
c. Variations in altitudes, directions, and spacing of flights do not form serious compilation hazards.
d. The need for geographic control is greatly reduced.
e. Per unit area, the requirements for personnel, planes, cameras, and film are much lower.
f. Much less time and money are required for compilation.
g. Excessive tilt of the photographs causes no serious problem or inaccuracy, in this method of compilation.

2. CAMERAS USED. Either the K-17 or K-3B type cameras are used in the trimetrogon assembly. In either case they employ the six-inch cone equipped with the Metrogon wide-angle lens and the A-5 vacuum back magazine exposing a 9×9 inch negative. These cameras have an angular coverage of approximately 74° as measured across the axis of the focal plane. Thus three of them provide more than enough coverage for the required 180° from horizon to horizon. While the lenses used in these cameras are called six-inch lenses, their focal lengths actually may vary from 149.2 mm to 156.6 mm.

3. INSTALLATION. In a trimetrogon assembly the two oblique cameras are mounted with their optical axes inclined at an angle of 30° downward from the horizontal; one pointing to the left of the aircraft and the other to the right, plus a third camera mounted with its optical axis vertical. This mounting provides for an overlap of approximately 14° between the fields of coverage of the

oblique and vertical cameras. With the aircraft in level flight the oblique cameras cover approximately 7° above the horizon.

The installation of the cameras of the trimetrogon assembly has been somewhat variable, depending upon the type of plane used and other conditions. No system of installation is in operation at present which will permit the cameras to be installed in the aircraft in exactly the same manner time after time. Generally, the installation is of a fixed nature with three cameras rigidly secured to the frame of the aeroplane. The vertical camera could be mounted on a floating type of suspension and controlled manually in order to maintain it in a vertical position; however, this condition would be undesirable since it is very important that the angular relationships existing in space between the three camera axes remain the same throughout the flight. With the cameras secured to the frame of the aircraft, it is not possible to correct them for the tilt or crab of the plane, which presents some difficulty in compilation. However, the only feasible improvement in installation would be to secure the three cameras to a single mount, adjustable for tilt and crab. The angular relationships between the three cameras may vary considerably between separate assemblies but this is of little consequence. The effect due to the distance between the cameras in the plane becomes negligible since the photography is performed at relatively high altitudes and it may safely be assumed that all three exposures are made from a common point.

4. EXPOSURES. The operation of the three cameras of the trimetrogon assembly is controlled by a single intervalometer. At the moment of exposure the intervalometer causes solenoids in the bodies of the three cameras to be actuated simultaneously. The action of the solenoids is translated through a mechanical linkage to the shutter release in each lens and thus opens and closes the three shutters. It is possible for the mechanical linkages in the three cameras to have varying degrees of free play which would thus cause exposures to be produced at slightly different times. This possible interval between normally simultaneous exposures can be but a small fraction of a second and probably is negligible.

5. FLYING. Flying for the trimetrogon photography is performed in practically the same manner as it is for vertical photography. For maximum efficiency exposures should be made in straight and parallel flight lines at an interval that will secure a sixty per cent overlap of the consecutive vertical exposures. A flying height of approximately 20,000 feet above mean terrain is generally accepted as a desired flying height. Occasionally, in inaccessible areas, it is desirable to photograph at lower altitudes in order to avoid layers of clouds However, the time required for compilation per unit area varies inversely almost as the square of altitude of the plane.

It is important that all three cameras be kept in satisfactory operation throughout the flight, unless one of the obliques is merely photographing water or dense clouds. It is most important that the vertical camera be in operation continuously as the obliques alone cannot be compiled without sacrificing much time and accuracy. In so far as possible the flight line of the aeroplane should be such that the vertical camera will not photograph large water areas, as it is practically essential that images appearing on each vertical photograph be identifiable on all adjacent photographs in the flight. When it is desired to photograph a large area the importance of keeping all the cameras in satisfactory operation cannot be overemphasized.

6. FLIGHT DATA. For filing and making use of trimetrogon photography certain information supplied by the camera crew should be shown on the negatives. Exposures at ends of strips and at ends of rolls are key exposures and should show the following data:

Flight number
Roll number
Camera mounting (left, vertical, or right)
Exposure number
Date
Project number
Serial number of lens
Focal length of lens
Approximate altitude of flight
Geographical coordinates of each end of flight

Photographs intervening the key exposures, commonly called intermediate exposures, should show the following information on the negatives:

Strip number
Roll number
Camera (L, V, or R)
Exposure number
Project number

7. PRINTS SUPPLIES FOR COMPILATION. *a. Type of prints.* Only contact prints are used in the photogrammetric compilation. To obtain a good degree of accuracy in mapping from trimetrogon photography, it is necessary that the prints to be used for the actual compilation be made on a good grade of double

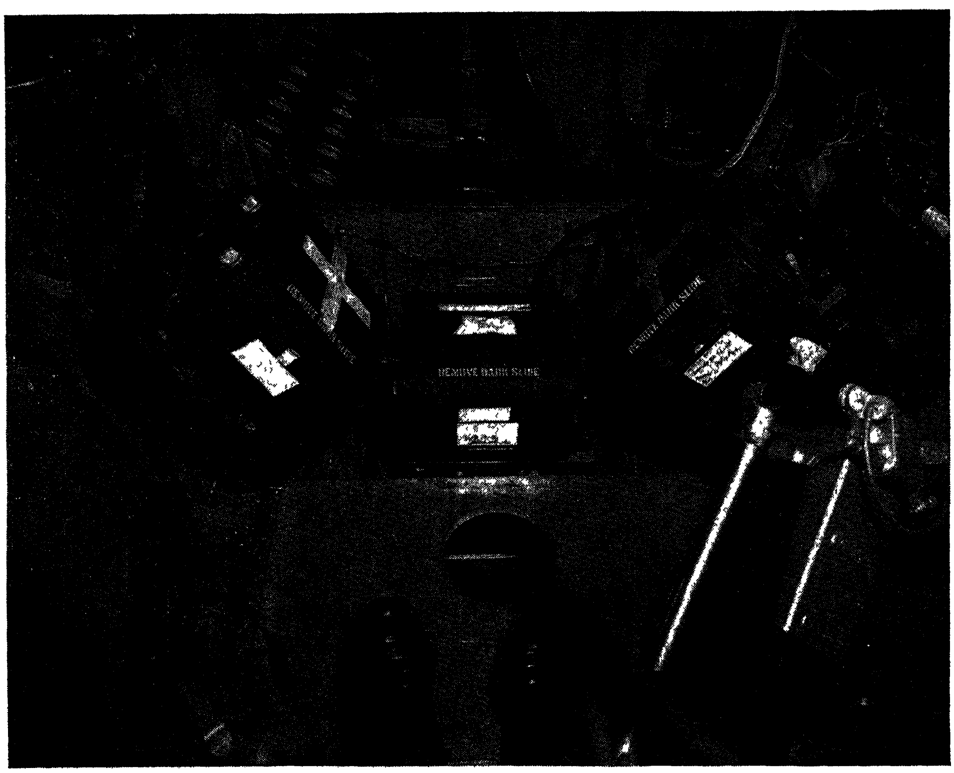


FIG. 1. Trimetrogon camera installation.

weight paper. Single weight prints can be utilized to a satisfactory degree of accuracy, but unless they are generally free from curling much time will be wasted during compilation. Prints should be washed sufficiently to be free from hypo stains. For best definition of detail, a medium to contrasty tone is desirable.

b. Fiducial marks. Contact prints of the photographs required for compilation should have all fiducial marks shown on each print. These marks on oblique photographs are often obscured because they appear against dark sky or portions of the aircraft. They can easily be made to show on the print by inserting a small piece of paper under their location on the negative while printing.

D. General method of compilation. The procedure for compiling small scale reconnaissance maps from trimetrogon photography may be conveniently broken up into at least twelve separate steps or stages. The procedure itself is based upon the principles employed in compiling vertical photography by the radial triangulation method.

When photographs are received from the photographic units they are indexed on small scale maps and suitable records are made. All geographic control points, if any, are identified on the photograph and shown on the project strip diagram. The relative orientation of all three of the cameras is determined. The tilts of all the vertical photographs is determined and the positions of the nadir points are designated. Pass points consisting of points common to the vertical and oblique photographs, distant red points common to successive obliques, tie points common to adjacent strips of photography, and detail points are selected and identified on the photographs. The azimuth lines on the verticals connecting the nadir points are established. Horizontal radial directions from the nadir points to all types of pass points on the oblique exposures are determined and represented on paper templets by means of the Lewis Rectoblique plotter. The paper templets sometimes are directly combined but generally they are used as the basis for making metal templets, which are assembled to form the main radial triangulation network that serves as the framework for the map compilation. When ground control is available it is plotted on the proper projection, scale approximately 1:80,000, and the metal templet layout adjusted to it Additional radial intersections to be used for controlling planimetric detail are established by orienting the paper templets to radial intersections previously established by the main radial triangulation network. The physical and cultural (planimetric) features to be shown on the compilation are delineated on the photographs with the use of the stereoscope and then transferred to the manuscript (planimetric base) by means of vertical or oblique Sketchmasters. Alternate photographs are used in proof reading the compilation for interpretation of features. The entire compilation is reduced from the usual compilation scale of approximately 1:80,000 down to 1:500,000 and adjusted to the best available base and geographic control plotted on the required projection. The necessary drafting incidental to completion of the compilation is then performed.

The method of compilation of reconnaissance maps described in the following pages can be used under a wide variety of conditions with considerable success, because:

1. Only a small part of the personnel need be experienced photogrammetrists, the rest being trained to do two or three simple operations.
2. The equipment needed is simple, portable, and easily obtained.
3. Maps can be compiled with great speed, using factory production-line methods.

The ability to produce reconnaissance maps quickly sometimes is most important. The trimetrogon process is particularly well adapted to speed of compilation. For example, it has been reported that 80,000 square miles (an area larger than that of such states as Nebraska or Missouri) was compiled and published a week after the photographs were received in the United States.

SECTION II

POINT IDENTIFICATION

A. Determinations affecting the point identification procedure.

Before the radial control points are picked on a strip of photographs there are two operations to be performed which are similar in any photogrammetric compilation process.

1. Suitable index maps or diagrams showing coverage and relative positions of the photographs should be prepared.
2. The availability of geographical control should be investigated. The Trimetrogon compilation method uses all usual sources and kinds of control. (It should be noted that the prismatic astrolabe has furnished much of the control used in trimetrogon compilation to date.)

The supervisor in charge of the point identification section must inspect all the photographs of a strip and make decisions regarding the following questions:

1. Have all the photographs needed for the compilation been supplied? (If any are missing they are requisitioned from the photographic laboratory.)
2. Can the entire strip of photographs be satisfactorily compiled without any breaks? (If two or three adjacent vertical photographs were exposed over large bodies of water or over cloudy areas, azimuth lines probably cannot be carried forward through those pictures. Confer with heads of other units as to the exact method of bridging the gaps. Sometimes it will be found that small breaks can be by-passed by tying in the rays of the metal templets to rays from adjacent strips.)
3. Are other strips of photography to be combined with this strip into a single compilation? (If so, tie points between the strips must be carefully selected.)
4. Is the entire strip to be compiled? (Some of the obliques may not be suitable for compilation, or some part of the strip may have already been compiled from previous photography.)
5. Which obliques are to be used in the compilation? (Decide whether to use the odd or even numbered photographs for the templets and sketching. Always be consistent within the strip. Never use the odd pictures of one oblique wing and the even pictures of the other.)

In order to make the instructions of this manual as clear as possible, those exposures which are to be used in the compilation procedure, are spoken of as "Alternates."

The decisions resulting from this inspection must be shown on a Strip Diagram (See paragraph D of this section) which is prepared by the point identification section.

B. Pass points. 1. GENERAL. A pass point is a strategically located and designated image point that appears on each of several overlapping photographs and which can be fixed in position on the manuscript by means of radial intersections. Pass points are picked *only* on those exposures for which paper templets will be made. In addition to certain other functions they perform, *all pass points are used to locate on the manuscript the correct position of the planim-*

etry appearing on the photographs. The kinds of control points used in the compilation procedure are listed below in the order in which they are usually picked:

Tie points between flights	Blue circles
Pass points common to verticals and obliques	Red circles
Distant red points	Red circles
Detail points	Yellow or black circles

Because of the larger number of points intersected and located in the trimetrogon method, it has been necessary to subdivide the general term "pass" point into the four classes noted above.

The picking of pass points in general means the selection, identification and designation of certain strategically located image points as they appear in each of several overlapping photographs, where.

Selection means the choosing of a suitable location within the area of common overlap.

Identification means the finding of a particular small image point on each of several overlapping exposures within the area selected.

Designation means distinguishing the character of the point by means of a colored circle. (Usually of 0.10 inch inside diameter.)

When the term "common image point" is used in this training manual reference is made to some particular small object on the ground as it appears in each of two or more overlapping photographs. The term "common image point" therefore always refers to two or more photographs and the corresponding photographic images of a given point on the ground.

2. PROCEDURE. *a. General instructions.* The proper selection and identification of the pass points is of foremost importance. In order to insure that this operation will be performed correctly, *the following instructions should be followed in detail when picking the various kinds of control points described in this chapter:*

1) The operator should never select and identify a point unless he is sure that it is correct. (If the operator cannot be sure that a certain point is correct, he should get help because a bad point is much worse than none at all.) The pass point is the image point at the center of the circle, and is not the circle itself.
2) In picking pass points, examine the photographs and determine the size and shape of the area within which a certain pass point must be located.
3) For the location of pass points (on both verticals and obliques) select wherever possible, very small, well-defined image points just about the size of a needle hole. The beginner will find that, as he gains experience, he will be able to find and clearly identify many small images on the exposures that he would not have noticed originally. This is important to the beginner who wants to learn how to do good work as quickly and as easily as possible.
4) Except as a last resort, it is not advisable to pick pass points on the sides of curves, such as lakes, islands, etc. Unless definite image points are selected, control points cannot be satisfactorily transferred to a number of other photographs.
5) On obliques, the beginner is cautioned not to use as the location of a pass point the "end" of some fairly small lake or island. In other oblique exposures this "end" point will actually be around on the side of the object, due to changes in the perspective of oblique photographs. On the other hand, the center of a very small lake can sometimes make quite a satisfactory location for a pass point.

6) If a pass point cannot be transferred put a cross in the approximate location to indicate that it was sought.
7) Pick the detail points on the top and bottom of slopes—never midway up the slope.
8) Do not pick any detail points on the vertical photographs.
9) There is a small triangular area between the principal point and the pass points on an oblique in which two rays cannot be obtained unless every oblique is used in the compilation. Therefore, the operator should not pick pass points in this area.
10) Great care must be exercised in selecting and identifying points on the obliques because the appearance of features change greatly from the first to the last exposure in which they may appear.
11) In picking pass points on oblique photographs covering mountainous terrain, it is desirable to pick the points either on high elevations or out in the middle of the valleys, so that the positions cannot be obscured by adjacent mountain tops on other photographs of the area.
12) The operator should be very sure that all pass points are transferred to all photographs on which they can be located. The time lost in locating a missing point is many times greater than it would have been to pick the point in the first place.

b. Tie points A tie point is an image point that can be identified in the common overlap between two adjacent strips of photography and which is designated with a blue circle on each of the several overlapping photographs in which it appears. Tie points are used in maintaining correct scale and direction between two adjacent strips in the metal templet plot. Selecting the location for a tie point is rather difficult, and great care must be used in correctly identifying the image points, not only on photographs of separate flights but also on all overlapping photographs of each strip. The index map, or strip diagram, will help indicate those portions of strips where tie points are needed. If possible, a sufficient number of such points should be selected so that there will be three well placed points appearing on each oblique for which a paper templet will be made. These points should be placed in such fashion that they will be equally spaced along an imaginary line lying equi-distant between the two flight lines. Tie points are numbered consecutively on both strips in an exactly corresponding manner.

c. Pass points. Ordinarily a pass point is located on an image point that appears in the area of overlap that is common to any two alternate obliques and their corresponding pair of verticals. Pass points should be picked as close to the outside edges of the vertical prints as possible consistent with positive identification. There are two kinds of pass points used in this compilation procedure:

Red pass points. Those which are used to form radial line intersections on the manuscript and to help accurately orient the paper templet with respect to the corresponding oblique.

Black auxiliary points. Those which are used only in helping to orient the paper templets with respect to their corresponding vertical exposures.

Under ideal conditions a red pass point can be located in each area of overlap that is common to two alternate obliques and their corresponding verticals. Sometimes, however, it is impossible to pick a pass point that will appear on both alternate obliques and their corresponding verticals because of insufficient overlap, crab, clouds, water, etc. In such cases, the pass points will be transferred to as many exposures as possible and then auxiliary pass points picked in such fashion as to satisfactorily fulfill the requirements illustrated in Figure 2.

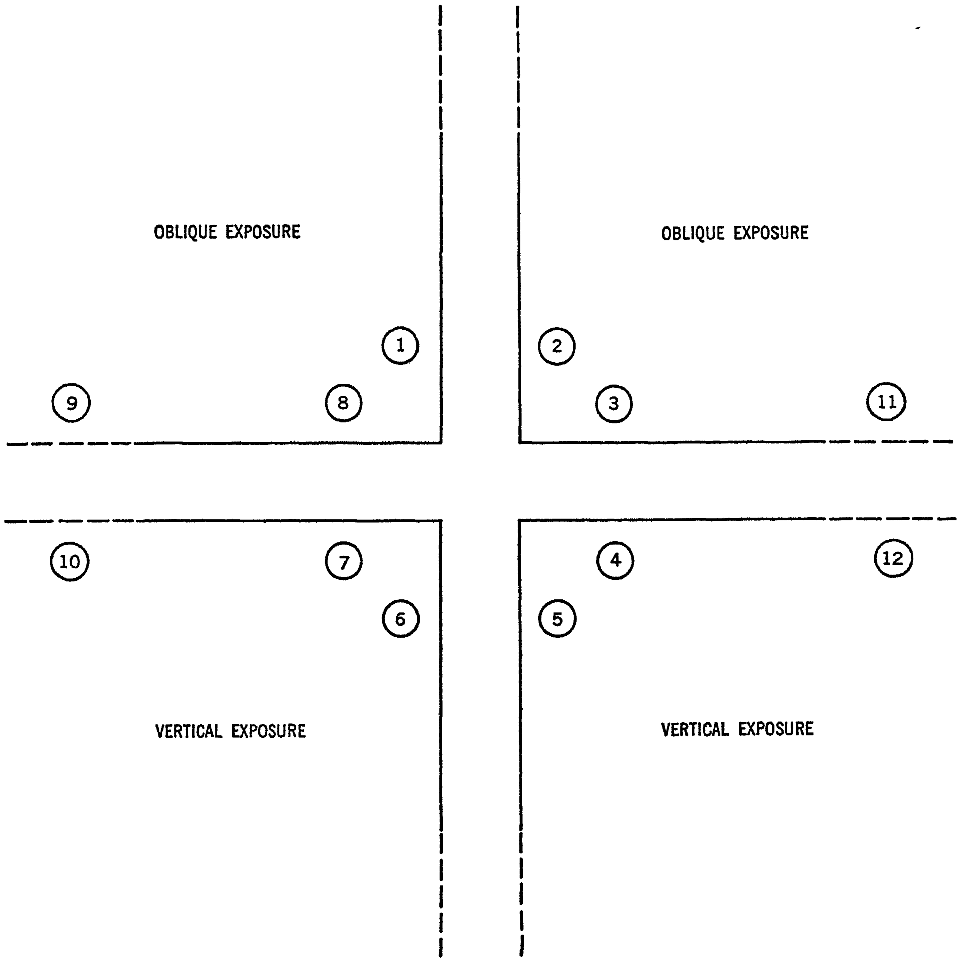


FIG. 2 Location of pass points.

1) In the above illustration 1-2, 3-4, 5-6, 7-8, 9-10, 11-12 are each located on identical pairs of image points
2) Auxiliary pass points such as 3-4, 7-8, 9-10 and 11-12 are necessary in order that paper templets may be checked for accuracy and azimuth lines may be transferred to them. These auxiliary points are circled with black ink, but do not appear on the manuscript.
3) Pass pionts such as 1-2 or 5-6 are necessary in order to provide rays for use in making radial intersections of these points on the manuscript. These points are circled with red ink.

It must be emphasized that, without exception, any pass points similar to 1-2 should be located on an image point located as near to the bottom edge of the obliques as possible. When this is done, it will sometimes be found that 1 or 2 can be transferred to the corresponding vertical, thus eliminating the need of the auxiliary pass point on that side. For similar reasons, it also must be emphasized that, without exception, any pass points similar to 5–6 should be located as near to the outside edge of the verticals as possible.

Under ideal conditions the auxiliary pass points 3-4 and 7-8 will be replaced by the red pass points appearing on the common image point 1-2, 5-6. Nevertheless, auxiliary points corresponding to 9-10 and 11-12 of Figure 2 must always be picked opposite the principal point of each alternate vertical.

The following procedure can be used to advantage in picking pass points:

1) Discard all photographs upon which pass points are not to be picked. (Usually every other exposure.) Stack separately the remaining vertical and oblique exposures at one side of the work space.
2) From the photographs prepared in step 1), take the first two obliques and the first two verticals and arrange them as illustrated by Figure 2. Pass points can be picked most easily with this arrangement of the photographs.
3) Arrange succeeding sets of four exposures in a similar manner to that described in step 2).

d Distant red points. On some obliques it will be impossible to select three tie points because:

1) The same detail cannot be identified in both strips.
2) The flights are too far apart.
3) The obliques have no parallel flight opposite them.

Nevertheless, each metal templet must have at least three long arms on each side, so well-spaced distant red points are selected to take the place of any missing tie points. The distant red points must appear on at least three of those obliques that are to be used in the map compilation. In order that these distant red points will not interfere with the metal arms to the tie points, they will ordinarily be located just inside of that imaginary line determined by the tie points. *In this manner the distant red points will not interfere with the metal arms to the tie points nor to the distant red points appearing on the opposite obliques.*

e. Detail points. Additional points are now selected on the obliques to aid the compilors in transferring important planimetric detail onto the manuscript. Figure 3 illustrates the approximate number and kind of detail points to be located on each oblique covering moderately flat country.

The kinds of pass points appearing in Figure 3 are described below:

NUMBER	COLOR	KIND
1, 2 and 3	Red or Black	Pass points
4, 5, 6, 7, and 8	Yellow	Detail points
9, 10, 11, 12, 14, 15, 17, 18	Black	Detail points
13, 16, and 19	Red or Blue	Tie points or distant red points

In extremely mountainous country about twice as many points must be picked as are shown in Figure 3.

Detail points are picked on important features of the planimetric detail in order that these features may be most accurately located on the manuscript. (For example, the points should be located on small towns, important mountain peaks, important road intersections, bends of large rivers, main ridge lines in mountainous country, etc.) The number of points to be picked will be determined by the accuracy desired, the skill of the men operating the Sketchmasters, the type of terrain, the amount of detail to be shown, and the need for speed in the compilation. Hard and fast rules cannot be laid down and the nature of the circumstances will decide the question. Nevertheless, if possible, it is highly desirable to select each detail point so that it will appear on at least three of the

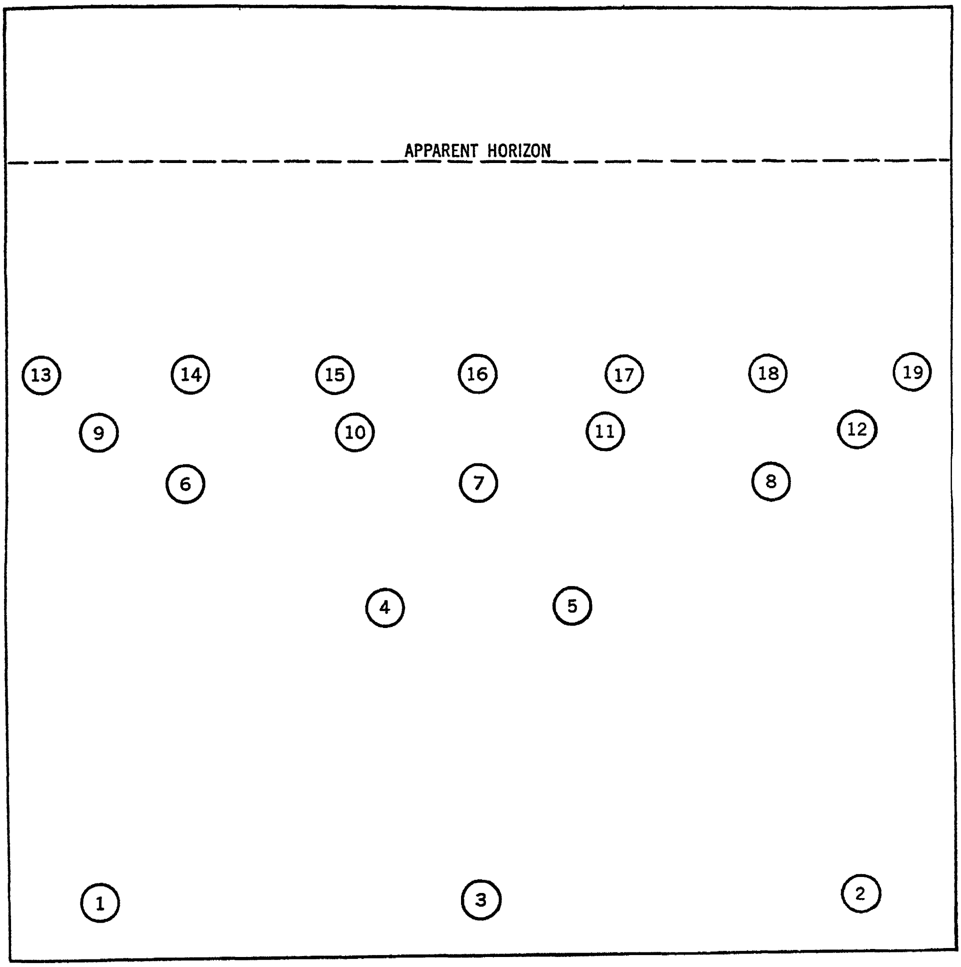


FIG. 3. Location of pass points on obliques.

obliques to be used in the compilation. Ordinarily, detail points should be picked at high points and low points—never on a slope.

An examination of Figure 3 will illustrate very clearly that pass points, distant red points, and tie points also serve as detail points in addition to their other functions. *For this reason all such points should be located so that they will fulfill the function of detail points as well as possible.*

f. The numbering of control points. Wherever considerable significant detail appears in the strip of photography, so that a large number of points are required, it may be found advantageous to number or letter the pass points in the following manner:

Tie points	T1, T2, T3, etc.
Distant red points	Capital letters
Detail points	Numerals (1 to 99)

The numbers help the operator transfer the pass points correctly to all the

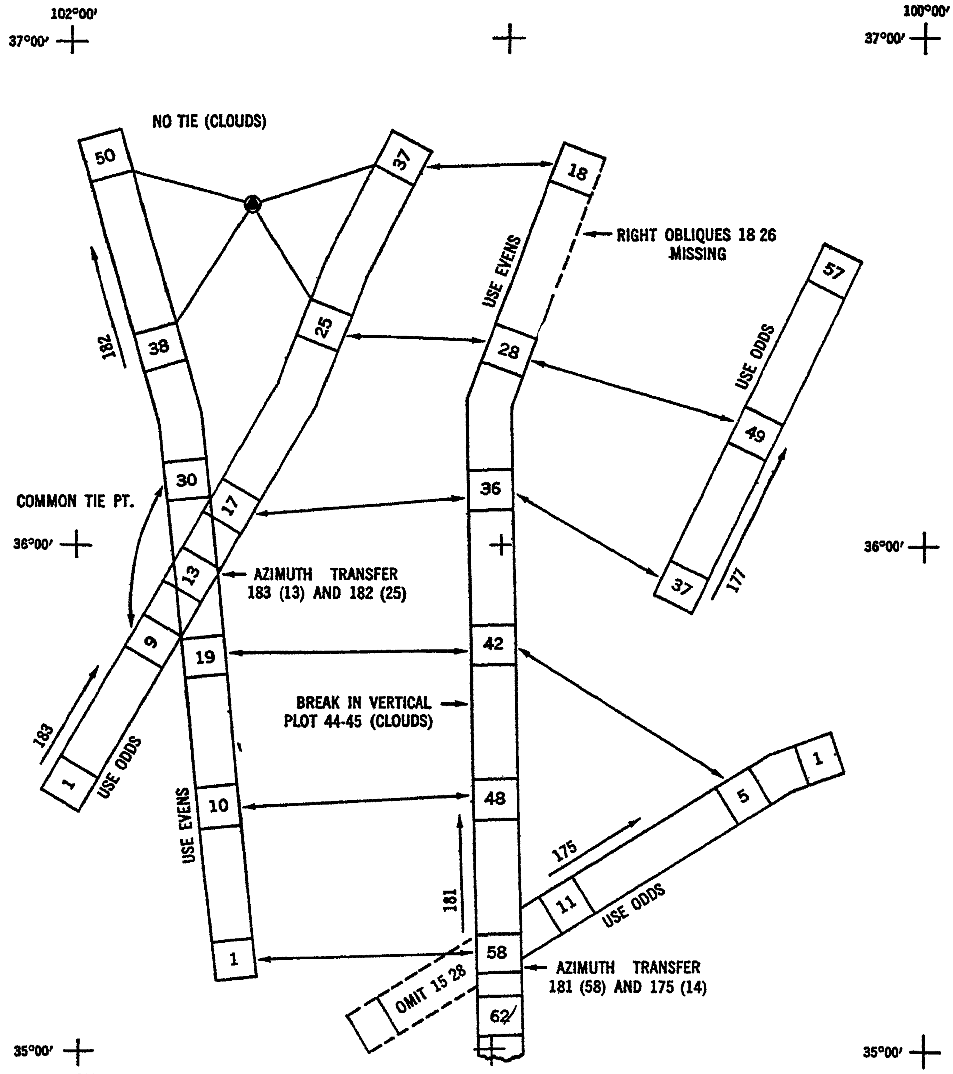


FIG. 4. Strip diagram.

prints on which they should be shown, and in addition, designate the direction rays appearing on the paper templets and the radial intersections appearing on the manuscript.

C. Methods of picking pass points. 1. IMAGE COMPARISON METHOD. Selecting, identifying, and transferring pass points may be done merely by the careful comparison of corresponding image points appearing on adjacent photographs. The instructions given in paragraph B-2-a apply to this method of picking pass points. In using this method the operator must be particularly careful to place each pass point on the same identical image point as it appears on each separate photograph.

2. STEREOSCOPIC METHOD *a. Advantages in the use of the stereoscope.* Although

it takes a little while to learn to use the instrument, the stereoscope is sometimes useful in picking control points because it has the following advantages:

1) Combining two images assists in clearing up foggy or indistinct prints and thus permits the positive identification of image points in areas where points could not otherwise be selected.
2) Prevents mis-identification of image points in the transfer of control points from one photograph to the next.
3) Gives an immediate check on the accuracy of each point as it is picked.
4) Permits the pass points, especially detail points, to be placed where they will be most effective in helping to correct for the displacement of planimetry due to relief.
5) Aids greatly in evaluating the difference in viewpoint so that identical points will be marked.

D. Strip diagram. For each area where the strips of photography can be tied together by common image points, the strips will be compiled as a unit and a strip diagram will be prepared giving pertinent information as to the characteristics of the photography and the relationships of the strip to each other. In a large compilation unit six copies of the strip diagram should be prepared for the use of the various sections. In a small unit, such as a task group, one or two diagrams are prepared and are simply passed along through the various sections. The positions of the strips of photography should be traced from the index map, or constructed proportionally at a slightly larger scale. A high degree of accuracy in the position of the strips is not needed for the strip diagram.

The strip diagram (see Fig. 4) must show:

1. Extent and direction of each strip designated by parallel lines having a spacing comparable to the lateral coverage of the vertical photographs.
2. File numbers of each strip.
3. The numbers, connected by an arrow, of photographs upon which appear image points that are common to two adjacent strips.
4. The exposure numbers of the end photographs of each strip, together with a few other well distributed photograph numbers.
5. Breaks in the photography (either vertical, left, or right) due to clouds, water, faulty camera operation, negative processing, etc , and those photographs having very poor definition of detail.
6. Azimuth transfers between overlapping vertical photographs from separate strips.
7. Whether odd or even obliques are to be compiled.
8. All available geographic control stations.
9. Intersections of latitude and longitude lines together with proper coordinate designations so that the strip diagram may be correctly oriented over a map of that area.

E. Locating principal points. The principal point is the foot of a perpendicular from the optical center of the lens to the plane of the photograph. It is located by the intersection formed by lines connecting the opposite fiducial marks on the photograph. The principal points of all the photographs used in the map compilation should be located in the following manner:

1. Lay a steel straight edge between the left and right fiducial marks on the photograph and make a fine scratch about one inch long with a needle at the center of the photograph.
2. Lay a straight edge along the fore and aft fiducial marks and make a short fine scratch with the needle across the first line.

Section III

DETERMINATION OF THE FIXED GEOMETRICAL RELATIONSHIPS OF TRIMETROGON ASSEMBLY

A. General. The three cameras of the trimetrogon assembly may be installed in a different manner in different types of aircraft. Even though separate cameras may be spaced as much as ten feet apart in the aeroplane, they may be assumed to be exposing from a single position in space since the photographed terrain is at such a relatively great distance from the aeroplane. Under normal circumstances the cameras are rigidly secured to the frame of the plane and an attempt is made to make the vertical camera point directly vertical in normal flight and to have the oblique cameras point exactly perpendicular to the line of flight with depression angles of 30°. As the installation of the cameras is rarely ever accomplished with the ideal geometrical relationships, it will be necessary to determine the slight variations from the ideal situation in order properly to use the photographs. These variations make it necessary to compute the geometrical relationships for each set of three cameras, as they are usually different for each set.

The determination of the fixed geometrical relationships of trimetrogon assembly involves computations and measurements that will determine the relative relationship of all three of the photographs with respect to each other. Normally it is expected that the rigid mounting of the cameras will be maintained throughout a single flight or perhaps several flights. Consequently it is not essential to make determinations for each set of three exposures but for merely enough exposures to ascertain the values of the relationships and the fact that the relationships are consistent.

In determining the geometrical relationships for a strip, computations and measurements should be made at the start and end of a strip as well as for about every tenth exposure. If there is a marked consistency the average is established as standard for the entire strip. As separate determinations are all angular values the data may be considered to be constant if there are variations not in excess of ten minutes. Where the variations are greater it will be necessary to make more determinations to establish whether the part of the strip is consistent or whether the discrepancy is great enough to make necessary separate determinations for each set of three exposures. Separate determinations for each set of exposures will only be necessary when it is apparent that the relationships change by more than ten minutes for successive exposures. Relatively large inconsistencies are attributable to loose mounting of the cameras or extreme vibration in the plane.

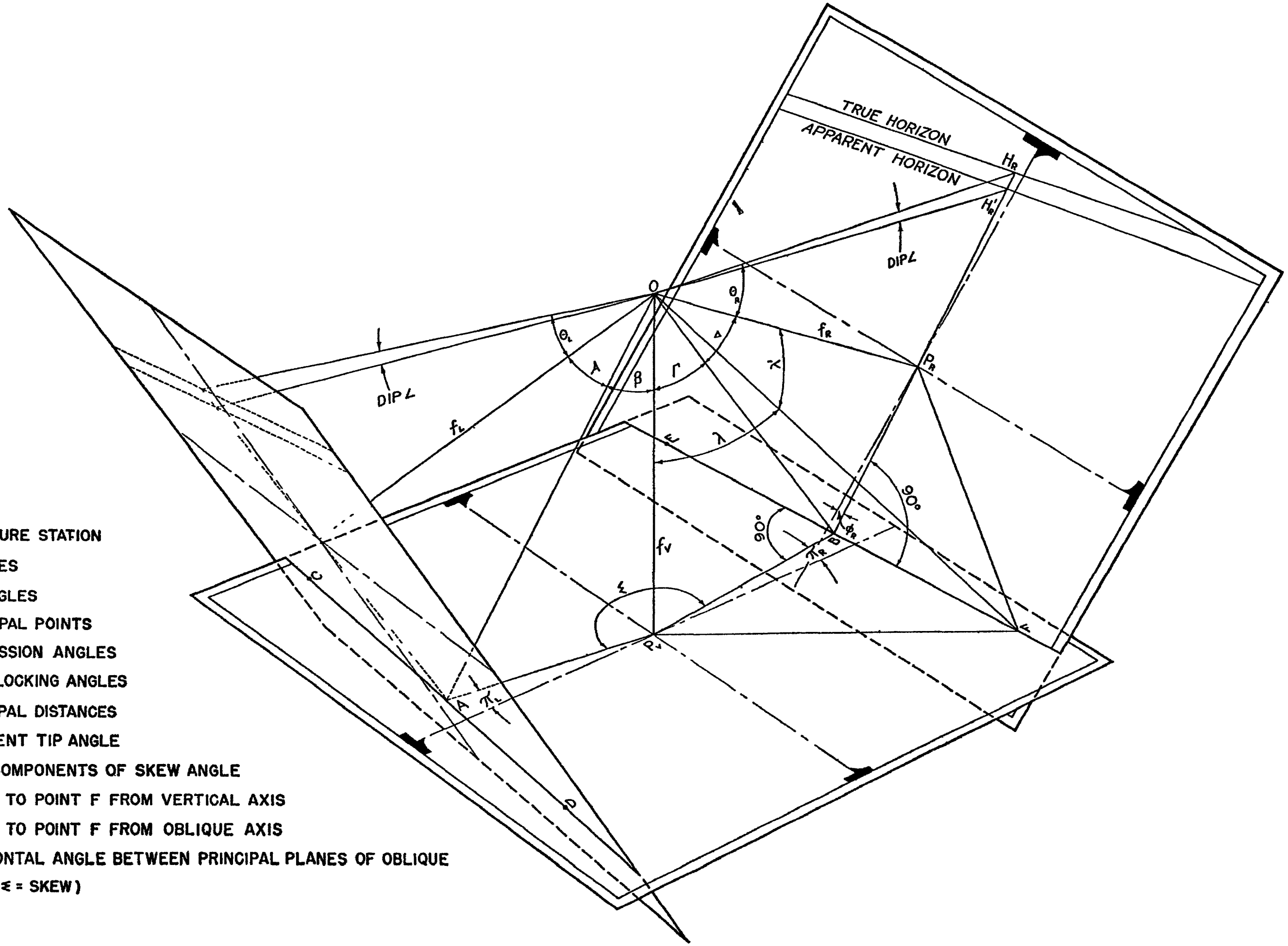
The actual geometrical relationships that are computed for each set of exposures are:

Interlocking angle
Inherent tip
Skew

These values are defined in the following paragraph together with other terms and determinations that are incident to obtaining the relationships. All terms are demonstrated in Figure 5.

B. Definitions:

Exposure station. When related to objective or ground coverage, it is the position



EXPLANATION

O	= EXPOSURE STATION
CD and EF	= ISOLINES
$H_L OH'_L$ and $H_R OH'_R$	= DIP ANGLES
P_L, P_V and P_R	= PRINCIPAL POINTS
Θ_L and Θ_R	= DEPRESSION ANGLES
$(A+\beta)(\Gamma+\Delta)$	= INTERLOCKING ANGLES
f_L, f_V and f_R	= PRINCIPAL DISTANCES
ϕ_R	= INHERENT TIP ANGLE
π_L π_R	= TWO COMPONENTS OF SKEW ANGLE
λ	= ANGLE TO POINT F FROM VERTICAL AXIS
λ'	= ANGLE TO POINT F FROM OBLIQUE AXIS
ϵ	= HORIZONTAL ANGLE BETWEEN PRINCIPAL PLANES OF OBLIQUE ($180° - \epsilon$ = SKEW)

FIG. 5. Trimetrogon assembly.

of the camera in space. When related to the photographic planes of cameras, it is the rear node of the camera lens or perspective center. (O)

Principal ray. Line joining exposure station with principal point and perpendicular to negative plane. (OP_R, OP_V, and OP_L)

Principal distance Distance from exposure station to principal point; equal to f, focal length. ($OP_R = f$)

Isoline. A line representing the inter-section of the planes of the oblique and vertical photographs or the line of common scale on which the same images appear on both photographs (EF)

Interlocking angle. Angle between principal rays of the vertical and oblique camera. ($A+B$), ($\Gamma+\Delta$)

Inherent tip (Relative Y-Tilt) The angle caused by rotation of the oblique camera about its own axis with respect to the plane of the vertical photograph and measurable on the oblique photograph between the isoline and a line parallel to the fore and aft fiducial markers (ϕ_R)

Skew. The deflection angle or 180° minus the angle between perpendiculars from the principal point on the vertical photograph to the isolines of the separate obliques Geometrically, it is the horizontal angle formed by the intersection of the two planes passing through the principal points of the opposite obliques, the principal point of the vertical photograph and the common exposure station. ($\pi_R+\pi_L$)

C. Procedure. The procedure set forth in the following paragraphs is used in computing the fixed geometrical relationships of a set of three exposures consisting of a vertical and its corresponding two oblique photographs

1. ISOLINES. Interlocking angle, inherent tip and skew are all determined from the position of the isoline as established on the vertical and corresponding oblique photographs. As the focal lengths for the vertical and oblique cameras do not vary by more than 1%, it is expedient to use them as equal and thereby determine a substitute isoline which is parallel to the true isoline, a procedure causing no discrepancy since the determinations are all angular. The location of the isoline is then a matter of finding common image points on both the vertical and oblique that are equidistant from the principal point. The following procedure is recommended:

a. Determine very carefully the position of the principal point on the vertical and oblique photograph or check the principal points if previously located

b Using a bow compass describe equal arcs of approximately 4.5 inches radii from the principal points to both corners of the area of common overlap of both the vertical and corresponding oblique photograph. As each pair of arcs from the different principal points pass through a single common image point they can be said to "intersect" each other Two intersections, one in each corner, are established by this operation corresponding to points E and F of Figure 5 and are points on the required isoline.

c. Since the common point on the two arcs may not fall on an identifiable image, an additional check is necessary. Identify a definite image point on both the vertical and oblique that will fall approximately on the line joining the two trial points previously selected. Using a scale divided in 100 divisions per inch, measure on each photograph the distance from the common image point to the principal point. These distances should be the same within .005 inch. If the difference exceeds .005 inch repeat the operation by selecting another small image point and making the required measurements until distances agree within the tolerance. The finally selected point will be considered

to fall on the isoline. Repeat the entire operation for the other intersection in the other corner of the photograph.

d. Using arcs of about 4 inch radius repeat the operation described in *a* and *b* above.

e Using a needle and straight edge, etch a line joining the four common image points on both the oblique and the vertical photograph. These lines are the required isolines All photographic image points intersected by the isoline on the oblique must likewise be intersected by the corresponding isoline on the vertical print.

f. Using the procedure described in steps *a* through *d* above, establish the position of the isoline between the vertical and the other oblique exposure of the set

2. INTERLOCKING ANGLES.

a Using a triangle draw a line in pencil perpendicular to the isoline from the principal point of one of the oblique photographs.

b. Identify a point on both the oblique and vertical photographs at the intersection of the perpendicular. Pick this point very carefully on both photographs It is not necessary that this point be exactly at the intersection but may be located within .05 inch from the perpendicular and as much as 1 inch from the isoline.

c On each photograph measure to the nearest thousandth of an inch the distance from the common point to the principal point.

d. Divide the distances from common point to principal point by the respective focal lengths of vertical and oblique photograph.

e. Obtain from the tables of natural tangents the correct angle to the nearest minute for each of the values obtained from step *d* above

f. Add the two angular values to obtain the interlocking angle. The angle determined from the oblique is A and from the vertical is B as shown in Figure 5.

g. Repeat operations *a* through *f* for the opposite oblique and the vertical giving the angle $(\Gamma+\Delta)$ as shown in Figure 5.

3. INHERENT TIP.

a. On the oblique photograph etch a line parallel to the fore and aft fiducial markers intersecting the isoline at the foot of the perpendicular from the principal point or at a point near the center of the isoline. Use a parallel rule if available.

b. Using a protractor oriented at the intersection of the isoline and the line parallel to fore and aft fiducial markers, measure the angle between the lines to the nearest five minutes by reading both edges of the protractor very carefully. This angle is the inherent tip.

c. Orient the oblique photograph so that the image of the horizon is at the top of the print and note whether the direction of flight is toward the left or right edge of the photograph. In all cases, consider the direction of flight or the direction in which the photograph numbers increase when the photographs are matched for common imagery. Determine whether the oblique photograph is tipped "forward" or "backward" with respect to the vertical photograph by finding whether the isoline at the forward edge of the print, is above or below the line parallel to the fore and aft fiducial markers. If the isoline is above, the photograph is tipped forward—if below, backward.

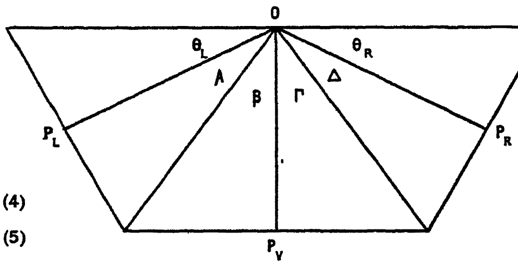
d. Denote the direction of inherent tip as forward or backward by F and B respectively following the angular value expressed in degrees. For example· 2° 00′ F.

e. Repeat steps *a* through *d* for the opposite oblique to determine its inherent tip with respect to the vertical photograph.

4. Skew.

a. On the vertical photograph, etch very carefully a line from the principal point perpendicular to each of the isolines and also draw a segment of this line at the opposite edge of the print.

b. Orient a protractor at the principal point and measure carefully the deflection angle or small angle formed by the intersection of the two etched lines to the nearest 5 minutes by reading both edges of the protractor. This is the skew angle.



File No. 918 (1)

Project 10 ALASKA (2)

Photo No 23 (3)

Lens No

L AF 1056 (4)

V AF 5867 (5)

R AF 9832 (6)

LEFT		RIGHT
F_L 6.044 (7)	F_V 6.032 (19)	F_R 6.071 (13)
I $T._L$ 1°10′F (8)	Σ 30′ 1) 2 (20)	I $T._R$ 45′B (14)
A 30° (9)	H_T 20,000 (21)	Γ 28°40′ (15)
β 30°20′ (10)	Dip 2°20′ (22)	Δ 31°10′ (16)
A+β 60°20′ (11)	Av List 1°00′ R (23)	Γ+Δ 59°50′ (17)
θ_L 29°40′ (12)	Av Tip 2°00′D (24)	θ_R 30°10′ (18)

Tip 3°30′ D (25) — Tip 1°35′D (28)

D A 28°40′ (26) — D. A. 31°10 (29)

Br 'LEFT 1°50′ (27)

Exceptions and Remarks: (30)

Computed by Shulman (31)

Date June 10th 1943

FIG. 6. Camera and tilt data sheet.

c. Observe which edge of the print is in the direction of the line of flight and then note whether the perpendiculars to the isolines form an angle less than 180° toward the direction of flight or in the opposite direction. If the angle less than 180° is toward the direction of flight, denote the direction of skew as $1<2$, if away from the direction of flight, $1>2$.

d. Denote the direction of skew following the value of the measured angle. For example: skew $=2°\ 05'\ 1<2$.

5. RECORDING DATA. The fixed geometrical relationships for the vertical and two oblique photographs are recorded on the "Camera and Tilt Data Sheet" (see Fig. 6). One copy of this form may be used for each separate determination of the geometrical relationships. The final average value for each of the relationships obtained from the several determinations will be recorded on this form and a file maintained of all forms on which the final average values are shown. This form also serves to record the results of tilt determination described in the following chapter. On Figure 6 are shown numbers following the space for the separate entries which serve merely to identify the spaces with the following instructions for recording the data:

(1) File number of strip of photography.
(2) Photography project—name or number.
(3) Photograph number appearing on all three prints.
(4)–(6) Lens number for the left oblique, vertical and right oblique respectively.
(7) Focal length of left oblique photograph. Obtain from data shown at end photo of strip or available records.
(8) Inherent tip for left oblique as determined from paragraph 3 above.
(9), (10) Angle A and B as determined from paragraph 2 above.
(11) Interlocking angle equal to the sum of angles A and B.
(12) $90°-(A+B)$.
(13), (18) Same as (7) through (12) except pertaining to right oblique photograph.
(19) Focal length of vertical photograph.
(20) Skew angle including direction as determined from paragraph 4 above.
(21)–(30) To be completed by procedure set forth for tilt determination in following chapter.
(31) Name of person performing the computation and measurements and the date accomplished.

NOTE: Entries for interlocking angle, inherent tip and skew should be recorded to the nearest five minutes and focal lengths should be recorded to the nearest thousandth of an inch.

6. SUPPLEMENTARY INSTRUCTIONS.

a. There are other methods that may be used for determining the position of the isolines and the angular values of interlocking angle, inherent tip and skew. However, the methods presented in the previous paragraphs are as simple and direct as any of the other methods.

b. The operations outlined in this chapter require more careful and accurate work than any other phases of the procedure. Consequently select only those photographs for use in computing and measuring the relationships on which the images are sharp and do not show large bodies of water. An inferior determination may indicate a variation between the separate exposures that does not exist at all.

Section IV

TILT DETERMINATION

A. General. Tilt determination generally means establishing the position of the plane of the photograph at time of exposure with respect to a horizontal plane A photograph taken with the negative plane level has zero degrees tilt. Oblique photographs purposely have large tilts, and in the trimetrogon assembly the tilts are approximately 60° for each oblique Contrary to the normal means of expressing tilt, the tilt of oblique photographs is sometimes expressed in terms of a depression angle which is the inclination of the principal axis of the camera from a horizontal plane rather than from a plumb line.

In this work the angle of tilt is broken down into two components, x-tilt and y-tilt, and each component is determined separately. y-tilt is generally the component in the approximate direction of flight, and x-tilt is the component approximately perpendicular to the flight line The two components may be combined at any time to give the actual tilt.

In the tilt determination, efforts are directed toward determining the position of the nadir point which, due to the simultaneous exposure of all three cameras, is at a common position and can be located on the vertical photograph. Since the three cameras have fixed relationships (see preceding section), the tilt determined for a vertical photograph can be considered as effective for the entire assembly. Flying conditions cause the tilts for individual exposures to vary. However, in much of the photography the variations between exposures will be less than 1°, yet the actual value of the tilt in the vertical photograph may be much larger. This is due to the fact that the camera is rigidly secured to the plane, permitting no leveling operations, and often the plane is not in level flight.

The effect of tilt displacements of images on photographs is to create slightly erroneous positions on the base sheet as located by the procedure dealing with templets described in the following chapter. However, for purposes of planimetric mapping the positions are not affected enough to necessitate individual tilt determinations for each exposure except in extreme cases. In actual practice an average tilt is determined for the strip, and photographs in the strip that have tilts exceeding the tolerance established for variations in tilt are listed as "exceptions," and the actual tilt angles are determined for them. The first part of this chapter deals with tilt determination of a single set of oblique photographs. The procedure for average tilt determination, together with the standards for the variations, are described in a later paragraph in this chapter:

There are many different methods of tilt determination. However, for this work there is no need from the standpoint of accuracy for a reference to ground measurements, nor is there time warranted for such work. The method described in this chapter is independent of any measurements of ground distances. The simplest method of tilt determination is based on measurement from the apparent horizon appearing on the obliques. Under normal circumstances this method can be used satisfactorily for 98% of the photography. Where the horizons are obscured other methods are available. This chapter will only describe specifically the method based on apparent horizons but will mention the features of the other methods and in what publications more detailed information is given.

Tilt determination involves the use of numerous technical terms that should be understood before tilt determination is attempted. In the following paragraph of this section these definitions are given and a reference to the graphic illustra-

tion, Figure 5 of the preceding chapter. It will be necessary to be acquainted with the procedure and terms used in the preceding chapter to perform tilt determination.

B. Definitions

Tilt. Angle generating from the exposure station between the principal ray and a plumb line, or the angle between the photographic plane and a horizontal plane.

X-axis (Lateral tilt axis) Line joining fore and aft fiducial marks on photograph.

Y-axis. (Tip axis) Line joining left and right fiducial marks on photograph.

X-tilt. (Lateral tilt) Angular component of tilt representing the rotation about the X-axis of the photographic plane from the horizontal position.

Y-tilt. (Tip) Angular component of tilt representing the rotation about the Y-axis of the photographic plane from the horizontal position.

*Raised.** Refers to that part of the photograph that was "raised" above a horizontal plane through the tilt axis at the time of exposure. Scale is larger on the raised side than at the tilt axis.

Depressed. Refers to that part of a photograph that was "depressed" below a horizontal plane through the tilt axis at time of exposure. Scale is smaller on the depressed side than at the tilt axis.

Nadir point. Intersection, on the plane of the photograph, of a plumb line extending through the exposure station.

Apparent horizon. The image of the horizon on the photograph; represents the intersection of rays tangent to earth's surface with the photographic plane.

True horizon. An imaginary line on the oblique photograph representing the in-intersection of a horizontal plane passing through the exposure station with the plane of the photograph. The true horizon is slightly above the apparent horizon and varying with the altitude of exposure.

Depression angle. The angle generating from the exposure station between the principal ray of the oblique photograph and a horizontal plane to the true horizon.

Dip angle. Vertical angle formed at the exposure station between a ray to the apparent horizon and the horizontal plane representing the true horizon, remains constant for given flying height.

Oblique tip. (Apparent Y-tilt) The angle formed on an oblique photograph by the intersection of the apparent horizon, and a line parallel to the fore and aft fiducial markers.

Induced tip. Tip of the vertical photograph expressed in the oblique plane, and also the oblique tip after the correction for inherent tip is applied.

Break angle. The deflection angle between the two vertical planes passing through the common Nadir Point and the principal points of the left and right obliques.

C. Tilt determination from apparent horizons. 1. LATERAL TILT. *a. General.* As the three cameras of the trimetrogon assembly are "locked" together by the fixed geometrical relationships, the lateral tilt may be determined for any one of the three photographs by measurements from the apparent horizon on either oblique photograph. On the oblique photograph the amount of lateral tilt is dependent on the distance from the apparent horizon to the principal point.

b. Dip angle. By definition, this is the vertical angle formed at the exposure station between a ray to the apparent horizon and the horizontal plane representing the true horizon. The value is dependent on the flying height, and is

* In fundamental photogrammetry "raised" and "depressed" refer to the negative plane, consequently the terms have reverse significance when applied to the photograph.

needed to obtain the correct depression angle. The flying height for a strip will be recorded on the first and last photographs of the strip. As this value is usually an altimeter reading above sea level, the value should be adjusted to represent the approximate flying height above mean elevation of the terrain The dip angle is determined from the following formula: Dip Angle $=59\sqrt{H}$, when "H" is the flying height and the dip angle is expressed in seconds. The dip angle should be converted into degrees and minutes.

c. Procedure for determining depression angle.

1) Locate the apparent horizon. This must be done very carefully, using a stereoscope in some instances to avoid confusion with cloud banks. Select three or more points on the apparent horizon, preferably one at the center and one at each edge. Align a straight edge along the points to determine that the point in the center is slightly above the line joining the outside points; this indicates the curving image of the horizon. Draw a short line coinciding with the apparent horizon at the approximate center of the horizon line or, more accurately, tangent to the horizon at a position where a perpendicular from the principal point would intersect the line at the point of tangency.
2) Measure the perpendicular distance from the principal point to the line at the apparent horizon.
3) Divide this distance by the focal length of the camera obtaining the tangent of the vertical angle between the principal ray and a ray to the apparent horizon.
4) Obtain the depression angle for the oblique photograph by adding the dip angle to angles obtained from (3) above.
5) Record the depression angle at the top of the oblique photograph.

d. Procedure for determining lateral tilt on vertical photograph.

1) For either the left or right oblique, subtract the depression angle from the respective depression angle for 0° tilt in vertical photograph (ϕ_L) as obtained from fixed geometric relationships. (See preceding chapter.) The difference is the angle of lateral tilt.
2) Determine whether the right side of the vertical photograph is raised* or depressed* by the following rules:
   a) When tilt is determined from the *right oblique*, if the depression angle is *greater* than the depression angle at 0° tilt in the vertical, the vertical photograph is *raised;* if less, *depressed.*
   b) When tilt is determined from the *left oblique*, if the depression angle is *greater* than the depression angle for 0° tilt in the vertical, the vertical photograph is *depressed;* if less, *raised.*
3) Record in red pencil the X tilt in the upper right hand corner (top of print is toward direction of flight) of the vertical photograph expressing depressed as "D" and raised as "R." For example, 2°00′ R.

e. Tilt from both obliques. For the first few photographs in a strip and for instances where the identification of horizons is difficult, it is advisable to determine the tilt by making determinations from both the left and right obliques. If the tilt determined from each oblique checks within 20 minutes, it is necessary to obtain the tilt from only one oblique. In every case it is advisable to make the determination on the oblique showing the best image of the apparent horizon.

2. TIP. *a. General.* The oblique tip component of tilt is the angle on the oblique photograph formed by the intersection of a line parallel to the apparent horizon and the line joining the fore and aft fiducial markers. Tip measured on

* In fundamental photogrammetry "raised" and "depressed" refer to the negative plane, consequently the terms have reverse significance when applied to the photograph.

the oblique photograph is oblique tip and on the vertical merely tip. Before converting oblique tip to tip, correction must be made for inherent tip and the fact that the induced tip is effective in the oblique plane.

b. Procedure for determining oblique tip.

1) Determine the direction of the apparent horizon by selecting such points on the horizon line as required.
2) Etch a line approximately two inches long on each side of the print parallel to the apparent horizon and passing through the principal point of the oblique photograph. This line is known as the orientation line. Use a parallel ruler if available.
3) Using a protractor measure the angle formed at the principal point betwean the line parallel to the apparent horizon and a line joining fore and aft fiducial markers. This is the angle of oblique tip.

c. Procedure for determining tip for vertical photographs.

1) The side of a vertical or oblique photograph referred to for raised or depressed tip is the side toward the direction of flight. For the vertical photograph this is the top of the photograph (see second paragraph p. 673), for the right oblique, the left side; and for the left oblique, the right side Determine the induced tip and whether it is raised or depressed* by the following rules, which are applicable to both the left and right oblique.
   a) If the line parallel to the apparent horizon is above the line joining fore and aft fiducial markers at the forward edge of the print, and the inherent tip is backward, add the oblique tip and the inherent tip; for these conditions the induced tip is *raised.*
   b) If the line parallel to horizon is above the line through the fiducial markers, at the forward edge of the print, and the inherent tip is forward, subtract the inherent tip from the oblique tip, and if the value is positive, the induced tip is "raised" and if negative, "depressed."
   c) If the horizon line is below and the inherent tip is backward, subtract the inherent tip from the oblique tip and if the value is positive, the induced tip is depressed, and if negative, raised.
   d) If the horizon line is below and the inherent tip is forward, add the oblique tip and the inherent tip; for these conditions the induced tip is depressed.
2) Convert the induced tip to tip for the vertical photograph by multiplying the induced tip expressed in minutes by .86, and express answer to terms of degrees and minutes. (This is a slight approximation, as more correctly the tangent of the induced tip should be multiplied by the cosines of the depression angle.) The same designation of raised and depressed apply to the tip as to the induced tip.
3) Record in blue pencil the value of the angle of tip in the upper right hand corner of the vertical photograph expressing Depressed as "*D*" and Raised as "*R*." For example, 3°00′ *R*.

d. Tip determination from both obliques. Generally the image of the horizon can be identified easier for purposes of tip determination than for lateral tilt. However, when the horizons are especially weak, it is advisable to make a few determinations from both the left and right oblique. Caution should also be exercised to avoid confusing horizon lines with cloud banks extending diagonally across the horizon. Sometimes, however, the actual horizon may not be identifiable but very distant cloud layers may be used to establish the direction.

* In fundamental photogrammetry "raised" and "depressed" refer to the negative plane, consequently the terms have reverse significance when applied to the photograph.

3. DEVICE FOR MEASURING TILT FROM HORIZONS. The work involved in making tilt determinations may be expedited by either preparing reference tables, giving tilt angles for measurements made, or by using a templet that can be placed over the photograph enabling the reading of tilt angles directly, the latter being the more practical. The device illustrated in Figure 7 measures both the depression angle and tip angle as determined from the position of the apparent horizon. It may be made from any transparent material such as cellulose acetate.

The adjustment in the center of the device, calibrated to hundredths of an inch, varies with focal length and dip angle (normally constants for a strip of photographs). In practice the adjustment is obtained from a table showing amounts of correction for these variables A small strip of acetate should be secured to the device at the proper reading on the scale such that the slot is left open at that reading toward the scale for depression angle. To read a depression angle a needle is placed through the slot and held at the principal point opposite the proper reading on the central scale permitting the depression angle to be read directly. To determine the tip, the device is oriented at the principal point and the top fiducial mark. The measured angle may be plotted by the lower protractor scale to obtain a line passing through the principal point and parallel to the apparent horizon.

The device incorporates a small negligible error for depression angles varying considerably from 30°. However, it does make possible rapid tilt determinations within a 15 minute accuracy.

D. Tilt determination by other methods. 1. GENERAL. The need for other methods of tilt determinations is brought about by situations where it is impossible to make the determination by measurements from the apparent horizon. In each case it is advisable to attempt to make tilt determinations from the apparent horizons, but if the horizons on both obliques are obscured, or extremely inconsistent results are obtained, then the other methods described below should be used. In making use of the method based on apparent horizons all determinations are made from the oblique photographs whereas the other methods involve working from vertical photographs and transferring tilt data through the established geometrical relationships to the oblique photographs. The same principle for converting lateral tilt and tip from oblique to vertical photographs, as described in the determination of tilt from apparent horizons, are applicable for the other methods.

2. ABSOLUTE TILT. The term "absolute tilt" is assigned to a particular method of tilt determination The method is based on selecting two lines on a vertical photograph that also appear on the adjacent photograph in the strip. Lines are so selected that by comparison of their lengths a ratio change is determined giving a function of actual tilt in a certain direction. The method is only reliable when it is possible to select lines, the terminals of which are at approximately the same elevation, or the elevations may be determined or accurately estimated. It has little practical value in rough terrain where it is impossible to establish values for the elevations of the terminals of the lines within 100 feet. This method is fully described in the publication titled "Scale Determination of Aerial Photography," compiled by East Central Division, Agricultural Adjustment Agency, U. S. Department of Agriculture.

3. FLIPPING SYSTEM. The term "flipping system" is given to a method of carrying forward tilt relationships from one vertical photograph to successive photographs in the flight. It does not enable actual tilt determination for a particular photograph, but it does permit establishing the relative tilts between successive exposures. Fundamentally, it treats tilt in two components—tip and

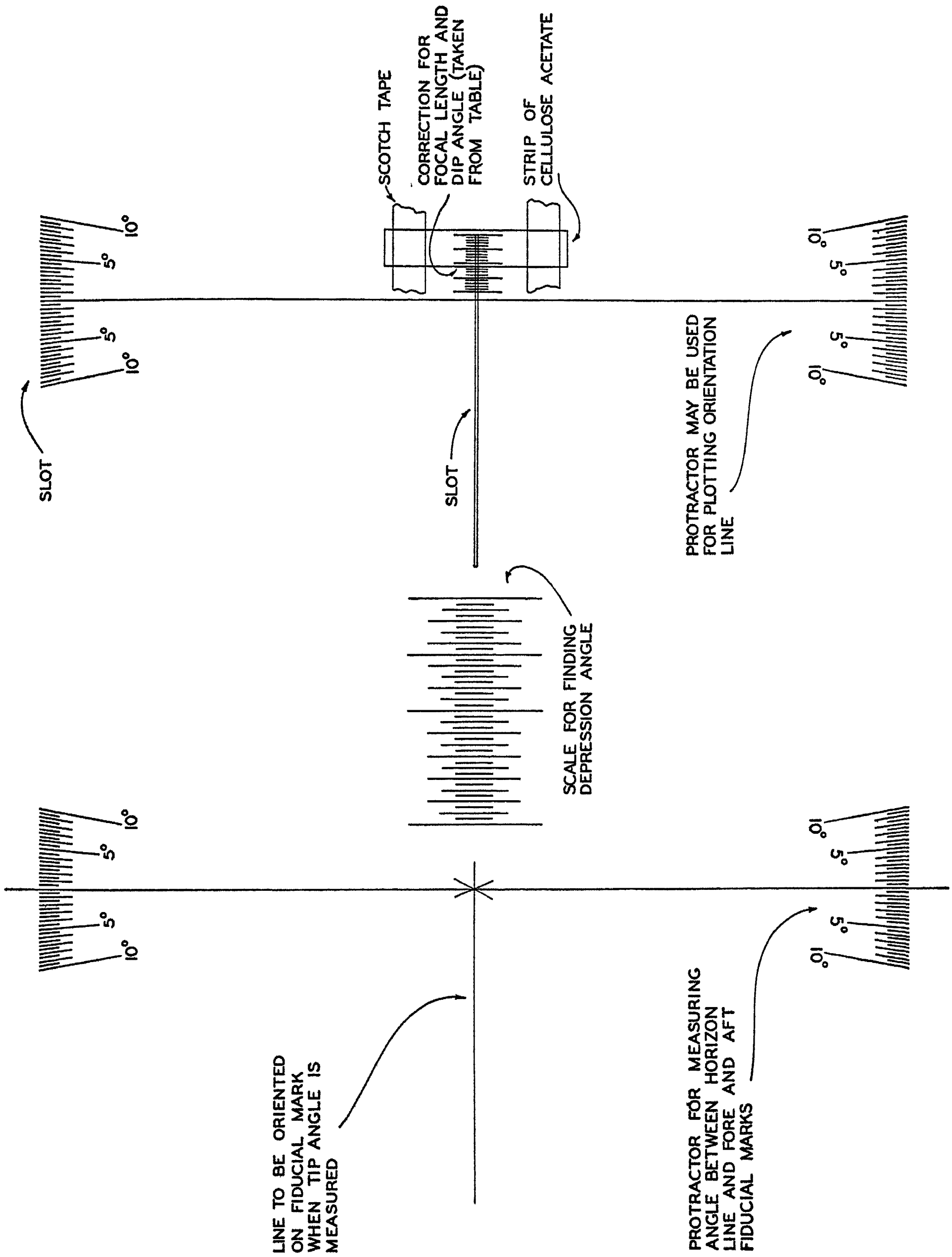


FIG. 7. Templet for measuring tilt from horizons.

lateral tilt—and carries the relationships separately from the successive photographs. Involved in the method is the matching of images along the line joining successive nadir points and making measurements of the differences in displacement, due to tilt at the edges of the prints. Its benefit lies in the fact that it permits a check of a tilt determination on one photograph against an independent check on another photograph several photographs removed in the flight line. This method is also fully described in the same publication as absolute tilt. (See preceding paragraph.)

4. AVERAGE INDICATIONS. Some types of photography may not lend themselves to either tilt determination from apparent horizons or the absolute tilt method. In such instances it may be possible to make use of data established for a particular strip where tilt determination was possible as a guide for a strip where either of the methods cannot be used. This is only applicable for photography taken by the same three cameras on the same day. As the vertical camera is mounted rigidly in the plane it can be expected to maintain approximately the same lateral tilt and tip for flights taken at about the same altitude. Therefore, on flights where it is impossible to obtain any reliable information the same values of lateral tilt and tip obtained for one strip may be used for the separate strips as a last resort. It should be borne in mind that the attitude of the plane will vary considerably with different loads.

E. Average tilt determination. Procedure for average tilt determination should be used for all photography where there is a marked consistency between the tilts for successive exposures. The extent of error caused by erroneous tilt determinations is rather small especially for tilts below 2° for relatively flat terrain. The procedure consists of making an approximate determination for a number of photographs in the flight and then selecting those having tilts varying considerably from the average and actually making individual tilt determinations for such photographs. The photographs that are selected for individual tilt determinations are called "exceptions" and their photograph numbers noted. In order for a photograph to be considered as an exception the tilt should vary from that established for the average tilt for the strip by the following limits.

In relatively flat terrain: Lateral tilt = 4°
Tip = 3°

In rough terrain: Lateral tilt = 2°30′
Tip = 1°30′

For photography varying between rough terrain and relatively flat terrain, the values of the limits should be adjusted accordingly.

Every flight should be tested to determine whether the average tilt can be used for most of the photographs. If average tilt determinations are used, the work in the following stages is made easier. A rough inspection of the oblique photographs, on which the best images of the horizon appear, will determine whether the distance from the apparent horizon to the edge of the photograph changes to a great extent. To determine the value of the average tilt actual tilt determinations should be made intermittently through the flight for those photographs indicated by the rough inspection to have an average amount of tilt. Those values should then be tabulated and the average determined for both lateral tilt and tip. As an aid to determine whether a photograph should be listed as an exception, the distance from the principal point to horizon should be established for the average tilt as well as those that would be on the border line as exceptions. Exceptions can then be immediately identified by mere scaling from the principal point.

For photographs that can be considered to have an average tilt no record of the depression angle, etc. should be made on the oblique photographs, nor the values of lateral tilt and tip recorded on the vertical photograph. For the exceptions the data should be recorded on the photographs as described in paragraph C of this section.

Photograph numbers usually appear on the photograph so that when reading the numbers, the top of the photograph indicates the direction of flight. However, in some instances, the numbers will be shown upside down and cannot be used as a guide. When average tilt is used, there will be no designation of tilt on most of the photographs, and the average tilt will be shown on the folders containing the vertical photographs. Therefore, in order to avoid confusion in later steps as to which edge is in the direction of flight, the designation "numbers correct" or "numbers reversed," whichever is applicable, should be shown on the folder.

F. Tilt data for making paper templets. 1. GENERAL. For each pair of oblique photographs used in the work a paper templet is made which represents true horizontal directions to the points selected on the photograph from the point identified as the nadir point on the vertical photograph The templet furnishes rays for both obliques. In order that the directions may be correct relatively it is necessary to determine the direction of pointing of the two oblique cameras. This difference in direction is called the "break angle."

2 BREAK ANGLE. The break angle represents the deflection angle between the vertical planes passing through the common nadir point and the principal points of the right and left obliques. For the sake of reference a line through the nadir point and the principal point of the left oblique is considered to have an established direction. If then the direction of the line from the nadir point to the right oblique is toward the direction of flight with respect to the established line from nadir point to principal point of left oblique, the break is to the right; if away, to the left Whether a break angle is to the left or right can be more easily visualized by considering the line of flight to be from left to right, then the line from principal point to right oblique appearing on the right side of the established direction represents a break angle to the right. The value of the break angle is dependent on the skew as determined from the fixed geometrical relationship and the induced tip. The induced tip will not be recorded on the photograph but the tip of the vertical photograph which is shown may be converted to induced tip by dividing the tip expressed in minutes by .86. It is advisable to prepare tables giving induced tip for different values of tip. The method of determining the value of the break angle produces a very close approximation for depression angles other than 30°, but is entirely practicable.

The following procedure is used to determine the value of the break angle as well as the direction of break.

a. If the skew is $1<2$ and the induced tip is depressed, add the skew angle and the induced tip; the break angle is to the right.
b. If the skew is $1<2$ and the induced tip is raised, subtract the induced tip from the skew and if the value is positive the break is to the right; if negative, to the left.
c. If the skew is $1>2$ and the induced tip raised, add the induced tip and the skew and the break is to the left.
d. If the skew is $1>2$ and the induced tip depressed, subtract the induced tip from the skew and if the value is positive the break is to the left; if negative to the right.

The rules may be visualized more easily if the skew angle on the vertical photograph is compared with the angle formed on the vertical photograph by

the intersection of the principal point rays from the oblique photographs at the plotted position of the nadir point, bearing in mind that the nadir point is plotted toward the raised* side with respect to the principal point.

3. INFORMATION FOR OPERATION OF RECTOBLIQUE. In order to make a templet from oblique photographs, the following information is needed and should be recorded on the top of each alternate photograph used in the compilation.

Depression angle.

Break angle. (Record on left oblique only.)

G. Recording data. 1. CAMERA AND TILT DATA SHEET. The camera and tilt data sheet will have been partially completed when the fixed geometrical relationships for each flight were determined. The result of the tilt determinations as well as the values recorded for making paper templets are also recorded on this form. (See Fig. 6.) In each of the spaces identified by number the following entries should be recorded:

(21) Flying height as determined, based on data appearing on the end photographs of the strips.
(22) The dip angle; determined from paragraph C, 1 of this section.
(23) Average lateral tilt determined for the strip.
(24) Average tip determined for the strip (vertical photograph).
(25) Average oblique tip for left oblique.
(26) Average depression angle for left oblique.
(27) Break angle; as determined from E, 2 of this section.
(28) Average tip on right oblique
(29) Average depression angle for right oblique.
(30) List the number of photographs for which individual tilt determinations are necessary and identify them as exceptions.
(31) The name of the person making tilt determination. If person is other than the one making the determination of the fixed geometrical relationships, record name above first signature.

2. FOLDERS FOR PHOTOGRAPHY. The following data, essential to making paper templets and plotting nadir points, should be recorded inside the respective folders for each strip when average tilt is used

Left oblique·	Oblique tip
	Depression angle
	Break angle
	Focal length
Vertical:	Tip
	Lateral tilt
	Photo number check for direction of flight
	Focal length
Right oblique	Oblique tip
	Depression angle
	Focal length

SECTION V

NADIR POINTS AND AZIMUTH LINES

A. General. An Azimuth Line is a line on the ground between two image points which represent the ground position of the nadir points of two separate

* In fundamental photogrammetry "raised" and "depressed" refer to the negative plane, consequently the terms have reverse significance when applied to the photograph.

exposures. This line may be plotted on the vertical photographs by the procedure given in the following paragraphs. In this procedure the term "alternates" will be used for the alternate prints which are used in the compilation, and the term "discards" will be used for the intermediate prints.

B. Procedure. 1. LOCATING NADIR POINTS.

a Observe the angle of lateral tilt recorded in red pencil in the upper right hand corner of the vertical photograph. When the tilt data appears in the upper right hand corner, the top of the print denotes the direction of the line of flight. Find the distance from the principal point to the nadir point for the pertinent angle of lateral tilt from the tables titled "Conversion Table for Tilt Angle and Nadir Point." (See Fig. 8) Using this distance plot a point along the lateral etched line used in locating the principal point always toward the raised side of the print. The raised side of the print will be denoted in the tilt data by the letter "*R*" preceding the recorded angle. "*D*" will indicate "depressed." As the information is written on the right side of the print, the letter preceding the tilt data will indicate that the right side is either raised or depressed For example, the notation ($D-2°\ 00'$) indicates that the corresponding distance should be plotted from the principal point toward the left side of the print.

b. Observe the angle of tip recorded in blue pencil in the upper right hand corner of the print. Likewise, select the proper distance from the tables and plot a distance from the point located by the procedure in paragraph (a) above parallel to a line joining the fore and aft fiducial markers always toward the raised side. As the information is recorded on the top of the print as either "*R*" or "*D*," it will denote that the value pertains to the top of the photo-

CONVERSION TABLE FOR TILT ANGLE, NADIR POINT, $F=6\ 00''$

Angle of Tilt Component	Dist Fr P P to Nadir	Angle of Tilt Component	Dist Fr P P. to Nadir	Angle of Tilt Component	Dist. Fr P P to Nadir
0° 05′	009	2° 05′	218	4° 05′	428
0° 10′	017	2° 10′	227	4° 10′	437
0° 15′	026	2° 15′	236	4° 15′	446
0° 20′	035	2° 20′	244	4° 20′	.455
0° 25′	044	2° 25′	253	4° 25′	463
0° 30′	052	2° 30′	262	4° 30′	.472
0° 35′	061	2° 35′	271	4° 35′	.481
0° 40′	070	2° 40′	280	4° 40′	490
0° 45′	079	2° 45′	288	4° 45′	.499
0° 50′	.087	2° 50′	297	4° 50′	508
0° 55′	096	2° 55′	306	4° 55′	516
1° 00′	105	3° 00′	314	5° 00′	.525
1° 05′	113	3° 05′	323	5° 05′	534
1° 10′	.122	3° 10′	332	5° 10′	.542
1° 15′	131	3° 15′	341	5° 15′	.551
1° 20′	.140	3° 20′	349	5° 20′	.560
1° 25′	148	3° 25′	358	5° 25′	569
1° 30′	.157	3° 30′	.367	5° 30′	578
1° 35′	166	3° 35′	376	5° 35′	587
1° 40′	.175	3° 40′	385	5° 40′	595
1° 45′	184	3° 45′	393	5° 45′	.604
1° 50′	.192	3° 50′	402	5° 50′	.613
1° 55′	.201	3° 55′	411	5° 55′	.622
2° 00′	.209	4° 00′	.419	6° 00′	631

FIG. 8. Conversion table.

graph. The terminal of this plotted position is the nadir point for the photograph. Using a drop-bow compass and red ink, draw a circle at the position of the nadir point.

c. Repeat this operation for each alternate print in the strip.

d. If the average tilt is used for the strip the data for tip and tilt will be recorded on the folder for the vertical photographs rather than on the individual prints, and the nadir point for all photographs will then be plotted in the same position with respect to the principal point. Therefore, this operation may be expedited by using an acetate templet showing the plotted position of the nadir point. This templet may be oriented on the principal point and a fiducial marker.

e. For some strips the average tilt will have been determined for most photographs, but for some the tilt will have been too great to use an average tilt value, necessitating individual attention. Such exposure will be listed under the space shown on folders for "exceptions" and will have the tilt data shown on the vertical photograph. Plot the nadir points for the "exceptions" in accordance with *a* and *b* above and for the rest of the photographs in accordance with step *d* above.

2. TRANSFERRING NADIR POINTS.

a. Transfer the nadir point, with the aid of a stereoscope, from the alternate print to the adjoining discards and circle the transferred points with a pencil or drop-bow pen. The images in the immediate vicinity of the nadir point should be fused perfectly under the stereoscope with the same images on the discard in order to transfer this point accurately.

b. Repeat this operation for each print in the flight.

c. When the above operation is completed, there will be two transferred nadir points on each discard. Etch a straight line between the two points on each discard by means of a steel straight edge and prick point. All lines should be etched on the prints over a light table

3 PLOTTING AZIMUTH LINES.

a. Using a stereoscope, fuse the common imagery of the alternate print and the adjoining discard so clearly that it is difficult to define on which print the etched line falls.

b. While the prints are fused, prick on the alternate print from six to ten well distributed and easily identified points coincident with the etched line.

c. Transfer as described in *b* above the azimuth line points from all of the discards to the alternate prints.

d. Align a steel straight edge with the transferred azimuth points on one side of the alternate print. These points should form a line passing through the nadir point. It will be noted that a few of the points may not fall exactly on the line. However, unless there are more than two points that fall off the line by more than .01 inch the straight edge should be placed in the best position to pass through most of the points. Where there are several points that fall off the line by more than .01 inch the photographs should again be checked and a new azimuth constructed. When the straight edge can be aligned satisfactorily and the line passes through the nadir point within .01 inch, etch a line from the edge of the print to the circle at the nadir point. Bear in mind that if the nadir point is transferred erroneously to the discard that the line from the other transferred nadir point is slightly in error in direction and that if the azimuth has been drawn on the adjacent alternate, it should be revised in line with the direction represented by the correct line on the discard.

e. Repeat the operation described in *d* above for the opposite side of the alternate print and for the remaining photographs in the strip.

4. AZIMUTH LINES WITH INSUFFICIENT OVERLAP. When there is insufficient overlap between consecutive prints in the flight to allow the direct transfer of a nadir point to the discard, the following procedure may be used:

a. Locate the nadir point for the alternate print by the procedure in paragraph 2.
b. Transfer an image point that falls as near the nadir point as practicable from the alternate to the discard. It should be in the approximate direction of the azimuth line.
c. Using a straight edge and needle, etch a fine line between the transferred image point and the transferred nadir point on the discard. (This line represents an approximately correct azimuth line.)
d. Transfer an image point to the outer edge of the alternate print that appears along the etched line of the discard.
e. Using a straight edge and sharp needle, etch a short line on the alternate from the nadir point toward the transferred image point. (This line represents the portion of the azimuth line that has the least amount of displacement.)
f. Transfer to the discard with the aid of a stereoscope several image points that appear on the short etched line of the alternate.
g Etch the azimuth line on the discard connecting the transferred nadir point and the row of transferred image points.
h. Transfer this azimuth line which should be reasonably correct to the alternate print. (If sufficient accuracy has not been obtained, repeat the steps above.)

If it is found that there is insufficient overlap to allow a direct transfer to either side of the discard, the procedure above will be applicable to both sides, and will necessitate the use of both adjoining alternates at once in order to obtain the correct azimuth for each.

5. AZIMUTH LINES OVER WATER AND CLOUDS. When it seems impossible to transfer an azimuth line from a discard to an alternate print due to appearance of clouds or water between the transferred nadir points, the following procedure will sometimes be found useful:

a. Select two or more pairs of points that are approximately equidistant from, and generally not in excess of one inch on either side of the azimuth line and transfer to the alternate.
b. Measure on the discard the perpendicular distance from each of these points to the azimuth line. Plot points at these distances from the respective transferred points on the alternate print toward the expected position of the azimuth line. Etch the azimuth line from the nadir point through the mean position of all points representing the plotted distance from the transferred points.

6. SUPPLEMENTARY INSTRUCTIONS. The experienced operator should never transfer an azimuth line unless he is sure of his accuracy, because it is extremely difficult to correct an azimuth line once it has been etched on the photograph. For this reason, the operator should ask for help with the difficult transfers. Care should be taken that the azimuth lines do not obliterate the nadir point appearing at the center of each vertical. The azimuth lines should be etched heavily enough so that they can be easily seen but not deep enough to cause the photographs to break or tear.

It is recognized that the method of locating azimuth lines described in this chapter does not always meet the theoretical requirements that image points should be transferred along a common radial line between two exposures in

order to remove distortion due to relief. Because of this, the results are not theoretically precise where a bend in the flight line occurs in rough or mountainous country. Under these conditions the transferred image points will not form a perfectly straight line because they do not fall on a common radial between the two consecutive exposures. However, for all practical purposes, this small distortion is neglibible.

In extremely rough terrain the discrepancy can be reduced by selecting transferred image points at elevations between the elevations of the nadir points on the discard and changing proportionately from high to low as the distance varies between the high and low nadir points.

SECTION VI

OPERATION OF THE RECTOBLIQUE PLOTTER

A. General. The Rectoblique Plotter is a mechanical adaptation of a graphical method for obtaining true horizontal angles from an oblique photograph. This instrument (see Fig. 9) consists of a table with two arms of different lengths mounted on it and connected with a bar free to slide parallel to the true horizon. The left arm, known as the photo arm, is made of transparent material and is pivoted at its lower end at what is known as the photo nadir point, which

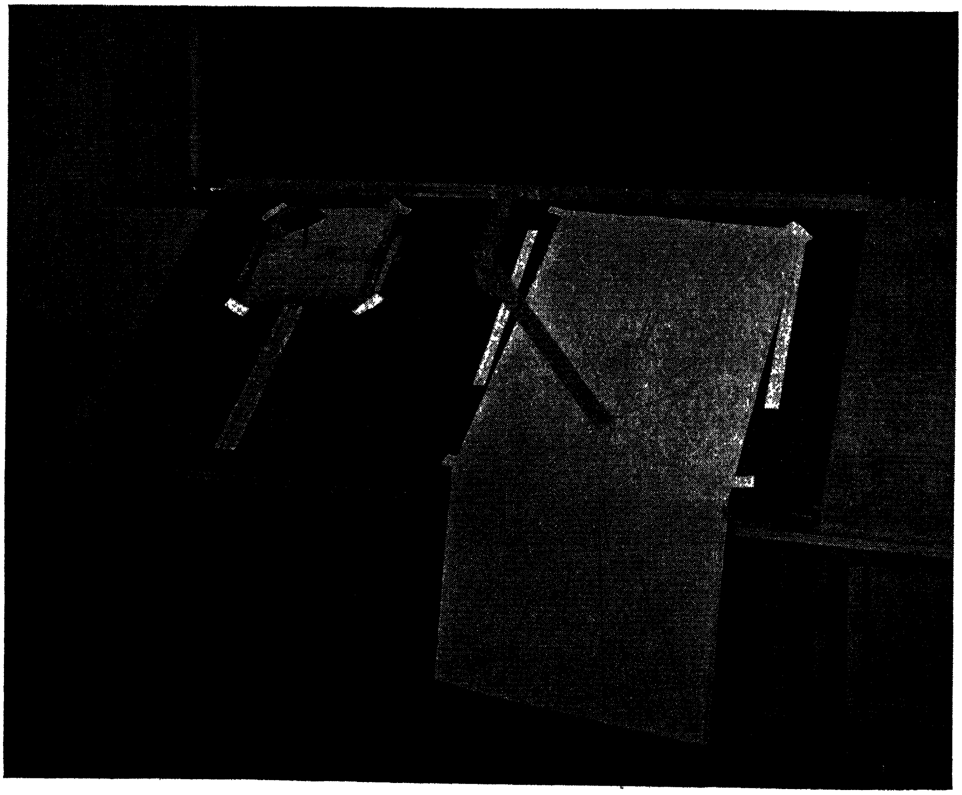


FIG. 9. Rectoblique plotter.

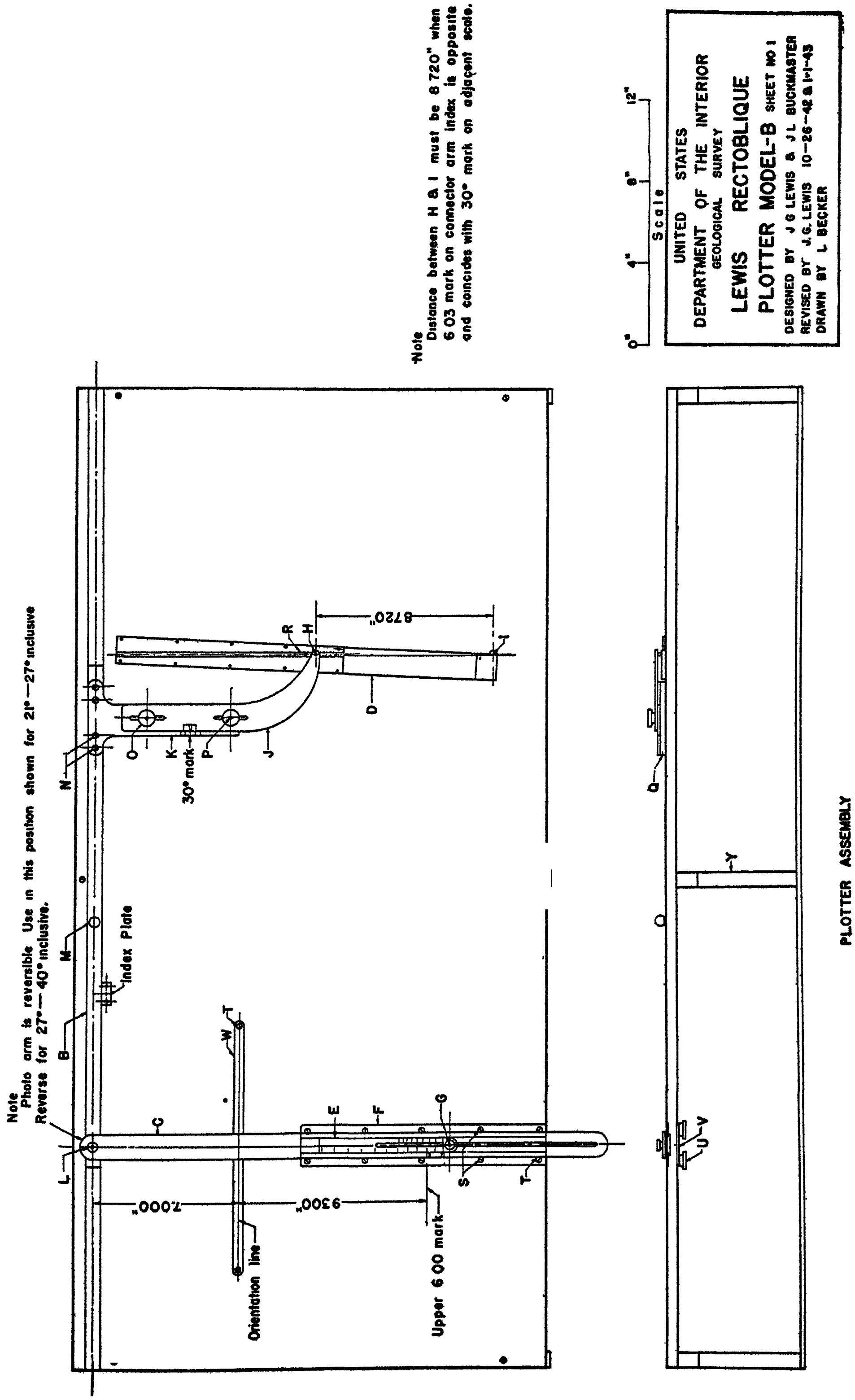


FIG. 10. Schematic diagram of rectoblique plotter.

corresponds to the nadir point of the oblique photograph. The right arm is known as the templet arm and rotates about a point called the templet pivot point which corresponds to the ground nadir point of an oblique exposure which has been rectified into a true horizontal plane. The templet arm describes angles in a horizontal plane that correspond to those described by the photo arm in the plane of the oblique exposure. The templet arm is connected to the horizontal bar by means of a rigid link which can be adjusted in length in order

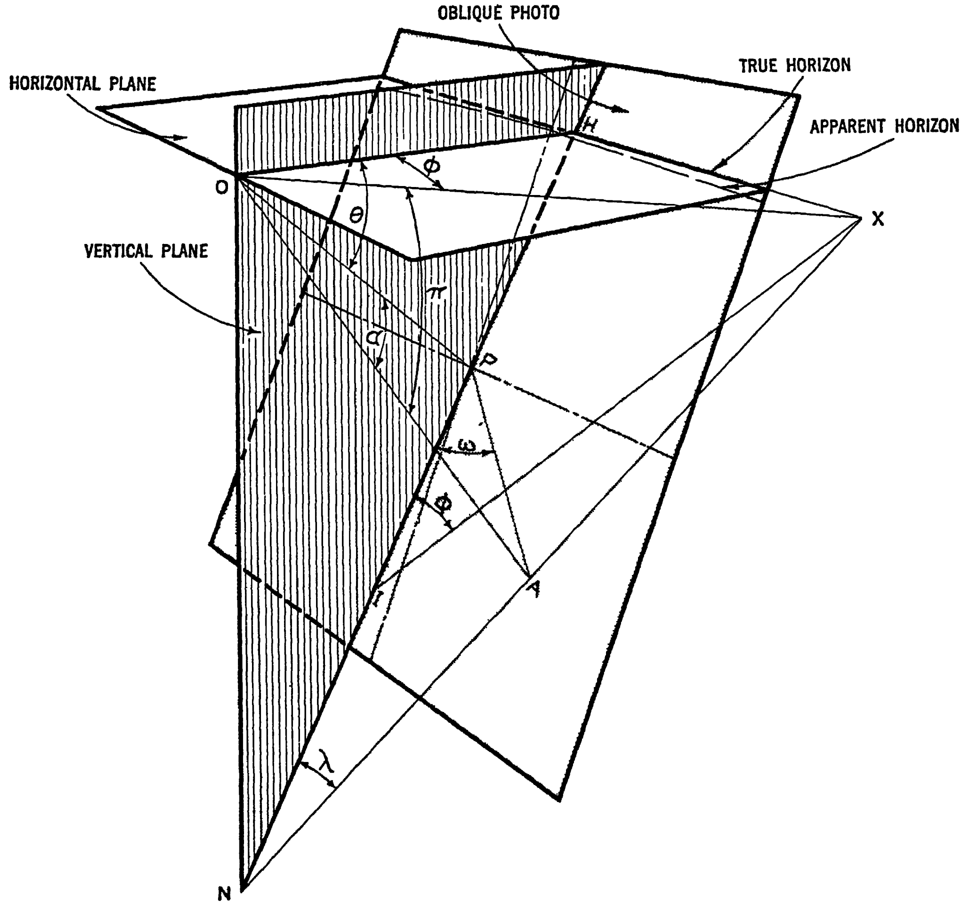


FIG. 11. Geometric relations of a single oblique photograph.

to allow for changes in tilt of the oblique photographs. On some rectobliques, the templet arm is pivoted at the middle point and rays can be drawn for both obliques without changing the position of the paper templet. However, a smallei model does not have the extension on the templet arm and the paper templet must be rotated 180 degrees in order to draw the direction lines for the second oblique photograph to be represented on the templet.

B. Theory. From an inspection of Figure 11 it will be seen that any image point A on an oblique photograph will determine a vertical plane passing through that point and the nadir point of the exposure. The angle between this plane $ANOX$ and the principal plane, measured in the plane of the oblique is λ; measured in the plane of the true horizon this angle is ϕ. These two angles are

related to each other by the equation tan $\lambda = \tan \phi \sin \theta$, where θ represents the depression angle of the oblique photograph. The Rectoblique Plotter is used to obtain a mechanical solution of this equation.

In Figure 11, O is the perspective center of the photograph, P is the principal point, XH is the true horizon, N is the nadir point, and I is the isocenter. The perspective center O and any given image point A form a line which represents the original light ray which exposed the image point A on the negative of the photograph. It will be shown below that when an oblique photograph is rotated about the true horizon XH, into the horizontal plane OXH, the perspective

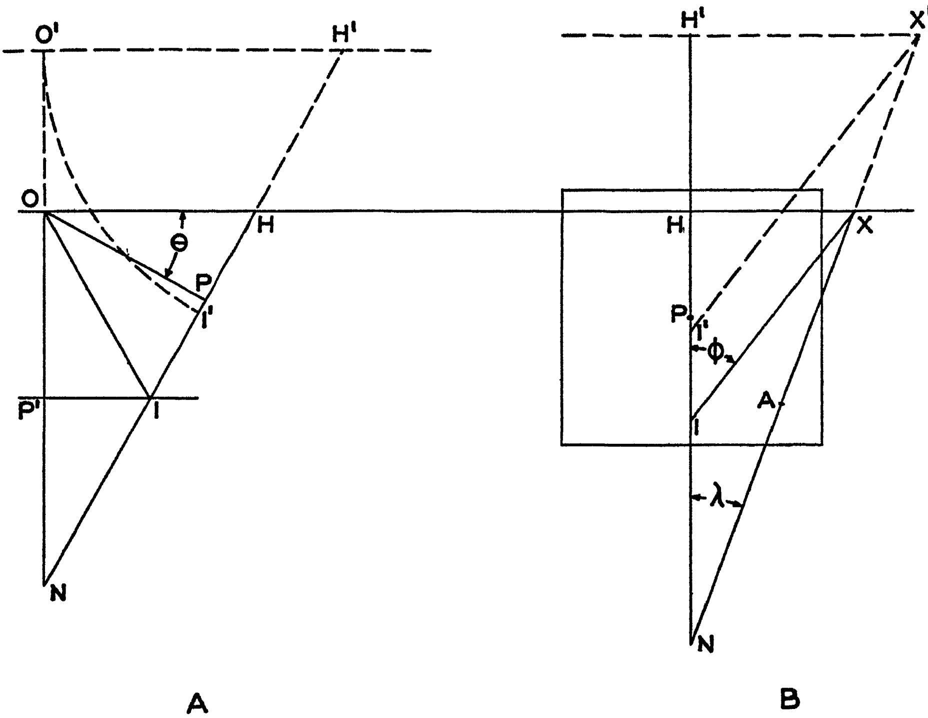


FIG. 12. Theory of rectoblique plotter.

center O will coincide with the isocenter I. The angle HIX is then equal to the true horizontal angle ϕ

Figure 12A illustrates a side view of the principal plane OHN of an oblique photograph, showing the depression angle θ, focal length PO, the true horizon H, the principal line HN, and the plumb line ON. (Compare Figure 11 with Figure 12A and Figure 12B.) The principal line is the trace of the principal plane on the photograph and contains the principal point P, the isocenter I and the nadir point N. In actual construction, the principal line may be identified on an oblique photograph by a line through the principal point and perpendicular to the image of the apparent horizon. Also, in actual construction, the isocenter may be identified by the intersection of the equivalent vertical photograph $P'I$ with the principal line of the oblique. The isocenter lies on the bisector of the angle $(90° - \theta)$.

Below is given a proof that: Tan $\lambda = \tan \phi \sin \theta$

1. Let the oblique photograph swing into such a position that the principal point occupies the position P' directly under the perspective center O. In this position the photograph becomes a truly vertical photograph. The line of intersection between the planes determined by the two positions of the oblique is called the isoline. Since the isoline is common to both planes, it is the only line of the original oblique which has the same scale after the oblique has been rotated into its new position in the plane $P'I$.
2. Let the interior tilt angle PON, which is equal to $(90°-\theta)$, be bisected by the line OI. Since OI is common to both triangles OIP and OIP', and OP and OP' are both equal to f, the triangles are equal and $PI=P'I$.
3. The distance from the horizon to the perspective center is equal to the distance from the horizon to the isocenter, as proved below:

$$\text{angle } HIO = \text{angle } INO + \text{angle } ION = \theta + \frac{90° - \theta}{2}$$

$$\text{angle } HOI = \text{angle } HOP + \text{angle } POI = \theta + \frac{90° - \theta}{2}$$

angle HIO = angle HOI and triangle HOI is isosceles and $HO = HI$.

4. Rotate the oblique photograph about the true horizon HX into the horizontal plane OXH. The isocenter I will coincide with the perspective center O, and true horizontal angles can now be measured, in the plane of the horizon, from the superimposed points I and O.

Figure 12B is a front view of an oblique photograph with the plane of the photograph falling in the plane of the paper. With reference to this figure, as well as to Figure 11.

$$\tan \lambda = KX \div HN$$

$$\tan \phi = HX \div HI.$$

From the above two equations:

$$\frac{\tan \lambda}{\tan \phi} = \frac{HX}{HN} \div \frac{HX}{HI} = \frac{HI}{HN} = \frac{HO}{HN}$$

$HO \div HN = \sin \theta$ (See Fig. 12A)

Therefore: $\tan \lambda = \tan \phi \sin \theta$ Q.E D.

The tangent of any angle measured from the nadir point of an oblique photograph, when divided by the tangent of its corresponding horizontal angle, becomes equal to the sine of the depression angle θ, which is a constant value for any given oblique exposure

For mechanical convenience in the construction of the Rectoblique Plotter, an artificial horizon is used which is parallel to the true horizon. However, true horizontal angles can still be obtained, following the theory outlined above The proof of the correctness of this construction is given below.

1. In the formula

$$\frac{\tan \lambda}{\tan \phi} = \sin \theta = \frac{HO}{HN}$$

The distance HN can be made any length $H'N$, provided that the length of the line HO changes in the same proportion, so that

$$\frac{HO}{HN} = \frac{H'O'}{H'N} = \sin\theta.$$

2. Triangle HNO is similar to triangle $H'NO$ so that

$$\sin\theta = \frac{H'O'}{H'N} = \frac{H'I'}{H'N}.$$

The new perspective center O' now falls in the new position I' when the oblique is rotated into the plane of the true horizon.

3. In Fig. 12B, triangle $NX'H'$ is similar to triangle NXH, so that

$$\frac{HX}{HN} = \frac{H'X'}{H'N} \quad \text{and} \quad \frac{HX}{HI} = \frac{H'X'}{H'I}$$

$$\frac{\tan\lambda}{\tan\phi} = \frac{HX}{HN} \div \frac{HX}{HI} = \frac{H'X'}{H'N} \div \frac{H'X'}{H'I'} = \frac{\tan\lambda}{\tan\phi} = \sin\theta$$

so that angle $H'I'X'$ is the same angle as the original angle HIX.

In the Rectoblique Plotter the ray NX' is replaced by a lucite arm which is called the photoarm. The ray $I'X'$ is replaced by a metal arm called the templet arm and the point X' moves along the artificial horizon by means of a horizon bar. Instead of placing the templet arm in the actual position $I'H'$ the motion is mechanically transferred by means of the horizon bar to a more convenient position on the right hand side of the device, where the horizontal angles are drawn on a paper templet.

C. Procedure for making paper templets.

1 Make the paper templet from a sheet of tracing paper approximately 36 inches long and 20 inches wide. In the upper right hand corner on each end of the paper templet record the following information: Project, flight number, exposure number, depression angle and focal length. Record the break angle under the information for the left oblique only. This information, with the exception of focal length, may be taken from each oblique photograph. The focal length may be found on the camera orientation sheet or on the first and last print in the flight.

2 Plot the break angle on the paper templet using the approximate center of the templet as the vertex. This point serves as the nadir point or point of radiation for the templet. Draw rays from the center (or vertex) lengthwise on the paper and extend to the edge. The angle and direction of break to be plotted is found in the upper left hand corner of the left oblique print and may be plotted on the templet by use of a large protractor made of cellulose acetate and graduated in 15 minute intervals.

3. Rays are then to be plotted on the paper templet from radial control points appearing on the oblique print. This is accomplished by means of the Rectoblique Plotter in the following manner:

a. Set the Rectoblique Plotter to the true depression angle and the focal length by sliding the photo arm slide (E), graduated in degrees and five or ten minute intervals, into position where the depression angle coincides with the desired focal length on the guide plate (S). Three index marks represent focal lengths of 6.00, 6.03 and 6.06 inches. Estimate intermediate focal lengths. For a range of depression angles from 21° to 32° use the upper set of focal length graduations. For a range of 32° to 40° take the slide out, reverse it, and match the depression angle to its corresponding

focal length on the lower set of focal length marks. When the depression angle and the focal length have been set, lock the plate tight by means of the stud.

b. Set the depression angle and focal length on the arms on the right hand side of the Rectoblique, making the depression angle on the connector arm (*K*) coincide with the focal length on the curved arm (*J*).

c. Move the horizon bar (B) into normal position making the index mark, located about 4 inches from the photo arm, coincide with the index mark on the guide. In this position the photo arm is perpendicular to the horizon bar.

d. Slide the photo under the photo arm and orient by making the orientation line on the photo coincide with the etched line on the orientation plate (*W*). At the same time place the principal point of the photo directly under the hairline of the photo arm. Now tape the photograph securely into position.

e. Place a piece of scotch tape on the underside of the paper templet, at the nadir point, to prevent the paper from tearing when placed on the pivot pin of the rectoblique plotter. Take the templet arm (*B*) off its pivot pin (*I*) temporarily, and place the paper templet in position pushing the pin through the nadir point on the paper templet, so that the paper templet lies flat. Replace the templet arm Check the normal position of the photo arm by looking at the index marks and then rotate the paper templet about its plumb point until the center line coincides with the ruling edge of the templet arm. Tape the paper templet securely in position.

f. Plot the rays on the paper templet representing lines of true direction to the pass points of one oblique. Extend the rays about 6 inches from the nadir point of the templet. Remove the templet from the Rectoblique, and test the accuracy of the plotted rays by superimposing the templet over the vertical photograph on the light table in such manner that the center of the templet coincides with the nadir point of the vertical, and the rays intersect their corresponding pass points as closely as possible.

This procedure serves as a preliminary check of the accuracy of the depression angle, and of the orientation line or tip angle. If the rays are not in coincidence with the pass points and the lack of coincidence is not greater than two or three hundredths of an inch (.02), the orientation data may be considered to be correct. If a discrepancy exists in the positions of the rays, first check as to whether the nadir point is plotted correctly and that the pass points are correctly transferred from the oblique print. After the check is completed the following rules may be used as a guide in determining the cause of any discrepancy.

1) If the center pass point ray intersects the center pass point on the vertical photograph, but the two outside rays fail to intersect and are diverged by the same amount, the depression angle should be decreased.
2) If the conditions are the same as described in 1), but the outside rays are converged, the depression angle should be increased.
3) If the outside rays intersect the corresponding points on the vertical photograph, but the center pass point ray fails to intersect, the orientation line or the tip angle is in error.
4) If the center pass point ray and one of the outside rays intersect the corresponding points, but the other fails to intersect, the discrepancy is due to a combination of the conditions described in 3) and 1) or 2).

This procedure for checking the accuracy of the paper templets will be used for only the first two or three prints of a strip of photography, or such prints of a strip that are necessary to determine that the orientation data (depression angle, tip, etc.), will permit making templets that will have the proper coincidence with the vertical photographs.

g. Plot rays to pass points (unless completed under f above), and all other radial control points appearing on the oblique photograph. As illustrated by Figure 13, the length of the direction rays and their distance from the center of the templet is determined by the position of the corresponding pass points on the obliques. Extend direction rays to the distant points out to the ends of the templet. Rays to near detail points may only need be about 4 inches long. Mark the ends of the rays on the templet with an arrow, using a colored pencil corresponding to the color of the points on the photograph. Point the arrow outward for points falling below the principal point to the oblique print, and inward for the points beyond the principal point. Number or letter the direction ray to agree with the number or letter used for the corresponding pass point on the print. Check the number of rays on the templet against the number of points on the print to avoid omitting any rays.

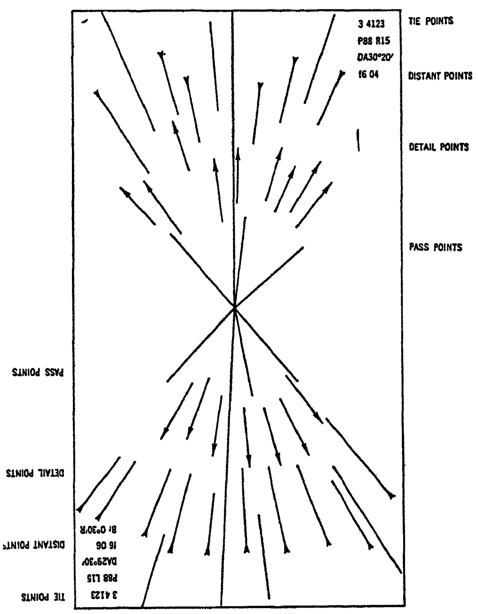


FIG. 13. Paper templet.

h. Rotate the templet around the pivot pin of the Rectoblique until the orientation mark for the break angle coincides with the ruling edge of the templet arm when the templet arm is in the normal position. Using a procedure similar to that outlined above, plot the ray direction lines for the other half of the templet from the other oblique photograph.

i. Remove the templet from the Rectoblique and orient it as accurately as possible over the corresponding vertical photograph, making the nadir point of the templet coincide with the nadir point of the vertical photograph and the pass point rays intersect the corresponding points. Any errors in construction of the templet that have not been checked in accordance with the procedure in paragraph f will be revealed in this operation. Such templets should be corrected in line with the tolerances prescribed in paragraph f.

j. Using the best orientation, complete the templet by tracing on it as accurately as possible the forward and back azimuth lines from the vertical photograph.

4. If it is determined that average values of tip and tilt can be used satisfactorily for a strip of photography, the tilt and camera data will remain constant for the entire strip and will be recorded on the appropriate folders rather

than the individual prints. The only information to be recorded on each templet, when average tilt values are used, is as follows: Project number, strip number and exposure number.

The following procedure may be used for an average tilt strip:

a. Since the break angle will remain constant for the entire strip, it may be plotted on all of the templets at one time. First, place the templets to be used for the strip in a neat stack and tape down the corners to avoid shifting. Plot the break angle on the top of the stack. Drive a very small nail or needle through the stack at the center and terminals of the break angle lines. The center point of the templet, marked by the needle, is then circled and the points at the edge of the templet may be used to orient the templet in the Rectoblique. Care should be exercised to avoid reversing the templets since this would also reverse the direction of break.
b. Since all of the orientation lines would fall in the same position for the left oblique prints, the constant angle of tip may be set in the Rectoblique by means of a protractor grid. The photographs can then be oriented by the fiducial marks and the line on the grid. This process eliminates the need of orientation lines on the oblique prints.
c. Since all of the left oblique prints will have the same depression angle and focal length, the Rectoblique can be set by that information and all of the rays from the left oblique prints plotted in order, through the strip. The first two templets should be checked for error in depression angle and tip angle. It will be unnecessary to check more than every fifth to tenth templet if the first two are found to be correct.
d. On completion of the left side of the templets the above procedure can be followed for the right.

Section VII

PAPER TEMPLET PLOT

A. General. In this operation the paper templets are combined into what is known as a paper templet plot. If the strip of photography represented by the paper templets is to be combined in the metal templet plot with several other parallel or intersecting strips, the paper templet plot is made in the manner described in Section B.2 below. If there is only one strip of photography in the project, the paper templets are combined as carefully as possible into the final radial plot in the manner described in Section B.1 below.

B. Procedure. 1. DIRECT COMPILATION FROM PAPER TEMPLETS. Where there is only one strip of photography in a project, the final radial plot is made directly from the paper templets without resorting to the use of the metal templets. In this case, the procedure to be used is given below:

a Determine the approximate scale of the paper templet plot by computing the desired distance between the centers of the first two paper templets. The following expression can be used to advantage:

$$\left(\begin{array}{c}\text{Desired}\\ \text{Distance}\end{array}\right) = \left(\frac{\text{Scale of vertical photo.}}{\text{Scale of manuscript}}\right) \times \left(\begin{array}{c}\text{Distance on photo. between}\\ \text{corresponding nadir points}\end{array}\right)$$

b. Place the first paper templet at one end of a light table with the azimuth line pointing down the center of the table. Fasten the templet in position with scotch tape at each end.
c. Place the second templet in place over the first in such fashion that the

centers of the templets are about two inches apart (use the distance computed in step a) and with the common azimuth line appearing on the two templets exactly coinciding.

d. Place the third templet over the second with the common azimuth exactly coinciding. Move the templet along this azimuth until the rays for each of the pass points pass exactly through the radial intersections for those points determined by the first two templets. If, because of mechanical errors, perfect radial intersections cannot be obtained with any one given orientation of the paper templet it will be necessary for the operator to select a position that gives the best average result for all the radial intersections. (See paragraph 3 at the end of this section.) However, the azimuth lines must be made to coincide even at the expense of creating small triangles of error in the intersections of the templet rays because the azimuth lines are supposed to be the most accurate part of the radial plot.

e. All the succeeding templets are oriented to the paper templet assembly in the manner described in step d. When the paper templet plot is completed all nadir points and radial intersections should be transferred to a sheet of cellulose acetate by the use of a sharp needle. (The sheet of acetate, which becomes the map manuscript, may be placed over the plot or under it, as the operator desires. If the acetate sheet is to be placed under the plot, the paper templets can be assembled directly on top of the acetate and fastened to it with scotch tape.) The pass points are circled and marked in ink with their proper designations.

Due to various small errors, it may frequently become necessary to make very slight adjustments in the orientation of the paper templets in order to secure a good over-all orientation of the templets. However, considerable caution should be used in this procedure. Occasionally it may be necessary to extend the rays in order to make them intersect with their corresponding rays on other templets. For some distant tie points the rays will intersect beyond the ends of the templets and the intersection can be made on the table outside the illuminated area.

2. PRELIMINARY ASSEMBLY FOR METAL TEMPLETS. *a. General.* The purpose of the paper templet is to serve as a model for the metal templet as to direction and length of the metal arms The rays on the paper templets give the correct direction for the metal arms but the length of these arms, at a particular scale, can be determined only after the paper templets are combined and the rays intersected in what is known as the paper templet plot. Each metal templet will have arms to the following points: Tie points, geographic control stations, and distant red points. The templet also will have two arms representing the forward and back azimuth lines.

b. Procedure. The following procedure is to be used:

1) Determine the approximate scale of the paper templet plot by computing the desired distance between the centers of the first two paper templets. The following expression can be used to advantage:

$$\left(\begin{matrix}\text{Desired}\\ \text{Distance}\end{matrix}\right) = \left(\frac{\text{Scale of vertical photo.}}{\text{Scale of manuscript}}\right) \times \left(\begin{matrix}\text{Distance on photo. between}\\ \text{corresponding nadir points}\end{matrix}\right)$$

2) Smooth out the first of the series of paper templets lengthwise along a light table and then place a second templet in place over the first in such fashion that the centers of the templets are about two inches apart—use the distance computed in step 1) and the common azimuth line appearing on the two

templets coincides exactly. (The azimuth line should be made to coincide even at the expense of creating small triangles of error in the intersections of the rays of the paper templets, because the azimuth lines are the most accurate part of this radial plot.) Carefully fasten the two templets together with scotch tape.

3) Place a third templet over the second with the common azimuth exactly coinciding. Move the templet along this azimuth line until the rays to each of the pass points passes exactly through the intersections for those points determined by the first two templets. Carefully fasten this templet to the other two with scotch tape.
4) All succeeding templets are oriented to the paper templet assembly in the manner described in step 3).
5) Identify by a short mark with a blue pencil across each of the templet rays the position of the radial intersection for each tie point, geographic control point, and distant red point.
6) After twenty or thirty paper templets have been assembled in a plot, carefully lay a smooth piece of tracing paper, about one foot in width, over the paper templet plot and transfer to it the plotted positions of all nadir points and geographic control points. Where these points will not fall on the tracing paper overlay of the paper templet plot, an extension to the side can be attached with scotch tape.

c. Supplementary instructions. There are certain conditions that sometimes exist within a strip of photography that tend to weaken the strength and accuracy of the metal templet plot. These conditions are:

1) The obliques on one side of the strip are of such quality that tie or distant red points cannot be identified on that side. This condition results in a series of paper templets possessing no tie or distant red points on one side of the flight line.
2) Vertical exposures of such quality that azimuth lines cannot be identified for one or more exposures.

Since a paper templet plot can be assembled more accurately than a corresponding series of metal templets, certain operations can be performed here that will improve the accuracy of the metal templet plot.

If insufficient tie or distant red points have been picked on the obliques of one side, the missing points can be replaced with purely imaginary and arbitrary distant red points that can be picked in strategic locations in the common overlap existing between three or four paper templets of the paper templet assembly (These imaginary points do not appear on any photographs nor on any other strip of paper templets.) Rays can then be drawn on the paper templets toward these imaginary distant red points. While these rays to the imaginary points do not increase the accuracy of the paper templet plot, they do help to maintain the strength of the metal templet plot and thus carry into the final compilation whatever accuracy existed in the original paper templet plot.

If satisfactory azimuth lines cannot be located on the vertical photographs because of clouds or water, a few paper templets of a series may be assembled by resection of the radial intersections Where the beginning of a paper templet plot exists, the rest of the series of paper templets may be combined into the paper templet plot by use of the following procedure:

1) Orient the next paper templet to be combined into the paper templet plot in its approximately correct position over the previously completed portion of the plot.
2) Move the templet to such a position that each of its rays passes *exactly* through the previously established radial intersections.

3) All the succeeding paper templets of the strip are combined into the plot in the manner described in step 2).
4) Draw lines between the centers of each pair of paper templets to represent the azimuth lines that had been missing previously.

The combining of paper templets into a paper templet plot by the resection process is a makeshift procedure at best and cannot be carried very far with any success. Added strength will be obtained if templets of each consecutive set of pictures are used instead of every alternate set as is the general procedure. This resection procedure, if very accurately performed, is useful in bridging paper templets over any short gap caused by missing azimuth lines.

3. TRIANGLES OF ERROR. All radial intersections should be carefully inspected as they are formed, because it is the task of the operator making the paper templet plot to analyze any triangles of error that occur and determine their cause, correct them, or make the decision that they are small enough to be accepted. It is not practical to suggest here a certain size of triangle which is small enough to be accepted Much will depend upon the shape and size of the triangle, its distance from the flight line, the number of rays involved, the importance of the point, and the time available for correcting or reducing the errors. There are listed below the most common causes of triangles of error:

a. Incorrect pass points. (It may be that a pass point has not been located on the same image point in each of the pictures in which it appears.)
b. Errors in tilt analysis.
c. Errors in the location of azimuth lines
d. Errors in paper templets (Templets may not be correct because of incorrect setting of the Rectoblique.)
e. Errors in paper templet plot (Triangles of error may result from faulty orientation due to lack of sufficient tie points and distant red points to check the orientation.)

SECTION VIII

METAL TEMPLET PLOT

A. General. The metal templet plot is very important because it provides the skeleton or framework upon which the entire map compilation can be made. It ties together two or more strips of photography by forming a network of radial control points located in their proper relative positions and at a constant scale even though there may be large scale changes between individual photographs due to changes in flying height, or changes in the elevation of the ground.

B. Procedure. 1. MANUSCRIPT. For the metal templet plot a large table, or group of tables joined together, is needed. Join a sufficient number of strips of cellulose acetate together to cover the area to be compiled. The edges between the strips are butt joined and secured together with long pieces of cellulose tape, placed on the upper surface along the joint. The resulting sheet of acetate is known as the manuscript.

2. PROJECTION. If geographical control is available it will be necessary to construct a projection on which it can be plotted. Any one of the usual forms of map projection that fits the specifications for the job at hand may be used.

3. PLOTTING GEOGRAPHIC CONTROL. The controlled laydown is one whose position is fixed upon the projection by the use of geographic control points. These control points are located most easily by the use of a graduated scale which is slightly longer than the distance between adjacent parallels or merid-

ians. If the interval between parallels and meridians is thirty minutes, the scale should be graduated into thirty major divisions each of which is divided into sixty parts representing seconds. The scale is held in such a manner that the zero and thirty minute marks coincide with the parallels or meridians, as the case may be, at each end of the thirty minute block. In this manner tick marks, representing the proper latitude and longitude of the control station, can be plotted above and below the approximate position of the control station, and at each side. The intersection determined by straight lines connecting these two pairs of tick marks exactly locates the position of the station on the manuscript.

Control points should be plotted on the manuscript carefully, with the original plotting being checked at least once. Geographic control stations are indicated by a small circle inscribed in a triangle. Triangulation stations are in red and are labeled with the name of the station and the initials of the bureau that established them. Astronomic stations are also in red and are labeled with the initials "A.P." (astronomic position). Map points are shown in green and are labeled with the initials "M.P." (map position). These points have been previously located on the photographs, so that when the templets are oriented to the geographic control points which have been plotted on the projection, the whole laydown assumes its proper position on the manuscript.

The uncontrolled laydown is one whose position is not fixed with respect to the map projection. In this type of laydown the paper templet plots should be checked carefully in order to determine the strongest plot to use as a base, and to determine the order in which adjoining flights are to be laid. The base strip is built to a predetermined approximate scale and the adjoining strips are then laid in the order previously selected.

Experience has seemed to indicate that if the trimetrogon compilation is to be incorporated into or combined with previously existing maps the metal templet plot should involve the use of enough map points selected from the other maps to insure that the two compilations can be combined smoothly along a previously selected line. This method is of real value because it provides a simple and mechanical procedure for evaluating any adjustment that must be made between the two compilations in the neighborhood of each map point. It should be emphasized that adequate geographic control should be used wherever possible and that the map points are used only to insure that the two compilations can be combined satisfactorily. The use of this procedure saves considerable time and trouble in the final compilation and adjustment operation.

4. ADJUSTMENT OF SCALE. It is usually not possible to make the paper templet plots at the exact scale of the corresponding map projection. For this reason, the lengths of metal arms indicated by the paper templet plot will have to be corrected at the time the metal templets are made, in order that the metal templets can be combined together at the desired scale. The amount by which the scale indicated by the paper templet plot must be changed, can be determined in one of the following ways:

If two or more geographic control points appear within a single strip of photography, the scale of the corresponding series of metal templets can be determined quite easily by the following procedure:

a. Measure the distance between the two geographic control points, as they appear in the overlay of the paper templet plot.
b. Measure the corresponding distance between the plotted positions on the manuscript.

c. Determine the correction factor necessary to change the scale of the paper templets (determined by the paper templet plot) into the required scale of the map projection, by the equation:

$$\text{Correction factor} = \frac{\text{scale of overlay}}{\text{scale of projection}}.$$

The exact scale of the metal templets may be determined by the use of geographic control positions that appear in each of several different strips. In this method a paper templet plot is made on the map projection itself, using a procedure that is similar to that given in the preceding chapter. The only paper templets that are used in this procedure are the ones that cover the area in between the geographic control stations to be used. Under these conditions the following procedure is recommended:

a. Select all those paper templets from a strip that have rays towards one of the geographic control positions. Assemble these templets on top of the projection at approximately their correct scale and position, with the radial intersection of the geographic control point placed above its plotted position on the map projection.
b. Select a few consecutive paper templets from the next adjacent strip in the direction of the other geographic control station. Assemble these templets so that the radial intersections of the common tie points fall at exactly the same positions on the map projection.
c. Repeat step 6 until templets can be assembled in this plot that provide a radial intersection of the other geographic control station. This intersection should fall fairly close to its plotted position.
d. Holding to all azimuth lines and radial intersections, adjust the scale and position of the paper templets until the radial intersections of the geographic control points fall very close to their plotted pos'tions on the map projection.
e. Determine for each separate strip pf paper templets the correction factor necessary to change the previously established scale of the paper tenplets into the required scale of the map projection To make this determination measure the original length of several different rays in each strip and their corresponding lengths as indicated by the above plot. The correction factor for each strip of templets can then be computed from the equation:

$$\text{Correction factor} = \frac{\text{Old length of ray}}{\text{New length of ray}}.$$

Where one or more strips of metal templets have already been assembled in the metal templet plot, the scale of the next adjacent strip of metal templets can be determined by combining, in their correct positions on the map projection, a series of six or eight paper templets into a paper templet plot in such manner that the radial intersections thus determined for the common tie points fall on their corresponding studs of the metal templet plot. From the lengths of the rays thus determined, the required correction factor can be computed in a manner similar to that described in the previous two paragraphs. In a similar manner the relative scale between two strips of photography can be determined by measuring between a number of different and widely separated tie points that are common to each pair of adjacent paper templet plots.

5. CONSTRUCTION OF METAL TEMPLETS. *a. Equipment needed.* A list of supplies needed for assembling metal templets can be found in section XII, C. In

order to be able efficiently to construct the metal templets from such a variety of equipment it is obvious that the material must be kept in some kind of systematic order. There are various ways of filing the metal arms, such as in pigeon holes, or in boxes. Perhaps one of the most simple ways is to hang them on the cross-bar of a small saw horse. (See Fig. 14.) A row of nails may be driven into the side of the cross-bar and the metal arms hung on the nails according to their

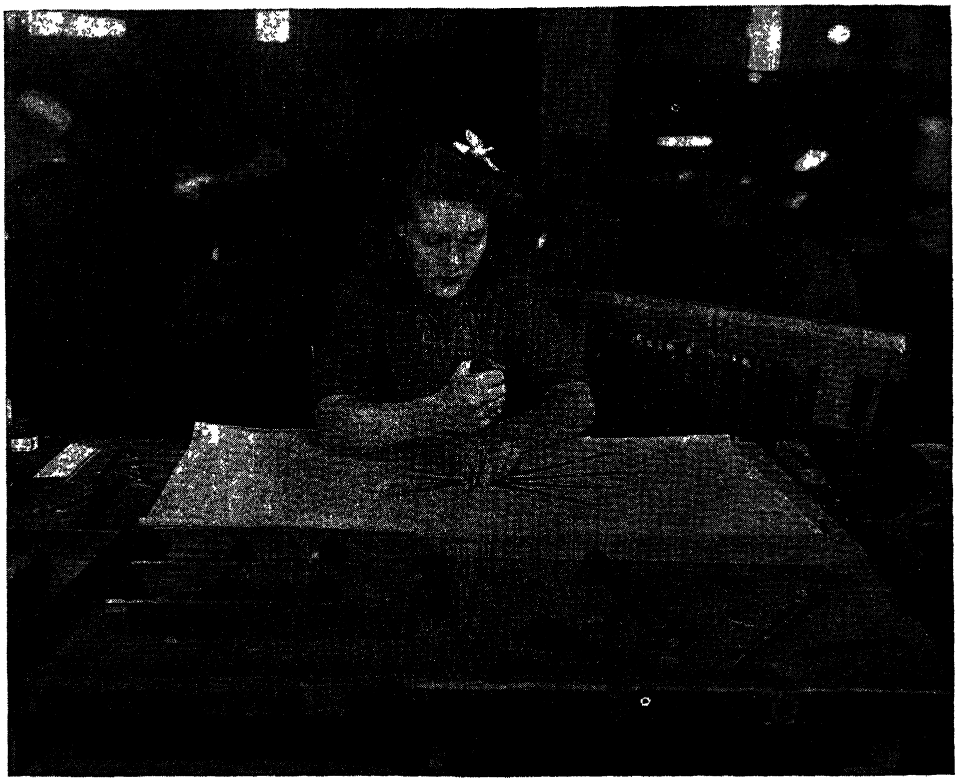


FIG. 14. Building metal templet.

length. Various types of material have been used for the surface of the work tables upon which the metal templets are made. A cork composition floor covering has been found to be the most satisfactory because it holds the pins firmly, and the holes in such material tend to fill up after the pins are removed. Soft pine boards are sometimes used but it is necessary to use pliers to remove the pins from such material and, after some use, this kind of table top becomes so full of holes that it must be replaced. If the table top is made of a material that is too porous the pins have a tendency to move laterally when the templet is being locked.

b. Procedure. The following procedure is recommended in constructing the metal templets:

1) Place a paper templet on the work table with the line of flight perpendicular to the front edge of the table. Eliminate all wrinkles.
2) A one inch pin is placed in a bolt and spotted carefully at the center of the templet. Hold the bolt firmly on the table and tap the pin with a hammer so that it is set securely into the top of the work table. In a similar manner

studs are placed at all the marks indicating the reduced length of the various rays for which metal arms are to be used, including the forward azimuth but not the back azimuth.

3) Place an end wrench over the metal bolt at the center of the templet. A short metal arm with no slot will represent the back azimuth. One end of this arm is placed over the center bolt, a stud with a pin through it is then placed through the last hole at the other end of the arm and oriented so that the pin is directly on the azimuth line. The pin is then driven into the table, thus fixing this arm in place. A short metal arm representing the forward azimuth is then dropped into place with the end hole over the center bolt, and the slot over the stud previously pinned on the azimuth line.
4) The long arms are put on next—the metal arms being selected so that the slots are centered on the studs as closely as possible.
5) After all the metal arms are in place, several washers are placed on top and a nut screwed down by hand. The left hand is then used to hold the arms carefully in place with the thumb placed on the handle of the end wrench to keep it from moving. (See Fig. 14.) A socket wrench is used to tighten the nut.
6) The templet is checked by the builder before removing it from the studs, by lifting each metal arm and letting it fall back over the stud to see if it will drop into its correct position. If any of the arms are in error, the nut must be loosened and then retightened.
7) All rays that are numbered or lettered on the paper templet are similarly designated on the metal templet by either placing a small piece of scotch tape on each of the arms upon which the correct symbol has been written, or by lettering the arms with a white or orange pigment ink.

The operator building the templet should make and letter the templets very carefully because an error in the templet may be very difficult to locate later and will thus introduce considerable error into the final compilation.

6. ASSEMBLING THE TEMPLETS. The first metal templet is placed on the manuscript in approximately its proper position; two studs are placed in the azimuth arm and one stud placed in the slot of each of the other arms. The second templet is then placed so that the azimuth arm falls over the studs in the azimuth arm of the preceding templet and so that the slotted arms fall over the studs in the corresponding arms of the first templet. The studs will have to be moved to their correct positions in the slots of the preceding templet before this may be done. The same process is repeated with each consecutive templet. The assembly must be shifted slightly, from time to time, to free the arms from any undistributed strain and to assist the system to reach its best adjustment. Do not force any arm to fit, and use a light touch for all movements.

In laying the templets it may be found occasionally that corresponding arms on two templets do not intersect satisfactorily, or that arms to tie points between flights do not intersect. In all such cases the cause of the error must be determined. In order to do this, the last two or three metal templets should be checked against their corresponding paper templets, the points on the photographs should be checked stereoscopically, and the combining of the metal templets should be checked. If no error is found it may be necessary to check the preparation of the paper templets themselves.

7. FINAL STEPS. When all the strips in the laydown are completed and adjusted, the azimuth bridges are removed. (See second paragraph p. 695.) Small nails are then placed in the center of each bolt and stud used in the plot, and are driven securely through the manuscript and into the table underneath. After all these points have been located on the manuscript in this manner, the metal

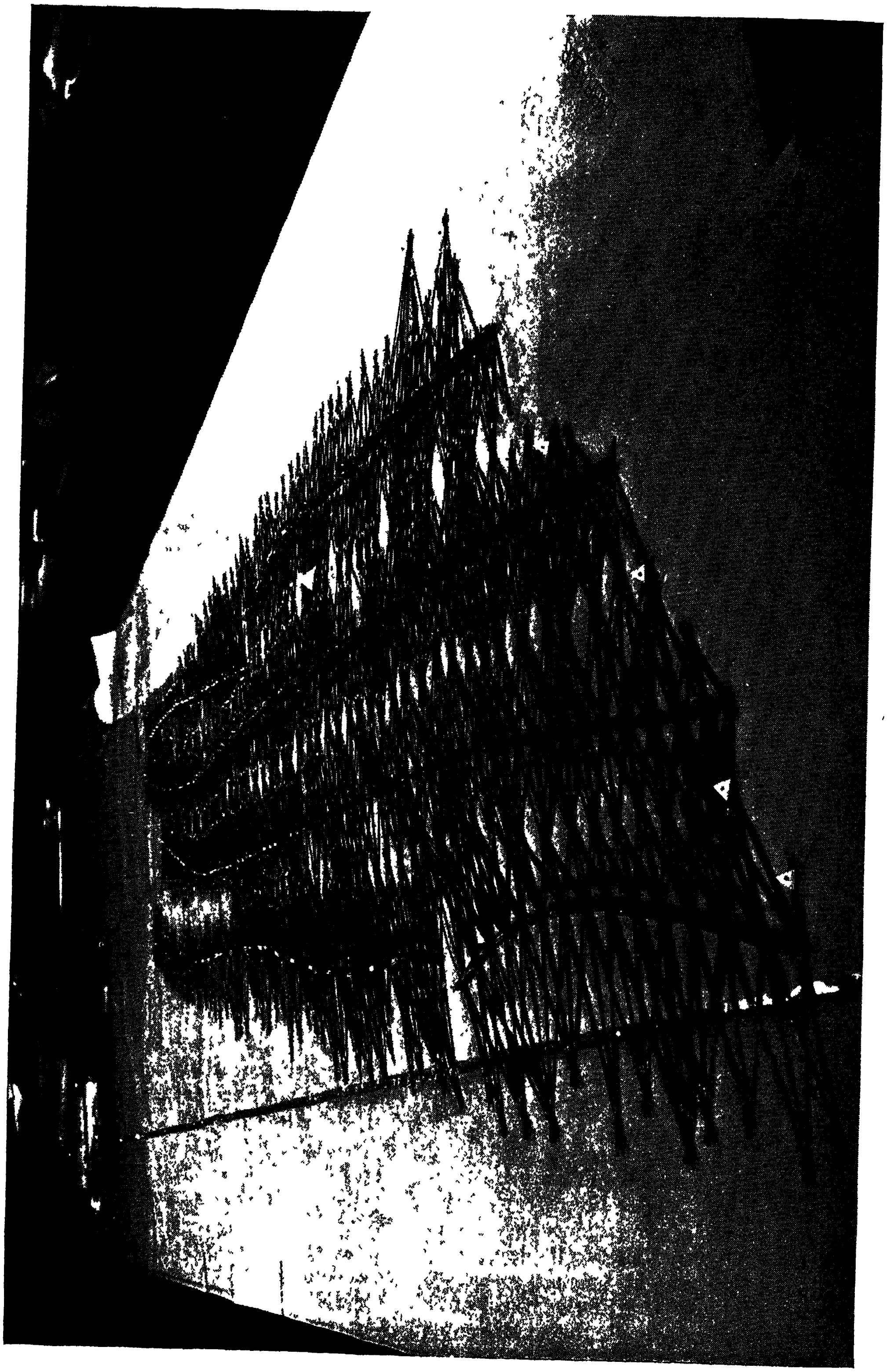
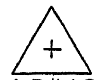


FIG. 15 Completed metal templet plot.

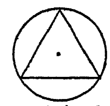
templets are then removed and taken apart. The nails are removed and the radial intersections thus located are marked with small inked circles and labeled with their proper designation in large enough letters to be legible after the manuscript has been reduced six times for the final compilation and adjustment. The plotted positions of the geographic control stations are designated with the following symbols in order to provide a permanent record on the manuscript of the manner in which the stations were used in the metal templet plot:

a. This symbol designates the plotted position of a control station which could not be held to in the metal templet plot because of an apparently faulty description.



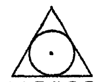
AP#42

b. This symbol, which must always be associated with the above symbol, designates the position of a control station determined by a radial line intersection of the metal templets.



A P#42

c. This symbol designates the plotted position of a control station which was held to in the metal templet plot.



M P#33

Both ends of each strip are lettered with the direction of the flight and the exposure numbers of the templets used. The legend on each end of each sheet of the acetate forming the manuscript shows the name of the region mapped, the type of the projection used, the scale of the projection, and the geographic control points to which the strips were tied. When more than one sheet of acetate is used for the manuscript, short numbered lines are placed across the joint at intervals of approximately two feet. These make it possible for the manuscript to be taken apart and reassembled at a later time.

8. SUPPLEMENTARY INSTRUCTIONS. *a. Extending the plot.* If the table is too small to contain the project all at one time, the plot is extended as far as possible After the best possible adjustment has been secured, the pins are tapped into place for about three-fourths of the templets. Remove one-half of the templets and mark the points. Remove the remaining pins and slide the manuscript and the remaining templets across the table and unroll the remaining portion of the manuscript sheet. Extend the projection and plot any additional geographic control stations. Replace the pins for the previously determined intersections and continue the work as before.

b. Azimuth bridges. As the work of assembling the metal templets progresses, the azimuth bridges are constructed. An azimuth bridge is two long metal arms rigidly bolted together like any other metal templet One arm represents a direction line towards the nadir point of approximately the sixth preceding templet and the other arm represents a direction line towards the nadir point of approximately the sixth following templet.

The overlay of the corresponding paper templet plot is used as a pattern in making the bridges. The nadir points of every sixth paper templet (approximately) are connected with a straight line and then the length of this line is reduced in the same proportion as the other rays of the paper templet plot. An azimuth bridge can then be made from this overlay, using two slotted metal arms of the proper length. Each azimuth bridge is labeled on the forward arm with the exposure numbers of the three photographs involved, in the order of their exposure. This prevents the mistake of getting the templet reversed.

Blunt pins are placed in the center of each of the proper metal templets, and studs are inverted and placed on the two outside pins. The strip of metal temp-

lets are then lined up by shifting them laterally until the templet bridge will fall exactly into place on the pins without any binding. These bridges which are used throughout the length of each strip of templets, serve to maintain the overall azimuth of the flight lines and thus improve the accuracy of the plot.

Section IX

INTERSECTIONS FOR DETAIL POINTS

A. General. After the metal templet plot has been completed, the detail points are intersected by means of the third and last radial plot. The plot of the detail points appearing on the obliques is made from the paper templets directly on the manuscript

B. Procedure. Radial intersections of all detail and pass points appearing on the oblique photographs may be obtained from the paper templets according to the following procedure:

1 Place the first of a series of paper templets under the manuscript in approximately its correct position and then orient the paper templet so that its rays to tie points and distant red points exactly intersect the plotted positions of those points on the manuscript. (See Fig 16 for an illustration of the arrangement of the work) If all rays to tie points and distant red points cannot be made to intersect their corresponding plotted positions on the manuscript, the very best possible average should be obtained.
2. Force a fine needle into the table through the center of the paper templet to serve as a guide for the straight edge that is to be used in transferring the rays to the manuscript.
3. Transfer onto the manuscript every ray appearing on the paper templet which represents the direction toward a pass point or detail point.
4. The first paper templet should then be removed and the next templet of the series correctly oriented and its rays transferred in the manner described in the above procedure.

If some of the rays thus located on the manuscript form triangles instead of actual intersections at a point, the work previously done must be checked. Possibly the wrong radial has been used or an error made in picking the point on the photograph. If all errors are corrected only very small triangles will be obtained at the intersections of the radials. Sometimes the points cannot be corrected and in these cases they should be rejected on both the photograph and the manuscript. If good three-cut intersections have been obtained, it is generally safe to accept the two-cut intersections for the points appearing in the foreground of the obliques If the location of an occasional point determined by only two cuts is questioned, a paper templet can be made for the intermediate pair of obliques and a third ray thereby obtained to check the previously established radial intersection.

Since all the rays are numbered on the paper templets, the correct intersections can be easily determined After completing the transfer of the rays appearing on any one templet, check the photographs to see that all the detail points have been intersected. This keeps the work complete as it progresses.

After all the detail points have been intersected, punch the center of each intersection with a needle, inscribe the point with green ink and, if the control points were numbered, label the radial intersection with the corresponding number appearing on the photograph. The radials are then cleaned off the manu-

FIG. 16. Locating intersections of detail points.

script sheet with absorbent cotton moistened with benzene The supervisor of this unit must review the plot before it is released to see that all the pass points appear to have been correctly located on the manuscript.

SECTION X

DELINEATION OF PLANIMETRY

A. General. The delineation of photographic detail, which is to be transferred to the map manuscript, is performed with the use of a suitable stereoscope. Because the transfer of this detail is accomplished with sketchmasters (see Sec. XI, Transferring Planimetry), brilliant, opaque colors, either pencil or ink, should be used. Suitable symbols, such as those in general use on maps are usually employed.

Since this operation is already familiar to photogrammetrists, only those parts of the work which apply particularly to the trimetrogon method will be explained.

B. Supplementary Instructions. 1. SCALE CHANGE ON OBLIQUES. It should be kept in mind that the scale of the oblique varies as the distance between the ground and camera changes. For example, with a 30° depression angle, the scale at the principle point of the oblique is one-half that of the vertical. A unit area

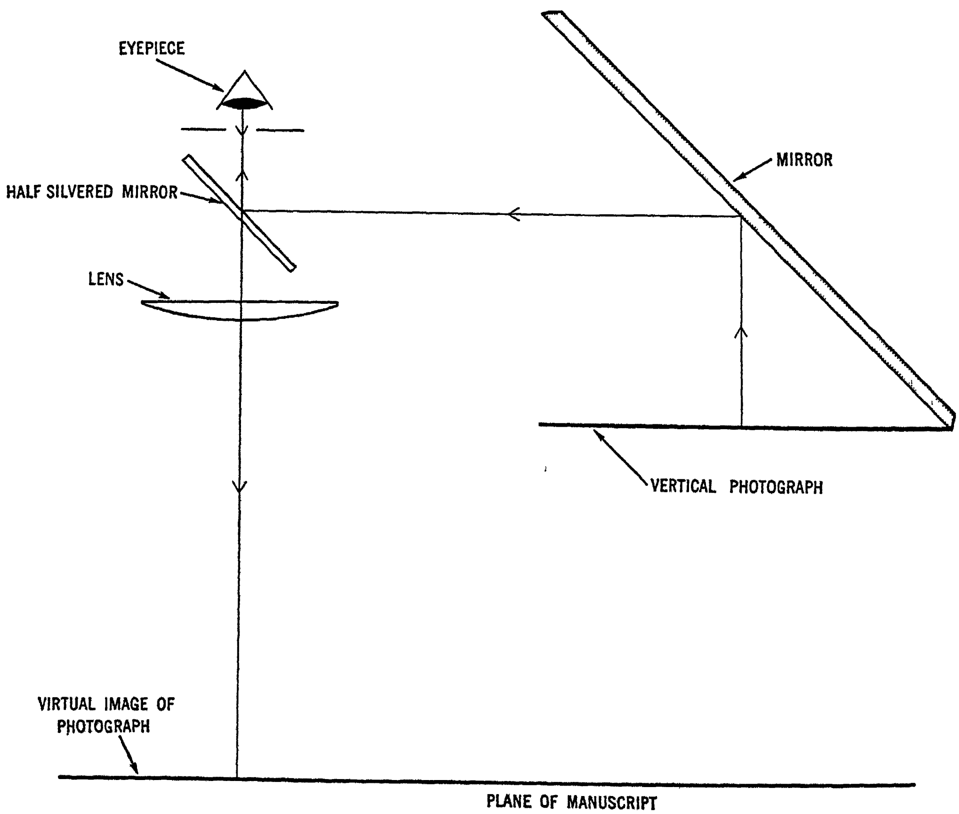


FIG. 17. Schematic diagram of vertical Sketchmaster.

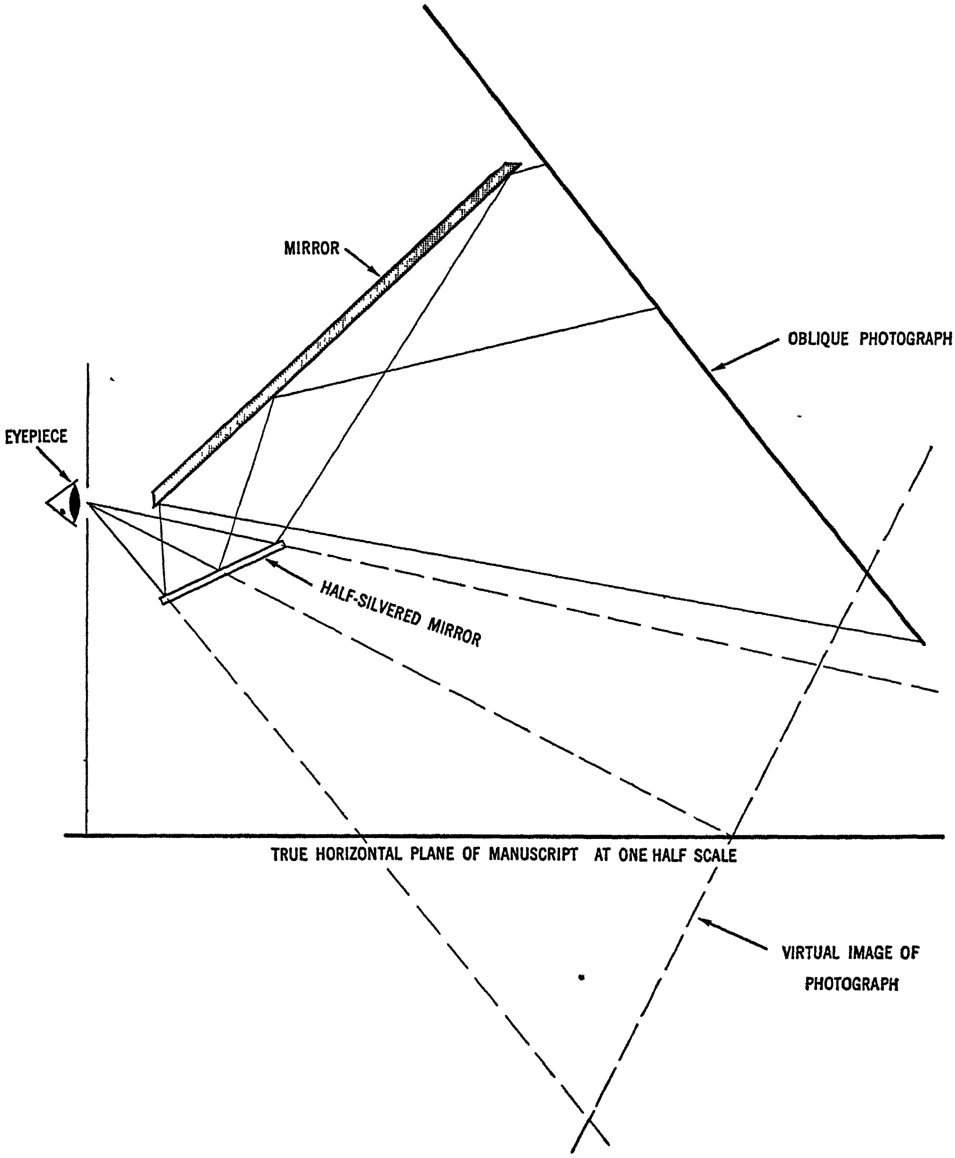


FIG. 18. Schematic diagram of oblique Sketchmaster.

at the principle point on the oblique covers four times the ground area that is covered by the same unit area on the vertical. Therefore, there is a greater concentration of detail and the need for closer delineation and careful examination of topographic features increases as the operator delineates out towards the horizon.

Delineation on the obliques should be carried towards the horizon only as far as pass points are carried.

2. OVERLAP. Operators using vertical photographs have a more or less regular amount of overlap in which to work. On trimetrogon obliques there is a larger overlap, which increases towards the horizon.

The process of joining the delineation on obliques from parallel strips presents another phase of the trimetrogon method which differs from the ordinary procedure on verticals. The angle of view in this case is so different that considerable care must be exercised in delineating features common to both flights.

3. STEREOGRAM The wide angle of the metrogon lens and the changing angle of view make it impossible to obtain a stereoscopic image of the whole area covered by the overlap of two obliques. The operator can fuse only small areas

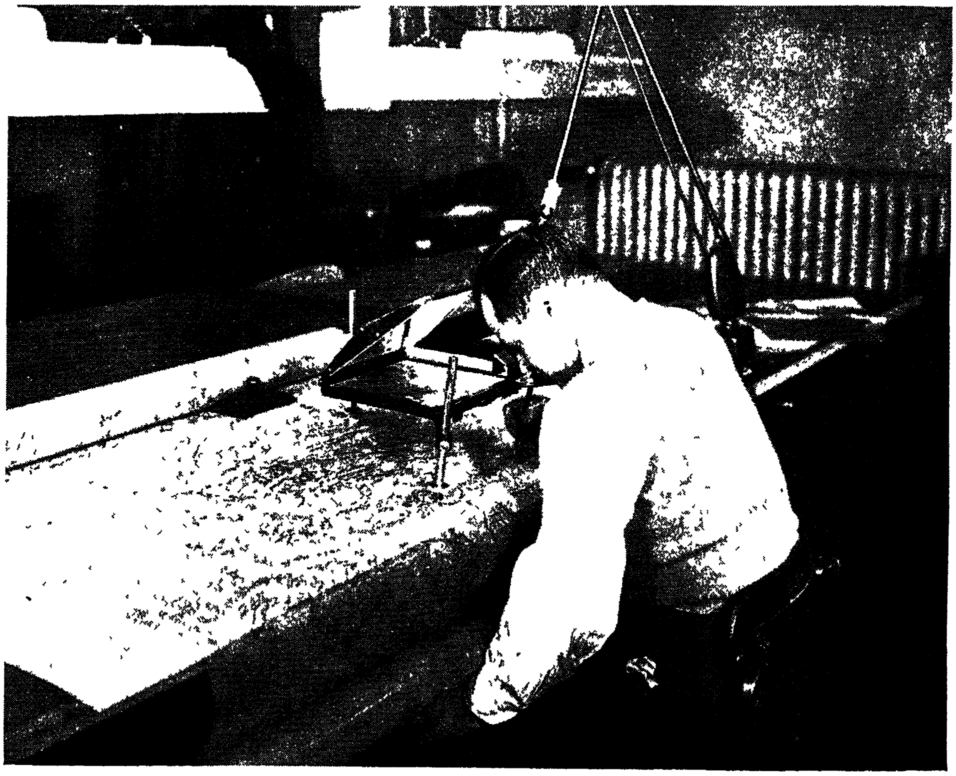


FIG. 19. Operation of vertical Sketchmaster.

in the foreground. These areas increase progressively in size as the work progresses away from the line of flight until it becomes possible to fuse the entire background of the two overlapping photographs at one setting.

SECTION XI

TRANSFERRING PLANIMETRY

A. Description of the Sketchmaster. 1. VERTICAL SKETCHMASTER. The vertical sketchmaster is a small portable instrument used for the transfer of planimetric detail from a vertical photograph to the manuscript. It consists of a tripod supported frame in which the photograph is mounted parallel to the table. As Figure 17 suggests, the operator looks down through a half-silvered mir-

ror mounted at the front of the instrument, and sees the image of the photograph superimposed upon the manuscript. The three legs can be adjusted in length to correct for tilt and differences in scale.

2. OBLIQUE SKETCHMASTER. The oblique Sketchmaster is a small portable instrument used for the transfer of planimetric detail from an oblique photograph to the manuscript. It consists of a frame, supported by three adjustable legs, in which the face of the photograph is mounted at a 60° angle with respect

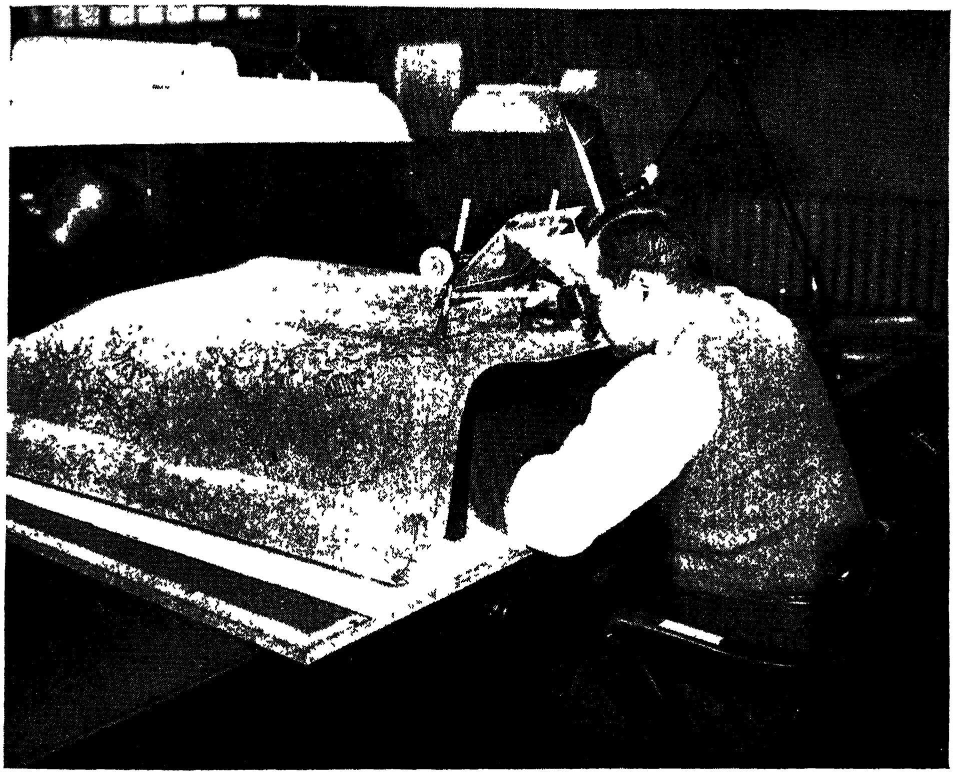


FIG. 20. Operation of oblique Sketchmaster.

to the table. Two mirrors, one of which is half-silvered, are set at the correct angles in order to place the oblique photograph in its correct position in space with respect to the manuscript. The operator looks through an aperture to see the image of the photograph superimposed above the manuscript. (See Fig. 18.)

B. Theory of the Sketchmaster. Figure 21 suggests in a very simple way the relations that exist between an aerial photograph and the original image. *ABCDEFGH* is a perfect square which has been subdivided into four equal parts, forming a simple grid. O'' is the exposure station of a vertical camera (which can also be thought of as the center of the camera lens or the perspective center of the photograph) which is directly above point P on the ground. $A''B''C''D''E''F''G''H''$ is an untilted vertical photograph. The grid appearing on this photograph is an exact copy, at a reduced scale, of the original grid. O' is the exposure station of a typical high oblique photograph, which appears at $A'B'C'D'E'F'G'H'$. However, the grid appearing on this photograph is quite

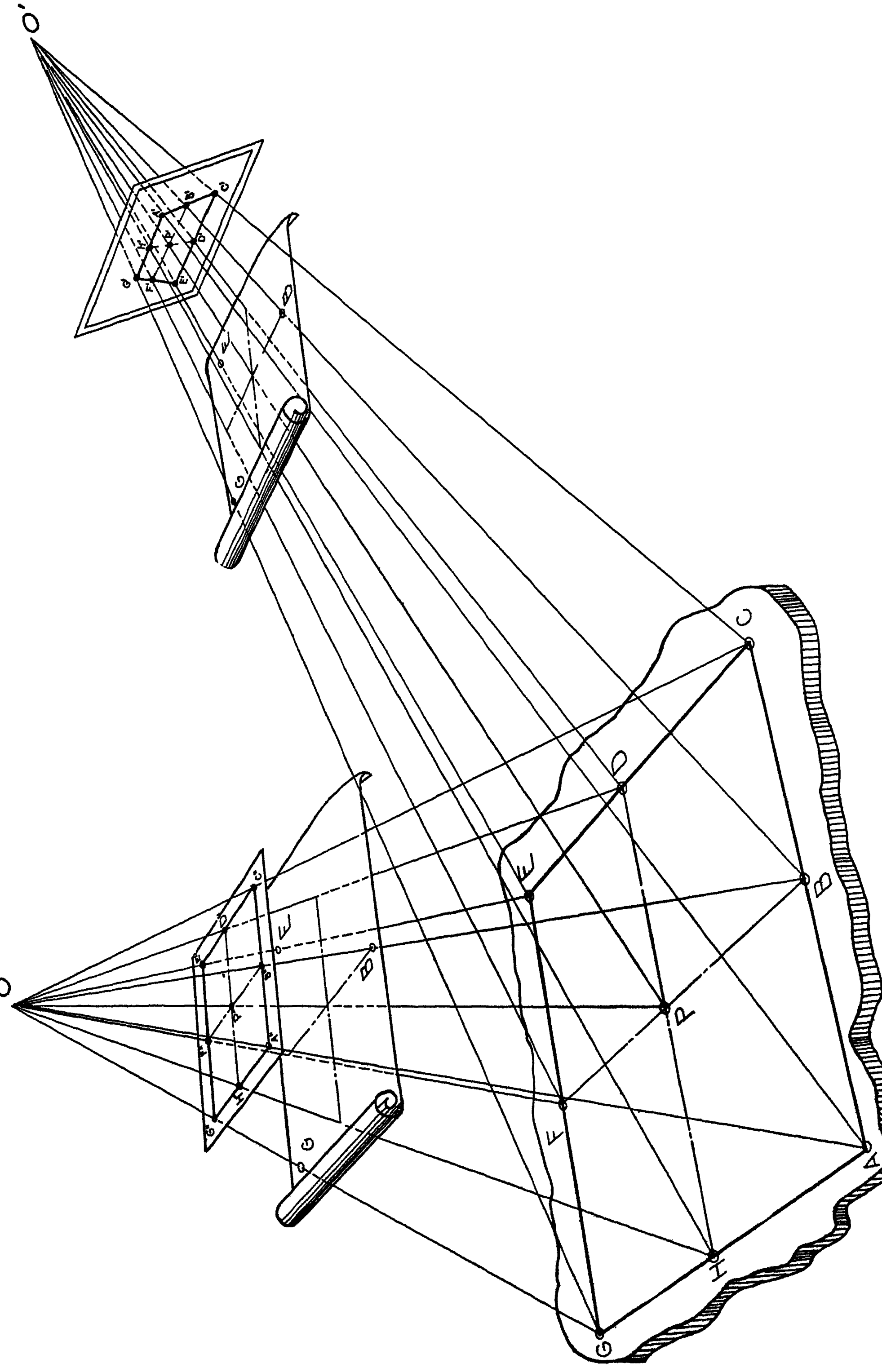


FIG. 21. Relation between vertical and oblique photographs and a simple ground image.

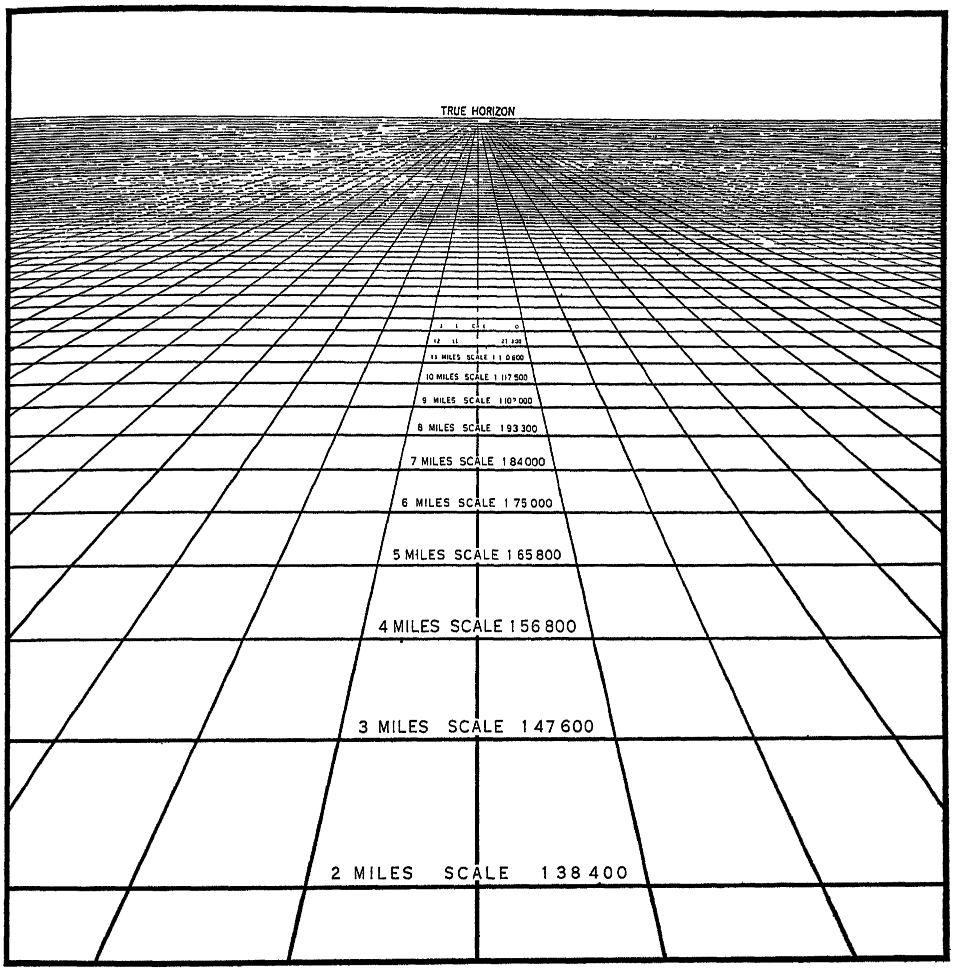


FIG. 22. Canadian perspective grid.

badly distorted. The manner of distortion is more clearly indicated in Figure 22 where each of the squares in the grid would be exactly the same size on a map of the area, and where each of the lines parallel to the flight line possesses a constant scale.

If points B, E, and G are plotted in their correct positions on a map manuscript, the manuscript can then be placed so that it intersects the bundle of rays coming from O'' in such a manner that the rays to the image points B'', E'', and G'' will pass through their plotted positions on the manuscript all at the same time. There is only one position in which the manuscript can be placed so that it will meet this condition. Likewise, the same manuscript sheet can be placed so that it intersects the bundle of rays coming from O' in such a manner that the rays to the image points B', E', and G' will pass through their plotted positions on the manuscript all at the same time. In this case too, there is only one position in which the manuscript can be placed so that it will meet this condition.

Figure 23 illustrates the conditions existing when a second high oblique photograph was exposed. Note the large hill at F'. On a map F' would appear at F. However, on the photograph F will be displaced away from the plumb point so that it falls at F' on the photograph rather than at F. This illustrates the statement that relief is displaced radially from the plumb point. If the eye were to be placed at O, as in an oblique Sketchmaster, such points as A, C would appear to fall at their correct positions on the datum plane. On the other hand, such points as B', D', E', and F' would appear to fall at incorrect positions on the datum plane, B'', D'', E'', and F'' respectively.

From the above discussion it can be seen that the Sketchmaster is essentially an aid to sketching It is impractical to place and hold the eye exactly at the perspective center of the photograph while sketching, and it is not possible to correct for displacement due to relief without adequate photogrammetric control However, when controlled by and adjusted to sufficient photogrammetric control, the device enables the planimetry to be complied in its reasonably correct rect shape and position. The Sketchmaster, has been designed and built for work with trimetrogon photography and cannot be used with other oblique photography where focal lengths and tilts are materially different from those of the trimetrogon exposures. The Sketchmaster can be used for sketching at scales ranging from one-half to twice the scale of the corresponding vertical exposures.

C. Procedure for transferring planimetry. 1. GENERAL. Standard map symbols are used in transferring photographic details to the manuscript. No lettering is made on the acetate, except notes to the proof-reader indicating conditions which he might not ordinarily detect. Where roads or railroads follow close to a stream, the distances between them must be exaggerated

It is important that care be taken not to touch the mirrors It has been found that the salt in the perspiration decomposes the chemical coating of the mirror and renders it useless. These mirrors are costly and difficult to replace.

An extremely important part in the transfer of planimetry is checking the detail on the photographs and making sure that it is transferred to the manuscript properly. Drainage and road systems must be connected properly, and the operator must also check to see that the delineators did not make any errors.

2. VERTICAL PHOTOGRAPHS. After all the pass points have been located on the manuscript and the photographs delineated, the planimetry can be transferred from the vertical photographs to the manuscript using the vertical Sketchmaster. The following procedure is recommended:

a. A portion of the manuscript is unrolled on a suitable table—both rolled up ends being held in position by cardboard cuffs. (It may be found that the long strips of acetate will be rather awkward to handle. A convenient holder can be made from the heavy cardboard tubes around which the acetate is wrapped. A piece of six inches long is sawed off and a lengthwise cut on the side is made. This cuff is then slipped over the roll of acetate, which prevents the roll from unwinding and falling off the table.)

b. By observing its corresponding points on the manuscript, the operator can determine which way the picture is to be inserted in the instrument. The picture is inserted within the frame so that it is perfectly flat, allowing all the pass points to be shown. The operator, looking through the eye piece, should approximately orient the instrument so that each pass point falls at least somewhere near its plotted position on the manuscript. The lights should be adjusted so that a correct balance of light is obtained between the manuscript and the photograph in order that the operator can see the picture points, the planimetry, the plotted control, and the pencil all at the same time.

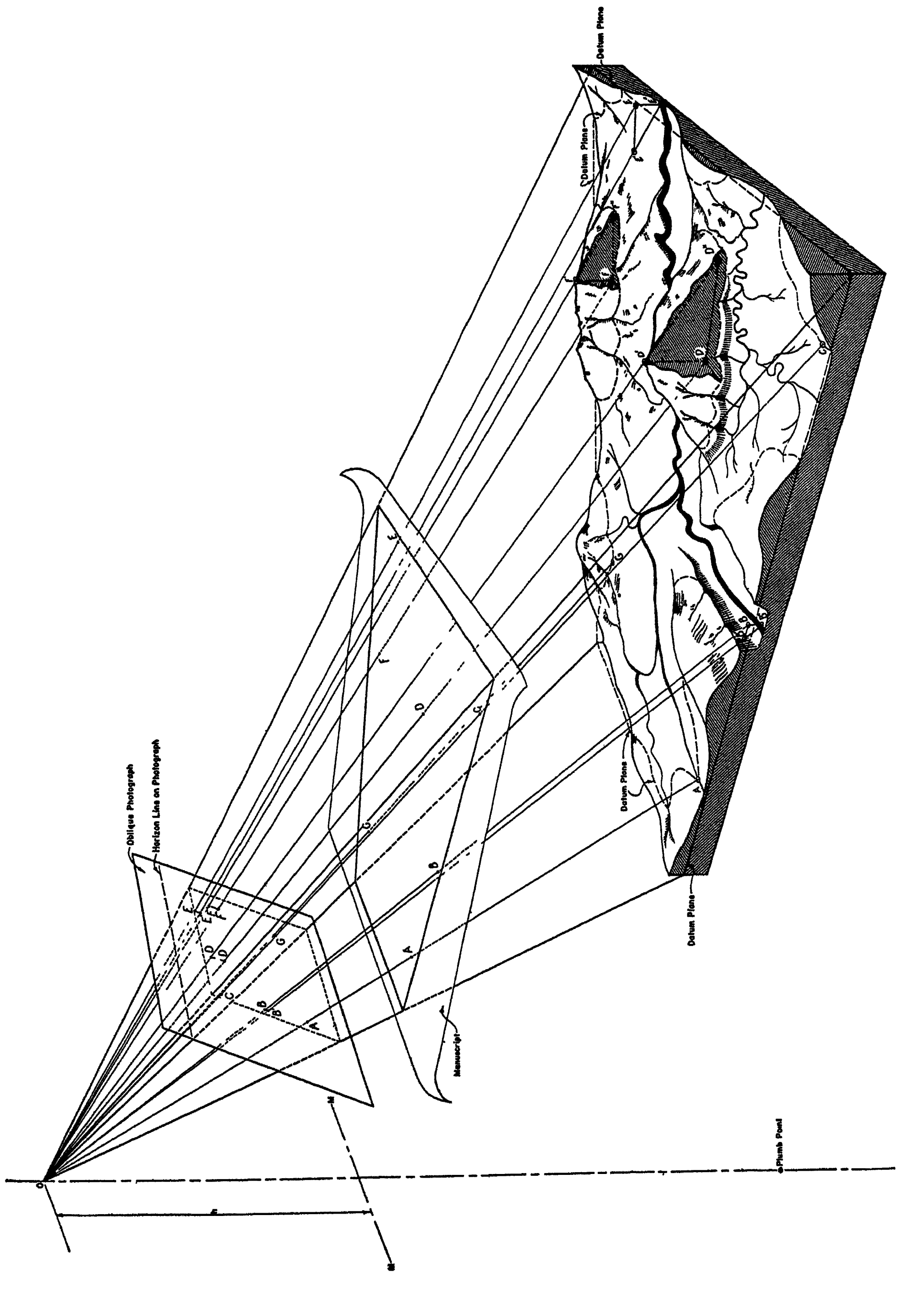


FIG 23. Effect of distortions due to relief.

c. Various lenses furnished with the instrument should be tested until one is found which removes practically all the parallax at the scale to be used. The following rules will help in the selection of the proper lens.
   1) For a low rig use a large numbered lens and vice versa.
   2) If a point on the manuscript moves in a direction opposite to that of the eye of the operator, use a smaller numbered lens.

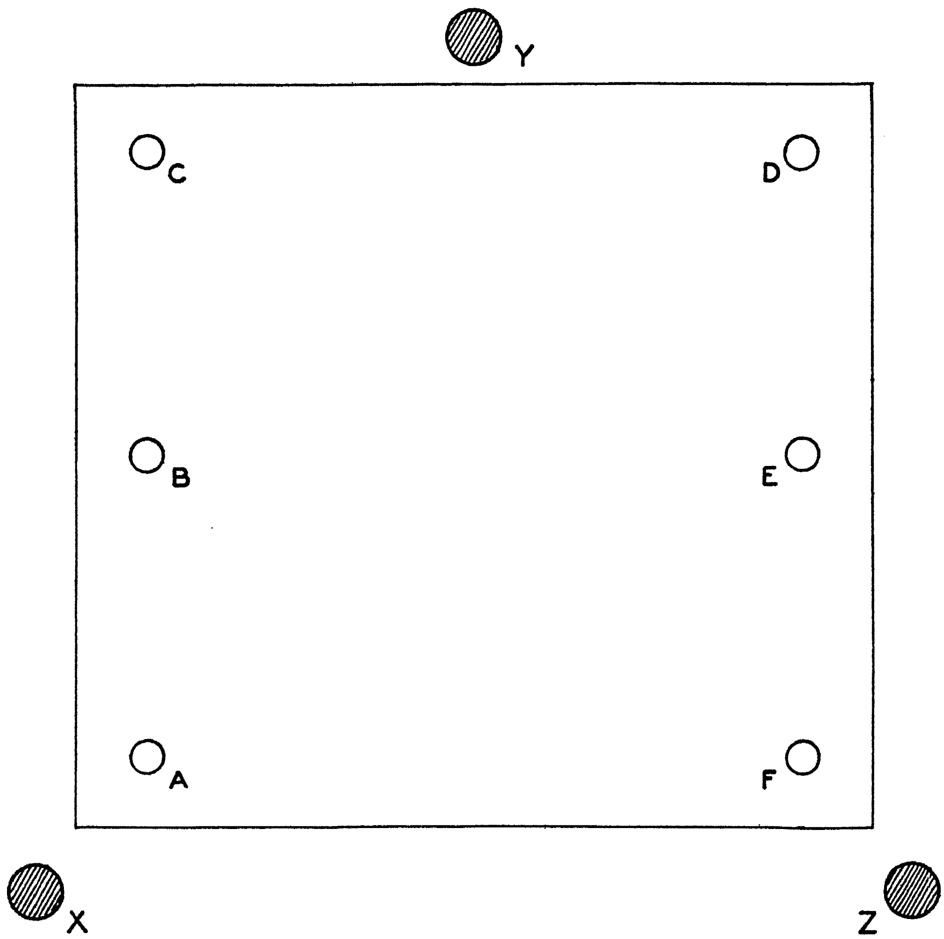


FIG 24. Orientation of vertical Sketchmaster.

d. The instrument is adjusted for scale by raising or lowering the frame on all three legs. Large movements are made by loosening the clamps and sliding the legs. The final adjustments are made with the screw feet. Adjustments on one or two legs are made to correct for tilt in the photograph. If the terrain has a very even slope, tilt can be introduced into the photograph until it is parallel to the ground surface. The adjustment is satisfactory and the instrument is correctly oriented when all the radial *lines* on the photograph appear to pass through their respective pass points on the manuscript, and the radial *points* coincide as nearly as possible Figure 24 can help to illustrate the many combinations of adjustments which can be made with the Sketch-

master If leg X is lowered, points A, B, and C will converge. When leg Y is lowered, points C and D will converge. When leg Z is lowered, points D, E, and F will converge. When legs X and Z are lowered, points A and F will converge. Inversely, when these legs are raised, the same sets of points in each of the above examples will then spread apart.

e. The planimetry is now ready for sketching. A hard blue pencil is used and all the planimetry that has been outlined on the photographs by the Delineating Unit is transferred onto the manuscript. The eye is shifted slightly to bring individual radial control points into exact register as the detail in their vicinity is being traced. For example, when tracing a stream, hold to the radial control points on or near that feature If there are no points on the detail, a skilled operator can properly orient nearby points in order to correctly locate the detail in question. The work of transferring the planimetry should be extended out to the edges of the vertical photographs or as far out as delineated.

3. OBLIQUE PHOTOGRAPHS The imaginary example suggested by Figure 23 illustrates very well the problem of transferring planimetry with the aid of an oblique Sketchmaster. In that example, when the eye of the operator is placed at O, which corresponds to the eye piece of the oblique Sketchmaster, a bundle of rays is established in space which corresponds quite closely to the bundle of light rays which exposed the original negative. In any photograph having considerable relief, such as this example, it cannot be expected that all the details of the planimetry will appear to fall in their right positions on the manuscripts at the same time; it is for this reason that detail points such as A, B, C, D, E, F and G must be plotted on the manuscript so that the planimetry can be properly corrected for displacements due to relief.

With regard to an oblique photograph, it can be demonstrated that, after the manuscript has been correctly oriented in the bundle of rays, any point on the manuscript can be made to fall on its corresponding ray by rotating the manuscript up or down about an axis M. (See Fig. 23) It is therefore the problem of the operator to orient the oblique Sketchmaster above the manuscript in such fashion that any plotted control point can be placed on its corresponding ray from the picture merely by small movements of the rack and pinion at the back of the instrument. Under these conditions, when any of the control points do not match their plotted positions because of differences in relief, the operator can match at least three adjacent control points at one time and trace everything within the imaginary triangle thus created, assuming that the area within the small triangle is all at the same scale.

In flat terrain, the points will match perfectly. In rough terrain, the relief displacement of the rivers and high ridges may measure several inches. In such cases, the drainage should be transferred first and then the ridges. The points which control the positions of the ridges are not usually picked at points of the same horizontal planes of elevation unless the point picking has been done with the aid of a stereoscope. Since their displacements are different, very few of these points will fit the control at any one time. Therefore, the operator should first locate one of the main ridges and then turn the rack and pinion movement so that at least one point on the ridge will match its corresponding manuscript point. It may be assumed that this point is one at the highest elevation on the picture. Now, in transferring the ridge line from this control point to the next control point appearing on the ridge and which is at lower elevation, the operator must move the rack and pinion up slowly as he sketches so that when he reaches the lower elevation point with his pencil the picture point will match the manu-

script point. Since the relief displacement from the first to the second point decreases as the elevation decreases, the gradual movement of the rack and pinion and the pencil will correctly reproduce the planimetry.

As another example, if the ridge line has but one control point on it, the operator should start from that point and, as he sketches, turn the rack and pinion up so that when he reaches the bottom of the ridge slope, the control points on the streams will fit their plotted positions on the manuscript.

The chance for error in plotting control points by means of the radial line method sometimes may be great, but the operator must be careful in arbitrarily deciding that a control point is in error just because he has difficulty in holding to it. In flat country, when ten or twenty points match and one or two cannot be placed upon their manuscript positions with any possible setting of the rack and pinion movement, it is then safe to assume that these two points are plotted in error The operator will mark them with an "X" and not use them in controlling the planimetry. However, whenever more than a very few appear to be wrong, they should be checked by the supervisor of the unit.

It should be remembered that, in rough country, the accuracy of points cannot be checked by the operator in the manner described above Much care then must be taken in observing their positions relative to their surrounding points.

The area transferred from an oblique photograph is much greater than that transferred from a vertical photograph, and therefore greater care must be taken in tracing the streams, ridges, roads, etc. Also, care must be taken in connecting the planimetry from the vertical with that of the corresponding oblique.

After the planimetry has been completely transferred from the vertical photographs, it can then be transferred from the oblique photographs by the use of the following procedure·

a Draw a blue pencil line along the row of nadir points of the completed vertical strip and between the two rows of pass points, to serve as a guide in cutting the manuscript in two pieces It may be necessary to bend this line occasionally but it has been found best to limit the number of bends as much as possible. When the line has been drawn, the number of the photograph should be written on that side of the line where it does not already occur If the guide line goes through an exposure number, that number should be placed on both sides of the line. At each end of the line and on both sides of it, a complete description of the strip should be printed with blue pencil Cut the manuscript along the guide line.

b. Place one of the manuscript strips on the special light table with the cut edge next to the operator. An oblique Sketchmaster is placed on the manuscript and a photograph placed in its frame so that all the radial control points are visible, and with the horizon line parallel to the top of the instrument. By moving the Sketchmaster on the manuscript and by using the rack and pinion movement, the operator can match the photograph pass points and the middle distant picture points with their corresponding manuscript points. When these points match, the operator can easily see how the other picture points fall with respect to their plotted positions on the manuscript. If the points at the right of the picture fall inside their corresponding manuscript points and the points at the left of the photograph fall beyond their corresponding manuscript points, he must necessarily raise the right rear leg. Having done this, the operator will find it necessary to lower the Sketchmaster with the rack and pinion movement. If the tilt is so great that the limit of the rear leg adjustment is reached and more tilt is still required, he must tilt the picture within the frame of the machine so that the right side is raised. By using a combination of these adjustments, he can reproduce the proper

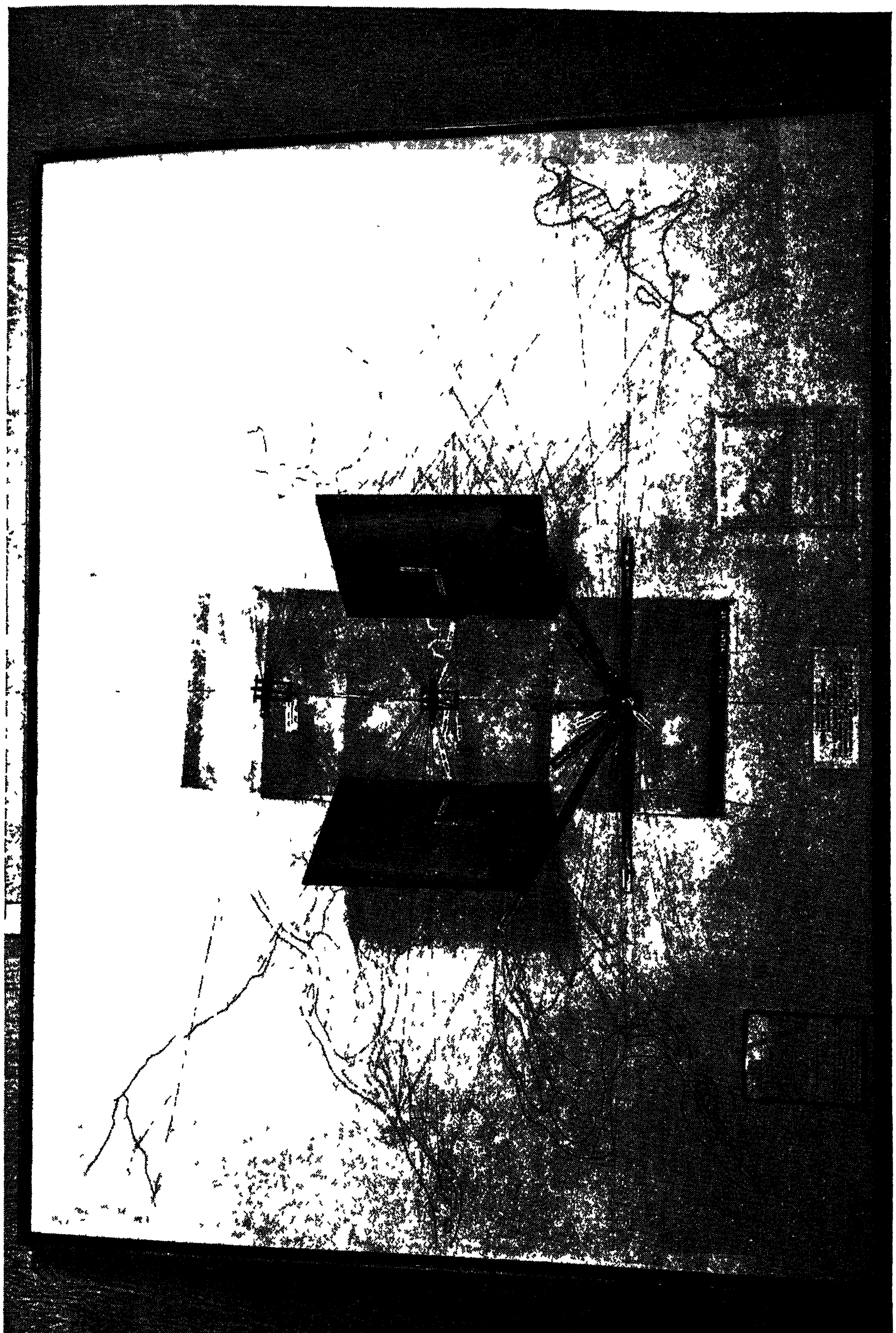


FIG 25 Trimetrogon compilation model.

tilt angle. Now, he might find that the picture points are too close together and should be spread out in order to match the manuscript points. Then, he must raise the Sketchmaster with the rack and pinion movement (When the Sketchmaster is raised, the points spread apart, and when it is lowered the points converge.) Points of different elevation on the photograph will appear to be above or below their respective points on the manuscript but they all can be brought into register, one at a time, by means of the rack and pinion movement, or by slight movements of the eye.

c. After the instrument has been correctly adjusted it will be found that all the picture points can be brought into correct orientation with the photogrammetric control on the manuscript, using the rack and pinion movement where necessary. The planimetry is transferred from the photograph to the manuscript A blue pencil is used for this transfer.

4 INSPECTION After all the planimetry has been transferred from the photographs, the two halves of the manuscript are fastened back together with transparent cellulose tape. The supervisor of the unit then checks the sheet to see that the work has been done correctly, using the following list of questions as a guide.

Has any detail been omitted?
Does the work on adjacent obliques join correctly?
Is the junction between the obliques and the verticals satisfactory?
Do adjacent strips join correctly?
Have all the details been labeled properly?
Have the proper symbols been used?
Have lakes been distinguished by cross-hatch lines?
Should the spacing between important features be exaggerated, in order that they will not run together on the reduction?

SECTION XII

SUMMARY

A. General. In the preceeding sections, the actual process of trimetrogon compilation has been outlined step by step and enough theory has been given to enable reasonably experienced operators to work out the problems which will arise from time to time However, no definite instructions can be given here for a method of organization and administration of a compilation unit, because each organization presents different problems. Nevertheless, in a general way it can be said that occasionally some strips of photography will require an unusual amount of time in certain of the compilation procedures because of special conditions. For example, much time may be required in the point section because of the wealth of topographic detail, while the tilt section may have relatively little to do. At another time this condition might be reversed. Therefore it will be found desirable to develop a sufficient number of employees with general training so that personnel may be shifted from section to section as the work demands.

Because of this constant fluctuation of the work load of each of the sections it will be found necessary to coordinate the activities of the various sections with considerable care so that bottlenecks or dead spots do not develop in any of the operations.

In special cases, where speed of compilation is of more importance than an

efficient flow of work, certain modifications can be made in the general compilation procedure. With careful planning the work may be scattered through several sections so that several different compilation procedures can be performed simultaneously. For example, paper templets might be started on a strip of photography before the radial control points were picked throughout the strip. Delineation can be sandwiched in between any of the operations after the radial control points are picked and before the sketching is started. Proof readers may follow closely on the heels of the sketching operation.

B. Allocation of personnel. The following tabulation based on a unit of 100 employees, will prove a useful guide as to the customary allocation of personnel.

Group	*Personnel*
Indexing	6
Geographic Control	4
Point identification	22
Tilt Determination	3
Azimuth Lines	3
Paper Templets	7
Combining Templets	5
Metal Templet Laydown	9
Intersection of Detail Points	8
Delineation of Planimetry	11
Planimetry	5
Proof Reading	9
Adjustment and Final Compilation	8
	100

C. Equipment needed. An organization assigned to reconnaissance mapping duties will be equipped with the following instruments:

- Stereoscopes, pocket, mirror, and prismatic
- Rectoblique plotters
- Sketchmasters, vertical
- Sketchmasters, oblique
- Light tables,
  - Azimuth lines
  - Paper templet plot
  - Sketchmaster operation
- Tables, plain top drafting and cork-composition top
- Lights, drafting
- Pantographs, ordinary
- Dividers, proportional
- Drawing instruments
- Pens, drop-bow
- Prickers
- Protractor, large circular, steel
- Straight edges, steel
- Triangles, assorted
- Glasses, magnifying
- Slide rule
- Beam compass, complete with assorted beams
- Scales, plotting

In addition to the instruments listed above, the organization will require all the usual stationery and supplies used by draftsmen, as well as assorted colored pigment links, scotch and transparent tape, and cellulose acetate in rolls forty inches wide.

The following list of tools and parts are required by the group to make up the metal templets:

No.	*Article*	*Equipment Length*	*Size*	*Material*
3	Pliers			
3	Hammer, tack			
3	Hex. Socket Wrench (Stevens-Spintite $3414)	$5\frac{1}{4}''$		Tool Steel
3	Closed end wrenches	$4\frac{1}{2}''$		Tool Steel
3	File, magneto			
3	Tin snips	8″		
1	Reamer			
600	Pins, center	1″		#22 Piano Wire
1000	Pins, for studs	$\frac{5}{8}''$		Steel
200	Pins, azimuth bridge	$1\frac{1}{2}''$		Steel
500	Hex bolts and nuts	9/16″		C R. Steel
1000	Large Studs	$\frac{1}{2}''$	$\frac{5}{8}''$	Brass
1000	Small Studs	7/16″	$\frac{3}{8}''$	Brass
1500	Washers		$\frac{1}{2}$×1/32″	Brass
200	Washers		$\frac{1}{2}$×1/32″	Fiber
1000	Arms	1-11/16″	$\frac{1}{2}$× 015	Clock spring steel
300	Arms	2-27/32″	$\frac{1}{2}$× 015	Clock spring steel
1000	Arms	$3\frac{1}{4}''$	$\frac{1}{2}$× 015	Clock spring steel
1000	Arms	$4\frac{1}{4}''$	$\frac{1}{2}$× 015	Clock spring steel
1000	Arms	$5\frac{1}{2}''$	$\frac{1}{2}$× 015	Clock spring steel
750	Arms	7″	$\frac{1}{2}$× 015	Clock spring steel
850	Arms	8″	$\frac{1}{2}$× 015	Clock spring steel
750	Arms	$9\frac{1}{4}''$	$\frac{1}{2}$× 015	Clock spring steel
300	Arms	$10\frac{1}{2}''$	$\frac{1}{2}$× 015	Clock spring steel
1000	Arms	$11\frac{3}{4}''$	$\frac{1}{2}$× 015	Clock spring steel
1500	Arms	13″	$\frac{1}{2}$× 015	Clock spring steel
250	Arms	$14\frac{1}{4}''$	$\frac{1}{2}$× 015	Clock spring steel
250	Arms	$15\frac{1}{2}''$	$\frac{1}{2}$× 015	Clock spring steel
1000	Arms	$16\frac{3}{4}''$	$\frac{1}{2}$×.015	Clock spring steel
1000	Arms	18″	$\frac{1}{2}$× 015	Clock spring steel

D. Final steps. In addition to the processes explained in the previous sections, compilation, obtained by use of the trimetrogen method should pass through the following stages which are the same as in any map compilation:

1. Proof reading manuscript map against pictures.
2. Inking manuscript
3. Photographic reduction to base map scale.
4. Map drafting and editing.

CHAPTER XIV

FIELD INSPECTION AND COMPLETION

COMPLETION SURVEYS

T. P. Pendleton

EDITOR'S NOTE The following material was written by the author to be included with his article "The Multiplex Instrument and Its Use," that appears in Chapter XI. In that no material was available for this chapter, Field Inspection and Completion, the Editor took the liberty to move it to this position. This chapter will be thoroughly covered in the next edition.

AMONG the many points of difference between the planetable method of mapping and the stereophotogrammetric method is the need of examining the latter type in the field before the map is released for publication. This need arises because the photogrammetrist is only able to represent on the map those features that he has been able to see in the stereoscopic model. Houses obscured by trees, trails and roads so overhung in wooded regions as to be invisible to the camera from above, boundary lines, section lines and survey markers must be located on the map by a field party. This examination, referred to as a completion survey, increases the value of the map because it permits the engineer not only to add those details that the photogrammetrist cannot be expected to obtain, but makes possible any necessary correction and deletion of copy that he considers advisable. It is probable that undesired barns and other out-buildings will have been shown, as well as farm roads and trails, as the operator works under instructions to indicate such features on the map if in doubt of their importance, because it is a relatively simple matter to delete, but costly to add it later if found to be important.

The time required to perform completion surveys will vary greatly from map to map, depending largely on the difficulty of recovering section lines and boundary lines of various kinds. The fact that all visible property lines are shown on the original map expedites the delineation of such section lines and boundary lines, particularly when county boundaries follow irregular property lines.

The collection and verification of place names and the exact location of bench marks on the map are the duties of the engineer in charge of the completion survey. He is required to classify roads as to construction and importance and to indicate the principal trails in those regions where they are the only routes of travel.

The purpose of the Completion Survey is to obtain all information that must necessarily appear on the map but which is impossible to secure by an office examination of the aerial photographs, and to delete all undesired information that has been indicated on the original drawing by the photogrammetrist. The engineer responsible for such surveys is charged with doing whatever may be needed to perfect the map but his duties may generally be briefed as follows:

(a) To classify all roads, add omitted roads, trails and houses; and delete all roads, trails or buildings not appropriate to the map.
(b) Delineate all State, County, reservation and other important civil division boundaries, as well as section lines.
(c) Secure all place names, and verify spelling of all names to appear on the map.
(d) Indicate and accurately locate all important detail, including bench marks, that may have been omitted from the map.
(e) Complete the mapping of such small areas within the body of the map that the photogrammetrist may have been forced to omit due to heavy shadows, clouds in the photographs, gaps between flights, defects in the negatives or heavy woodland growth.
(f) Check and re-draw the map in all places where the delineation is erroneous.

CHAPTER XV

MISCELLANEOUS APPLICATION

CLASSIFYING FOREST AND OTHER VEGETATION FROM AERIAL PHOTOGRAPHS

A. E. Wieslander and R. C. Wilson

INTRODUCTION

SINCE 1937 in connection with a vegetation mapping project,[1] the California Forest and Range Experiment Station[2] has been carrying on studies with the use of air photographs for the purpose of developing improved mapping techniques. Preliminary results of this research were reported in 1939.[3] These and more recent results have now been crystallized in classifications and procedures that are proposed as a basis for future mapping work in the Forest Survey of California and western Nevada.[4] The classifications and procedures, together with proposed map presentation, are described here for their possible value in similar or related surveys elsewhere.

KIND AND EXTENT OF THE VEGETATION BEING CLASSIFIED

The gross area it is proposed to classify from air photographs in California and western Nevada approximates 70 million acres. Of this area about 19 million acres support conifer timber stands; 3 million acres are deforested land, largely shrub-covered; 10 million acres are nontimberland supporting cordwood stands of various oaks and other broadleaved trees as well as piñon pine and juniper; 8 million acres are nontimberland supporting chaparral; and an unestimated acreage supports grass, sagebrush, or subalpine vegetation. As would be expected in a region where elevations range from below sea level to nearly 15,000 feet above and annual precipitation from less than 5 to more than 90 inches, the vegetation is varied and complex, comprising many species.

PURPOSE OF THE CLASSIFICATIONS

The objectives of vegetation classification in California and western Nevada are twofold:

1. To provide maps showing the kind, character, and distribution of the natural vegetation of forest and related lands as a basis for wise management of timber, range, water, recreation, and wildlife resources.
2. To facilitate estimation of the volume of timber and cordwood in the region as part of the Federal Nation-wide forest survey and to localize such estimates by county units.

AIR PHOTOGRAPHS AND BASE MAP REQUIREMENTS

Most of the area covered by air photographs in California and western Nevada, as in the rest of the United States, is by an approximate scale of 1:20,000 or about 3 inches = 1 mile (original negatives are either 7×9 or 9×9 inches in size). To date (July 1942) about 60 per cent of the forests and related lands have

[1] Wieslander, A. E. First steps of the Forest Survey in California. Jour. Forestry 33. 877–884. 1935.

[2] Maintained by the U. S Department of Agriculture at Berkeley, Calif., in cooperation with the University of California.

[3] Burks, G. F., and R. C. Wilson. A vegetation inventory from aerial photographs. PHOTOGRAMMETRIC ENGINEERING 5(1): 30–42. Jan.–Mar. 1939.

[4] Wieslander, A. E., H. A. Jensen, R. C. Wilson, and G. F. Burks. Proposed plan for the vegetation type and inventory phases of the Forest Survey in California and western Nevada. 1942. (Unpub. ms.)

been flown, most of them by the U. S. Department of Agriculture. This flying was done mainly during summer months of the past 3 or 4 years. The 1:20,000 photo scale is large enough to show a fair amount of detail, and summer-flight photographs are free from long shadows even on rather steep north slopes. In a Nation-wide survey this type of photograph seems to be the best adaptation to the variety of requirements of engineers, agriculturists, foresters, timber cruisers, and range surveyors. Therefore the techniques of forest and vegetation classification described here were developed for use with the 1:20,000 scale photograph and for summer flying, even though in some predominantly forested areas either larger or smaller scales and other flying seasons are probably desirable for economy and amount of forest detail required.

Accurate base maps are necessary for adequate presentation of the classifications; and it is assumed that planimetric or topographic base maps prepared from air photographs will be available. During the past several years, the Division of Engineering of the United States Forest Service in California has been compiling planimetric base maps from air photographs by the radial-line-templet method. These are $7\frac{1}{2}$-minute quadrangle units on a 2-inch = 1-mile scale; they show all the planimetric features of the landscape, including the drainage pattern, landmarks, culture, and section lines. Since a fair amount of vertical control is established at the time of the planimetric survey, the topography may be drawn in later with little additional control when the extra cost is justifiable.

CLASSIFICATION AND TECHNIQUES

General Basis of Classification

In this scheme the natural vegetation is classified according to prescribed percentages of the ground space covered by component elements individually or in various pattern combinations. Other features such as cultivated, urban, industrial, and barren areas are classified and delineated directly as they appear on the photographs.

Method of Photo-Interpretation

The classifying is done by trained technicians who study pairs of photographs through magnifying stereoscopes. Experience has shown that foresters with some training in engineering are best adapted for this work. Skill in photo interpretation can be acquired only after considerable field experience in checking photographic details. This background of experience and a general knowledge of the region to be mapped are the keys to recognition of vegetation elements and other features on air photographs Their photographic appearance depends not only on the topographic situation in a region, but also on the season and time of day the photographs were taken. Film or filters, or both, used in the camera, quality and focal length of the lens, techniques of printing and developing, and perhaps most of all, scale of the original negative also affect the appearance of photo details. Therefore it would be very difficult, if not impossible, to define the photographic characteristics of the vegetation or other features to fit all cases. However, certain characteristics hold true in general for the usual 1:20,000 scale many-purpose photographs. Although these characteristics alone will not always allow positive identification of an element, they seem important enough to mention as illustrative of the kinds of contrasts that must be considered in the interpretation of vegetation on air photographs. The key that follows is intended to give a few of the outstanding contrasts. Symbols shown below are those used to illustrate classification of vegetation (Fig. 4).

Natural Vegetation

Basic Elements and Features and Their Identification

1. Timber trees. All conifer tree species of timber or pulpwood value.

Identification: Trees, both timber and cordwood, down to minimum sizes shown in Figure 1 usually appear more irregular in pattern and darker in tone than other vegetation. Pattern and tone in timber-tree canopy are more regular (suggestive of stippling) and often darker than in cordwood stands. Individual tree crowns are longer and more pointed or tapering, as revealed by shadows. Stereoscopic images of individual crowns are more nearly circular and narrower than images of cordwood trees of comparable heights.

2. Cordwood trees Hardwoods and nontimber conifers such as Digger pine, piñon pine, and juniper

Identification: Pattern and tone of hardwood canopy are usually irregular, tree crowns blending together so that individual crowns are indistinguishable in dense stands. Individual crowns in open stands are usually broader and have a more irregular spread than timber trees of comparable heights, and the crowns are more definitely rounded or flat on top, as revealed by shadows. Nontimber conifers are sometimes difficult to distinguish from timber conifers and may be included with them pending field checking in areas where their ranges meet. However, these areas are so restricted that the amount of field checking thus required will be small. Stands of dead trees (snag areas), either timber or cordwood, are also designated, and may be identified by their very light tone

3. Shrubs. Shrubs together with trees of shrub stature below the sizes illustrated in Figure 1.

Identification: Pattern varying in regularity, with various tones of gray, which depend largely on species composition. Stereoscopic height of shrubs is negligible if apparent at all.

4. Herbs. Herbs, grass, or other herbaceous plants. Meadows are given a separate designation.

Identification: Upland grass has an even light gray tone. Meadows exhibit an even dark gray tone but not usually as dark as open water.

Age-Class Elements (Illustrated in Fig. 2)

1 2 3 4 5

1. Small immature timber trees.

Identification: Crowns are very narrow and distinctly circular in horizontal outline, darker than cordwoods of the same height. Trees are short, and crown shadows are pointed.

2. Large immature timber trees.

Identification: Crowns are narrow to intermediate in width, circular in horizontal outline, and intermediate in height. Crown shadows taper to a point or are slightly rounded.

3. Mature timber trees.

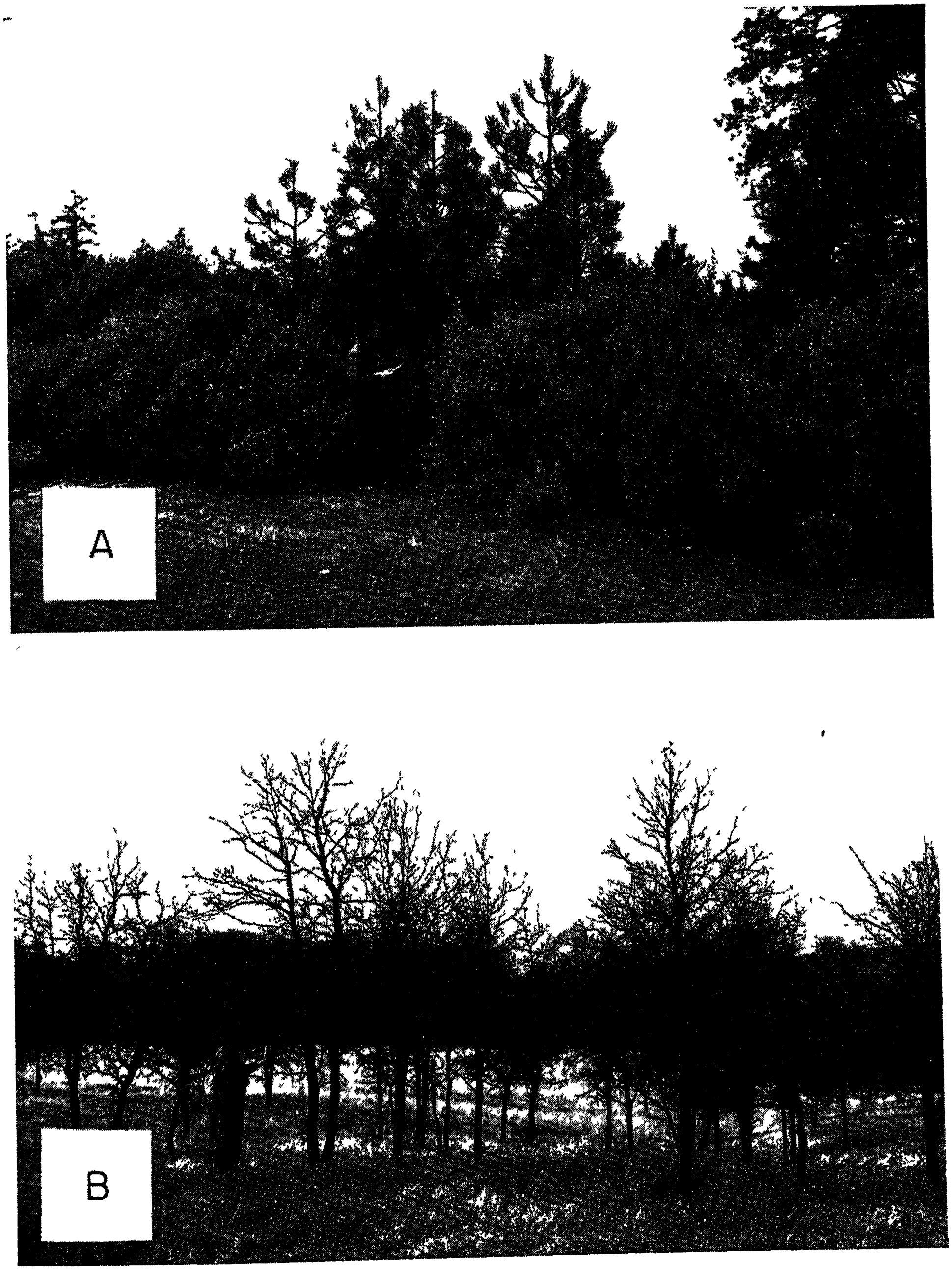


FIG. 1. Minimum sizes of timber trees (A) and cordwood trees (B) visible in 1:20,000 scale air photograph. Below these sizes they are indistinguishable from shrubs.

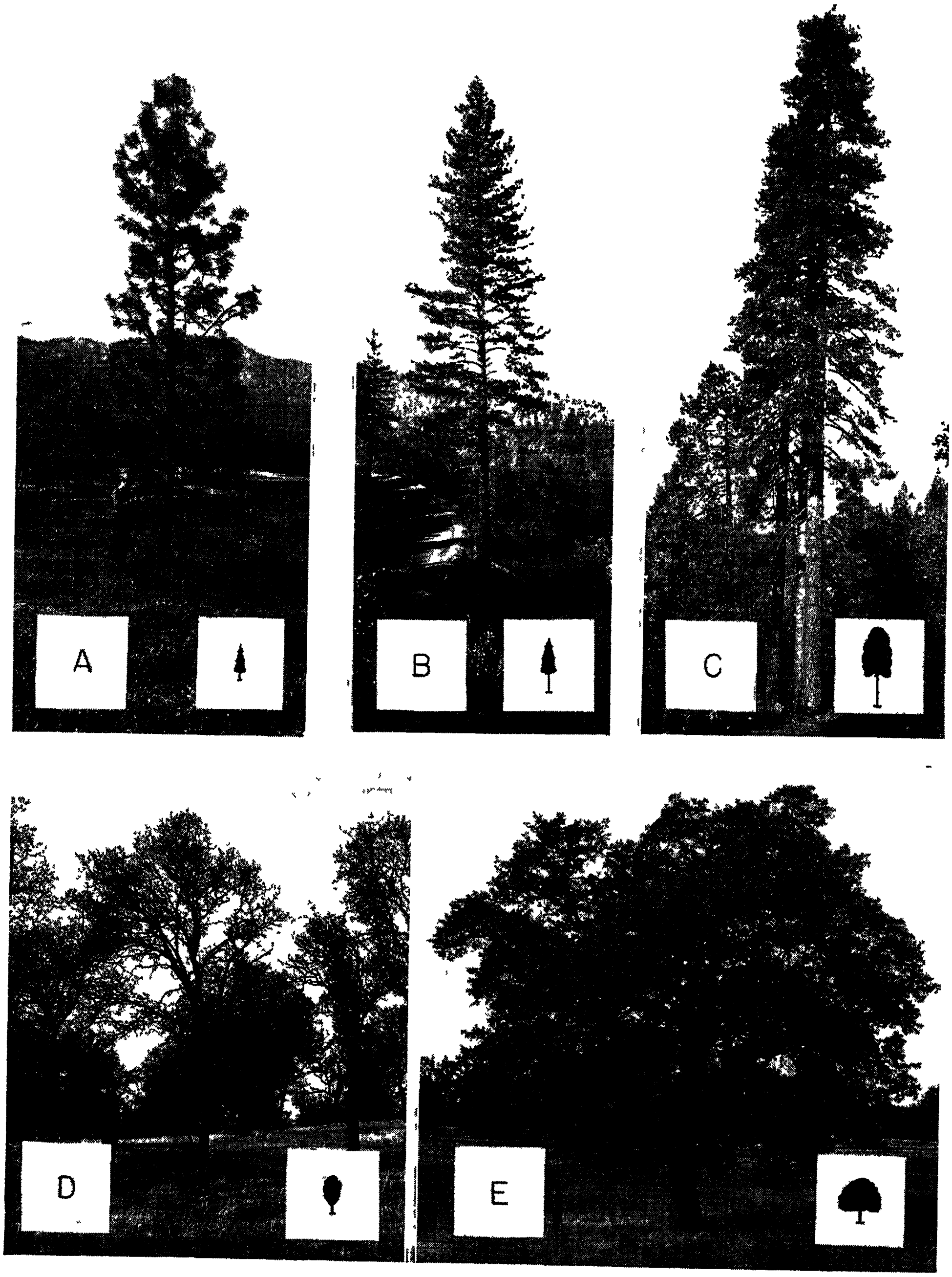


FIG. 2. Age-class elements visible in 1:20,000 scale air photographs: (A) Small immature timber tree—under sawlog size; (B) Large immature timber tree—sawlog size; (C) Mature timber tree; (D) Immature cordwood tree; (E) Mature cordwood tree.

Identification: Crowns are wide and circular to slightly irregular in horizontal outline. These trees form the tallest tree group. Crown shadows are rounded to flat on top.

4. Immature cordwood trees

Identification: Crowns are narrow to intermediate in width. Images on 1:20,000 scale photo usually are 0.1–0.25 mm. wide (denoting crown spread of about 10–20 feet), and are irregularly rounded in horizontal outline. Trees are short. Crown shadows are rounded or flat on top.

5. Mature cordwood trees.

Identification· Crowns are widest in these trees. Images on 1:20,000 scale photo usually are 0 4–0.7 mm wide (denoting crown spread of about 30–50 feet), and are irregularly rounded. Trees are intermediate in height with flat-topped crown shadows.

Thus the factors of most aid in differentiating the above age classes are relative tree heights and tree-crown characters. Since magnified stereoscopic images or crown shadows, or both, provide the criteria for recognition of these factors, the extent to which these criteria may be used depends largely on stand densities, topographic situation, and season or time of photography.

Other Features

1. Barren. Apparently devoid of vegetation, separate designations are given to barren ground and rock

Identifications: rock shows very light gray to white tone, barren ground slightly more irregular in pattern and slightly grayer in tone than grass.

2. Open water.

Identification: usually very dark to inky black tone except where sunlight reflected into the lens gives a white or silvery tone. Muddy water and shoals also appear lighter in tone Despite these variations in tone, bodies of water can scarcely be confused with other features because of contrast in slope and appearance of the shore line.

3 Cultivated land. Segregated by major type of use. Areas no longer cultivated are designated as abandoned, although classified according to present cover.

Identification obvious from regularity of various cultivated patterns.

4 Urban and industrial areas. Identification obvious.

5. Plantations of trees.

Identification: Uniformity in spacing and size of trees.

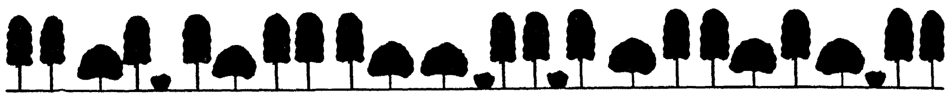
The vegetation elements given above and the barren areas are the basis for the following classifications. The percentages used in defining them, although checked in borderline cases by micrometer measurements along line samples drawn on the photographs, are estimated and applied as approximate guides instead of precise divisions.

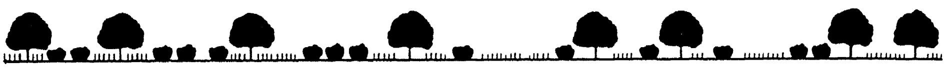
The Classifications

Vegetation Classes

Applied to all lands except cultivated urban, and industrial areas and apparently barren areas, which are classified and delineated directly as they appear on the photographs.

1. Areas essentially composed of a single vegetation element.



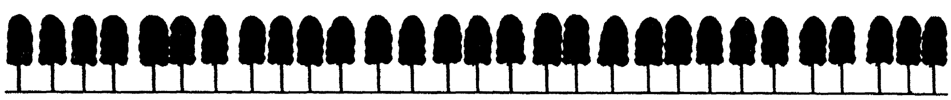


2. Areas in which two or more vegetation elements are significant

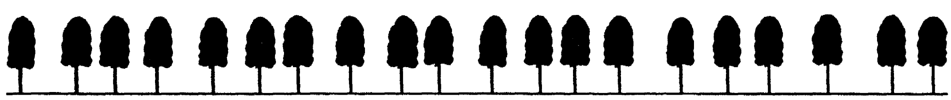
Ordinarily a vegetation element is not considered significant and is not designated in the classification unless it occupies 20 per cent or more of the ground space or forms 20 per cent or more of the total vegetation cover. However, timber trees, cordwood trees where they are the only trees present, and shrubs in treeless areas are considered and designated where they occupy 5 per cent or more of the ground space. The barren element is designated in both vegetation classes 1 and 2 where it occupies 20 per cent or more of the ground space.

Density Classes

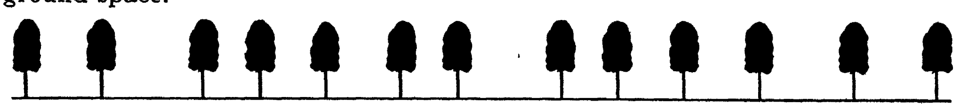
Applied to the combined tree and shrub cover on all lands, to the timber trees alone, and to the cordwood trees on nontimber sites.



1. Dense. Stands in which the crowns cover 80 per cent or more of the ground space.



2. Semidense. Stands in which the crowns cover from 50 to 80 per cent of the ground space.

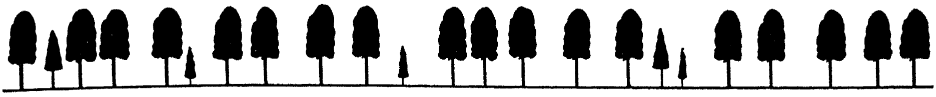


3. Open. Stands in which the crowns cover from 20 to 50 per cent of the ground space.

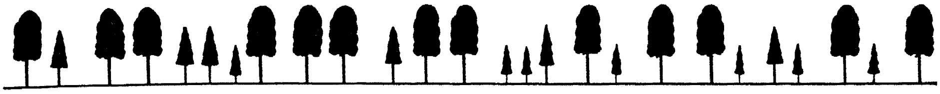
4. Very open. Stands in which the crowns cover from 5 to 20 per cent of the ground space.

Age-structure Classes

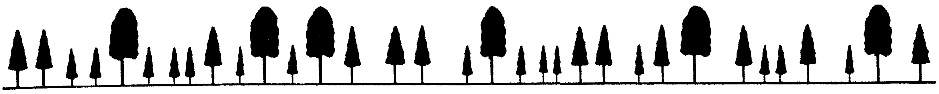
Applied to timber trees on timber sites and to cordwood trees on nontimber sites. In timber stands the age-class elements present, as well as the composite structure, are recorded as illustrated in Fig. 4C.



1. Old growth. Stands in which mature trees form 80 per cent or more of the crown space.



2. Old growth—young growth. Stands in which mature trees form from 50 to 80 per cent of the crown space.



3. Young growth—old growth. Stands in which mature trees form from 20 to 50 per cent of the crown space.

4. Young growth. Stands in which mature trees form less than 20 per cent of the crown space. Stands in which the percentage of mature trees is between 5 and 20 are indicated by a special symbol.

Method of Recording

The delineation and designation of these classifications are made on the photographs, as illustrated in Figure 3. Where two or more vegetation or age-class elements occur in a single classification, the symbols are listed in order of the relative area occupied by each element. Although the entire photograph in Figure 3 has been classified in order to illustrate a wider range of conditions, only the central portion of the photograph within the effective area boundary (the same portion used in constructing the base map) would be classified in actual practice. Classifications from the central portions can be transferred to the base map by means of a reflecting projection machine with the least correction for relief displacement and for tilt distortion and so with greatest accuracy, speed, and ease.

MAP PRESENTATION

The classifications illustrated in Figure 3 form the basis for four series of maps, sample profile sections of each being shown in Figure 4. All the information indicated in the profile samples is shown on the maps by appropriate symbol. These maps are prepared on a scale of 2 inches = 1 mile, and it is planned to issue them as $7\frac{1}{2}$-minute quadrangles in the form of blue-line prints. There follows a brief discussion of each map and of the techniques involved in the supplementary field work required for two of the maps.

Vegetation Classes and Densities

The maps of this series show vegetation classes for all lands according to composition by vegetation elements and densities of tree and shrub cover. This information is obtained entirely from office study of the air photographs. These

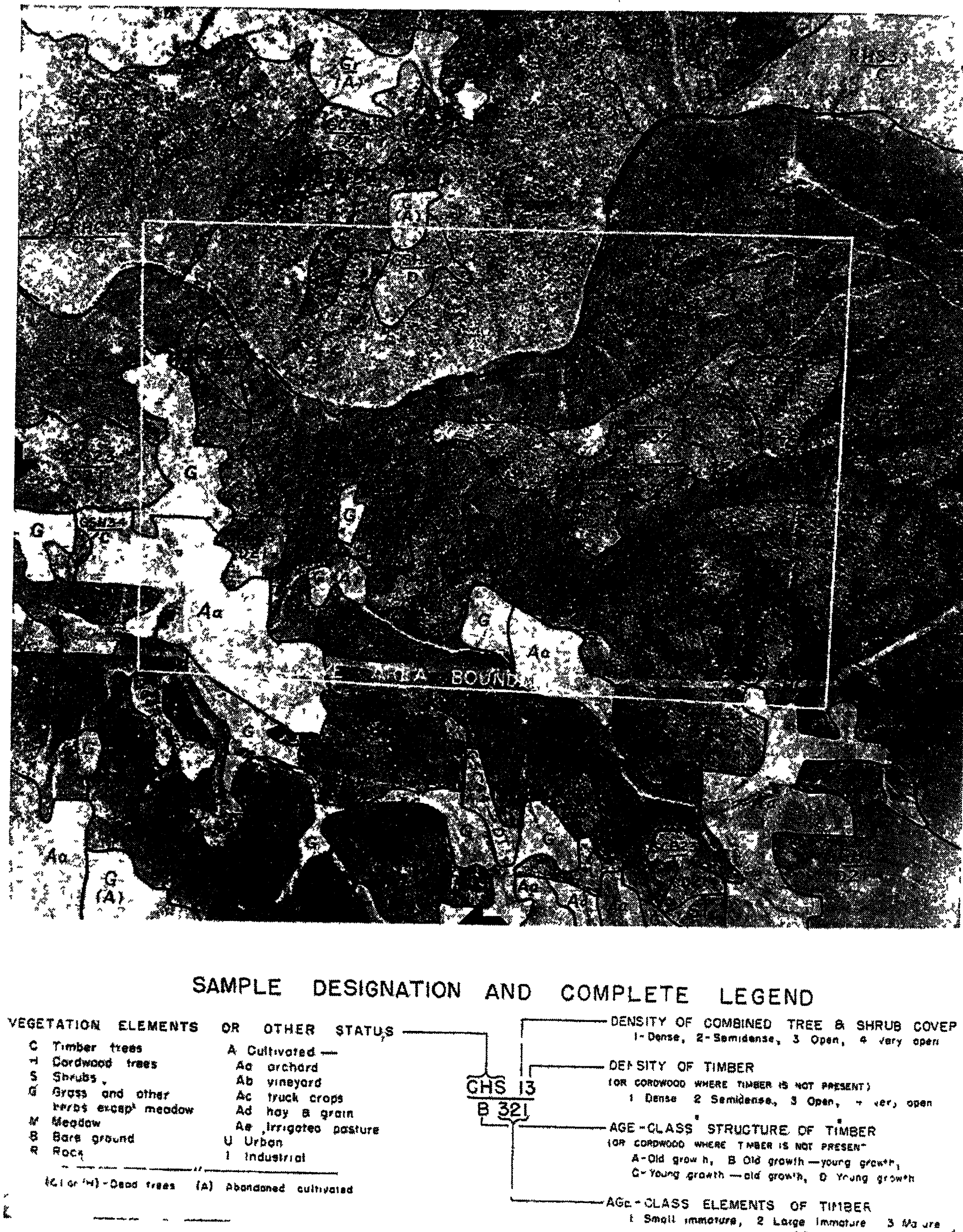


FIG. 3. Example of classified air photograph showing kind of information obtained by office study under magnifying stereoscope. Note effective area boundary referred to above.

maps should be more useful for some purposes than the maps showing species composition heretofore prepared by ground work alone. Their utility may be enhanced by coloring in various ways according to the use in mind. For example, if a vegetation map is desired for general purposes the vegetation can be grouped by color legend into the broad classes of timber, cordwood, brush, and grassland, shown in Figure 4A. On the other hand, if the map is to be used for planning action

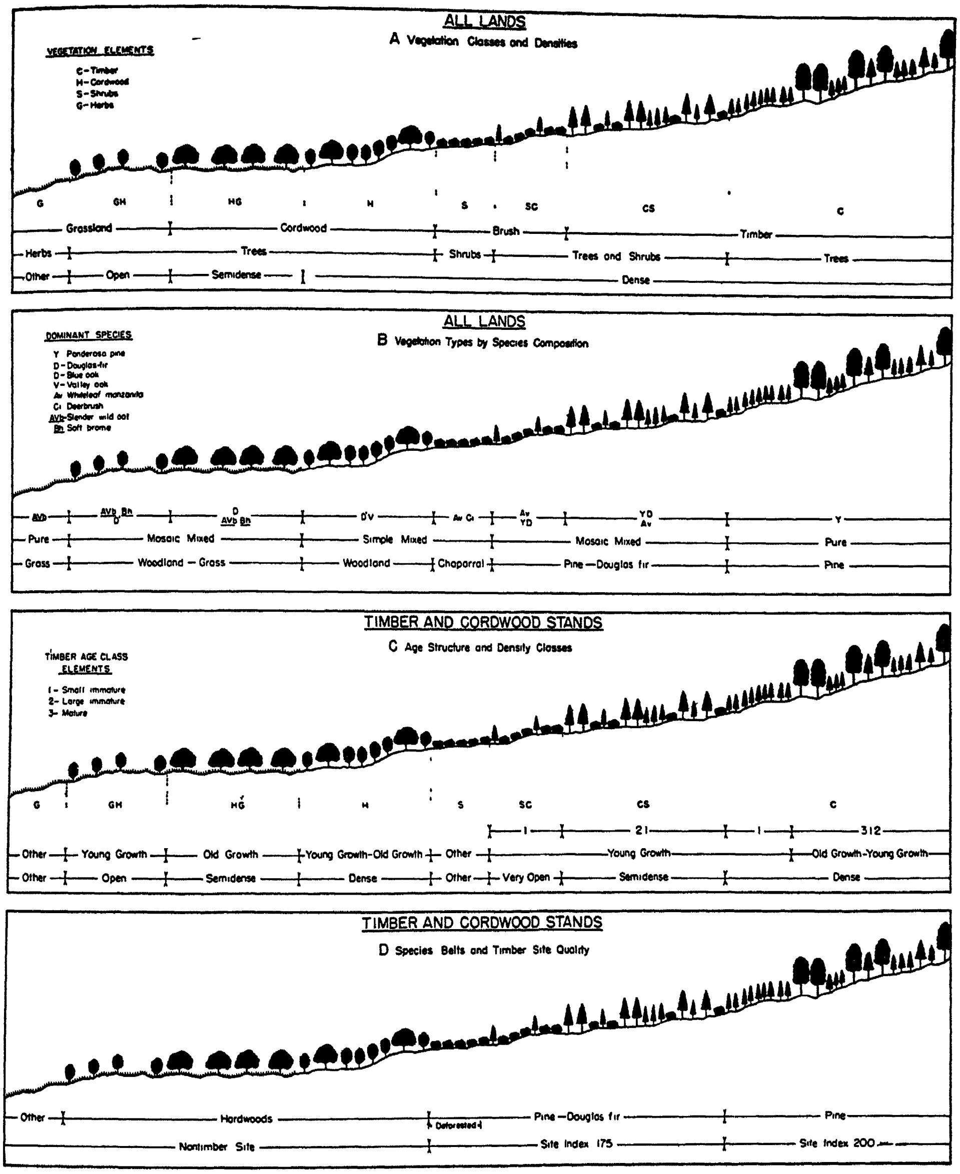


FIG. 4. Profile samples illustrating the kinds of maps that can be prepared for a given area. Information presented in maps A and C is obtained from office study of air photographs. That in B and D requires supplementary field work.

programs in fire and flood control, or to emphasize concealment or maneuverability values for military purposes, or for estimating the amount of tree and shrub cover to be removed in clearing rights-of-way for roads or power lines, color legends would accentuate the trees and shrubs and their density, shown in Figure 4A.

Vegetation Types by Species Composition

These maps show vegetation types and their composition by dominant species as indicated in Figure 4B. For general purposes these maps can be colored to

show types such as grass, chaparral, and pine, and for more specific uses to show the occurrence of a single species of tree or shrub that assumes importance because of economic value or scientific interest.

The preparation of this series of maps entails supplementary field and office work as follows:

(a) The air photographs showing vegetation classes and densities are taken into the field for further classification of the vegetation according to natural plant associations. In this classification three kinds of associations, (1) pure, (2) simple mixed, and (3) mosaic mixed stands, as illustrated in Figure 4B, are recognized and delineated on an overlay of transparent cellulose acetate. Only species exposed to the sky are considered. The first kind of associations comprises those in which a single species forms more than 80 per cent of the cover; the second, those that are wholly arborescent, shrubby, or herbaceous and that are made up of two or more species, each occurring to the extent of 20 per cent or more; and the third, those that are neither distinctly arborescent, shrubby, nor herbaceous but mosaics of two or more of those elements. A pure stand is designated by the single important dominant. In a simple mixed stand each dominant forming 20 per cent or more of the stand is designated, the dominants being listed insofar as practicable in the order of relative abundance, and in a mosaic mixed stand the dominants designated are those that form 20 per cent or more of each class of vegetation. For example, in a timber-shrub formation each timber species forming 20 per cent or more of the timber cover and each shrub species forming 20 per cent or more of the shrub cover are designated as a dominant.

The delineation of these plant associations may or may not require subdivision of the vegetation classes that have been determined by office study of the photographs. For example, ground examination may reveal that one timber area includes only ponderosa pine and therefore requires no subdivision. Another timber area may be composed of ponderosa pine and white fir occurring in three distinct plant associations—pure pine, pure fir, and mixed pine and fir—and thus require subdivision. Delineations such as these are often facilitated by further study of the photographs under a portable field stereoscope. Although the dominants of associations are usually directly observable, their determination will frequently need to be verified by sample plots. In herbaceous formations however, segregation beyond that of marsh, meadow, grass, and bushy herb associations is not attempted, and the determination of dominants when shown is made from sample plots that may or may not typify the entire association outlined. The sample plots also provide information for the proper description of the various plant associations—details of species composition, size of trees and shrubs, and depth of leaf litter.

Age Structure and Density Classes

In this series of maps only areas supporting timber and cordwood stands are classified. These maps show for timbered areas (1) composition by vegetation elements, (2) timber-tree composition according to age classes present, (3) age-structure classes, and (4) stand density of timber trees; for nontimbered areas supporting cordwood stands (1) composition by vegetation elements, (2) age-structure classes, and (3) stand density of cordwood trees. All this information is obtained from office study of the air photographs. The maps may be colored to emphasize either stand density or age-structure classes. The classifications they contain were designed primarily to facilitate and improve the quality of the timber and cordwood volume estimate to be obtained by a sampling system

in the Nation-wide forest survey. The maps of this series however will be useful to both public and private owners of timberland not only as an aid in timber cruising and evaluation but also in many phases of forest management.

Species Belts and Timber Site Quality

The maps of this series show species belts and site-index classes for all timber sites, and species belts of cordwood stands on nontimber sites. A color legend may be superimposed to accentuate either species belts or site index classes. Since species belts are derived directly from the map showing vegetation types by species composition, the data for site-index classes are obtained in connection with the field work required for that map.

The species belts segregate timber stands and cordwood stands according to important species groups. Both existing stands and deforested areas are included in species belts of timber, existing stands being classified by key species forming 20 per cent or more of the crown space, and deforested areas being classified by remnant trees, indicator plants, or the adjacent stands and their topographic relationships.

The site-index classes, which grade timberland according to relative capacity for growing timber crops, are based on the age-height relationship of average dominant trees expressed in terms of height attained (in 25-foot classes) at 300 years. In general they coincide with broad climatic belts except where soil or other habitat factors are markedly different from those typical of the locality. The delineation of these classes requires the following steps:

(a) In the field, selection of suitable dominant trees spaced not less than two to a 36-square-mile township and measurement of their height, their location being referenced by number on the air photograph. Ages are then determined by annual-ring counts of cores extracted from these trees, and site-index values are obtained by reference to age-height curves.

(b) In the field, delineation on air photographs of the boundaries of site-index classes that are clearly evident on the ground, such as those caused by soil differences or by depressed height in windswept areas.

(c) In the office, determination of the boundaries between site-index classes that are not visible on the ground, such as those caused by gradual climatic change with change in elevation. Rainfall and other climatic data are used as guides in locating these boundaries.

Other Use of Classifications

The age structure and density classification of timber and cordwood stands have a use for forest-survey purposes in addition to the map presentation described above. Before the completion of a long-time mapping program, reliable area tables of the timber and cordwood classification illustrated in Figure 4C can be quickly obtained for watersheds, counties, or other desired units by office line sampling of the air photographs. Then by applying to these classifications average stand per acre figures procured through a system of ground line plot sampling, an inventory of the kind, quality, and volume of the timber is obtained.

AERIAL PHOTOGRAPHS IN GEOMORPHIC STUDIES[1]

H. T. U. Smith

Introduction

THE FIRST aerial photograph on record was made in the year 1860, from a balloon over the city of Boston (Taft, 1938).[2] The wet plate process was used. During the ensuing half century, sporadic experimentation with photography from balloons and kites was carried on, but it was not until the first World War that much progress was made. At that time, the value of aerial photographs for studying terrain was quickly recognized, and the foundations were laid for systematic procedure in taking and using photographs. Since then, planes, cameras, film, mapping instruments, and technique have steadily improved. A new science of mapping—aerial photogrammetry—has evolved, and enormous areas have been photographed from the air for map making and other purposes. With the advent of the second World War, the use of aerial photographs has been given a second great impetus, and it might be said that at least ten years of normal progress have been telescoped into a few brief years

The potentialities of aerial photographs for geologic and geomorphic studies were early recognized by geologists serving in the first World War, and soon thereafter were admirably illustrated by Lee (1922) in a pioneer volume. In the ensuing decade, the value of aerial photography in the various fields of economic geology was gradually established, and the use of aerial photographs became standard practice. For many research workers in pure science, however, the benefits of developments described above were long withheld by prohibitive costs. This limitation was finally withdrawn when large areas of the United States were photographed by the U. S. Department of Agriculture in the middle thirties, and the resulting pictures were made available at the mere cost of reproduction. A powerful new tool was thus placed in the hands of the geomorphologist and geologist, and promises to be fully as important as was the introduction of the thin section and the microscope in studying rocks and minerals.

Geomorphology is a branch of geology devoted primarily to the description and analytical study of the earth's surface features, in all their variations and details. The approach is basically genetic, being concerned with origins and with modes of development, for it is these which are fundamental in a rational description and classification of landscapes, and in the many applications of the science having to do with land utilization problems. The factual basis of geomorphology lies in all available data on the lineaments of the land surface and their relations to soil, rock, and climate. In the past, such data have been supplied mainly by topographic maps and by individual field studies. To these sources of information aerial photographs constitute an addition of major importance, effecting enormous economies of time and effort, and providing a wealth of qualitative detail virtually unattainable in any other way.

In the following pages, a brief review of the various ways in which aerial photographs are of service to the geomorphologist is presented. Obviously, no very comprehensive treatment of the subject is possible in a paper of this length; for a general background in the principles of geomorphology, the interested reader is referred to standard textbooks on the subject, such as those of Lobeck (1939), von Engeln (1942), and Cotton (1941), and for a more extended discus-

[1] Abridged and in part revised from a paper previously reprinted in *Photogrammetric Engineering* from the *Journal of Geomorphology*, with the permission of the editor of that journal.

[2] Citations, given by author and date, are to reference list at the end of this section.

sion of geomorphic interpretation, to another work by the present writer (Smith, 1943a). It is believed that an appreciation of the geomorphic approach should be of real interest to many of the workers who study aerial photographs for other purposes. Thus viewed, the wealth of topographic detail so clearly shown on photographs cannot but assume a vastly increased significance, regardless of the particular objectives to which the interpreter is working.

Availability of Aerial Photographs

To date, roughly two-thirds of the continental area of the United States has been photographed from the air (Fig. 1), mainly by or for the following Federal agencies:

- Geological Survey
- Coast and Geodetic Survey
- Army Corps of Engineers
- Tennessee Valley Authority
- Department of Agriculture:
  - Soil Conservation Service
  - Agricultural Adjustment Administration
  - Forest Service

Of the photographs made for the Federal government, the great majority belong to the Department of Agriculture, and are single-lens verticals on a scale of approximately 1:20,000, made mostly with cameras of 8.25 inches focal length. The size is generally about 7×9 in or 9×9 in. In some areas, however, multi-lens cameras have been used, producing larger photographs on a smaller scale. Those for large areas in the southwest are 4-lens composites, on a scale of about 2 inches to the mile, and are about 10×10 in. in size. All pictures are marked with serial numbers, and many also are dated. Negatives for many photographs are filed in central offices at Washington, D. C, and others in regional offices elsewhere. Department of Agriculture prints, enlargements, and mosaics are generally available for examination, in peace time, in district offices, which are to be found in nearly every county within the agricultural areas of the country.

Index mosaics, mostly of the uncontrolled type, are available for much of the area photographed. They range in scale from about 0.5 to 2 in per mile. Despite inaccuracies, these constitute the best base maps available for many areas otherwise unmapped or poorly mapped.

In normal times, reproductions of aerial photographs and mosaics belonging to the Department of Agriculture may be obtained from the various offices of the Department at very moderate cost. Information on purchasing procedure may be had from district offices, or from central offices in Washington, D. C. General information on areas covered by photographs may be obtained from the Map Information Office, Room 7206, North Interior Department Bldg, Washington, D. C. At present, however, war-time restrictions apply to all purchases of photographs and to the release of specific information about them. The sale of photographs is limited to persons, agencies, or institutions contributing directly to the war effort, and specific military authorization for contemplated purchases must first be obtained from the Army Defense Command Headquarters for the area concerned.

In Canada, large areas are covered by both vertical and oblique photographs. Subject to war-time restrictions, these may be examined at and purchased from the National Air Photographic Library in Ottawa.

Where aerial photographs of the desired type or scale are not otherwise available, it is entirely feasible for the geomorphologist to take his own pictures, pro-

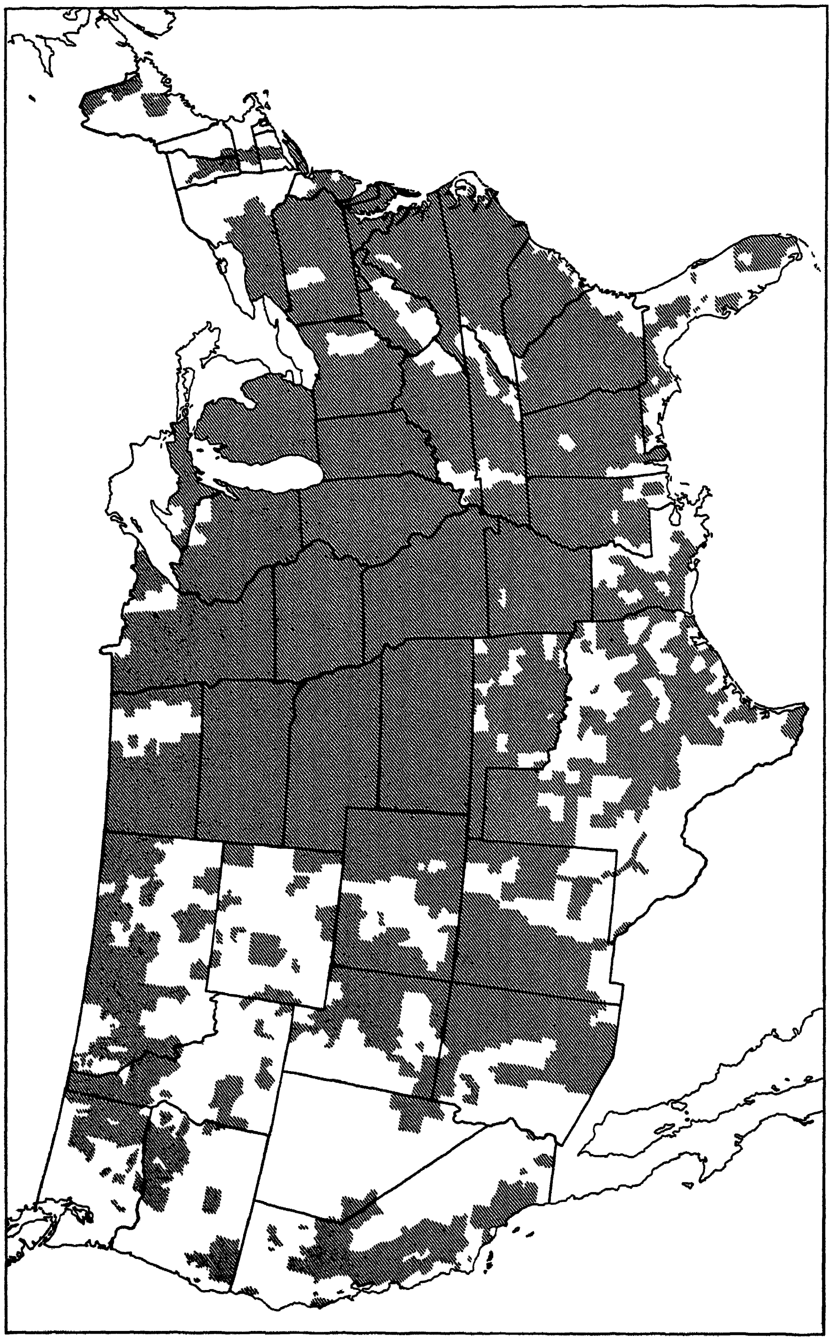


FIG. 1. Map showing extent of aerial photography for the U. S. Department of Agriculture up to Feb. 1, 1941. Additional areas are in project and areas in New England and along the Atlantic and Pacific coasts and elsewhere have been photographed by other agencies. Redrawn from a map issued by the Department of Agriculture.

viding that photogrammetric precision is not required. Any well-constructed camera having a sufficiently fast lens and shutter, and suitable film capacity, may be used. Those using 35 mm. film, such as the Leica and Contax, are particularly well qualified. Excellent examples of photographs made in this way have been published Rich (1939, 1941, 1942). Individual verticals may be taken from the side of the airplane by banking steeply, but for systematic vertical photography a plane with an opening in the floor of the cabin is desirable. Oblique shots are best made through an open window of the cabin, although good results may be obtained through a closed window if the glass or plastic is clean and free from scratches Stereoscopic obliques may be taken simply by making two successive exposures with the camera pointed at right angles to the line of flight, so that the two pictures overlap in area about 70 to 90 per cent. In many cases, Kodachrome photographs are particularly valuable to the geomorphologist, and stereoscopic views in full color may be said to rêpresent the ultimate in amateur aerial photography. For further information on procedure, reference is made to the publications of Gaty (1937), Ashworth (1937), and Rich (1941).

Geomorphic Interpretation of Aerial Photographs

Geomorphic interpretation involves, first, the identification of cultural, drainage, and relief features, and, second, a recognition of the significance of the latter in terms of bedrock geology, and erosional and depositional history. The first of these steps is common to all types of photo interpretation, and is purely empiric in character. The second step, however, is of an explanatory nature, being concerned with the proper genetic classification of the topographic features, or landforms, which are present, and with their gradual evolution through geologic time. The landscape, in other words, is viewed not as a fortuitous assemblage of meaningless forms, but as the product of a definite series of geologic processes working on a particular set of geologic materials through an orderly sequence. Individual features are seen not as isolated and unrelated units, but as significant parts of a continuous and systematic pattern. When the interpreter's observations are guided by this viewpoint, his attention is directed to many important details of the topography which might otherwise escape unnoticed, and he gleans many additional data which the empirical approach could obtain only more slowly and less directly.

The recognition of drainage features—streams, ditches, canals, lakes, ponds, and swamps—presents no particular problems. Stream patterns are readily seen in all their detail, and, furthermore, the ghost-like traces of older channels, since abandoned, are much clearer on photographs (Fig. 2) than they are to the observer in the field. Stream systems may be classified as to type—dendritic, trellis, rectangular, angulate, radial centrifugal, radial centripetal, annular, etc.—and this leads at once to certain conclusions as to bedrock geology and other relations. Analysis of the channel characteristics of individual streams provides the basis for an appraisal of their permanence and their behavior in flood.

Relief features, on individual prints and on mosaics, are outlined roughly by the drainage pattern itself, and are represented also, to a greater or lesser extent, by shadow effects, by contour cultivation where present, by rock outcrop patterns, and by differences in color tone related to variations in soil, vegetation, and moisture conditions. Under favorable conditions of topography and lighting, the photograph may have the appearance of a shaded relief map. Under unfavorable conditions, however, the appearance may be flat, and show relief indifferently if at all. In such cases, and to no less a degree with others as well, it is only

by stereoscopic examination that the wealth of topographic detail contained in the photograph may be realized

For routine examination of photographs, a reflection or refraction type of stereoscope, or both, is convenient For detailed work, the magnification afforded by the refraction type is indispensable. For rapid scanning of photographs, or for field use, it is more convenient to dispense with the stereoscope, and cultivate stereovision with the unaided eyes. With some practice, it is possible for nearly all persons to do this. It is required only that convergence and accommodation (or focus) of the eyes be disassociated, so that the former may be zero and the latter be adjusted for objects at a short distance. There are various types of ex-

FIG. 2. Vertical (single-lens) photograph of the Kansas River valley in Douglas County, Kansas. Note truncation of an earlier meander plain by a later sand-bar plain, and the truncation of the latter by a still younger sand-bar flat. Photo courtesy of U. S. A. A. A The black bar indicates scale, and is approximately one-half mile in length.

ercises for cultivating this faculty, but the following is perhaps the simplest. A stereo pair of photographs is correctly aligned, some sharply defined feature near the edge of one photo is selected, and is brought close to its counterpart on the other. The photographs are then viewed with a card held between the two eyes, so that the left eye sees the selected feature only on the left photo, and the right eye only on the right. It should then be possible, after a few trials, to obtain a fused image in stereoscopic relief This done, the separation between the matched points is gradually increased, and the card is removed. After from a few hours to a few weeks of practice, it should be possible to attain a distance of separation equal to the interpupillary distance, or about 2.5 in. Many persons easily attain a much greater "spread" of the photographs, ranging up to 6 in. At first, there may be some sense of eye strain in doing this, but it gradually disappears.

Viewed stereoscopically, aerial photographs have the semblance of a relief model (Fig. 3), and the third dimension, in fact, appears to be considerably exaggerated, so that even slight topographic irregularities are discernible. Without stereoscopic examination, it is impossible to exploit fully the wealth of detail which the photographs contain, and the recognition of the less obvious topographic features is tremendously handicapped.

Geologic interpretation goes hand in hand with geomorphic interpretation, and a clear picture of bedrock geology is requisite for the proper understanding of geomorph c features. The interpretation of geologic features is based partly on the topographic expression of rock bodies, as viewed stereoscopically, and

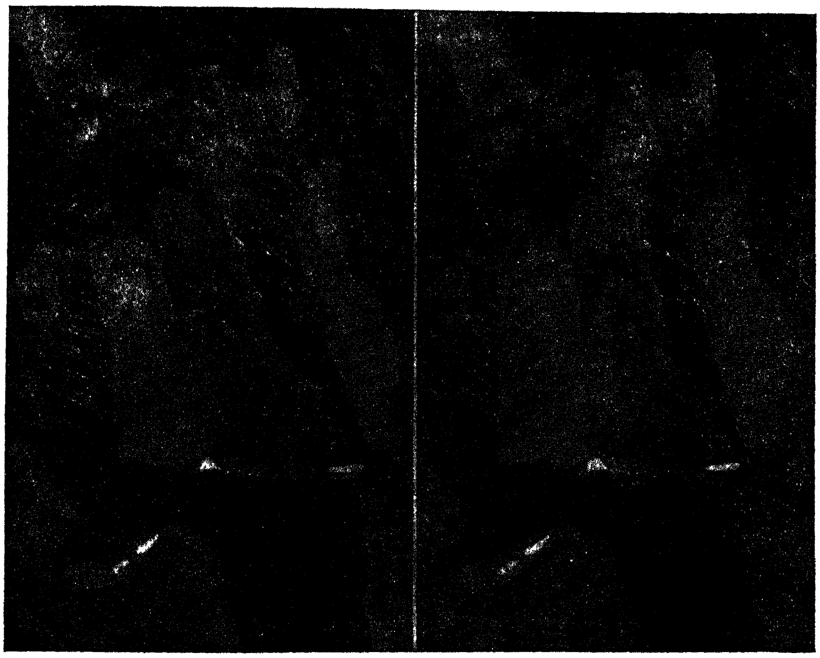


FIG. 3. Stereoscopic pair of photographs showing cirque and glaciated valley in the Culebra Range of southern Colorado, west of the Spanish Peaks. Each picture is approximately 1.1 miles in width. These photographs may be viewed either with a simple lens type of stereoscope, or with the unaided eyes. Photos courtesy of U. S. Forest Service.

partly on differences in color tone related to soil and rock coloration, to vegetal zoning, and to soil moisture conditions (Fig 4). Faults, folds, dikes, veins, unconformities, and other structural features commonly have a distinctive appearance from the air, even where their relations may be none too clear to the observer on the ground. Illustrations are given by van Nouhuys (1937), by Loel (1938), and by the present writer in another work (1943a). The clarity with which geologic features are displayed is conditioned by climate, being greater in arid and semiarid regions. Other variations in clarity of surface expression are related to: (1) the season of year at which the photograph was taken; (2) the time of day at which the exposure was made; (3) antecedent weather conditions; and (4) the extent to which natural features have been modified or obliterated by the work of man.

Having ascertained the relation of bedrock geology to topography, the interpreter proceeds next to identify the characteristic erosional and depositional landforms associated with the work of streams (Melton, 1936; Rich, 1939; Van Tuyl and Lovering, 1935), glaciers (Sproule, 1939; Wilson, 1939; Thwaites, 1943), wind (Melton, 1940, Smith, 1940, Hack, 1941), waves and currents (Lucke, 1934; Nichols and Marston, 1939, Putnam, 1942), and ground water (Dicken, 1938; Morgan, 1942, Smith, 1940). The recognition of these varied types of landforms obviously rests on a thorough familiarity with the principles of geomorphology, and cannot be detailed here. Further discussion may be found in the papers referred to above and in a book by the present writer (Smith,

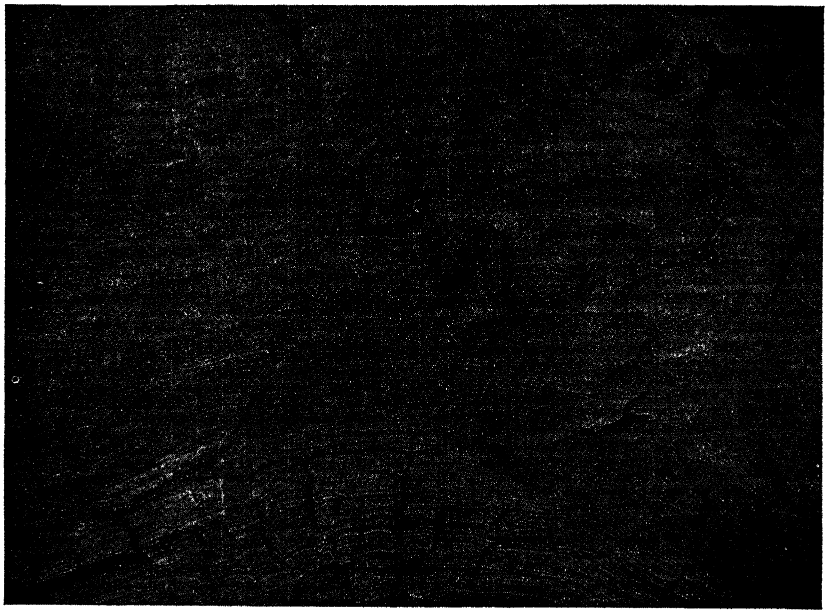


FIG 4. Ancient surface beveling steeply dipping beds of Arbuckle limestone in the Arbuckle Mountains of Oklahoma. Note minor faults, surface expression of beds, and indifference of drainage to structure. The black bar indicates scale, and is about one-half mile in length. Photo courtesy of U. S. Soil Conservation Service.

1943a). Representative examples of some of the main types of landforms are shown in Figures 2 to 11, and may be briefly commented on as illustrative. Figure 2 shows a series of episodes in the history of a river valley marked by faint channel scars. Three stages of slight downcutting by the river are indicated. Figure 3 represents a type of blunt, box-like valley head produced only by the erosion of alpine glaciers. Figure 4 shows a very well developed erosion surface on which the projecting ledges of steeply inclined rock were bevelled by the erosive work of streams. Figure 5 shows one of the many forms in which landslides may occur. The sliding here is of a gradual type. Figure 6 displays an ideal example of an alluvial fan. The shifting character of the stream responsible for the fan is clearly seen, and stream incision toward the apex of the fan may be noted. A considerable thickness of alluvial detritus may be inferred here. Figure 7 shows a

variety of types of sand dunes which remained virtually unknown in this country until aerial photographs were made Certain of the details of form are yet to be explained, and illustrate the way in which a research problem may be suggested by photographs In Figure 8, a more common type of dune is shown. The probable near-ultimate development of a dune topography, after wind action has ceased, is represented in Figure 9; interpretation of such topography, however, is more speculative than for the other examples described above. From a

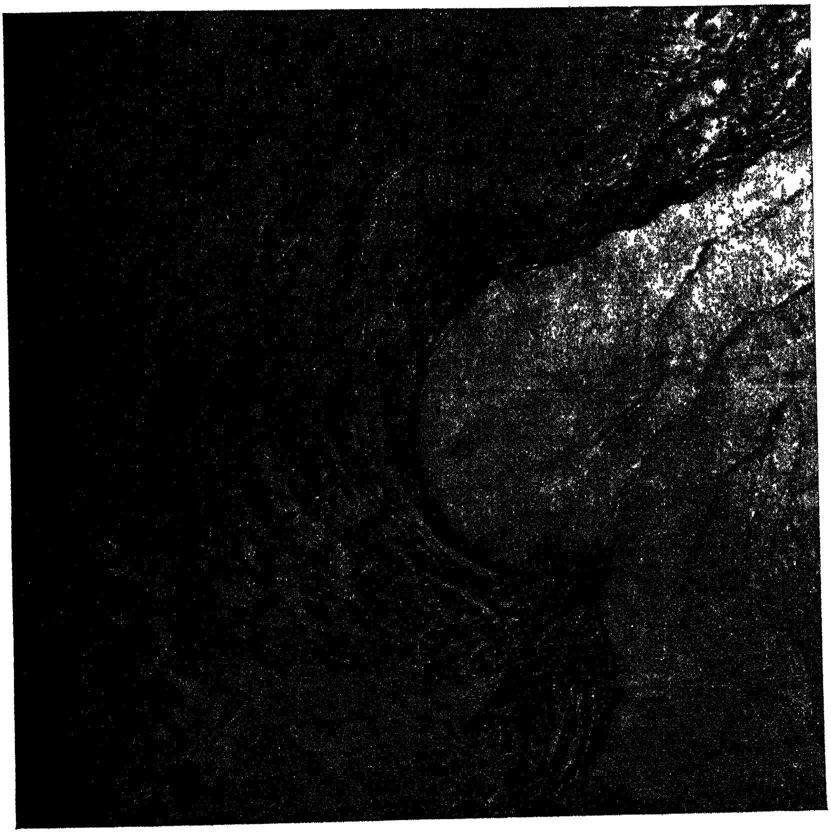
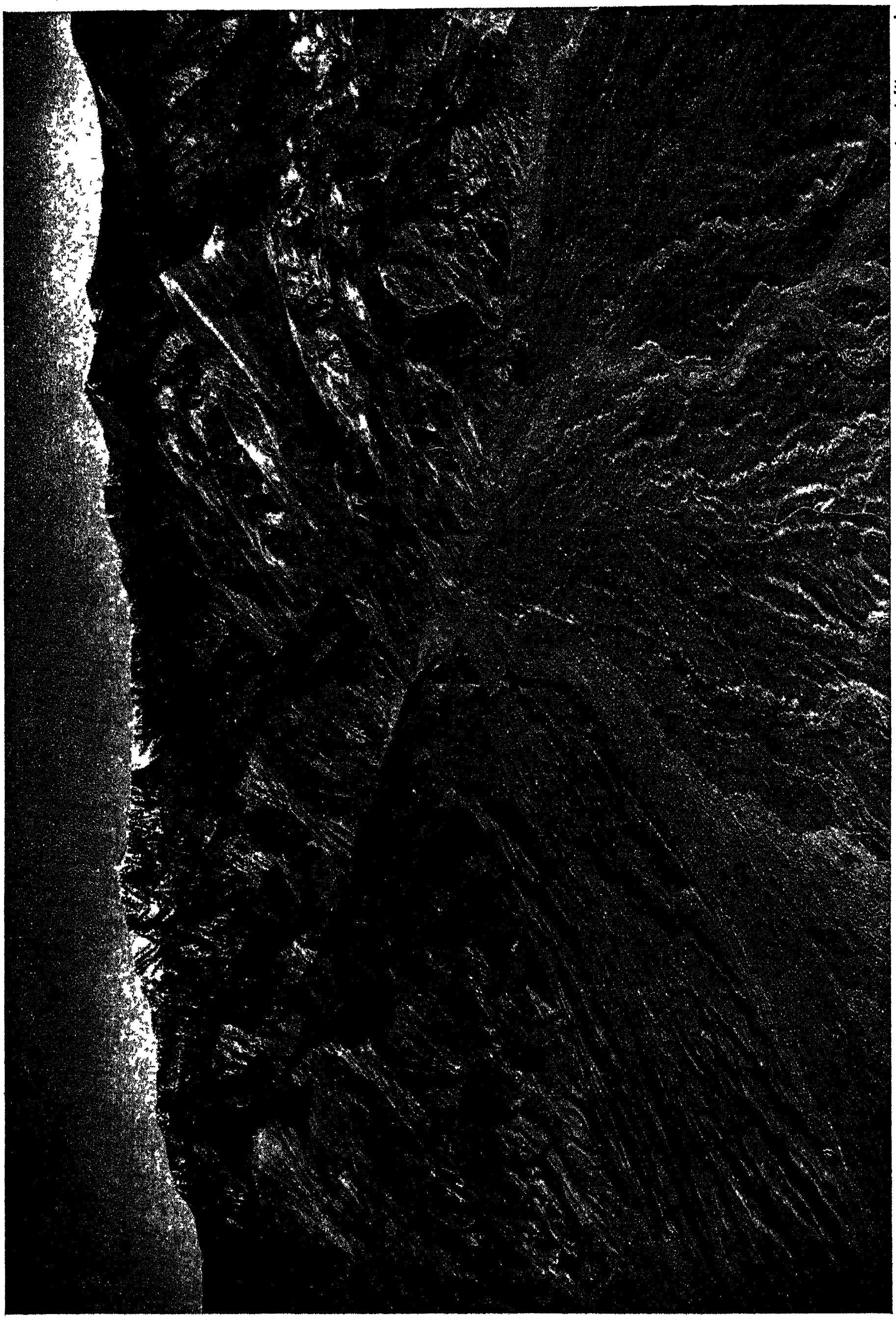


FIG. 5. Four-lens composite photograph of landslide slopes on Lucero Mesa, near Suwanee, N. M. The black bar indicates scale, and is approximately one-half mile in length. Photo courtesy of U. S. Soil Conservation Service.

study of Figure 10, the geomorphologist concludes that for a considerable period of time the shoreline was being extended lakeward, as indicated by the beach ridges, or "growth lines," but that more recently the shoreline has been pushed landward, as evidenced by the truncation of the beach ridges at the present shoreline.

Following the identification of the various landforms, comes the analysis of their chronologic development and their climatic environment. Figure 11 is suggestive in this connection. The intricate dissection points to erosion of weak rock under arid conditions, and the sequence of bench-like dissected pediments indicates that there have been repeated intervals of lowering of local base level and stream downcutting, with intervening periods of stability and lateral erosion. More is said on this topic in a later section.



Aerial Photographs

FIG. 6. High oblique photograph of alluvial fan at mouth of Hanaupah Canyon Panamint Range, California. Copyright Spence Air Photos.

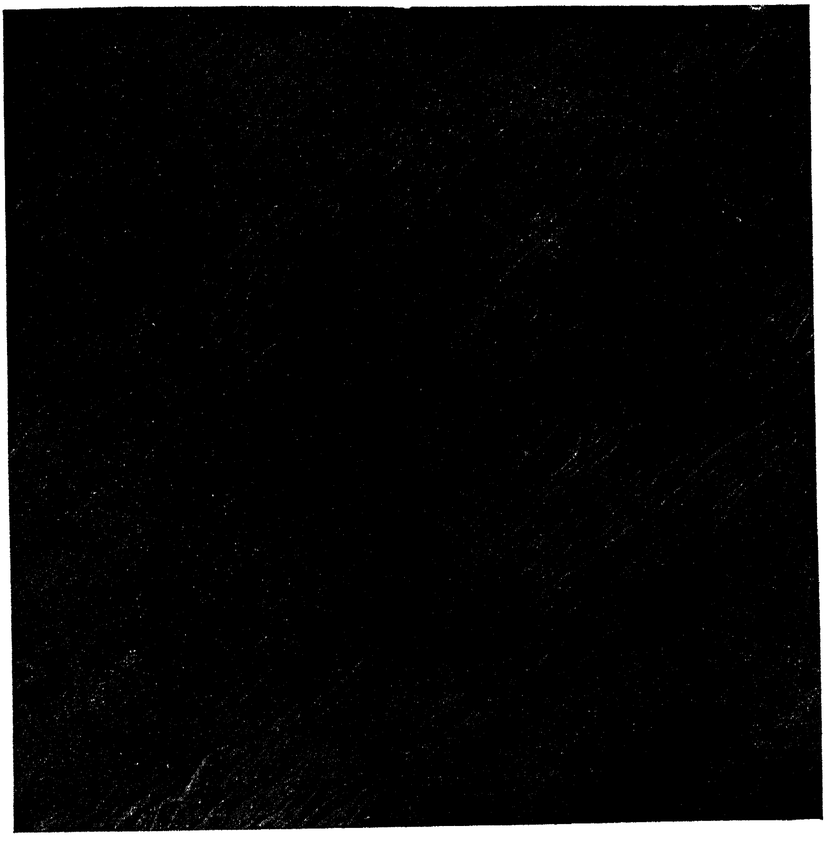


FIG. 7. Dunes northeast of Tuba City, Arizona. Longitudinal, transverse, "parabolic," and complex types are represented. The black bar indicates scale, and is about one-half mile in length. Photo courtesy of U. S. Soil Conservation Service.

In all phases of geomorphic interpretation, the importance of continued cross-reference between photograph and terrain in the field cannot be too strongly emphasized. It is only in this way that judgment is sharpened for correctly evaluating the innumerable minor variations in form, color, and pattern so faithfully recorded by the camera.

Preparation of Geomorphic Maps

Maps play a very important part in geomorphic studies and in the presentation of results. The use of aerial photographs in this connection is twofold: first, in the preparation of base maps, and, second, in the addition of particular geomorphic features to base maps previously made from photographs or obtained from other sources.

Base maps are necessary both in field studies and in office compilations. They may be prepared from either vertical or high oblique photographs, the former being generally used in this country because of their ready availability. Cartographic methods best suited for the practicing geomorphologist are those having maximum simplicity and speed, and requiring a minimum of special equipment. In many cases, correction for the various types of distortion is unneces-

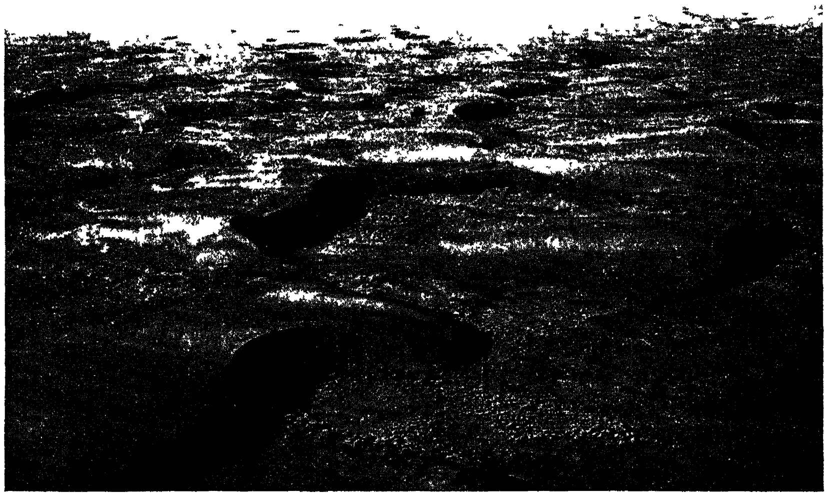


FIG. 8 High oblique photograph of sand dunes south of Moses Lake, Washington. It is impossible to photograph such features as these very satisfactorily from the ground. Photo courtesy of U. S. Soil Conservation Service and 41st Division Aviation, Washington National Guard.

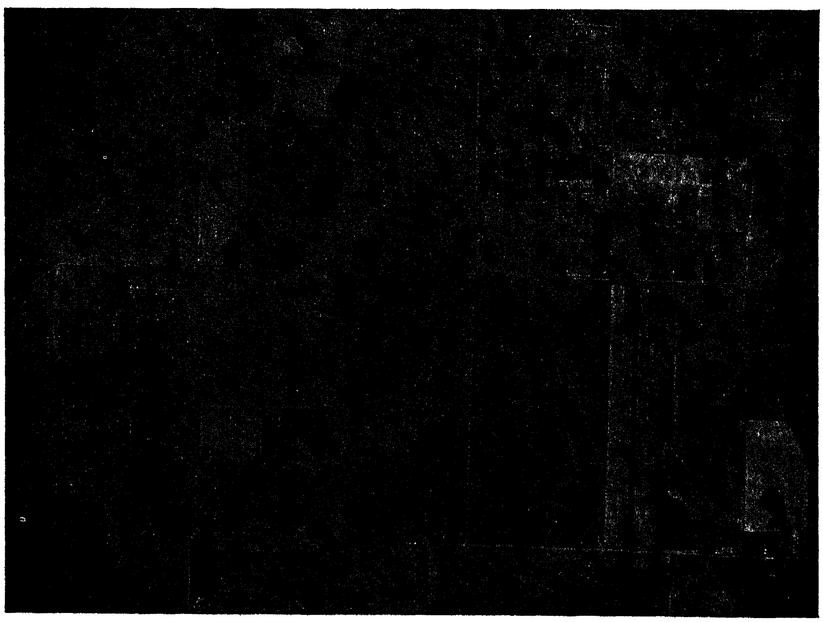


FIG. 9. Undrained depressions in southeastern Washington County, Colo. Probably an old-age dune topography, although possibly solution and collapse may have played a part. The section-line roads indicate scale. Photo courtesy of U. S. Soil Conservation Service.

sary, and sketch maps may be prepared from contact prints or enlargements by: (1) direct tracing; (2) inking and bleaching; or (3) the use of the camera lucida (Smith, 1943b). When enlargement or reduction is desired, this may be effected by: (1) the pantograph, (2) the camera lucida; (3) the grid method, or (4) photographic or photostatic methods. Where uncorrected maps are to be prepared for larger areas, requiring sets of photos, detail may be traced from individual photographs on to an overlay of some transparent material, such as frosted celluloid, using selected photo control points to tie in from one picture to the next. Where the overlap exceeds 50 per cent, center points and transposed center points of the photographs are best as controls. In areas where the township, range, and

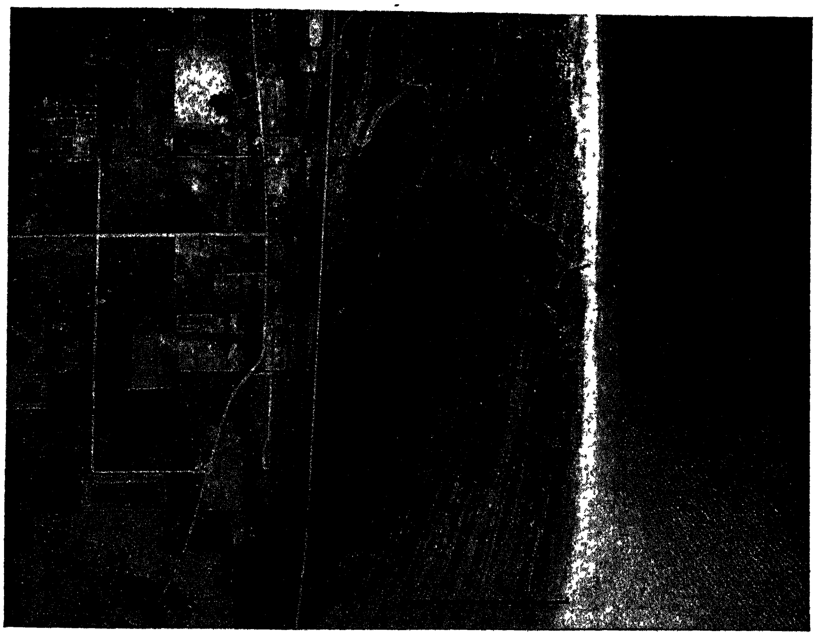


FIG. 10. Beach and dune ridges along the shore of Lake Michigan between Waukegan and Zion City, Ill. These features fail to register on the 10-foot contours of the Waukegan, Ill.-Wis., topographic sheet. The black bar indicates scale, and is about one-half mile in length. Photo courtesy of U. S. A. A. A.

section system of land subdivision is used, and roads and fences follow section lines, the section line grid itself may be used as a framework for the compilation of detail. Where mosaics of satisfactory quality are available, they may be used directly as base maps for some purposes, or line maps may be traced from them. Similarly, uncontrolled mosaics may be prepared simply by "shingling" the photos, matching marginal detail, and fixing them together with scotch tape. From such an assemblage, a tracing may be prepared, or a map on a reduced scale may be drawn with the pantograph.

Where correction for distortion is necessary, the radial method must be employed. Either the overlay method or the hand-templet method is easily used, preference between the two being governed by circumstances or by individual preference.

Relief features may be represented on maps prepared as above by hachures, form lines, or contours. The latter may be added by any of the following methods: (1) field sketching with the aid of the plane table and alidade, Brunton compass, aneroid barometer, or other surveying instruments; 2) freehand sketching under the stereoscope, using spot elevations obtained in the field as guides; or (3) the use of one of the simpler contouring machines, such as the stereocomparagraph.

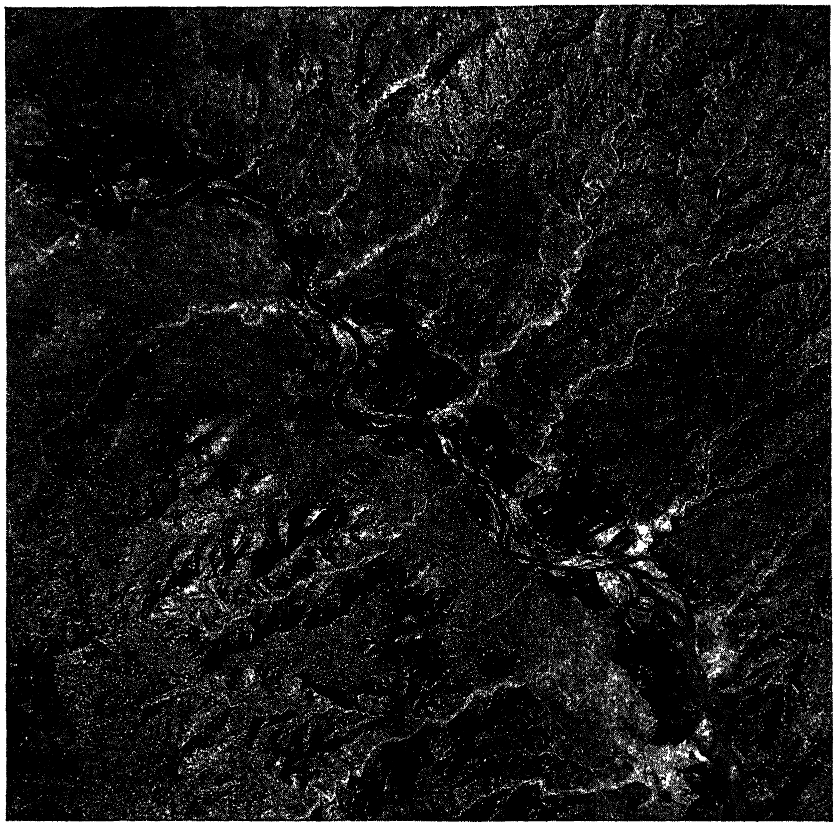


FIG. 11. Dissected pediments along the Chama Valley in the Abiquiu quadrangle, New Mexico. The scale bar indicates a distance of about one-half mile. Photo courtesy of U. S. Soil Conservation Service.

Where only oblique photographs are available, the perspective grid method is perhaps best adapted for the amateur cartographer. The accuracy of the results will be governed by the ratio of topographic relief to flight altitude. For reconnaissance work, oblique photographs taken by the worker himself, using any good hand camera, may be used with this method.

The transfer of particular geomorphic detail from vertical aerial photographs or mosaics to preexisting base maps may be affected in various ways. For some purposes, freehand sketching, aided by geographic grid lines, such as meridians and parallels, may suffice. In other cases, more exact methods, as by the use of the pantograph, proportional dividers, etc , may be in order. In this way, such features as sand dunes, sink holes, glacial landforms, the outlines of dissected

erosion surfaces, etc., may be mapped in varying degrees of detail at different scales, either in preparation for anticipated field study, or in the light of field or office study already completed. Examples are found in the sand dune maps of Melton (1940) and of Hack (1941). Transfer of detail from oblique photographs may be affected by the use of perspective grids, or by graphic triangulation. Typical of the use of obliques for this phase of geomorphic mapping was Wilson's (1939) study of the eskers northeast of Great Slave Lake in Canada.

Aerial Photographs for Purposes of Illustration

For illustrating geomorphic discussions and descriptions, both aerial photographs and maps prepared from them may be used. For showing the broader relations of many geologic and geomorphic features, high obliques are ideal. Over ground photos they have the advantage of unlimited choice of vantage point, and over verticals they have the advantages of covering much larger areas, and of showing the landscape in familiar perspective. By judicious choice of altitude, angle of inclination, direction, time of day, and season of year, features difficult or impossible to photograph satisfactorily on the ground may be shown to best advantage (as Figs. 6 and 8). Excellent examples may be found in the publications of Cooper (1935), Van Tuyl and Lovering (1935), Washburn (1935, 1941), Rich (1939, 1942), Melton (1936, 1940), and others. The use of stereoscopic obliques is particularly effective, but thus far has been given little attention. It is to be noted that oblique photographs, by themselves, are primarily illustrative and suggestive. Although providing a bird's-eye view of certain field relations, their value as evidence on moot questions of geomorphic interpretation is subject to definite limitations. The picture which they provide is not necessarily complete or true to proportion, unless certain elaborate cartographic techniques are first applied. If fact, their representation of the field facts may be decidedly selective in some instances, for the vantage point best suited to show some features may entail the suppression or distortion of others equally significant, particularly in areas of strong relief. Linear perspective is sometimes misleading, distances, directions, and elevations being confused through foreshortening. Oblique photographs may supplement maps and profiles, but cannot substitute for them. Used, however, with due appreciation of these qualifying factors, they may contribute notably to the clarification of geomorphic description.

Vertical photographs and mosaics may be used for illustrative purposes where a plan view is preferred, or where the effect of a pictorial map is desired. If data on elevations and gradients are important, these may be shown by contours drawn on the photo. Also, topographic or geologic features of particular interest may be emphasized in ink. Where the effect of a relief model is necessary to do full justice to the landforms concerned, stereo pairs of verticals may be used (Fig. 3). In many cases, vertical and oblique views of the same area supplement one another, and both may be supplemented by ground views of the same area also.

Aerial Photographs for Purposes of Record

In studying the progress of erosion, deposition, volcanism, and current diastrophism, accurate data on changes in the shape, size, and position of geomorphic features are necessary. This requires the comparison of detailed records of the feature in question at successive points in time. For such records, topographic maps, ground surveys, and ground photographs have been used. The record provided by aerial photographs, however, is more accurate and more

complete than that afforded by any save the most elaborate maps, thus anticipating fortuitous changes and allowing an unlimited number of points for future comparison. The same area may be rephotographed at any time desired, even under conditions, such as flooding, which might render ground methods impracticable. Records of certain processes in action—floods, storm waves, dust storms, igneous extrusion—may be included in the sequence. Thus for studying shifts in stream channels, the growth and migration of dunes, glacial advance and retreat, changes in the position of shorelines, slow landslide movements, etc , aerial photographs provide an ideal medium. Photographs such as those shown in Figures 2, 7, and 10 might provide starting points for studies of this type. Either vertical or oblique photographs may be used, the former being best for quantitative comparisons Photographs made at both the beginning and the end of the observation period are naturally desirable, but if the latter are not feasible, ground measurements made from points identified in the former may provide an adequate basis for comparison. Fortunately, however, the photographic program of the U. S. Department of Agriculture provides for the rephotographing of many areas at more or less regular intervals, and for some areas a second set of photographs is already available.

Aerial Photographs in Teaching

Aerial photographs provide a valuable aid in teaching geology and geomorphology, both in the classroom and in the laboratory (Melton, 1939). In the classroom, lantern slides made from both oblique and vertical photographs provide excellent illustrative material Stereoscopic lantern slide projection promises to be particularly helpful in this connection. With the new Vectograph lantern slides recently developed by the Polaroid Corporation, of Cambridge, Mass., any standard projection lantern may be used for stereoscopic effects, provided that the spectators are equipped with the proper polaroid viewing spectacles. Another method, involving "double-barreled" projection of stereo pairs, has recently been described by Fisher (1942), and permits stereoscopic projection in full color

In the laboratory study of geomorphology, attention is commonly centered on the areal relations of landforms. Topographic maps have been used traditionally as the basis for this mode of approach. For elementary classes, however, aerial photographs provide a more vivid and naturalistic picture of topographic features. Their pictorial character minimizes the problems of interpretation, permitting a more direct and rapid acquaintance with selected examples of the various types of landforms.

For courses of a more technical character, aerial photographs may be used in many ways. Exercises of the familiar question-and-answer type may be used, and familiarity with the detailed form and pattern of erosional and depositional features may be acquired through the preparation of planimetric and contour maps from photographs Some elementary work in photogrammetry should be a prerequisite for advanced courses in geology and geomorphology under this plan (Smith, 1942).

For the more advanced studies in geomorphology, the possibilities disclosed through the use of aerial photographs are unlimited. Photographs of new areas repeatedly present new problems of interpretation, suggest new hypotheses, and lead to new field projects. Although the basic principles of interpretation are quickly mastered, skill in the recognition of innumerable minor forms, patterns, and color tones continually enlarges with experience. A broad new field of geomorphic interpretation awaits the master of this new technique, and with its

more widespread application we may anticipate that present concepts in the science will be greatly refined, if not in some measure revised.

Aerial Photographs as an Aid in Field Studies

Throughout the course of a field project, from the laying of plans through the field survey itself to the completion of the manuscript, aerial photographs may render invaluable service. During the preparatory stage, preliminary study of photographs in the office provides a basis for more effective planning of the field work to follow. The general nature of the problem is scrutinized, working hypotheses are evolved where possible, specific questions to be answered in the field are raised, the accessibility of field areas is noted, and base maps showing features of geomorphic interest may be prepared. As a result, the actual time spent in the field is used more efficiently, and each step in the work becomes more purposeful.

Field reconnaissance frequently constitutes the initial stage in a geomorphic project. It may be devoted to a delimitation of the problem, to the formulation of working hypotheses, and to the selection of place and methods of attack During this stage of the work, aerial mosaics are especially helpful, and even the rough, uncontrolled type is valuable. Where topographic maps are available, they may be used in conjunction with the mosaics. On entering the field, the mosaics provide a guide in planning traverse routes, and serve as base maps for subsequent observations. Thus the writer, in studying the extensive dune fields of western Kansas, was enabled, through a study of aerial mosaics, to find critical localities which otherwise could have been discovered only by hundreds of miles of traversing requiring many weeks of time. In some cases, the study of aerial photographs and mosaics may itself take the place of preliminary reconnaissance, either wholly or in part. Upon completion of the reconnaissance, mosaics are of further assistance in interpolating between traverse lines, and thus in integrating widely scattered observations. Places most promising for detailed examination are indicated, and the need for contact prints of specific localities is suggested.

Helpful though they are, aerial mosaics are by no means unequivocal as indicators of geomorphic features. Frequently they are merely suggestive. Forms having low relief and gentle slopes may not be distinguishable without the stereoscopic view afforded by overlapping contact prints, although in many instances they are more readily seen on mosaics than on the ground. The interpretation of mosaics, in general, is facilitated by continued correlation between photographic detail and topography in the field.

In detailed field study, overlapping contact prints are preferable to mosaics. Careful stereoscopic study of the prints for critical areas on beginning field examination leads to a much clearer picture of local relations, suggests specific questions to be answered on the ground, and indicates places most promising for investigation—places where sedimentary deposits are exposed, where structural affiliations may be noted, where topographic unconformities may be appraised, where the relations of ground water, soil and vegetation may be observed, where obscure features shown on the photographs may be interpreted, and where detailed maps or profiles might be desirable. During the prosecution of field study, the prints are invaluable as guide maps and location maps. Desired data may be plotted either on the prints themselves, on a transparent overlay, or on planimetric maps previously prepared from the prints. For interpolation between the localities thus detailed, the prints provide either a detailed field guide along the contacts or boundaries to be traced, or a basis for tracing

these lines in the office upon completion of field work. In either case, a considerable economy of time is effected, much random traversing is avoided, and the accuracy of the results is increased.

Detailed studies of limited areas frequently provide a key for the interpretation of regional geomorphic features. In applying the results of the former to the latter, reconnaissance methods are commonly employed, and under favorable conditions critical studies of aerial photographs or mosaics alone may suffice. The implications of this proceeding are discussed below.

Aerial Photographs as a Direct Basis for Geomorphic Interpretation

It is a truism to state that field facts constitute the broad foundation of present geomorphic knowledge. In the gathering of these facts, however, there has arisen a certain division of function Where adequate topographic maps or aerial photographs, or both, are available, the essential field work, insofar as morphology alone is concerned, may be said to have been carried out by the topographer or by the aerial photographer. The need for much laborious traversing by the individual investigator is removed, and one very important aspect of the field is literally brought into the laboratory Furthermore, it would appear that, in many areas, the published results of previous field surveys, either geologic or geomorphic, and made without benefit of aerial methods, might contribute such requisite information of an other than morphologic nature as to provide a sufficient foundation for subsequent studies based immediately on aerial photographs. Thus the results of earlier investigators might be refined, enlarged, extended, or modified. Is it to be inferred, then, that the need for original field study is minimized or obviated when aerial photographs are available? The possibilities challenge attention, and demand a critical examination of the part which may be played by aerial photographs in geomorphic interpretation. Such an appraisal is essayed in the following pages.

The validity of aerial photographs as a basis for geomorphic analysis rests, essentially, on two questions (1) to what extent does surface form alone constitute an adequate basis for analysis, and (2) to what extent do aerial photographs provide an adequate picture of form? Considering first the latter of these two questions, it is obvious that form is a three-dimensional quality, comprising outline in ground plan and profile in cross section. It is only the first of these which is directly represented in quantitative terms on aerial photographs. This, however, is shown in much greater detail than is possible on contour maps of comparable scale. Delicate shadings of color tone, together with stereoscopic exaggeration of minor relief features, display intricacies of form beyond the capacity of contour lines to resolve. This is especially true in areas of low relief, where forms and patterns which fail to catch a contour line, and which may be obscure even to the observer on the ground, are shown with striking clarity—meander scrolls (Fig. 2), undrained depressions (Fig. 9), shore features (Fig 10) dune forms (Fig. 7), intricate ramifications of drainage patterns, and innumerable other features. What might be termed a "microgeomorphic" approach to the study of landforms is provided, and problems which might otherwise pass unnoticed are brought to light. In areas of rugged mountainous topography, however, distortion of scale by parallax may be objectionable, and for the representation of the larger landforms *in gross*, contour maps where available, have certain advantages. But on the whole, insofar as geomorphic analysis may be based on the study of landforms and drainage features *in ground plan*, vertical aerial photographs provide an ideal medium.

It is in their representation of the third dimension that aerial photographs

may be found wanting. Unless special instrumental equipment is used and ground control established, it is only relative slope and relative relief that are shown. Where exact data on gradients, slopes, and elevations are required, as in certain studies of erosion surfaces, aerial photographs alone are not adequate, and recourse must be had to topographic maps or to measurements in the field.

The factor of scale may introduce still another limiting factor. Certain important minor features, as weathering forms, glacial scorings, eolian fluting, niches, stone nets, etc., are too small in size to be shown adequately on aerial photographs of standard scale. For the study of these, the field approach is obviously necessary. And at the other limit of scale, it is sometimes the regional rather than the local picture which is desired In such cases, the wealth of detail shown on aerial mosaics may be of no particular advantage, and the lack of vertical control might be a serious handicap. Contour maps or profiles prepared from them, might be preferable. However, there are large areas in the United States, well covered by excellent aerial photographs and mosaics, for which topographic maps either have not yet been made, or are so generalized as to be of little value.

The adequacy of surface form as a basis for geomorphic interpretation is a more searching question, too far-reaching to be discussed very fully in this paper. In brief, however, it may be said that geomorphic studies are of two general types, (a) those concerned with the formulation of principles, and (b) those involving the application of established principles to specific areas or problems, as to the unraveling of erosional chronologies In practice, the two are sometimes combined, but not uncommonly the second is separated in place and in time from the first. In both, however, the emphasis is on surface form. In the first, the approach is essentially inductive Form is correlated with genetic processes and with environing factors, through the synthesis of direct observations on processes in action, indirect chains of circumstantial evidence, experimental findings, evidences from internal constitution, comparative verification of consequences deduced from working hypotheses, and scattered data drawn from the body of geologic knowledge. Gradational variations in form, moreover, are correlated with successive stages in an ideal sequence through which the process or processes in question operate. In this type of study, clear delineation of surface form is an essential starting point, but is only a starting point. Aerial photographs may thus have an important auxiliary role.

For many types of landforms, studies of the above type are already history. The major generalizations have probably been made. Nevertheless, there are many questions which have not yet passed beyond the stage of bland generalization. The genesis of such familiar phenomena as graded slopes, for example, is still debated. Certain of the criteria for establishing erosional history likewise remain a source of contention. Many of our concepts have yet to be set forth in quantitative rather than qualitative terms. For nearly all types of landforms, there remains the need of defining detailed conditions of genesis more critically. Not until form is thoroughly understood in terms of process, stage, and environmental factors does it constitute a valid criterion for interpreting geomorphic history.

The application of genetic principles to the interpretation of a given landscape is based primarily, but not exclusively, on form. The approach here, however, is essentially deductive. Form is used as a criterion of process, stage, climatic environment, and tectonic background. Attention is focused on the distribution, orientation, and interrelations of landforms, on the transection of older forms by younger, and on the recognition of ancient forms which have suffered modification or partial destruction. It is this method of approach which is followed in studying successive erosion surfaces, stages in shoreline develop-

ment, changes in wind direction and potency as recorded in dune fields, etc. It presupposes the accurate differentiation and identification of forms—the recognition of superficial differences between forms genetically similar, and of superficial similarities between forms genetically different. It is to be noted, however, that form is not always unequivocal Forms genetically diverse may be morphologically indistinguishable. In such cases, it is to the evidence of internal constitution—lithology, bedding, cross-bedding, soil zones, disconformities, etc. —that appeal must be made. Certain depositional forms are distinguishable from erosional forms, and from one another, only on this basis. This applies particularly to glacial features, as in the distinction between morainal topography and heavily pitted outwash. It applies similarly to the distinction between rock-cut and cut-and-fill terraces. The recent controversy as to the origin of the Carolina Bays presents still another case in point (Melton, 1933; Johnson, 1942). And insofar as internal evidence is requisite for a solution of the problem, aerial photographs can lead but halfway to the goal. They cannot take the place of hammer, shovel, and drill. They can suggest working hypotheses, and may narrow the probabilities, but cannot point to the final decision. Where form is diagnostic, however, and does provide a reliable indicator of process and stage, aerial photographs, within the limits already outlined, may provide a satisfactory basis for interpretation in problems of limited scope, such as the distribution and orientation of dune forms, the study of simple terrace sequences, the analysis of detailed drainage patterns, etc Similarly, when a clearer and more detailed knowledge of morphology is all that is needed to build upon the results of previous field studies, aerial photographs alone may suffice. It is in problems of a less specialized character that additional field study may be necessary to refine the given picture of form, or to make collateral observations of a more purely geological character. In deciphering the complete geomorphic history of any very large area, for example, stratigraphic as well as geomorphic methods must be employed A part of the erosional history is recorded only in correlative sedimentary deposits, and it is in these that the answers to many questions must be sought.

In summary, the adequacy of aerial photographs as a direct basis for geomorphic interpretation depends on the type or types of topography to be studied, on the scale of the investigation, on the importance of accurate vertical control, on the nature and scope of the study, on the extent of previous field studies on the same problem, and on the reliability of purely morphologic criteria for the given problem. Frequently the photographs are suggestive rather than definitive. In some cases, their primary role is to reveal the existence and nature of the problem. In many types of investigation, they need to be supplemented by contour maps on the one hand, and by field study on the other. To evaluate their potentialities and limitations in specific projects is a problem for trained judgment. Error in the one direction may lead to unwarranted speculation, and error in the other direction may result in a needless waste of effort. Used, however, with a discrimination commensurate to the advantages attainable, aerial photographs provide a new and revitalized approach to many fields of geomorphic inquiry.

Applied Geomorphology

In closing this paper, it is fitting that the preceding theoretical discussion be followed by a consideration of the more practical aspects of the subject, which in fact, are actually rooted in the more general and basic topics discussed up to this point. In the following paragraphs, some specific applications are briefly noted.

In mining geology, the localization of certain types of ore deposits is directly related to geomorphic processes and to geomorphic history. Residual ores of iron and manganese, for example, are found to be associated with remnants of dissected erosion surfaces. Placer deposits of various minerals are formed by the action of streams, waves, and currents, past and present. The discovery and exploitation of such deposits is partly a geomorphic problem, and one in which the type of information provided by aerial photographs is particularly valuable.

In engineering geology, it is important to ascertain the nature of the substratum, the depth to bedrock, and the presence of actual or potential obstacles, such as landslides. Correct analysis of geomorphic features may lead directly or indirectly to the necessary information, and the use of aerial photographs contributes to the speed and efficiency of such investigations.

Land classification and description is partly a matter of applied geomorphology, whether deliberately or unknowingly The recognition of a stabilized sand dune topography on aerial photographs, for example, points at once to certain limitations of the use of the land concerned The identification of various types of glacial deposits, again, throws considerable light on soil conditions.

The study of the pressing problems of soil erosion demands the application of geomorphic methods, and draws heavily on the data supplied by aerial photographs. The progress and effects of erosion by wind and by water may thus be studied, their causes analyzed in terms of established geomorphic principles, and remedial measures planned and tested.

At the present time, military applications of geomorphology are probably foremost in interest and importance. There is ample reason to believe that much is being done in this field, but details have not been made public, and probably will not be until the end of the war. The general mode of approach, however, is well outlined in a recent paper by Erdmann (1943).

Applied geomorphology differs from "pure" geomorphology only in its objectives, not in its principles or methods. As methods are improved, more widespread applications may be expected. The introduction of aerial photographic methods marks so radical an advance as to entail far-reaching effects on both the practical and the theoretical aspects of the science.

References

Ashworth, F. L. (1937), Aerial mapping: Leica Photography, Vol 6, No 9, pp. 7 and 16

Cooper, W. S. (1935), The history of the upper Mississippi River in late Wisconsin and postglacial time· Minn. Geol Surv. Bull. No. 26

Cotton, C. A. (1941), Landscape as developed by the processes of normal erosion. Cambridge University Press, 301 pp.

Dickens, S. N., and Brown, H. B. (1938), Soil erosion in the Karst Lands of Kentucky: U. S. Dept. Agric. Circular No. 490

Erdmann, C. E. (1943), Application of geology to the principles of war Bull Geol. Soc. Amer., Vol. 54, pp. 1169–1194.

Fisher, D. J. (1942), Projector for stereoscopic pictures. Amer. Jour. Physics, Vol. 10, pp. 46–49.

Gaty, J. P. (1937), Aerial photography: Chap. 27 in Morgan, W. D., and Lester, H. M. The Leica Manual, 2nd Ed., Morgan and Lester, N. Y.

Hack, J. T. (1941), Dunes of the western Navajo Country. Geog. Rev., Vol. 31, pp. 240–263.

Johnson, D. W. (1942), The origin of the Carolina Bays, Columbia Univ. Press, N. Y., 341 pp.

Lee, W. T. (1922), The face of the earth as seen from the air. Amer Geog Soc Spec. Publ. No. 4.
Lobeck, A. K (1939), Geomorphology, McGraw-Hill, N Y , 731 pp
Loel, Wayne (1938), Use of aerial photographs in geologic mapping. Amer. Inst Min & Met. Eng Tech. Publ No. 890, 54 pp.
Lucke, J. B (1934), A study of Barnegat Inlet, New Jersey, and related shoreline phenomena. Shore and Beach, Vol. 2, No 2, 54 pp
Melton, F. A., and Schriever, W. (1933), The Carolina "bays"—are they meteorite scars? Jour Geol., Vol. 41, pp 52–66.
Melton, F. A. (1936), An empirical classification of flood-plain streams Geog. Rev., Vol. 26, pp. 593–609.
——— (1939), Aerial photographs and the first course in geology. Photogram Eng., Vol. 5, pp. 74–77
——— (1940), A tentative classification of sand dunes, its application to dune history in the southern High Plains: Jour. Geol , Vol 48, pp. 113–174.
Morgan, A. M (1942), Solution-phenomena in the Pecos Basin in New Mexico: Trans. of 1942 of Amer Geophys Union, pp 27–35.
Nichols, R. L., and Marston, A. F (1939), Shoreline changes in Rhode Island produced by hurricane of September 21, 1938 Bull Geol Soc. Amer , Vol 50, pp 1357–1370
Putnam, W. C. (1942), Geomorphology of the Ventura Region, California· Bull. Geol. Soc Amer., Vol 53, pp 691–754
Rich, J. L. (1939), A bird's-eye cross section of the central Appalachian Mountains and Plateau: Washington to Cincinnati Geog. Rev., Vol 29, pp. 561–586.
——— (1941), The aerial traverse, an application of aerial photography to geographic studies. Ohio Jour. Sci , Vol. 41, pp 212–224.
——— (1942), The face of South America: Amer. Geog Soc. Spec Publ. No 26, 299 pp.
Smith, H. T. U (1940), Geologic studies in southwestern Kansas. Kans St Geol. Surv Bull. No. 34, Pls. 24, 25, 27, 28, and 30.
——— (1942), Aerial photos in the geological curriculum (abstract) Bull Geol Soc. Amer., Vol. 53, p. 1852.
——— (1943a), Aerial photographs and their applications, Appleton-Century, N Y., 372 pp.
——— (1943b), The camera lucida as an aid in aerial photographic mapping: Military Eng., August, pp 410–411.
Sproule, J. C. (1939), The Pleistocene geology of the Cree Lake region, Saskatchewan: Trans. Roy. Soc. Can , Vol 33, sec. 4, pp. 101–110.
Taft, Robert. (1938), Photography and the American scene, Macmillan, N. Y., pp 186–188.
Thwaites, F. T. (1943), Pleistocene of part of northeastern Wisconsin. Bull Geol. Soc. Amer., Vol. 54, pp. 87–144.
Van Nouhuys, J. J. (1937), Geological interpretation of aerial photographs: Amer. Inst. Min. & Met. Eng , Tech. Publ. No 825, 18 pp.
Van Tuyl, F. M., and Lovering, T. S. (1935). Physiographic development of the Front Range. Bull. Geol. Soc. Amer , Vol. 46, pp. 1291–1350.
Von Engeln, O. D. (1942), Geomorphology, Macmillan, N. Y., 655 pp.
Washburn, Bradford. (1935), Morainic bandings of Malaspina and other Alaskan glaciers: Bull. Geol. Soc. Amer., Vol. 46, pp. 1879–1890.
——— (1941), A preliminary report on studies of the mountains and glaciers of Alaska. Geog. Jour. (London), Vol. 98, pp. 219–227.
Wilson, J. T. (1939), Eskers north-east of Great Slave Lake: Trans. Roy. Soc. Can., Vol. 33, sect. 4, pp. 119–130.

CHAPTER XVI

TRAINING AND EDUCATION

TEACHING OF BASIC PHOTOGRAMMETRY

Reynold E Ask

I. Introduction

DURING the past three and one-half years the writer has been instructing a War Training Course in Aerial Photogrammetry at George Washington University. The purpose of this article is to discuss the manner in which the course was presented with the hope that the information be of some value to those who may be called upon to teach similar courses in the future. Due to the increase in the use of aerial photographs for engineering and military purposes, it is most probable that courses in photogrammetry will soon be offered in all progressive engineering colleges.

The scope of any particular course necessarily will have to be adjusted to fit the needs and qualifications of the students In this discussion we will assume that the students are familiar with topographic drafting, trigonometry, and plane surveying, and that they are interested in obtaining a basic knowledge of the entire field of aerial photogrammetry, with special emphasis on practical map compilation. The order of presentation of the various subjects will be as follows: Equipment required, suggested course outline, and general textbooks.

II Equipment Required

Generally speaking, schools starting courses in photogrammetry have a limited budget for equipment. Although the cost of many photogrammetric instruments runs into thousands of dollars, those necessary to teach the basic principles can be obtained at a reasonable figure. A suggested list of equipment for a class unit of fifteen students will be given, together with the approximate cost of each item. A list of known manufacturers of these instruments will be found at the end of this section.

Many of the instruments to be included in this discussion are used extensively by the armed forces, and therefore are rather difficult to obtain at the present time. Even with a high priority rating, the purchaser may have to wait months for delivery For this reason, suggestions are given on substitute equipment which can be constructed in any school workshop and which require the use of very little critical material.

Lens Stereoscopes (15 required)

Illustrated in Figure 1 are several types of lens stereoscopes. Type "A" is convenient for carrying in the pocket, but has the disadvantage of resting directly on the photographs, thereby interfering with their adjustment. Type "B" is less portable than "A" but has the advantage of completely clearing the photographs. The cost of both of these types is approximately ten dollars. Type "C" is a simple improvised folding stereoscope equipped with "dimestore" lenses. The total material cost of this unit is about seventy-five cents. Another type of lens stereoscope (not illustrated) is mounted on a small plywood base. Such an arrangement is very convenient for field use. Although lens stereoscopes give considerable magnification (about 3×), their field of view is rather small.

Mirror Stereoscopes (3 required)

The price of mirror stereoscopes of all-metal construction ranges from forty to two hundred dollars. Although the cheaper type (having back-surfaced mir-

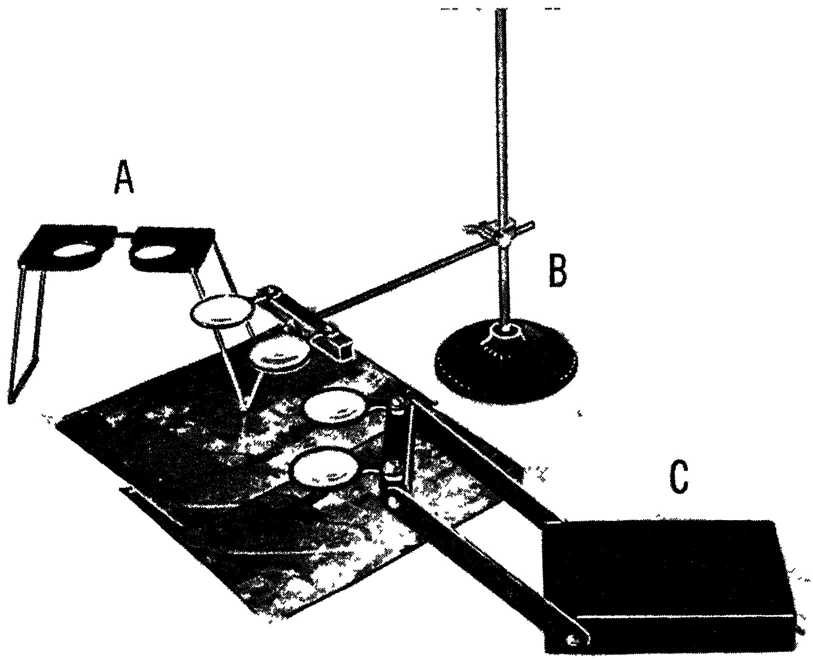


FIG. 1. Types of lens stereoscopes. (A) Folding pocket model. (B) Desk model which completely clears the photographs. (C) Improvised wood model.

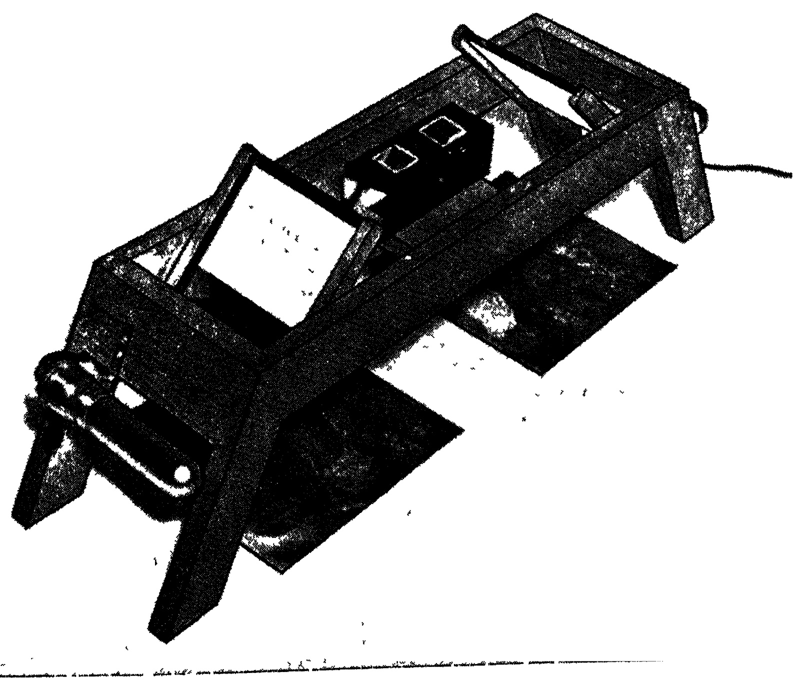


FIG. 2. Mirror stereoscope of simple wood construction.

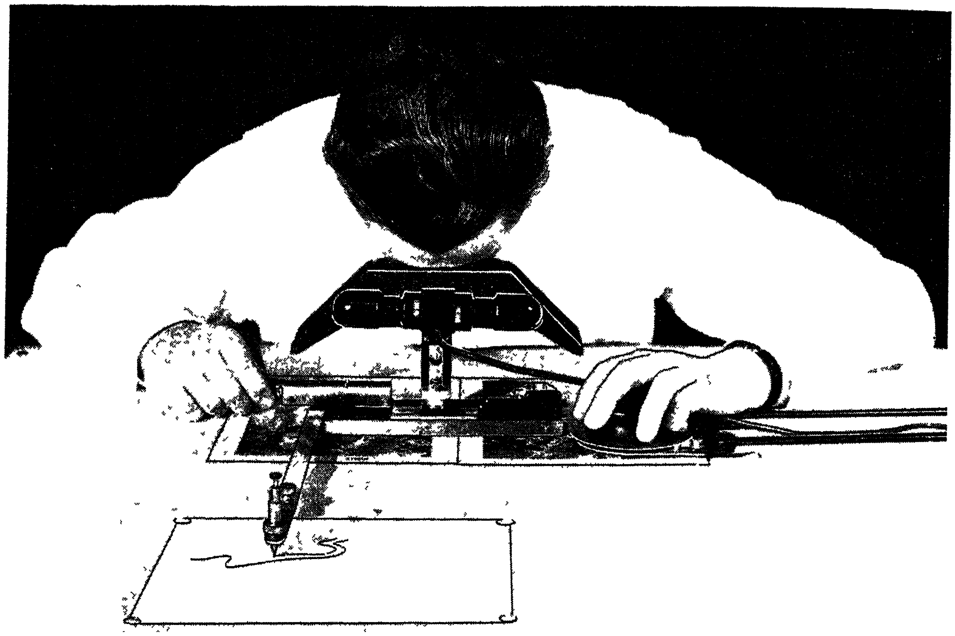


FIG. 3. Fairchild Stereocomparagraph in operation.

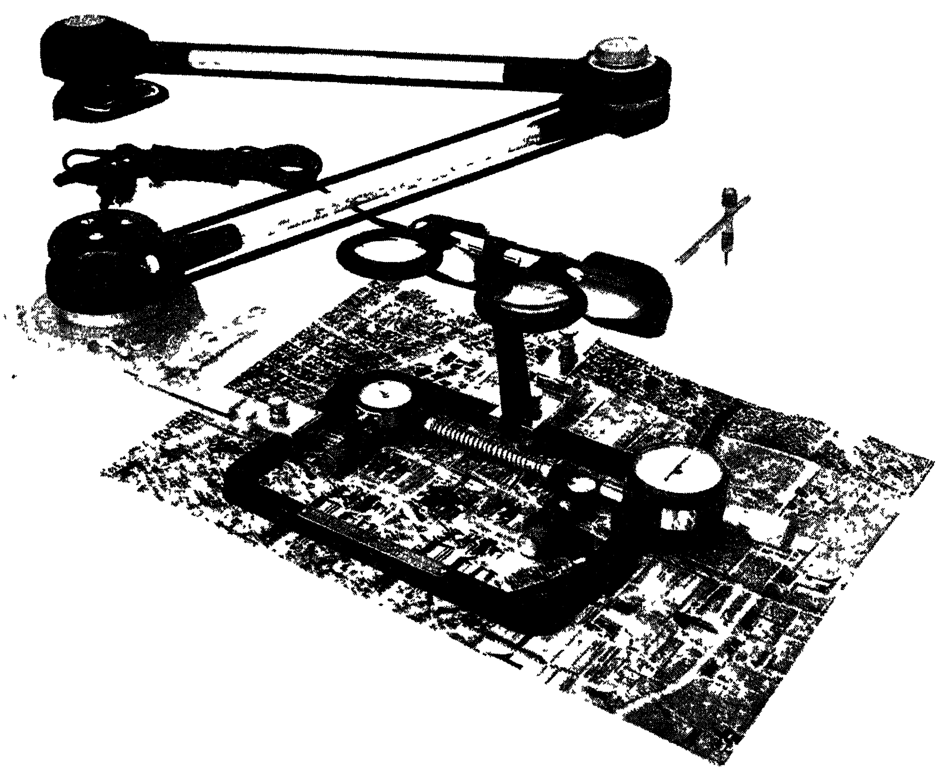


FIG. 4. Abrams Contour Finder ready for operation.

rors) produces a slight ghost image, it probably is less apt to be injured by students than the front-surfaced type. To obtain higher optical performance the more expensive stereoscopes are equipped with front-surfaced mirrors, and right angle reflecting prisms often replace the small eye mirrors. Also provision is sometimes made for adding a magnifying lens system (see stereoscope shown later in Figure 5) Shown in Figure 2 is a wood stereoscope of rather simple con-

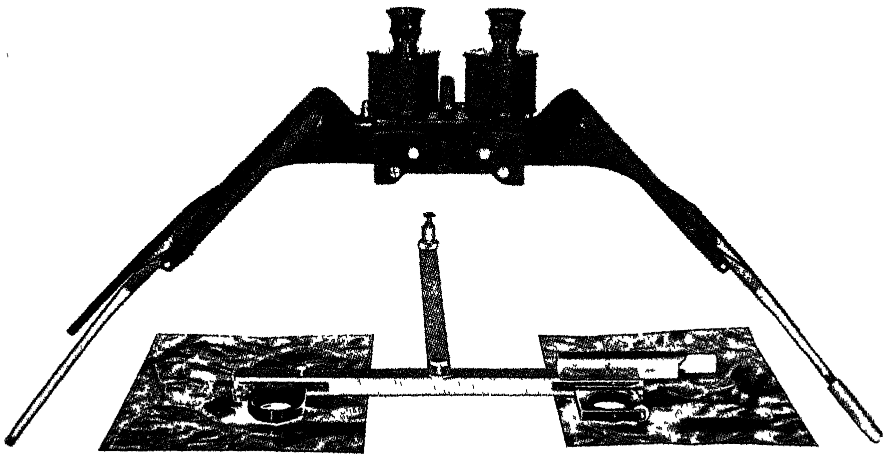


FIG. 5. Fairchild Parallax Bar with magnifying stereoscope.

struction. The model illustrated is equipped with front-surfaced mirrors (cost of 4 mirrors about five dollars) and an attached illuminating system. Mirror stereoscopes must be used when large photographs are to be examined, or when a large field of view is desired. Their magnification is usually less than unity when used without a lens system.

STEREOCOMPARATORS (2 required)

In this discussion will be included only those simple types in which the floating marks rest directly on the photographs (see Figures 3 and 4). Instruments of this nature are used for the determination of elevations and the drawing of contours on overlapping vertical photographs. It is the cheapest and simplest instrument available for training persons in stereoscopic plotting. However, the following limitations must be considered: (1) Accurate results cannot be obtained from tilted photographs unless rather tedious correction factors are applied to the parallax values; (2) an error is caused by a difference in scale of the photographs; (3) the contours drawn show a perspective view of the terrain rather than an orthographic or map view; (4) the distortion of the photographic paper often introduces considerable error.

When drawing contours it is highly desirable that the stereocomparator be attached to an alignment mechanism (such as a standard drafting machine) in order that the eye base of the instrument remain parallel to the air base on the photographs as the observer moves the floating mark over the stereoscopic model.

An instrument related to a stereocomparator is the "parallax bar" (Figure 5) or "tracing stereometer." This is a parallax measuring device similar to the

floating mark assembly on the above stereocomparators. It can be used with almost any mirror stereoscope, thus keeping the cost of the complete unit to a minimum.[1] The cost of a stereocomparator (also called stereocomparagraph or contour finder) ranges from 250 to 300 dollars Drafting machines to use with these instruments cost from forty to ninety dollars. The cost of a parallax bar is about eighty dollars. In case a stereocomparator cannot be obtained, it is still possible to instruct students in the theory of parallax measurements with no more equipment than a twelve-inch engineer's scale (see page 760, item 11)

METAL TEMPLETS (75 units—optional)

Various names such as radial intersectors,[2] slotted mechanical templets, and mechanical triangulators have been used to describe this equipment. For instructional purposes in radial triangulation, these metal templets have several advantages over other templet systems (see page 759, item 8) since they can be used repeatedly without any material loss. The cost of 75 complete units (for 9″×9″ photographs) including arms, bolts, nuts, studs, and pins is approximately two hundred dollars

REFLECTING PROJECTORS (one—optional)

This type of instrument is used for transferring detail from the photograph to the plotting sheet, with or without a scale change as desired. Although the common type used in most mapping offices is too expensive for class use, a satisfactory low-priced substitute is found in the "vertical sketchmaster" illustrated in Figure 6. This portable instrument, based on the camera lucida principle, was designed for field use by J. L. Buckmaster of the U. S. Geological Survey. Its adjustments are arranged to take care of scale changes (from 0.25 to 1.5) and approximate tilt removal. Its cost is about one hundred dollars. An oblique sketchmaster has also been designed for transferring detail from oblique photographs.

Some instructors have built reflecting map projectors from parts of discarded enlarging or copying cameras, utilizing the lens (if available) and purchasing a suitable front-surfaced mirror.

MULTIPLEX EQUIPMENT (optional)

Although this type of equipment is highly desirable for instructional purposes, its relatively high cost places it outside the budget of most educational institutions (unit complete with three projectors costs about five thousand dollars.) After the war it is quite possible that excess or obsolete equipment of this type may be obtained from the armed services. The writer had a wood model of the multiplex equipment constructed which has proved very valuable for lecture use (see Figure 7). The cords (red and green) represent light rays from various ground points. Six of these points have been chosen in the common overlap area, and are arranged in a manner similar to those chosen for relative orientation in the actual multiplex equipment. When not in use, the model can be readily dismantled and packed in a 27-inch fiber luggage case.

MISCELLANEOUS EQUIPMENT

Each student should be required to have the usual drawing instruments needed in any mechanical drafting class, also a magnifying glass and a needle point. The following additional equipment should be available for class use: beam compass, drop bow pen, assorted scales and straight edges. A lantern slide

[1] See Chapters VII and XI for further illustrations and descriptions of above mentioned stereoscopic instruments.

[2] See Chapter IX.

FIG. 6. Vertical Sketchmaster in operation.

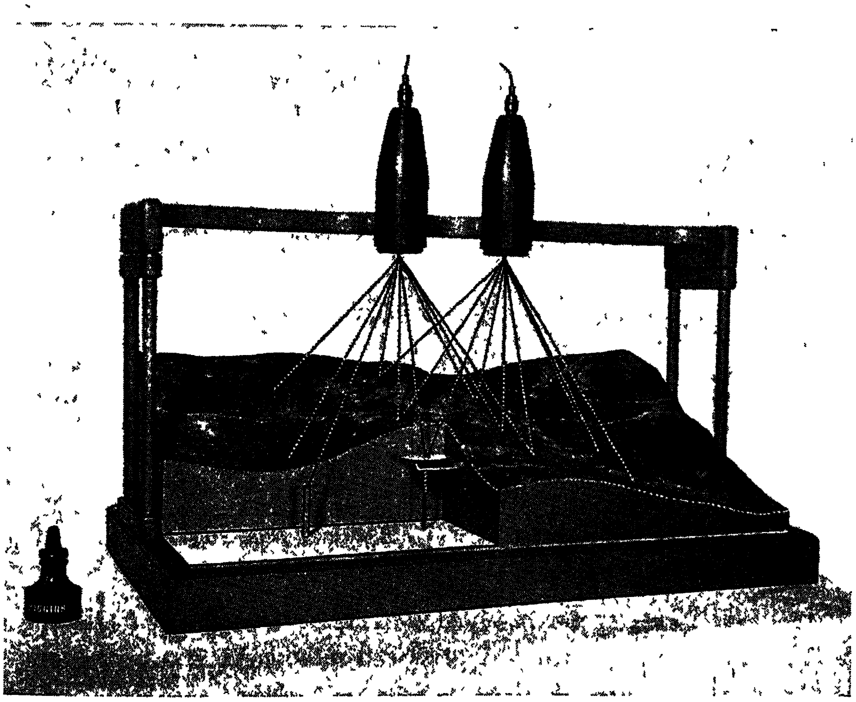


FIG. 7. Model of multiplex equipment for lecture use.

projector and a variety of slides should be available for class use. Also samples of aerial photographs of many types should be secured Due to present war restrictions, it is difficult to secure aerial photographs from the usual peace-time sources. Among these were commercial aerial survey organizations, and various Federal Government agencies such as the Agricultural Adjustment Administration, Forest Service, Geological Survey, etc. Instructors of War Training Courses in Photogrammetry are usually able to obtain permission from local military authorities to use certain available aerial photographs covering non-strategic areas.

EQUIPMENT MANUFACTURERS

Following is a list of concerns (known to the writer) which manufacture the above described equipment No attempt is made to list all the photogrammetric products of each concern, but only those items pertinent to the present discussion:

Abrams Instrument Company, Lansing, Michigan.
Lens stereoscopes, contour finders, mechanical triangulators, vertical and oblique sketchmasters.

Aero Service Corporation, 236 East Courtland St., Philadelphia, Pennsylvania.
Vertical and oblique sketchmasters.

Bausch and Lomb Optical Company, Rochester, New York.
Multiplex equipment.

Evaporated Metal Films Corporation, Ithaca, New York.
Front-surfaced and semi-transparent mirrors

Fairchild Camera and Instrument Corporation, 88-06 Van Wyck Blvd., Jamaica, New York.
Mirror stereoscopes, stereocomparagraphs, parallax bars.

Graves, H. W., 4535–18th St., North, Arlington, Virginia.
Relief models, instrument models (see Figure 7).

Kalart Manufacturing Company, Inc., Stamford, Connecticut.
Slotted mechanical templets, oblique sketchmasters.

Keuffel and Esser Company, Hoboken, New Jersey.
Drafting machines, general drafting supplies.

Ryker, Harrison C., Inc., 1000 Ashby Ave., Berkeley, California.
Lens and mirror stereoscopes.

III. SUGGESTED COURSE OUTLINE

Since photogrammetry is a rapidly changing science, any plan of study should frequently be modified to bring it up to date. Consideration should also be given to special circumstances in some localities which make it desirable to stress one particular branch of photogrammetry, in which case the outline given below will have to be so modified. This outline is based on 60 two-hour class periods (30 lectures and 30 laboratory or drawing room sessions). In a regular college course this is equivalent to six semester hours credit. It might be possible to cut this time in half if only the elementary aspects of the subject are to be presented. The sequence of class periods as given below has been arranged so as to coordinate the various subjects as much as possible.

1. GEOMETRIC CHARACTERISTICS OF AERIAL PHOTOGRAPHS[3]

(4 hours lecture and 6 hours laboratory) Scale relationships, coverage, perspec-

[3] See Chapter VI and XII Also Anderson, R. O , "Applied Photogrammetry," (1941), 190 pages, $3.00 Copies may be obtained from author whose address is 401 Pound Bldg., Chattanooga, Tenn.

tive qualities, relief and tilt displacements, methods of tilt determination for nominally vertical photographs (Note: The use of a three-dimensional model is very helpful in explaining the geometry of the photograph. A model of this nature, printed on white cardboard, has been designed by O. M. Miller of the American Geographical Society, New York)[4]

Problem A: Construction of a tilt diagram The example shown in Figure 8 may be used for several purposes, (a) it furnishes a simple graphical method of introducing the subject of tilt, (b) the tilt displacement at the corner of the square may be mathematically computed to check the graphical solution, (c) computations can be made for an optical rectification which will restore the distorted shape to a true square.

Problem B: Tilt determination on a photograph to be furnished each student. If sufficient photographs are not available, the instructor can work up a tilt problem from a selected photograph, and then transfer the ground points and principal point to a sheet of drawing paper for each student. If time is available it would be preferable to require the problem to be solved by more than one method, so as to compare their relative accuracy.

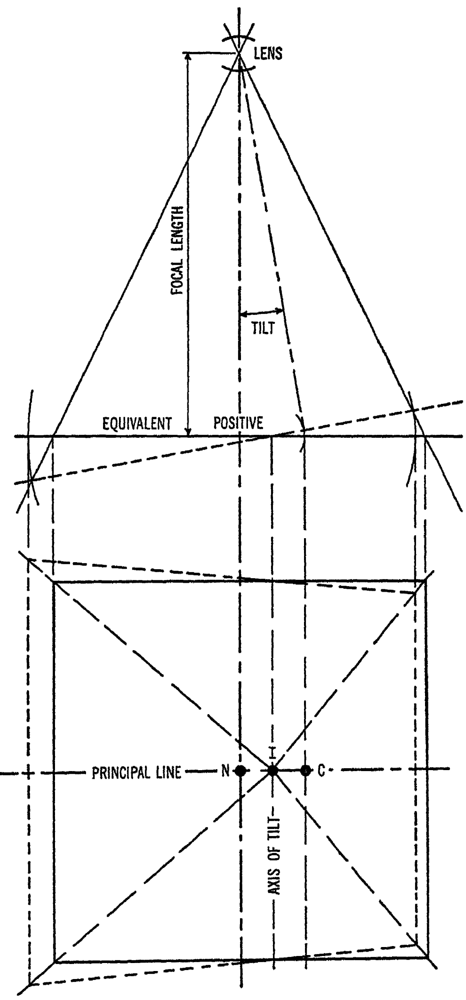


FIG. 8. Diagram showing the effect of tilt on the image of a square object. Principal point denoted by "C," isocenter by "I," and nadir point by "N."

2. PHOTOGRAMMETRIC OPTICS[5]

(*4 hours lecture and 2 hours laboratory*) Introduction to lenses, photographic lenses, optical rectification, errors caused by glass plates, simple optical systems, color filters, prisms, mirrors.

Problem C: Optical rectification of a tilted photograph. Compute rectification data for removing the tilt from a given photograph. If desired, the distorted shape shown in Figure 8 can be rectified back into a true square. Draw to scale a schematic view of the rectifying camera [6] It would be very helpful if the students could visit a photogrammetric laboratory where a rectifying camera was in actual operation.

[4] Other interesting models are described under the second paper of this chapter, page 768

[5] See Chapter II.

[6] See Chapter II, Fig. 21.

3. Aerial Cammera and Accessories[7]

(*2 hours lecture and 2 hours laboratory*) Single and multiple lens cameras, types of shutters, film flattening devices, film winding mechanisms, recording devices, view finders, intervalometers, mounts, auxiliary horizon cameras, statoscopes, equipment for night photography. One laboratory period should be devoted to a demonstration of aerial camera equipment. Arrangements for such an exhibit can often be made with a nearby camera user or manufacturer. In some localities it may be possible to inspect a camera installation in an aircraft.

4. Flying for Photography[8]

(*2 hours lecture and 2 hours laboratory*) Job specifications, preparation of flight map, cost estimates, aircraft requirements, special navigating instruments, making the exposures.

Problem D: Preparation of a flight map and estimate of time and material needed for a specific job. It is desirable that each student be furnished with a 1:125,000 scale quadrangle map for this problem.

5. Photographic Materials, Chemistry, and Laboratory Technique[9]

(*2 hours lecture and 2 hours laboratory*) Properties of photographic emulsions, types of film base, color filters used with various emulsions, processing of aerial film, characteristics of photographic papers, contact and projection printing, preparation of photo-index map. If possible, arrange an inspection tour through an aerial photographic laboratory.

6. Introduction to Stereoscopy[10]

(*2 hours lecture and 2 hours laboratory*) The human eye and binocular vision, stereoscopic vision, simple lens and mirror stereoscopes, orientation of photographs for stereoscopic observation, pseudoscopic effect, stereoscopic training and testing, anaglyphs, vectographs. The purpose of this lecture is to present the fundamentals of stereoscopy and to train the students in the use of simple stereoscopes. A thorough discussion of parallax measurements will be presented later under item (11). The laboratory period should be devoted to practice in the use of various types of stereoscopes, and stereoscopic vision tests. In regard to the last item, it is obvious that only those individuals possessing normal binocular vision can operate a stereoscopic plotting instrument. Photogrammetric laboratories using such instruments usually require vision tests of prospective employees.

7. Review of Surveying Fundamentals[11]

(*2 hours lecture and 2 hours laboratory*) Map projections, grid systems, ground control, representation of planimetric and topographic details, and map reproduction.

The reason for devoting such a short time to above subjects is with the as-

[7] See Chapter III. Also: Talley, B. B., "Engineering Applications of Aerial and Terrestrial Photogrammetry," pages 42–133.

[8] See Chapter IV.

[9] See Chapter V. Also: Mack, J. E., and Martin, M. J., "The Photographic Process" (1939), McGraw-Hill Book Co, Inc., New York. Neblette, C. B., "Photography" (1942), D. Van Nostrand Co, Inc, New York.

[10] See Chapter VII. Also: Judge, A. W, "Stereoscopic Photography" (1935), American Photographic Publishing Co., Boston, Mass. Polaroid Corporation, 730 Main Street, Cambridge, Mass (specimen vectographs, made from aerial negatives, may be obtained from this organization).

[11] See Chapter I.

sumption (previously stated) that a knowledge of surveying be a prerequisite to this course in photogrammetry.

Problem E: Construct projection and plot ground control for radial triangulation problem which follows. Grained cellulose acetate sheeting should be used. This sheeting may be obtained from the Eastman Kodak Co., Rochester, N. Y., or from The Lustro Co., 117–125 East 13th St., Chicago, Ill.

8. RADIAL PLOTS[12]

(4 hours lecture and 18 hours laboratory) Control requirements, field inspection, preparation of the photographs incluing determination of average scale, discussion of various types of templets, adjusting of templets to fit ground control, compilation of detail, use of various types of map projectors, drafting procedure.

Problem F: Construction of a radial plot. Each student should be supplied with a strip of at least five 9″×9″ photographs printed on low-shrink paper. It is desirable that the terrain covered by these photographs be rather rugged (elevation differences of at least 1000 feet on 1:20,000 scale photographs) in order to vividly demonstrate the theory of the method. A reasonable amount of culture should be present for detailing purposes. Properly distributed horizontal ground control should also be available. It is suggested that three well distributed control stations be located on the first photograph, and one or two on the last. In order to check the student's work it is convenient to have an accurate map of the same region. Some instructors may find it desirable to have the class as a whole work on a much larger problem than suggested above. For instance, five strips of fifteen photographs may be used, each student being responsible for a certain section of the job. The advantage of a problem of this type is that a study of the effect of control distribution can be made. A word of caution is necessary, however, in that careless work by one student can upset the accuracy of the entire plot.

In regards to templets used for this problem, three types are available: (a) clear cellulose acetate templets—requires no special equipment for their construction and is therefore the easiest and cheapest to obtain, although the least desirable from a practical standpoint, (b) slotted cardboard templets—requires the use of a special patented slot cutter, (c) metal templets—have a high initial cost but can be used repeatedly with no material loss, and is considered the best type for instructional purposes. They can be purchased at a reasonable price from several different sources.

In compiling detail, standard map symbols should be used throughout.[13] The use of a vertical sketchmaster (shown in Figure 6) is very helpful for this operation. In regard to lettering, the use of printed type on transparent gummed tissue is preferred. The style of type used for each feature should conform with standard mapping practice. Prepared lettering of this nature can usually be secured from the larger printing establishments.

9. MOSAICS[14]

(2 hours lecture and 2 hours laboratory) Uses, classification as regards control, preparation of the photographs, laying technique, finishing, reproduction. The laboratory period should be devoted to a demonstration of mosaic construction by a well qualified person.

[12] See Chapter IX.

[13] War Department Field Manual, "Conventional Signs, Military Symbols, and Abbreviations," FM 21–30 (1939). Copies may be obtained from Superintendent of Documents, Washington, D. C., $0.20.

[14] See Chapter X.

If the instructor desires to emphasize mosaic work in the course, the compilation of detail in the above radial triangulation problem can be omitted. In its place can be substituted the construction of a controlled mosaic, using the radial plot as a base.

10. INTERPRETATION OF AERIAL PHOTOGRAPHS[15]

(2 hours lecture and 2 hours laboratory) Study of objects in regard to size, shape, shadows, tone, and texture. Identification of military installations, detection of camouflage, use of infra-red and color films, night photography with flash bombs. It is highly desirable that the students be furnished with photographs of nearby areas, so that actual field interpretation studies can be made.

Although a lecture presentation of the subject of interpretation will prove very useful to the student, a complete familiarity of the subject can only be obtained through the actual compilation of a map from the photographs. During this compilation procedure, many additional problems in interpretation come up which require further clarification.

11. STEREOSCOPIC PLOTTING METHODS[16]

(12 hours lecture and 12 hours laboratory) Absolute and differential parallax equations and tables, the stereocomparator, the Brock process, the multiplex equipment, the aerocartograph, the stereoplanigraph, the autograph, etc., relative merits of various types of stereoscopic plotting instruments.

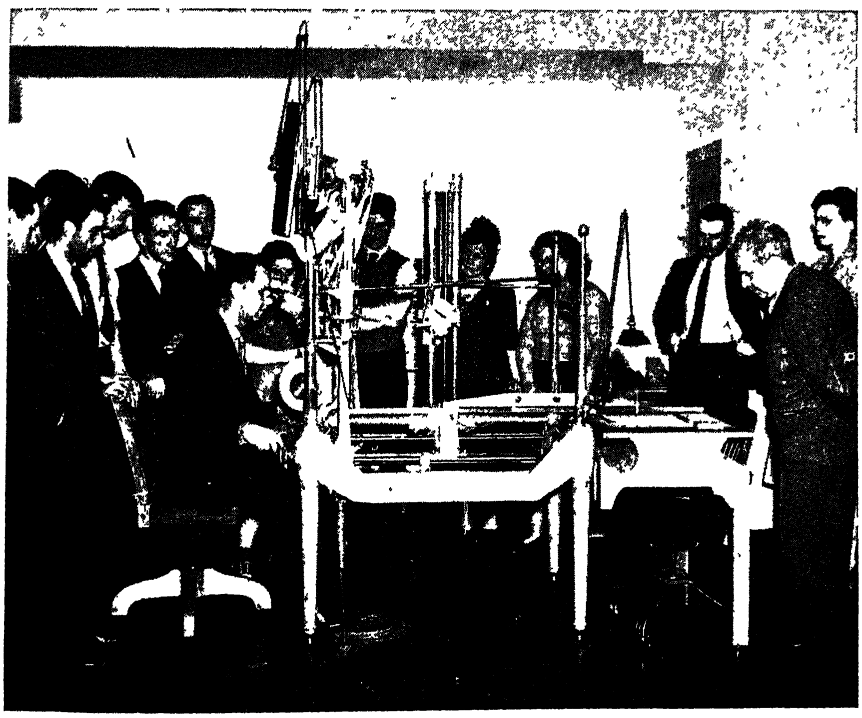
Several inspection tours (see Fig. 9) of photogrammetric laboratories should be arranged in order that the students see practical applications of the various instruments and methods discussed in class lectures.

Problem G· Contouring of vertical photographs. Contours are to be drawn on the same vertical photographs as were used for the radial plot. A stereocomparator is to be used if available. In case this instrument cannot be obtained, it is still possible to instruct students in the theory and practice of parallax measurements with no more equipment than a twelve-inch scale (preferably one with 0.01″ subdivisions). In regard to contour accuracy, it is not likely that the lack of precision of this method will be much greater than that caused by tilt or paper distortion in stereocomparator work. However, the use of a scale cannot compete with a stereocomparator for the determination of heights of relatively small objects such as buildings, trees, etc The following procedure is suggested for using a scale: (a) with the aid of a straight edge, line up the principal point bases of two overlapping photographs, separating corresponding points any convenient distance but slightly less than the length of the scale and tape in position; (b) measure the distance on the photograph between corresponding points of known elevation (at least three well distributed points are desired in the overlap region); (c) plot a graph with elevations on vertical axis and measured distances on horizontal axis (Note: The difference in distance between any two pairs of corresponding points is the difference in parallax of these points. However, it is not necessary to make this subtraction if the graph is plotted as mentioned above); (d) measure the distance between any pair of corresponding points whose elevation is desired, enter this distance on the graph and read off the

[15] See Chapter VIII Also: Eardley, A. J., "Aerial Photographs—Their Interpretation and Use" (1942), Harper and Brothers, New York, $2.75 War Department Field Manual, "Military Intelligence—Role of Aerial Photography," FM 30-21 (1940)

[16] See Chapter XI. Also: Talley, B. B , "Engineering Applications of Aerial and Terrestrial Photogrammetry," pages 484-551. Von Gruber, O , "Photogrammetry," (1932 English translation), pages 276-375. War Department Technical Manual, "Multiplex Mapping Equipment," TM 5-244 (1943).

(a)



(b)

FIG. 9. Class inspection tour of Clarendon, Virginia, office of the U. S. Geological Survey. (a) Multiplex equipment. (b) The Aerocartograph.

corresponding elevation; (e) contours may be sketched in after the elevation of a sufficient number of critical points have been determined. In order to secure a better representation of the relief, it is very desirable to use a stereoscope when selecting the critical points and also when sketching the contours. The contours as sketched on the photographs represent a perspective view of the terrain. Their true plan position can be found as follows: (a) Locate by intersection, on the previously run radial plot, the plan position of the critical points (spot heights) on the photographs, (b) adjust the contours to these points in the same manner as was previously used in tracing off detail such as crooked roads or streams.

12. MAPPING FROM OBLIQUE PHOTOGRAPHS[17]

(4 hours lecture and 10 hours laboratory) Uses, geometrical relationships, graphical determination of horizontal and vertical angles, perspective grids, and tri-metrogen mapping. Instrumental methods including the Wilson photo-alidade, the Miller single eyepiece plotter, and others

Problem H· Construction of perspective grid, using a high oblique photograph of flat terrain. Using this grid, transfer the detail from the photograph to the plotting sheet.

Problem I: Graphical determination of horizontal and vertical angles from oblique photographs. An instrument of recent development for obtaining horizontal angles from oblique photographs is the "Rectoblique Plotter" developed by J. G. Lewis of the U. S. Geological Survey. This plotter mechanically determines horizontal angles to various points on an oblique photograph and records them graphically on a paper templet.

13. FURTHER APPLICATIONS OF PHOTOGRAMMETRY[18]

(4 hours lecture) Geology and mining, soil surveys, forestry, route planning, water supply and sanitation, tax studies, city and regional planning, etc.

14. EXAMINATION AND DISCUSSION PERIODS

(12 hours) It is suggested that three 2-hour examinations be given during the course. Each examination should be followed by a 2-hour discussion period. The writer has found that the students benefit considerably from such discussions.

Given below are a number of suggested questions and problems on various phases of photogrammetry. Other problem lists can be found elsewhere in this manual.

1 Using a camera with a 100.00 mm. focal length, at what height in feet should it be flown over flat terrain in order to obtain vertical photographs with a scale of 1:48, 000. Ans: 15,748 feet.
2. On a vertical photograph of flat terrain, the distance between two points is scaled at 5.28 inches. On a map (whose scale is 4 inches to the mile) the distance between the same two points is 4 64 inches. Compute the scale of the photograph. Ans: 1:13,920.
3. A camera with a focal length of 6.54 inches is to be used to obtain vertical photographs of an area whose average elevation is 900 feet above sea level. The desired scale of the photographs being 1·48,000. (a) At what altitude above sea level should the plane be flown? Ans: 27,060 feet. (b) If the negative size is 9×9 inches, how

[17] See Chapter XIII.

[18] See Chapter XV. Also: Smith, H. T. U., "Aerial Photographs and Their Applications," (1943), D. Appleton-Century Co., New York (this book gives emphasis to geologic applications).

many square miles does a single exposure cover? Ans· 46.5. (c) If the forward lap is 60 per cent, compute the spacing of photograph centers in line of flight. Ans: 2.73 miles. (d) If the plane speed is 140 miles per hour, compute the exposure interval in seconds Ans: 70 sec. (e) If the side lap is 30 per cent, compute the flight spacing. Ans: 4 77 miles (f) What will be the scale of the photographs for an area whose elevation is 653 feet? Ans 1:48,453. (g) What will be the scale of the photographs for an area whose elevation is 1132 feet? Ans. 1:47,574.

4. Given the following conditions· The scale of a vertical aerial photograph is 1:6,000 applies to valley floor; the focal length of the lens used is 8 inches, the image of the top of a hill appears on the photograph 3.98 inches from the principal point; the height of the hill is 150 feet above the floor of the surrounding valley. What is the amount and direction of the relief displacement with respect (a) to the valley floor, (b) to sea level, assuming that the valley floor is 250 feet below sea level (Death Valley). Ans (a) 0.149 in. outward, (b) 0.103 in inward.
5. From what point on a tilted photograph do: (a) relief displacements radiate, (b) tilt displacements radiate; (c) both tilt and relief displacements radiate?
6. An air camera whose focal length is (F), has been tilted (T) degrees from the vertical. What is the distance on the resulting photograph from principal point to isocenter, and from principal point to nadir point. Ans: $F \tan T/2$, $F \tan T$.
7. Draw a sketch showing the relative position of the following items on a tilted photograph; fiducial marks, principal point, principal line, nadir point, isocenter, axis of tilt, upper side, lower side Select one point on the upper and one point on the lower side and indicate in what direction each point must be displaced to obtain its tilt-free position.
8. Define the following lens terms; distortion, curvature of field, resolving power, f-number.
9. It is desired to compile a map of an area on a scale of 600 feet per inch. Available aerial negatives of the area were made with a 6 26 inch focal length aerial camera flown at a height of 10,000 feet. (a) At what magnification or ratio will an enlarging camera have to be set in order to obtain photographic prints at the desired compilation scale? Ans: 2.662. (b) If the focal length of the enlarging camera lens is 10 inches, what will the object and image distances be for this magnification? Ans: $O=13.76$ in., $I=36.62$ in.
10. Explain the meaning of the following terms used in photogrammetry. (a) equivalent focal length; (b) calibrated focal length; (c) principal distance.
11. What special requirements are desired in a photographic airplane to be used for: (a) military reconnaissance, (b) large scale topographic mapping?
12. An aerial camera with a focal length of 8 inches is flown in an airplane traveling 150 miles per hour at a height of 5000 feet. What is the slowest allowable shutter speed if the image movement on the film during exposure is not to exceed 0.002 inch? The camera is equipped with a between-the-lens shutter. The 0.002 inch value above was selected so as to utilize the average resolving power of the lens. Ans: 1/176 sec.
13. Given the following conditions: Focal length of aerial camera is 10 inches, negative size is 9 ×9 inches; focal length of ground glass type view finder is 8 inches. Compute the spacing of exposure interval lines on the view finder for a forward lap of 60 per cent. Ans: 2.88 in.
14. Explain the general construction of a continuous-strip aerial camera. What possible uses does it have in military photography?
15. Discuss the variation in illumination on the film of a wide-angle single lens aerial camera (including necessary mathematical relationships). What difficulty does this variation cause?
16. Define the following aerial camera terms; crab, maximum angular field, view finder, intervalometer.

17. In photography it is customary to divide the visual spectrum into three general regions. What is the approximate color and wave length limits of each region? What terms are used for the regions at either end of the visual spectrum? Answers to be shown on a sketch Also indicate the range of sensitivity of the following types of film; orthochromatic, infra-red, colorblind, panchromatic.
18. What film and filter combination is generally used for taking aerial photographs through the average atmospheric haze By haze we do not mean fog or smoke. Explain reasons for selecting this combination Assume plane flies at an altitude of about 15,000 feet.
19. Discuss the general theory involved in film development and fixation
20. List the relative advantages of simple lens and mirror stereoscopes.
21 Discuss the anaglyphic principle as applied to projection apparatus (such as the multiplex)
22. For practical purposes a straight line relationship is said to exist between difference in elevation and difference in parallax as measured on a pair of overlapping vertical photographs With the aid of a sketch, explain why this relationship is not absolutely correct. Also indicate on the sketch how a tilted photograph would affect parallax measurements
23 Discuss a method of adjusting a pair of overlapping vertical photographs for use in a stereocomparator.
24. Explain the operation procedure for a simple type of stereocomparator
25. A camera with a focal length of 6 000 inches is flown at an altitude of 18,000 feet above a rugged seacoast. The average length of the air base as measured on two overlapping vertical photographs was found to be 2.700 inches The difference in parallax between a point (A) at sea level, and another point (B) on top of a hill was found by measurement to be 0.192 inches Compute the elevation of point (B). Assume that the principal points of both photographs are at sea level Ans: 1195 feet.
26. Discuss the chief characteristics of the following map projections; Polyconic, Lambert conformal conic, Mercator, transverse Mercator.
27 What are the advantages of using a rectangular coordinate system instead of a map projection?
28. In regards to ground control, what order of accuracy is required for maps compiled by photogrammetric methods?
29. What special precautions need be taken in establishing ground control for a photogrammetric mapping project?
30. Discuss the construction and operation of a prismatic astrolabe.
31. Discuss the amount and distribution of horizontal ground control needed for a radial plot.
32. List relative merits of the different types of templates used in laying down a radial plot.
33. How does tilt affect the accuracy of a radial plot? What methods are used to overcome these difficulties?
34. If the principal point of a photograph used in a radial plot falls in a body of water, describe a method of transferring this point to adjacent overlapping photographs.
35. State the principal steps necessary in the preparation of photographs preliminary to making a radial plot
36. List in proper sequence about 10 major steps used in laying down a controlled aerial photographic mosaic. Assume that you start with the original aerial negatives and finish with a photographic print of the completed mosaic.
37. What difference in procedure is necessary in laying down a controlled mosaic in hilly terrain as compared with flat terrain?
38. What difficulty is caused by using tilted photographs in laying down a controlled mosaic? How would you remedy this difficulty?

39. The principal distance of a multiplex diapositive is 30 mm. The focal length of the projector lens is 27.7 mm. (a) Compute the distance from the projector lens to the optimum image plane. Ans: 361 mm (b) Practically, the useable image field has a depth of about 150 mm What relationship does this depth have to the image distance computed in part (a)?
40. Each projector of the multiplex equipment has 6 motions. Name and describe each one.
41. In the multiplex equipment what is meant by; interior, relative, and absolute orientation?
42. Assuming that all orientation adjustments on the multiplex have been completed, describe procedure to be followed in drawing; (a) contours, (b) planimetry
43. Discuss the amount and distribution of ground control needed for topographic mapping with the multiplex equipment.
44. Assuming that all orientation adjustments have been completed for a pair of multiplex projectors, what happens when you· (a) increase their separation along the air base; (b) raise one projector upwards vertically, (c) rotate one projector about the vertical axis?
45. Given the following conditions concerning a high obliuqe aerial photograph focal length of air camera is 6 inches; flying height is 20,000 feet, distance from principal point to visible horizon on the photograph is 2 78 inches. Compute the true depression angle Ans· 27° 11′.
46. Explain a simple graphical method of obtaining true horizontal angles between image points on a high oblique aerial photograph of rugged terrain.
47. Explain the principles of the photo-alidade.
48. Explain the construction and use of an oblique sketchmaster.
49. Explain the theory of the Rectoblique Plotter.
50. List about 10 major steps in the compilation of small scale maps by the Tri-Metrogon method.

IV. General Textbooks

It is the opinion of the writer that this manual is the most complete book on photogrammetry published in the United States up to the present time. It should prove very suitable as a class text.

Information concerning other current books on photogrammetry is given below. Included in this list are several surveying books with one or more chapters on the subject. Each student should be required to have a copy of the selected class text. It is also desirable that one copy of each of the books listed be obtained for class reference use. A more complete list of books may be found in the reference list at end of manual.

Bagley, James W., "Aerophotography and Aerosurveying," (1941), 324 pages, $3.50, McGraw-Hill Book Co., Inc., New York. This book has been widely used as a class text. However, the following changes would probably increase its usefulness: Revision of terminology to agree with general usage; revision of chapter on photographic transformation, emphasizing the practical rectification of nominally vertical single lens photographs; inclusion of more information on stereoscopic plotting instruments; inclusion of problems at the end of each chapter.

Breed and Hosmer, "Higher Surveying" (1940), $3.50, John Wiley and Sons, Inc., New York. The section of the book devoted to photogrammetry (152 pages) is intended to form a part of a typical surveying course and does not contain enough material for a course entirely devoted to photogrammetry, unless supplemented by considerable outside material. Considering the limited space available, too much emphasis is placed on terrestrial photogrammetry.

Church, Earl, "Elements of Aerial Photogrammetry," (1944), 95 pages, $2.00, Syracuse University Press, Syracuse, N. Y. This book was prepared by Prof. Church for use as a text at Syracuse University Part I concerns fundamental mathematical theory, while Part II gives a brief description of practical mapping methods.

Davis and Foote, "Surveying" (1940), $5.00, McGraw-Hill Book Co., Inc., New York. Contains a chapter of 63 pages on "Photogrammetric Surveying" written by B. B. Talley. The chapter gives a brief but excellent discussion of practically all branches of the subject, and therefore should prove useful as an introduction to photogrammetry in a regular surveying course.

Hart, C. A., "Air Photography Applied to Surveying" (1943), 366 pages, $7.50, Longmans, Green and Co., New York. This book, written by an English author, gives a well balanced presentation of the entire field of photogrammetry with special emphasis on methods and equipment used in the British Empire. The book should prove valuable as a class reference.

Hotine, M., "Surveying from Air Photographs" (1931), 250 pages, printed in Great Britain but copies can be secured through G. E. Stechert and Co., New York. This book contains excellent basic information on all phases of photogrammetry except photography, optics, and types of aerial cameras. Due to the many new developments made in this field in the past twelve years some parts of the book are now obsolete.

McCurdy, P. G., "Manual of Aerial Photogrammetry" (1940), 102 pages, printed by Hydrographic Office, Navy Department. Describes the graphical methods used by this organization for mapping from vertical and oblique photographs. Of special interest is the manner in which the construction of a radial plot is presented, excellent diagrams and halftone reproductions of sample photographs being included to illustrate each step.

Raynor, W. H., "Advanced Surveying" (1941), $3.25, D. Van Nostrand Co. Inc., New York. Contains a chapter of 60 pages devoted to a general discussion of photogrammetry. Should prove useful as an introduction to the subject in a regular surveying course.

Sharp, H. O., "Photogrammetry" (1943), 129 pages ($8\frac{1}{2} \times 11$), $3.50, John Wiley and Sons, Inc., New York. This book was prepared by Prof. Sharp for use as a text at Rensselaer Polytechnic Institute. It is believed that an improvement could be made in this book by rearranging the subjects in a more logical manner. Also information on mosaics, photographic materials, and photographic laboratory technique might be included.

Talley, B. B., "Engineering Applications of Aerial and Terrestrial Photogrammetry," (1938), 612 pages, $10.00, Pitman Publishing Corp., New York. This was the first complete book on photogrammetry published in the United States. It is hoped that a revision will be made some time in the near future so as to include recent developments. The book is useful as a class reference, especially in the fields of aerial cameras and stereoscopic plotting instruments.

Von Gruber, O., "Photogrammetry" (1932 English Translation), 454 pages, $8.00, American Photographic Publishing Co., 428 Newbury St., Boston, Mass. The greater part of the book is devoted to a thorough discussion of the theory and operation of stereoscopic plotting instruments, and therefore serves as an excellent reference book in this field. Chapters are also included on the geometry of the photograph, aerial camera lenses, and shutters. Practically no information is given on graphical methods of compilation.

Whitmore, G. D., "Elements of Photogrammetry" (1941), 136 pages, $1.75, International Textbook Co., Scranton, Pa. An excellent book on elementary

photogrammetry. It is written in such a style that beginners will find no difficulty in understanding the principles presented.

War Department—The following technical and field manuals have been prepared by this organization for use as training texts for army personnel. These manuals may be obtained from the Superintendent of Documents, Washington, D. C., at the indicated prices.

"Advanced Map and Aerial Photograph Reading," FM 21–26 (1941), 190 pages, $0.20. Entirely devoted to reading of maps and photographs. It is a desirable text for beginners.

"Basic Photography," TM 1–219 (1941), 342 pages, $0.50. Contains desirable material for those interested in photographic laboratory procedure.

"Topographic Drafting," TM 5–230 (1940), 302 pages, $1.00. In addition to discussing topographic drafting, more than one-half of the text is devoted to map preparation from aerial photographs, with emphasis on drafting procedure. Considerable space is given to the operation of the stereocomparagraph, including parallax tables.

"Aerial Phototopography," TM 5–240 (1944), 111 pages, $0.30. Gives an excellent presentation of practical mapping methods in current use. Numerous illustrations and diagrams are included. If this book were supplemented by other material, it would be very satisfactory as a text in elementary photogrammetry.

AIDS IN TEACHING PHOTOGRAMMETRY

H. T. U. Smith

THE study of photogrammetry is partly a matter of solid geometry, and clear visualization of the space relations of aerial photos is essential. The attainment of that goal is greatly facilitated by the use of various mechanical devices and three-dimensional models which represent essential relations in idealized form. The purpose of this paper is to describe certain devices which have been found particularly helpful for teaching purposes in the aerial photographic laboratory of the University of Kansas.

Models to Show Effects of Parallax

The various effects of parallax in introducing distortion on both vertical and oblique photos may be demonstrated effectively with the following equipment.

(1) A regular cone on which contours are painted at definite vertical intervals (Fig. 1 A). The cone used by the writer is made of wood, is 4 inches high, and has sides sloping 35°. It was fashioned on a wood-turning lathe, and shallow grooves were cut for the contours. The completed cone was painted dull black, with the contours in white.

(2) A wooden hemisphere of the same height as the cone, and with contours shown in the same way.

(3) Small wooden posts made of dowel pins 3 inches high and 0 5 inch in diameter, and mounted on circular discs of clear celluloid about 2 inches in diameter. These also are painted black.

(4) A large sheet of drawing paper on which grid lines are ruled at right angles with spacing at any convenient interval.

By setting the above models at various places on the grid, and viewing from different points and angles, the effects of parallactic displacement in distorting straight lines, contours, and slopes is made clear. A particularly effective method of viewing is to use a camera of fairly large size having a ground glass back The camera is best mounted on a tripod with a tilting head, and is used with the shutter open. The effect of changing the position and inclination of the camera may then be observed on the ground glass screen. Some of the effects which may be shown in this way are pictured in Figure 1B–F. *B* represents a vertical photo from a point directly above the apex of the cone The asymmetrical distortion of the spacing of the contours is imperceptible, but the displacement of the upright posts toward the corners of the photo is evident, and the radial character of the displacement is made clear. *C* represents a similar photo of the hemisphere. Here the symmetrical or concentric displacement of the contours is obvious, the two lower contours being cut off from view. In *D*, the eccentric distortion of the contours is shown, together with the distortion of lines crossing the two figures. *E* and *F* represent oblique views, *F* having the larger angle of tilt. Here the contours are seen to be distorted in form as well as being displaced in position, and the convergence of the upright posts illustrates the principle that distortion is radial from the plumb point.

A Model to Show the Geometry of an Oblique Photo

To illustrate the descriptive geometry of an oblique or tilted photo, the model shown in Figure 2 is particularly helpful. Detailed plans for constructing such a model are given in Figure 3. Clear celluloid of about 0.1 inch thickness is the material; acetate celluloid is preferred. Guide lines are drawn or cut with a scriber, and parts are then cut out with a jeweler's hand saw. The various grid

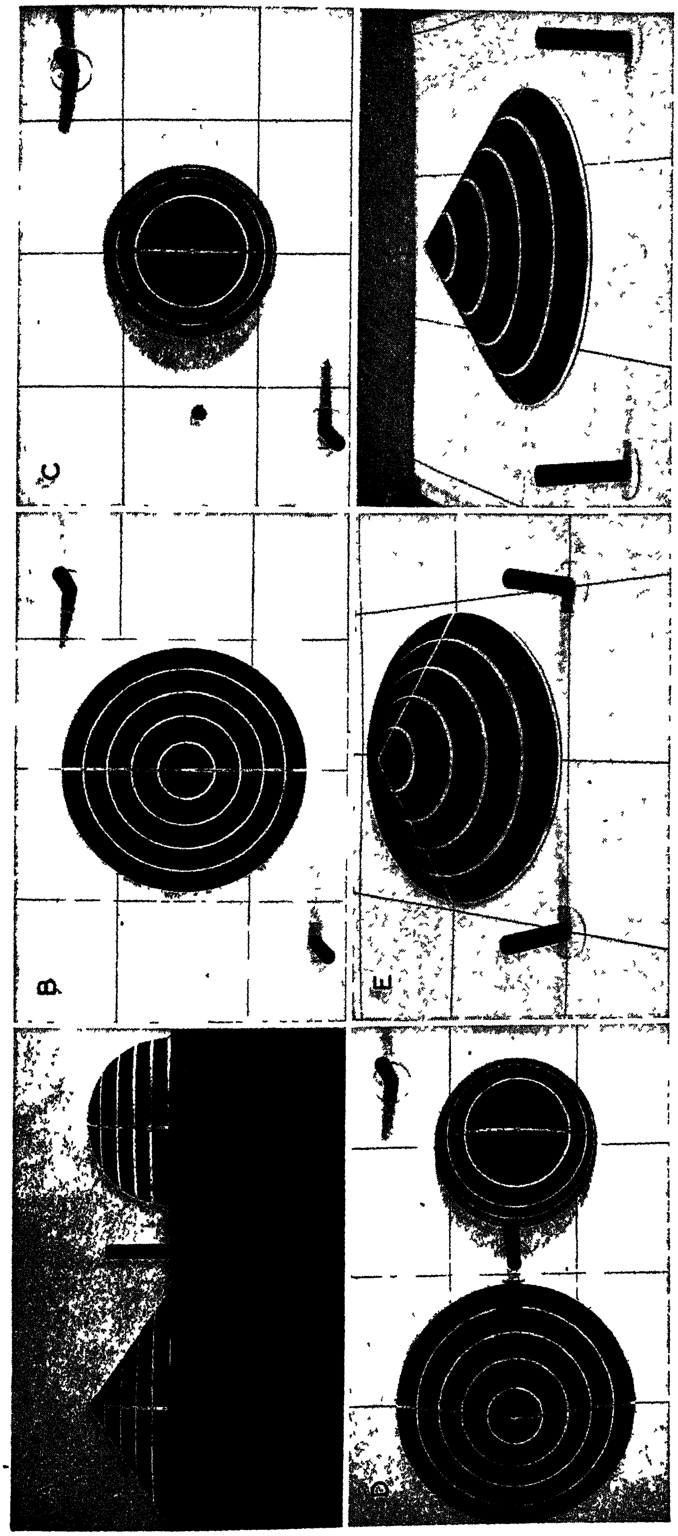


FIG. 1. Photos of models to illustrate parallactic displacement. A shows the models in profile; B, C, and D represent vertical photos, and E and F represent oblique photos.

lines and reference lines are similarly engraved with the scriber, and then are filled with india ink, using different colors to emphasize different sets of lines. The dimensions given in Figure 3 are those for the original model shown in Figure 2; for some purposes a much larger model might be desirable.

The horizontal plane is represented by *acef*, with *acml* showing the field of view of an oblique photo tilted 30°, and *jkef* indicating the field of view of a vertical photo taken from the same exposure station. A small hole is drilled at *x*, and a somewhat larger hole at *y*. At the latter point, a hollow upright post 0.45 inch high is affixed. The principal plane is represented by *pb'i'n*. The lines converging toward the perspective center at *p* have their origin along grid lines equally spaced in both directions from the isocenter, and lying in the horizontal

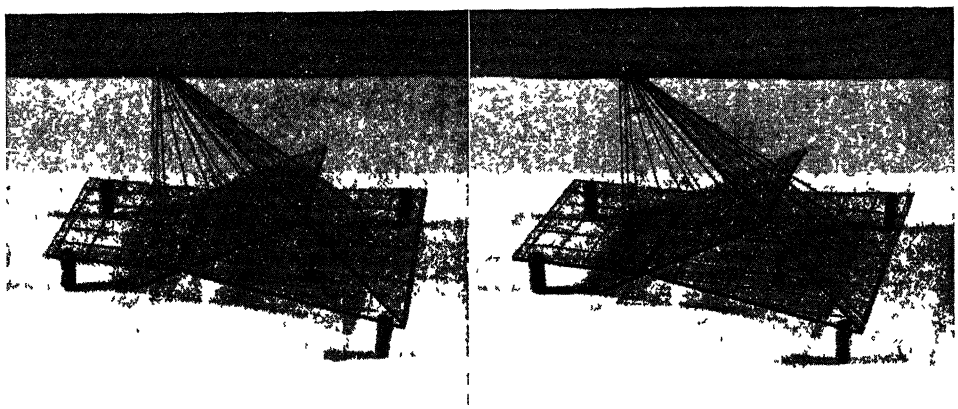


FIG. 2. Stereogram of celluloid model to illustrate the geometry of an oblique or tilted photo.

plane parallel to the axis of tilt. The intersections of these rays with the plane of the oblique photo determine the locus of corresponding grid lines on that photo. The plane of the oblique photo is shown by *qrst*, with its perspective grid corresponding to the rectangular grid on the horizonal plane. *C* marks the center point or principal point, *I* the isocenter, and *V* the plumb point, which falls just beyond the border of the photo itself. The triangular piece *p'd'c'* shows how points for controlling the converging set of grid lines on *qrst* are determined by the intersections of rays from the grid lines on *abml* on the plane of *qrst*.

In assembling the model, *qrst* is first fitted to *acef* by engaging the slots *id* and *g'i''*. The next step consists in fitting *pb'i'n* to the two pieces already assembled, with the slot *i'o* engaging the slot *o'u* and the triangular projection *i'nh'* fitting into the slot *ih*. All parts are now carefully aligned, and are then cemented in position, using a cement made by dissolving scraps of celluloid in acetone. Particular care is necessary at this step to check the alignment of the parts; when all is in good order, a line of sight from point *p* through any intersection of grid lines on the oblique plane should cut the corresponding intersection on the horizontal plane. When cementing is finished, a colored thread or string is passed from point *x* through point *x'* to *p*, and thence back through *y*, to the top of the hollow post at *y*. The string from *x* to *p* illustrates the projection of a given point from the datum plane or from the plane of the vertical photo on to the plane of the oblique photo. The string from the post at *y* to *p*

represents the projection of a point above the datum plane, and illustrates the combined effects of tilt and parallax.

The final step in constructing the model consists in fitting $p'd'c'$ to the part already assembled, and cementing it in position.

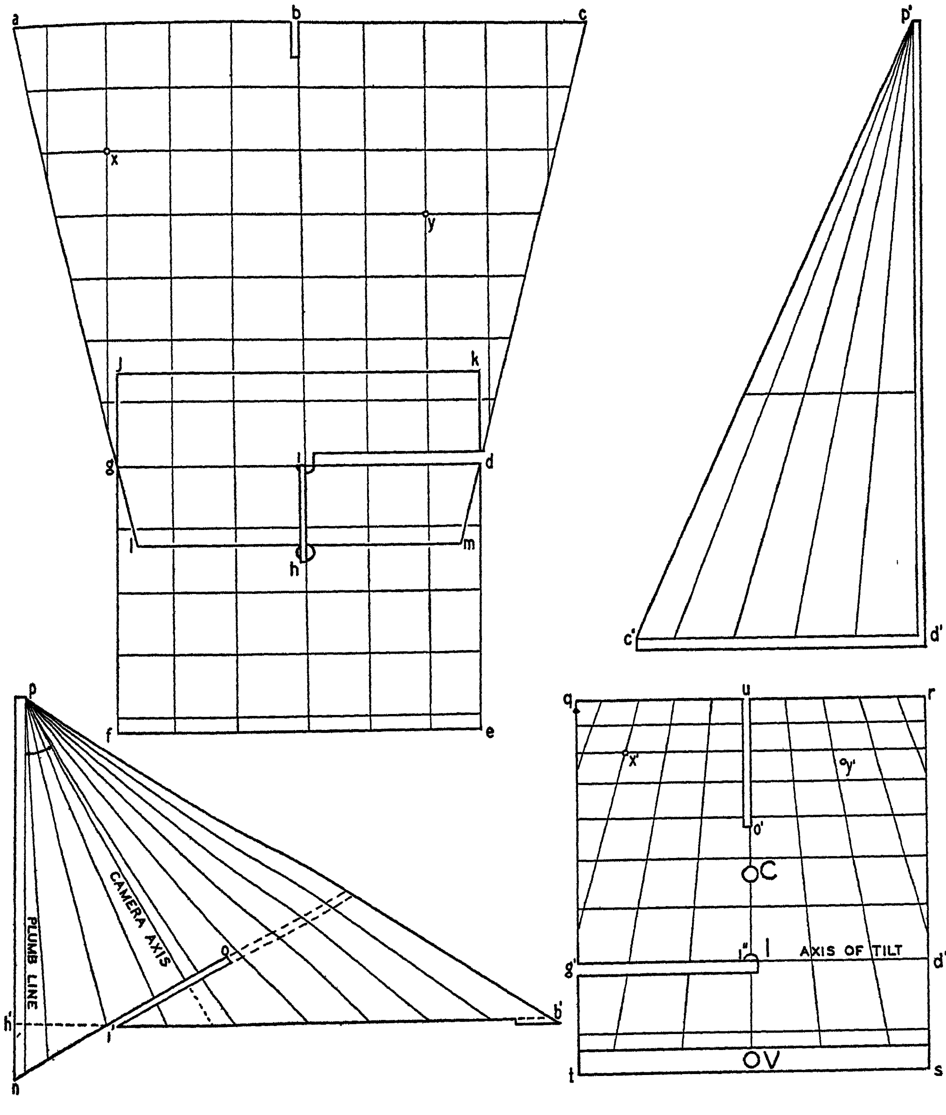


FIG. 3. Plans for the model shown in Fig. 2. Corresponding parts are indicated by the same letters.

Parallax Grids to Aid in Contour Sketching

The parallax grids shown in Figure 4 may be used both to illustrate the principles of stereoscopic parallax and as an aid in stereoscopic sketching of con-

tours. This instrument is patterned after the Barr & Stroud topographical stereoscope, but is designed to be used under a separate stereoscope, which may be of either the reflection or the refraction type. The grid plates themselves are prepared by first drafting a set of diagonal grid lines on drawing paper, exercising particular care to space the lines evenly and make them of uniform weight; this drawing should be several times larger than the final grid plates, and the spacing of the grid lines should be such as to reduce to about 6 or 7 mm. This drawing is then photographed, and two identical positives are made on lantern

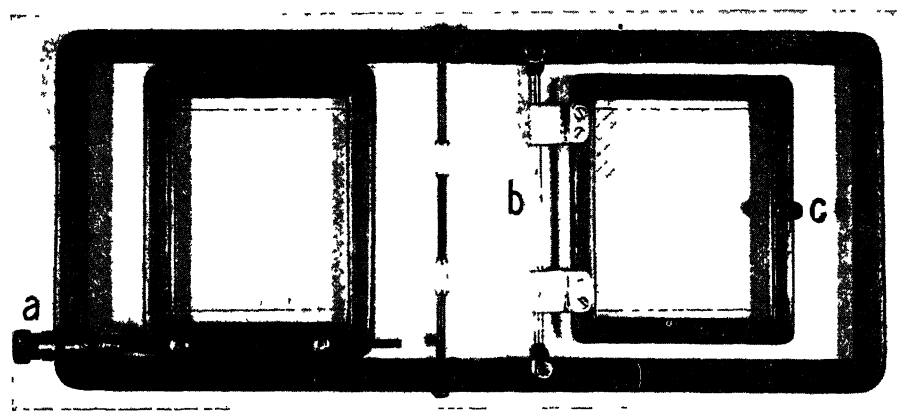


FIG. 4 Photograph of parallactic grids used for illustrating stereoscopic parallax and for contour sketching

slide plates (3.25×4 in.), or, if desired, on glass plates of larger size. The plates are then mounted, emulsion side down, in wooden frames, with the bottom of each plate flush with the bottom of its frame. The mounted plates are now mounted in a larger wooden frame, as shown in Figure 4. The left-hand grid is arranged so as to be adjustable by means of the knob at *a*, which is attached to a threaded rod. The right hand grid is arranged so as to be rotated around the rod *b* as an axis. It may be brought into stereoscopic correspondence by sliding along this rod, and by loosening the screws at the ends of the rod, allowing the latter to move freely back and forth in grooves. When correct alignment is attained, the ends of the rod are clamped in position by means of the screws.

When the grids are correctly adjusted to a stereo pair of photos, the grid lines are seen in stereoscopic fusion, and provide a reference frame for ascertaining all topographic points having the same amount of stereoscopic parallax, or, if the photos are without tilt, the same elevation. By changing the setting of the grid with the knob at *a*, topographic points at a different level may be found, and, if adequate ground control is available, contour lines or form lines may be sketched directly on the right-hand photo by swinging up the grid plate on that side by the knob at *c*. An additional feature which would extend the usefulness of the instrument would be the use of a vernier scale on the left-hand grid plate, in order to measure parallax difference.

CHAPTER XVII

NOMENCLATURE AND DEFINITIONS

DEFINITIONS OF TERMS USED IN PHOTOGRAMMETRIC SURVEYING AND MAPPING

Revised April, 1944

Prepared by the Committee on Nomenclature of the American Society of Photogrammetry

PREFACE

IT is important that every art and science have a clearly defined and accepted nomenclature if ambiguity and misunderstanding are to be avoided. This is particularly applicable to photogrammetry due to its rapid development in recent years. So much effort and thought have been given to this new subject that it is not strange that carelessness in the use of terms peculiar to the art has arisen or that new and ambiguous terms have sometimes been proposed for older ones which were entirely satisfactory.

It was for the purpose of avoiding this confusing situation that the American Society of Photogrammetry has had its Committee on Nomenclature engaged for several years in defining those terms that were acceptable for use. This study has now progressed to a point where it seems advisable that the results be made generally available as a first and important contribution toward the eventual standardization of photogrammetric terms.

In preparing these definitions it has been the primary purpose to select and define the more fundamental photogrammetric terms and eliminate many duplicate and superfluous ones. The approval of certain terms and the disapproval of others has been based on a careful study of general photogrammetric usage, usage in other fields of endeavor and the relative simplicity of the terms when used in oral and written discussion. In some cases it was necessary for the Committee to be somewhat arbitrary in selecting a particular term in preference to another but this has seemed preferable to the continued use of several terms where one would suffice.

In many cases definitions have been amplified to include somewhat detailed explanations of the meaning and use of the terms. This has been done deliberately with the thought that such explanation will be useful to the student or practitioner, particularly until such time as the terms used in photogrammetric textbooks are more nearly standardized.

In addition to the strictly photogrammetric terms, a number of words and terms in allied fields of endeavor such as Optics, Cartography, Surveying, etc., have been included. In general, these are terms most often encountered in the various operations of practical photogrammetry and for which definitions are not readily available. It is believed that the inclusion of these terms will make the report on nomenclature more useful to all who may have occasion to refer to it.

In many cases the definitions of related terms have been grouped with the thought that this arrangement will serve to emphasize the different shades of meaning as between closely related terms. This correlation has the effect of cross-referencing an appreciable number of terms but on the whole it is felt that the advantage gained by so grouping them more than outweighs the disadvantage accruing from cross-referencing.

These definitions represent a very considerable amount of research and study on the part of the Committee. It is realized that the list is not complete nor are the definitions final and absolute. The membership of the American Society of Photogrammetry is urged to familiarize itself with the definitions that have been

recommended and make it a point to use them as defined. It is very important that all comments on the list be forwarded to the Committee for its future guidance. These comments should include not only the terms listed but also all others that may have been overlooked or which may be introduced in the future.

The Committee on Nomenclature should be continued from year to year for the purpose of receiving these suggestions and modifying and expanding the list as seems necessary from time to time, as only by so doing can an eventual standardization of photogrammetric terms be attained.

The Committee is deeply indebted to the Committee on Definitions of Surveying and Mapping Terms of the Federal Board of Surveys and Maps and to the committee on photographic definitions and nomenclature of the American Standards Association, whose reports were placed at the disposal of the Chairman of the Committee. Acknowledgment of the definitions furnished by these committees is indicated in the text.

T. P. PENDLETON,
President, American Society of Photogrammetry

Definitions recommended to the Federal Board of Surveys and Maps by its Committee on Nomenclature have been marked with an asterisk (*)

Definitions prepared by the same Committee but not formally reported to the Board of Surveys and Maps have been marked with two asterisks (**)

Definitions submitted by Dr Walter Clark are indicated with a dagger (†)

Definitions being considered for approval by Subcommittee No 9 of Photographic Committee Z-38 of the American Standards Association, of which Dr. Walter Clark is chairman, are indicated with a double dagger (‡).

Figures referred to in the definitions will be found at the back.

Aberration—Optics: A defect of an optical image caused by the fact that essentially no lens system can form a perfect image. ***Astigmatism***—An aberration affecting the sharpness of images caused by the fact that for objects off the axis the rays passing through different meridians of the lens come to a focus in different planes. Thus an extra-axial point object is imaged as two mutually perpendicular short lines located at different distances from the lens. ***Lateral Chromatic Aberration***—An aberration which affects the sharpness of images off the axis because different colors produce different magnifications. ***Longitudinal Chromatic Aberration***—An aberration which affects the sharpness of all parts of the image because different colors come to a focus at different distances from the lens. ***Spherical Aberration***—Rays from various zones of a lens focus at different places along the axis. This results in an object point being imaged as a blur circle. ***Coma***—An aberration affecting the sharpness of images off the axis—caused by the fact that rays from an object point off the axis passing through a given circular zone of the lens come to a focus in a circle rather than a point, and the circles formed by rays through different zones are of different sizes and are located at different distances from the axis. The image of a point object is comet shaped. ***Curvature of Field***—An aberration affecting the longitudinal position of images off the axis in such a manner that objects in a plane perpendicular to the axis are imaged in a curved or dish-shaped surface. ***Distortion***—An aberration affecting the position of images off the axis, caused by the fact that objects at different angular distances from the axis undergo different magnifications; frequently referred to as ***Lens Distortion.***

Absolute Ceiling—Air Navigation: The height in the standard atmosphere at which the rate of climb of an aircraft is reduced to zero. The maximum height above sea level at which a given airplane can maintain horizontal flight. Not CEILING. See also SERVICE CEILING.

Absolute Fore and Aft Tilt—Not approved. See Y-TILT under TILT.

Absolute Humidity—See under HUMIDITY.

Absolute Lateral Tilt—Not approved. See X-TILT under TILT.

Absolute Orientation—See under EXTERIOR ORIENTATION.

Absolute Stereoscopic Parallax—See under PARALLAX.

Accommodation—The faculty of the human eye to adjust itself to give sharp images for different object distances.

*Accuracy***—Degree of conformity with a standard. Accuracy relates to the quality of a result, and is distinguished from precision which relates to the quality or refinement of the operation by which the result is obtained The standard of reference may be (1) an exact value, such as the sum of the three angles of a plane triangle is 180°, (2) a value of a conventional unit as defined by a physical representation thereof, such as the international meter, defined by the international prototype meter (bar) at the National Bureau of Standards; (3) a survey or map value determined by refined methods and deemed sufficiently near the ideal or true value to be held constant for the control of dependent operations, such as the adjusted elevation of a permanent bench mark. For example, the accuracy attained by field surveys is the product of the instructions to be followed in the execution of the work and the precision with which those instructions are followed.

Achromatic Lens—Refers to a lens which has been partly corrected for chromatic aberration Such a lens is customarily made to bring green and red light rays to approximately the same point focus. Also called an *Achromat.*

Aerial Camera—See under CAMERA.

Aerial Map—Not approved Maps are not classified by the method of obtaining the data. See SURVEY and MAP.

Aerial Mosaic—See MOSAIC.

Aerial Photogrammetry—See under PHOTOGRAMMETRY.

Aerial Photograph—A photograph made from an aircraft.

Aerial Photography—The taking of photographs from aircraft.

Aerial Photographic Mosaic—Not approved. See MOSAIC.

Aerial Survey—See under SURVEY.

Aerial Triangulation—Not approved. See RADIAL TRIANGULATION.

Aeronautical Chart—A map especially designed for the aviator, on which, in addition to essential topography, are shown obstructions, aids to navigation, and other information to assist the aviator in navigating

Air Base—*Photogrammetry.* The line joining two air stations or the length of this line. See AIR STATION under CAMERA STATION.

Air Base Inclination—Not approved. See BASE TILT under BASAL ORIENTATION.

Air Coordinates—See under COORDINATES

Air Speed—The velocity of an aircraft relative to the surrounding atmosphere.

Air Station—See under CAMERA STATION.

Altimeter—*Air Navigation:* An instrument which utilizes relative pressure of the atmosphere to indicate the vertical distance above a specified datum plane.

Altitude—*Aerial Photography:* Vertical distance above the datum, usually mean sea level, to an object or point in space

Amici Prism—A prism which deviates the rays of light through 90° and, because of its shape, inverts the image. An amici prism is often called a ROOF PRISM. See also definition of ROOF PRISM.

Anaglyph—A picture printed or projected in complementary colors and combining the two images of a stereoscopic pair, and which gives a stereoscopic image when viewed

through spectacles having filters of corresponding complementary colors See also VECTOGRAPH.

Analytic Nadir Point Triangulation—See under RADIAL TRIANGULATION

Analytic Radial Triangulation—See under RADIAL TRIANGULATION

Anastigmatic Lens—A lens which has been corrected for astigmatism and curvature of field See ABERRATIONS

Aneroid Barometer—See BAROMETER.

Angle of Convergence—See under PARALLAX.

Angle of Incidence—*Optics* The angle measured from the normal at which a ray of light strikes a surface

Angle of Reflection—*Optics:* The angle measured from the normal at which a reflected ray of light leaves a surface

Angle of Refraction—See under REFRACTION.

Ångstrom Unit (Abbreviated A. U. or Å)—A standard of measurement of wave length of light equal to one ten-millionth of a millimeter, for example, the visible spectrum is said to extend from about 4000 Å to 7000 Å (0.0004 to 0 0007 mm)

Angular Calibration Constants—In a multiple lens camera, the interior orientation of the plate perpendiculars of the several lens-camera units to a common origin of direction.

Angular Magnification—See under MAGNIFICATION

Angular Parallax—See under PARALLAX.

Angular Parallax Difference—See under PARALLAX

Ansco Color—*Photography* A color process in which the film and paper are each coated with three layers which respond respectively to blue, green, and red light The dye-forming couplers are incorporated in the layers, which are developed directly to give dye images of colors complementary to the spectral regions to which the layers respond. Colorless dye-forming substances in the emulsion combine with reaction products of a single developer to form the dye images in the three emulsion layers.

Antihalation Coating†—*Photography*· A light-absorbing coating applied to the back side of the support of a film or plate, or between the emulsion and the support, to suppress halation. See HALATION.

Aperture—See RELATIVE APERTURE and APERTURE STOP.

Aperture Stop—*Optics:* The physical element (such as a stop, diaphragm or a lens periphery) of an optical system which limits the size of the pencil of rays traversing the system. The adjustment of the size of the aperture stop of a given system regulates the brightness of the image without having any necessary effect upon the size of the area covered. ***Field Stop***—The physical element (such as a stop, diaphragm, or lens periphery) of an optical system which limits the field of view covered by the system. ***Entrance Pupil***—The image of the aperture stop formed by all the lens elements on the object side of the aperture stop. ***Exit Pupil***—The image of the aperture stop formed by all the lens elements on the image side of the aperture stop. ***Entrance Window***—The image of the field stop formed by all the lens elements on the object side of the field stop. ***Exit Window***—The image of the field stop formed by all the lens elements on the image side of the field stop. ***Field of View***—1. *Optics*· The angular coverage of a lens system. It is equal to the angle subtended by the diameter of the entrance window at the center of the entrance pupil. 2. *Photogrammetry*· The angular coverage of a photograph is equal to twice the angle whose tangent is one-half the length of the diagonal (or diameter) of the photograph giving satisfactory definition, divided by the calibrated focal length of the camera.

Apochromatic Lens—A lens which has been corrected for chromatic aberration for three colors.

Apparent Horizon—See HORIZON.

Aspherical Lens—A lens in which the surfaces depart from a true spherical shape For instance condenser lenses are sometimes ground with a parabolic surface, thus making possible a practical elimination of spherical aberration near the outer zones. With such a condenser it is possible to concentrate the light within a very small aperture. One type of multiplex projector is equipped with a condenser system of this nature.

Astigmatism—See under ABERRATION.

Astronomic Station—A point on the earth at which observations are made on heavenly bodies to determine latitude, longitude, or azimuth

Asymmetrical Lens—A lens which is not symmetrical. See SYMMETRICAL LENS.

Autopositive Film and Paper†—*Photography*. A material which gives a positive from a positive transparency (or a negative from a negative) by direct development.

Axis—See OPTICAL AXIS.

Axis of Tilt—See under PRINCIPAL PLANE—*Photogrammetry*.

Azimuth Line—*Photogrammetry:* A radial line from the principal point, isocenter, or nadir point of a photograph which represents the direction to a similar point on an adjacent photograph, used extensively in radial triangulation

Back Focal Length—See EQUIVALENT FOCAL LENGTH.

Background—*Photography*. That part of the landscape imaged in a horizontal or oblique photograph which is more distant from the camera station

Barometer—An instrument for measuring the pressure of the atmosphere. ***Aneroid Barometer***—A hollow corrugated metal box from which the air has been partially exhausted and the walls of which are so thin that it will change its form when the air pressure changes. Most aneroids have two scales, one graduated to correspond to height of a mercury column and the other to feet of altitude. ***Mercury Barometer***—Basically a vertical glass tube containing mercury; the upper end of the tube is closed so as to form a vacuum above the mercury and the lower end rests in a suitable cup, the column of mercury being sustained by the pressure of air against the mercury in the cup. A suitable scale measures the height of the mercury column

Barometric Elevation—An elevation which has been determined with a barometer. See also ELEVATION

Basal Coplane—See under COPLANAR.

Basal Orientation—The establishment of the position of both ends of an air base with respect to a ground system of coordinates. In all, six elements are required. These are essentially the three dimensional coordinates of each end of the base. In practice, however, it is also convenient to express these elements in one of two alternative ways. (1) The ground rectangular coordinates of one end of the base and the difference between the ground rectangular coordinates of the ends of the base (2) The ground rectangular coordinates of one end of the base, the length of the base and two elements of direction such as Base Direction and Base Tilt. ***Base Direction***—The direction of the vertical plane containing the air base which might be expressed as a bearing or azimuth. ***Base Tilt***—The inclination of the air base to the horizontal.

Basal Plane—See under EPIPOLES.

Base Direction—See under BASAL ORIENTATION.

Base Map—See under MAP.

Base Tilt—See under BASAL ORIENTATION.

Beam of Light—See under RAY OF LIGHT.

Between the Lens Shutter—See under SHUTTER.

Binocular Vision—See under STEREOSCOPY.

Bridging—*Photogrammetry* The extension and adjustment of photogrammetric surveys between bands of ground control.

Brightness Scale—*Photography:* The ratio of the brightness of the brightest highlight to the deepest shadow in the actual terrain as measured from the camera station.

Cadastral Map—See under MAP.

Cadastral Survey*—A survey relating to land boundaries and subdivisions, made to create units suitable for transfer or to define the limitations of title. Derived from "cadastre," meaning register of the real property of a political subdivision with details of area, ownership, and value. The term cadastral survey is now used to designate the surveys of the public lands of the United States, including retracement surveys for the identification, and resurveys for the restoration of property lines; the term may also be applied properly to corresponding surveys outside the public lands, although such surveys are usually termed land surveys through preference.

Calibrated Focal Length—See under EQUIVALENT FOCAL LENGTH

Calibration—The act or process of determining certain specific measurements in a camera or other instrument or device for comparison with a standard ***Camera Calibration***—*Photogrammetry.* The determination of the calibrated focal length, the location of the principal point with respect to the fiducial marks, and the lens distortion effective in the focal plane of the camera and referred to the particular calibrated focal length In a multiple lens camera, the calibration also includes the determination of the angles between the component perspective units. The setting of the fiducial marks and the positioning of the lens are ordinarily considered as "adjustments" although they are sometimes performed during the calibration process. Unless a camera is specifically referred to, distortion and other optical characteristics of a lens are determined in a focal plane located at the equivalent focal length and the process is termed "lens calibration " See also ANGULAR CALIBRATION CONSTANTS and COLLIMATE.

Calibration Constants—*Photogrammetry:* Results obtained by calibration which give the relation of the principal point to the fiducial marks of a camera and the calibrated focal length of the lens-camera unit.

Calibration Data—A record of the measurements obtained by calibration See CALIBRATION.

Calibration Plate—See CALIBRATION NEGATIVE which is preferred

Calibration Templet—*Photogrammetry:* A templet of glass, celluloid, or metal made in accordance with the calibration constants to show the relation of the principal point of a camera to the fiducial marks. Used for the rapid and accurate marking of principal points on a series of photographs. Also in the case of a multiple lens camera a templet prepared from the calibration data and used in assembling the individual photographs into one composite photograph.

Camera—A chamber or box in which the images of exterior objects are projected upon a sensitized surface. ***Aerial Camera***—A camera specially designed for use in aircraft. The prefix aerial is not essential where the context clearly indicates the use of an aerial camera rather than a ground camera. ***Ground Camera***—A camera designed for use on the ground. See also PHOTOTHEODOLITE. ***Surveying Camera***—A camera specially designed for the production of photographs to be used in surveying. The prefix surveying indicates that the camera is equipped with mechanism to maintain and to indicate the interior orientation of the photographs with sufficient accuracy

for surveying purposes. A surveying camera may be an aerial surveying camera or ground surveying camera *Mapping Camera*—The term Surveying Camera is preferred *Photogrammetric Camera*—A general term applicable to any camera used in any of the several branches of photogrammetry *Precision Camera*—A relative term used to designate any camera capable of giving results of a definite high order of accuracy. *Single Lens Camera*—A camera having only one lens. *Multiple Lens Camera*—A camera with two or more lenses, the axes of the lenses being systematically arranged at a fixed angle in order to cover a wide field by simultaneous exposures in all chambers. In most such cameras the oblique lenses are arranged symmetrically around a central lens. In some multiple lens cameras the axes of the lenses are all vertical and images are projected onto a single film, the obliquity of the side photographs being obtained by mirrors or prisms in front of the side lenses Several types of multiple lens cameras are the three, four, five, seven, and nine lens aerial cameras. Prints are made from the oblique negatives of a multiple lens camera by projection in a transforming printer which projects the oblique images into a common plane. The transforming printer is usually designed specially for the particular multiple lens camera. The transformed photographs are assembled to form one composite photograph equivalent to a photograph taken with a single wide angle lens. In some cases the transformation and assembly of the separate photographs are both performed by the transforming printer. *Horizon Camera*—*Aerial Photography* A camera used in conjunction with an aerial surveying camera in vertical photography to photograph the horizon simultaneously with the vertical photographs. The horizon photographs indicate the tilts of the vertical photographs.

Camera Axis—In a single lens camera, the photograph perpendicular In a multiple lens camera the photograph perpendicular of the central perspective unit or the photograph perpendicular of the transformed photograph.

Camera Calibration—See under CALIBRATION.

Camera Station—*Photogrammetry*: The point in space, in the air or on the ground, occupied by the camera lens at the moment of exposure Also called the *Exposure Station.* In aerial photogrammetry the camera station is called the *Air Station.*

Cant and Torque—*Photogrammetry:* Elements of tilt in the mechanical relative orientation of a pair of photographs which are the declination and polar bearing respectively of the photograph perpendicular of one photograph in a system of spherical coordinates in which the polar axis is the air base. The reference basal or meridional plane can be arbitrarily chosen but is usually vertical In a measuring stereoscope based on the Fourcade principle of orientation the *Principal Torque*, sometimes called the *Lateral Level Element,* is the torque of the photograph perpendicular of the left hand photograph and the *Differential Torque* is the difference of torque between the left and right photographs See Fig. 2 and also BASAL ORIENTATION.

Cantilever Strip—A term used principally in Great Britain and Canada for a strip of vertical air photographs with the usual forward lap extending over a known area into an area of no control, the plot of the portion of the strip taken over the known area serving to fix the scale and aximuth of the plot of the cantilever portion, or the portion over the area of no control

Cardan Link—A universal joint An optical cardan link is a device for universal scanning about a point.

Cartographer—One who practices the science or the art of cartography.

Cartographic—(Not cartographical.) Of or pertaining to cartography.

Cartography—The science and art of expressing graphically, by means of maps and charts, the known physical features of the earth's surface, and often including the works of man and his varied activities; specifically, cartography is the art of map

construction and the science upon which it rests. It combines the achievements of the astronomer and mathematician with those of the explorer and the surveyor in presenting a picture of the physical characteristics of the earth's surface. It invariably includes assembly, evaluation, selection and rejection of data.

Celluloid Templet—See under TEMPLET.

Characteristic Curve†—*Photography.* The curve showing the relationship between exposure and resulting density in a photographic image. It is usually plotted as the density against the log exposure in candle-meter-seconds. Called also the "H. and D. curve" and the "sensitometric curve." ***Density***—A measure of the degree of blackening of an exposed film, plate or paper after development, or of the direct image in the case of a print-out material. It is defined strictly as the logarithm of the optical opacity, where the ***Opacity*** is the ratio of the incident to the transmitted (or reflected) light. It varies with the use of scattered or specular light. ***Gamma***—The tangent of the angle which the straight-line part of the characteristic curve makes with the log exposure axis. It indicates the slope of the straight line part, and is a measure of the extent of development and the contrast of the photographic material. ***Speed***—(film, plate or paper). The response or sensitivity of the material to light, often expressed numerically according to one of several systems, e g , H. and D , D I N , Scheiner, and A.S.A. gradient speed See also RELATIVE APERTURE ***Gradient***—The slope of the characteristic curve at any point. ***Gradient Speed***—The speed of a photographic material determined on the basis of the exposure corresponding to a particular gradient of the characteristic curve. See also CONTRAST, EXPOSURE, EXPOSURE SCALE, BRIGHTNESS SCALE

Chart—See NAUTICAL CHART and AERONAUTICAL CHART.

Chromatic Aberration—See LATERAL and LONGITUDINAL CHROMATIC ABERRATION under ABERRATION.

Circle of Confusion—*Optics*· The circular image of a distant point object as formed in a focal plane by a lens. A distant point object (e.g., a star) is imaged in a focal plane of a lens as a circle of finite size which may be caused by (1) the focal plane not being placed at the point of sharpest focus, (2) the effect of certain aberrations, (3) diffraction at the lens, (4) the grain of a photographic emulsion, or (5) poor workmanship in the manufacture of the lens, etc.

Clinometer—A simple instrument used for measuring the degree or per cent of slope.

Coated Lens—A lens whose air-glass surfaces have been coated with a thin transparent film of such index of refraction as to minimize the light loss by reflection. This reflection loss of uncoated lenses amounts to about 4 per cent per air-glass surface.

Collimate—*Physics and Astronomy:* To render parallel to a certain line or direction; to render parallel, as rays of light; to adjust the line of sight of an optical instrument so that it is in its proper position relative to the other parts of the instrument. See also COLLIMATOR. ***Collimate***—*Photogrammetry:* To adjust the fiducial marks of a camera so that they define the principal point. See also CALIBRATION

Collimating Axis—Not approved for FIDUCIAL AXIS which see.

Collimating Line—Not approved for FIDUCIAL AXIS which see.

Collimating Marks—Not approved for FIDUCIAL MARKS.

Collimator—An optical device for artificially creating a target at infinite distance (a beam of parallel rays of light) used in testing and adjusting certain optical instruments. It usually consists of a converging lens and a target (a system or arrangement of cross hairs) placed at the principal focus of the lens.

Coma—See under ABERRATION.

Common Scale Strip—A term used in Great Britain and Canada in graphical methods

of plotting aerial photography to define the plot of a strip of vertical aerial photographs whose scale is adopted as standard for compilation purposes Plots of adjacent parallel strips are then brought to this standard scale by means of the extreme lateral control points in the common overlap.

Comparator—An optical instrument, usually precise, for measuring rectangular coordinates of points on any plane surface, such as a photographic plate.

Complementary Colors—*Optics:* Two colors are said to be complementary if when added together, such as by projection, they produce white light.

Composite Photograph—*Aerial Photography:* A photograph made by assembling the separate photographs made by each lens of a multiple-lens camera during the same simultaneous exposure into the equivalent of a photograph taken with a single wide angle lens. See also MULTIPLE LENS CAMERA under CAMERA.

Condenser—*Optics:* A lens or lens system so designed as to concentrate the illumination from a light source upon a limited area.

Conjugate Distance—For every position that an object may occupy with respect to a lens, there is a corresponding position for the image. The distances of object and image from the nodal points of the lens are called conjugate distances. A convenient formula for computing the conjugate distances "O" and "I" for use in photographic enlarging is:

$$O = +\frac{F}{M}; \; I = F + FM$$

in which

F = focal length of lens
M = enlarging ratio

Note: the total distance from object to image equals the sum of the two conjugate distances plus or minus (depending on lens design) a small distance called the nodal point separation.

Conjugate Focal Point—Not approved for the ISOCENTER, which see.

Conjugate Image Point—The images on two (or more) overlapping photographs of a single object point.

Conjugate Image Rays—Rays connecting each of a set of conjugate image points with its particular perspective center.

Constant Tilt—*Aerial Photography:* In vertical aerial photography, the condition where the photographs in a flight strip or section of a flight strip are tilted in nearly the same direction.

Contact Glass—See FOCAL PLANE PLATE under FOCAL PLANE.

Contact Plate—See under FOCAL PLANE.

Contact Print—See under PRINT.

Contour (line)—An imaginary line connecting the points on a land surface that have the same elevation; also the line representing this on a map or chart. ***Depression Contour***—A closed contour inside which the ground is at a lower elevation than outside.

Contour Map—See under MAP.

Contrast†—*Photography:* The actual difference in density between the highlights and the shadows on a negative or paper. Contrast is not concerned with the magnitude of density but only with the difference in densities Also the rating of a photographic material corresponding to the relative density difference which it exhibits. See also DENSITY under CHARACTERISTIC CURVE.

Control—A system of relatively accurate measurements to determine the distances and

directions or differences in elevation between points on the earth, as triangulation, traverse, or levels, and upon which depends a system of lesser accuracy. The accuracy of the control is usually described as first-order, second-order, third-order, or fourth-order. ***Horizontal Control***—Control which determines horizontal positions only, as with respect to parallels and meridians, or to other lines of reference ***Vertical Control***—Control which determines positions with respect to elevations only, specifically leveling. ***Geodetic Control***—Control which takes into account the size and shape of the earth. Geodetic control implies a reference spheroid representing the geoid and horizontal and vertical control datums. ***Ground Control***—In photogrammetry, control obtained by ground surveys as distinguished from control obtained by photogrammetric methods As for example, first, second, or third-order triangulation or traverse are used to control the photogrammetric plot which in turn establishes more intensive but less accurate control for the map detailing. ***Photogrammetric Control***—Control established by photogrammetric methods as distinguished from control established by ground methods. Also called ***Minor Control.***

Control Point—*Photogrammetry:* Any station in a horizontal and/or vertical control system that is identified on a photograph and used for correlating the data shown on that photograph.

Control Strip—*Aerial Photography:* A strip of aerial photographs taken to aid in planning and accomplishing later aerial photography, or to serve as control in assembling other strips.

Controlled Mosaic—See under MOSAIC.

Convergence—See ANGLE OF CONVERGENCE under PARALLAX.

Converging Lens—See POSITIVE LENS.

Convertible Lens—Usually refers to a photographic lens containing two or more elements which can be used individually or in combination. Seldom used in photogrammetric instruments.

Coordinates—*Surveying and Mapping*· Linear or angular quantities which designate the position which a point occupies in a given reference frame or system. Also used as a general term to designate the particular kind of reference frame or system, as Plane Rectangular Coordinates, Spherical Coordinates, etc. ***Plane-Rectangular Coordinates*** (also called simply ***Plane Coordinates***)—A system of coordinates in a horizontal plane, used to describe the positions of points with respect to an arbitrary origin by means of two distances perpendicular to each other. The two reference lines at right angles to each other passing through the origin are called the coordinate axes. The distances parallel with the true, or arbitrarily assigned, North-South axis are called the ordinates, the y coordinates, or the total latitudes. The distances parallel with the true, or arbitrarily assigned, East-West axis are called the abscissas, the x coordinates, or the total departures A plane-rectangular coordinate system is used in mapping areas of such limited extent that the errors introduced by substituting a plane for the curved surface of the earth will be within the required order of accuracy. In mapping, the North and East directions are positive and the South and West directions are negative In practice, therefore, in order to avoid the use of negative coordinates the origin of the system is usually chosen to be a point to the southwest of the area being mapped, or its coordinates instead of being zero are assigned large positive numbers. The great merit of a rectangular-coordinate system is that computations involving positions of points thereon may be performed by the use of plane trigonometry. Plane-rectangular coordinates may or may not be adjusted to a map projection. ***Grid Coordinates***—A plane rectangular coordinate system based on and mathematically adjusted to a map projection in order

that geographic positions (latitudes and longitudes) may be readily transformed into plane coordinates and the computations relating to them made by the ordinary methods of plane surveying. ***State Plane Coordinate Systems***—A series of Grid Coordinate Systems prepared by the U. S Coast and Geodetic Survey to cover the entire United States, there being a separate system for each State. Each State system consists of one or more zones The Grid Coordinates for each zone are based on and mathematically adjusted to a map projection. The Lambert Conformal Conic Projection with Two Standard Parallels is used for zones of limited North-South extent and predominant East-West extent The Transverse Mercator Projection is used for zones of predominant North-South extent and of limited East-West extent. ***Polar Coordinates***—A system of coordinates used to describe the position of a point in space with respect to an arbitrarily chosen origin by means of two directions and one distance, i e., the vectorial angles and radius vector Any plane containing the polar axis may be called a meridional plane, and the plane perpendicular to the polar axis containing the origin is called the equatorial plane or equator. As any point must lie on a meridional plane, one coordinate of a point in this system is the angle formed by the intersection of its meridional plane with the reference meridional plane. This is called the polar angle or polar bearing The second coordinate of a point is the angle in its meridional plane subtended at the origin between the line to the point and the polar axis This angle is called the polar distance and its complement, the angle between the line to the point and the equator, is the declination The third coordinate is the distance between the origin and the point (See Fig 4) ***Plane Polar Coordinates***—A system of polar coordinates in which the points all lie in one plane In the phraseology of analytical geometry the distance from the origin to the point is the radius vector and the polar distance is the vectorial angle. ***Spherical Coordinates***—A system of polar coordinates in which the origin is the center of a sphere and the points all lie on the surface of the sphere The polar axis of such a system cuts the sphere at its two poles. In Photogrammetry, spherical coordinates are useful in defining the relative orientation of perspective rays or axes and make possible the stating and solving in simple forms of many of the problems connected therewith. For example, as used in the determination of the exterior orientation of a single photograph, the origin is the air station and the polar axis is the vertical. The polar bearing is the horizontal bearing (azimuth) of the principal plane and the polar distance is the tilt. (See Fig 1.) In the determination of the relative orientation between pairs of photographs by the method originated by Fourcade, the polar axis of the coordinate system is the air base and the origin is one of the air stations. A meridional plane in this case is called a basal or epipolar plane and the reference meridional plane may be arbitrarily chosen but is usually the vertical. (See also definitions under Tilt and Fig. 2) ***Geographic Coordinates***—A system of spherical coordinates for describing the positions of points on the Earth. The declinations and polar bearings in this system are the latitudes and longitudes, respectively. ***Photograph Coordinates***—*Photogrammetry*. A system of coordinates either rectangular or polar to describe the position of a point on a photograph If a two dimensional system is used the origin is usually the principal point but it may be the nadir point, isocenter, one of the fiducial marks, or in high oblique photography the intersection of the horizon and principal line. The coordinate axes are usually either the fiducial axes, or the principal line and a photograph parallel. If a three dimensional system is used the origin is either the principal point or the perspective center ***Space Coordinates***—*Photogrammetry:* A three dimensional system of rectangular coordinates in which the x and y coordinates lie in a reference plane tangent to the earth at a selected point and the z coordinate is perpendicular to that plane. Used in the extension of horizontal and vertical control through a series of overlap-

ping vertical photographs from an initial point which is the point of tangency of the reference plane When these coordinates are corrected to allow for the curvature of the earth they cease theoretically to be true space coordinates because the x and y coordinates become distances along great circles at right angles to each other and the z coordinates are distances perpendicular to the vertical control datum. The use of the term space coordinates therefore should be strictly limited to a three dimensional rectangular coordinate system which has not been adjusted to the vertical and horizontal control datums ***Air Coordinates***—*Photogrammetry* The space coordinates of any point imaged on an overlapping pair of photographs which define its position with reference to the air base. They correspond in respect to the position of origin and direction of axes to a system of spherical coordinates in which an air base is the polar axis (See Fig 2.) Consequently one such system as suggested by Fourcade can be defined as follows.

Origin The left-hand air station.

X-axis The line of the air base to the right.

Z-axis. The line perpendicular to the X-axis, in the basal plane containing the principal point of the left-hand photograph The ground is considered as being in the negative direction.

Y-axis: The line perpendicular to the X and Z axes. The positive direction is towards the top side of the strip when viewed as running from left to right.

Strip Air Coordinates—The air coordinates of any point in a strip, whether on the ground or actually an air station, referred to the origin and axes of the air coordinate system of the first overlap.

Coplanar—Lying in the same plane. ***Basal Coplane***—*Photogrammetry* The condition of exposure of a pair of photographs whereby the two photographs lie in a common plane parallel to the air base. If the airbase is horizontal the photographs are said to be exposed in HORIZONTAL COPLANE.

Correspondence—*Stereoscopy.* The condition when corresponding images on a pair of photographs lie in the same epipolar plane.

Course—*Air Navigation:* The direction in which a pilot attempts to fly an aircraft, the line drawn on a chart or map as the intended track. Its direction is measured in degrees from the true meridian and the true course is always meant unless it is otherwise qualified, as a magnetic or compass course See also TRACK

Crab—1. *Air Navigation:* Any turning of an airplane which causes its longitudinal axis to vary from the track of the plane. 2. *Aerial Photography* The condition caused by failure to orient the camera with respect to the track of the airplane as indicated in vertical photography by the sides of the photographs not being parallel to the principal point base lines. See DRIFT for a special condition of crab.

Crabbed Photograph—*Aerial Photography:* See CRAB.

Critical Angle—*Optics:* When a ray of light is passing from a medium of higher refractive index to a medium of lower refractive index (such as from glass to air) there is a certain angle of incidence beyond which total internal reflection of the ray takes place. Such an angle is termed the CRITICAL ANGLE and is expressed by the equation $\sin A = 1/N$ where A is the critical angle and N the index of refraction of the glass.

Culture—*Mapping:* Those features of the terrain that have been constructed by man such as roads, trails, buildings, and canals; also, boundary lines; and all names and legends.

Curvature of Field—See under ABERRATION.

Cylindrical Lens—A lens in which the surfaces are segments of cylinders.

Datum—See under GEOID.

Datum Level—See under GEOID

Datum Plane—See under GEOID.

Declination—*Geometry*: In a system of polar or spherical coordinates the angle at the origin between a line to a point and the equator, measured in a plane perpendicular to the equator Also the arc between the point and the equator measured on a great circle perpendicular to the equator. See also MAGNETIC DECLINATION and Fig 4

Definition—*Optics.* A term used to express the ability of a lens to record fine detail. Also see RESOLVING POWER.

Delineation—*Surveying and Mapping:* The accurate representation on a map of physical and cultural features of the earth, or a section thereof, by means of lines and symbols

Density—See under CHARACTERISTIC CURVE

Depression Contour—See CONTOUR

Details—*Mapping*: The small items, or particulars of information shown on the map by lines, symbols, and lettering which when considered together, as a whole, furnish the comprehensive representation of the physical and cultural features. The greater the omission of details the more generalized is the map

Develop and Development†—*Photography:* To subject to the action of chemical agents for the purpose of bringing to view the invisible or latent image produced by the action of light on a sensitized surface, also, to produce or render visible in this way.

Diapositive—*Photogrammetry:* A positive photographic print on a transparent medium, usually on glass. In photogrammetry the term is generally used to refer to a transparent positive on a glass plate used in a plotting instrument, or a projector

Differential Shrinkage—*Mapping*: The difference in unit contraction along the grain structure of the material as compared to the unit contraction across the grain structure; frequently applied to photographic film and papers and to mapping papers in general.

Differential Torque—*Photogrammetry.* See under CANT.

Diffraction—*Optics.* The bending of light-waves around the edges of opaque objects. Due to diffraction a point of light seen or projected through a circular aperture will always be imaged as a bright center surrounded by light rings of gradually diminishing intensity Such a pattern is called a diffraction disk.

Diffuse Reflection—The type of reflection obtained from a relatively rough surface (such as a matte photographic print) in which the reflected rays are scattered in all directions.

Diopter—A unit of measurement of power of lenses, especially spectacle lenses. The power in diopters equals the reciprocal of the focal length in meters, thus a lens whose focal length is 20 cm. has a power of 5 diopters

Direction of Flight—See COURSE and also TRACK.

Direction of Tilt—The direction (azimuth) of the principal plane of a photograph. Also the direction of the principal line on a photograph.

Direct Radial Triangulation—See under RADIAL TRIANGULATION.

Direct Radial Plot—See under RADIAL TRIANGULATION.

Dispersion—*Optics:* A characteristic of optical glass which enables it to form a spectral band. Also used in place of "dispersive power," to mean a specific characteristic of optical glass having to do with the relative width of the spectral band formed.

Distant Line—The trace of the apparent, or visible, horizon on an oblique photograph used as an approximation to the terrestrial horizon trace when the latter is not identifiable. See also HORIZON.

Distortion—Optics: See under ABERRATION.

Diverging Lens—See NEGATIVE LENS.

Dove Prism—A prism which performs a reversion of the image but does not deviate nor displace the beam. A given angular rotation of the prism about its longitudinal axis causes the image to rotate through twice the angle. Also called a ***Rotating Prism.***

Drift—1 *Air Navigation:* The horizontal displacement of an aircraft under the action of the wind from the track it would have followed in still air. 2. *Aerial Photography.* Sometimes used to indicate a special condition of crab wherein the photographer has continued to make exposures oriented to the predetermined line of flight while the airplane has drifted with the wind, in such instances the edges of the successive photographs are parallel but sidestepped.

Elevation—Vertical distance above the datum, usually mean sea level, to a point or object on the earth's surface. Do not confuse with altitude which refers to points or objects above the earth's surface.

Emergent Nodal Point—See under NODAL POINT—*Optics*

Emulsion†—*Photography·* A suspension of a light-sensitive silver salt, especially silver chloride or silver bromide, in a colloidal medium, usually gelatin, used for coating photographic films, plates or papers.

Enlarging Lens—See PROCESS LENS.

Entrance Pupil—See under APERTURE STOP

Entrance Window—See under APERTURE STOP.

Epipolar Plane—See under EPIPOLES.

Epipolar Ray—See under EPIPOLES.

Epipoles—In the perspective set up of two photographs (two perspective projections), the points on the planes of the photographs where they are cut by the air base (extended line joining the two perspective centers). In the case of a pair of truly vertical photographs the epipoles are infinitely distant from the principal points. ***Epipolar Plane***—Any plane which contains the epipoles; therefore, any plane containing the air base. Also called ***Basal Plane. Epipolar Ray***—The line on the plane of a photograph joining the epipole and the image of an object. Also expressed as the trace of an epipolar plane on a photograph.

Equator—Geometry: In a system of polar or spherical coordinates, the great circle of a sphere cut by a plane which passes through the center of the sphere and is perpendicular to the polar axis.

Equator Trace—The trace on a photograph of an equatorial plane passing through the former's perspective center.

Equivalent Focal Length—The distance measured along the lens axis from the rear nodal point to the plane of best average definition over the entire field used in the aerial camera. (In general usage the term also applies to the distance from the rear nodal point to the plane of best axial definition but in photogrammetry this meaning is rarely used and will not be understood unless the term is accompanied by a qualifying phrase) ***Back Focal Length***—The distance measured along the lens axis from the rear vertex of the lens to the plane of best average definition. This value is used in setting the lens in the aerial camera. ***Calibrated Focal Length***—An adjusted value of the equivalent focal length so computed as to distribute the effect of lens distortion over the entire field used in the aerial camera. Also stated as the distance along the lens axis from the interior perspective center to the image plane; the interior center of perspective being selected so as to distribute the effect of lens distortion over the entire field. The calibrated focal length is used when determining the set-

ting of diapositives in plotting instruments and in photogrammetric computations based on linear measurements on the negative (such as those made with a precision comparator) ***Principal Distance***—The perpendicular distance from the internal perspective center to the plane of a particular finished negative or print. This distance is equal to the calibrated focal length corrected for both the enlargement or reduction ratio and the film or paper shrinkage or expansion and maintains the same perspective angles at the internal perspective center to points on the finished negative or print, as existed in the taking camera at the moment of exposure. This is a geometrical property of each particular finished negative or print.

Equivalent Principal Distance—Not approved See under EQUIVALENT FOCAL LENGTH.

Exit Pupil—See APERTURE STOP.

Exit Window—See APERTURE STOP.

Exposure†—*Photography* The total quantity of, light received per unit area which may be expressed as the product of the illumination and exposure time such as meter-candle-seconds Also used to mean the act of exposing, a section of film, and exposure time.

Exposure Scale—*Photography*· The useful exposure scale is the ratio of the maximum exposure to the minimum exposure between which the emulsion yields satisfactory reproduction.

Exposure Station—See under CAMERA STATION.

Exterior Orientation—*Photogrammetry* A set of quantities which fixes the position of the camera station and the angular orientation of the photograph Such a set consists of three elements of position and two elements of angular orientation. The position is usually expressed in terms of three rectangular coordinate distances—x, y, and z. The elements of angular orientation are essentially the tilt of the photograph perpendicular and the azimuth of the principal plane ***Relative Orientation***—The reconstruction of the same perspective conditions between a pair of photographs which existed when the photographs were taken. In a stereoscopic pair this is achieved when each pair of conjugate image rays lie in an epipolar plane. ***Absolute Orientation***—Following relative orientation which establishes the model, absolute orientation fixes the scale, position, and orientation of the model with reference to the ground coordinates

Exterior Perspective Center—See PERSPECTIVE CENTER.

Eye Base—The distance between the centers of rotation of the eyeballs of an observer

Eyepiece—In a telescope or microscope, the lens group nearest the eye, with which the image formed by preceding lenses is viewed.

Fiducial Axes—*Photogrammetry:* The lines joining opposite fiducial marks on a photograph.

Fiducial Lines—Not approved. See FIDUCIAL AXES

Fiducial Marks—*Photogrammetry*· Index marks rigidly connected with the camera lens through the camera body and forming images on the negative which define the principal point of the photograph.

Field Inspection—*Photogrammetry*. The process of comparing aerial photographs with conditions as they exist on the ground and of obtaining information to supplement or clarify that not readily discernible on the photographs themselves

Field of View—See under APERTURE STOP.

Field Stop—See under APERTURE STOP.

Film Base†—*Photography:* A thin, flexible, transparent sheet of cellulose nitrate or acetate or similar material which is coated with a light-sensitive emulsion and used for taking photographs.

Film Pressure Plate—See PRESSURE PLATE.

Filter—Any transparent material which absorbs a certain portion of the spectrum, such as for use in the optical path of a camera lens to prevent certain portions of the spectrum from reaching the sensitized negative.

Five Lens Camera—See under CAMERA

Fix or Fixation†—*Photography* To render a photographic emulsion permanent by removing the unaffected light-sensitive material

Flat—See OPTICAL FLAT.

Flight Altitude—The vertical distance above a given datum of an aircraft in flight or during a specified portion of a flight In aerial photography the datum is usually the mean ground level of the area being photographed.

Flight Line—A line drawn on a map or chart to represent the track over which an aircraft has been flown or is to be flown

Flight Ceiling—See SERVICE CEILING.

Flight Map—The map on which are indicated the desired lines of flight and/or the positions of exposure previous to the taking of air photographs, or the map on which are plotted, after photography, selected air stations and the tracks between them.

Floating Mark—*Photogrammetry*· A mark seen as occupying a position in the three dimensional space formed by the stereoscopic fusion of a pair of photographs and used as a reference mark in examining or measuring the stereoscopic model The mark may be formed (1) by one real mark lying in the projected object space as in the case of projection instruments such as the Multiplex, (2) by two real marks lying in the projected or virtually projected object spaces of the two photographs as in the case of certain other types of stereoscopic plotting instruments; (3) by two real marks lying in the planes of the photographs themselves as is usually the case in simple mirror stereoscopes, (4) by two virtual marks lying in the image planes of the binocular viewing apparatus as is the case in certain other types of stereoscopic plotting instruments ***Index Mark***—*Photogrammetry* A real mark such as a cross or a dot lying in the plane or the object space of a photograph and used singly as a reference mark in certain types of monocular instruments or singly or as one of a pair to form a floating mark as in certain types of stereoscopes. ***Reticle***—*Photogrammetry:* A mark such as a cross or system of lines lying in the image plane of a viewing apparatus and used singly as a reference mark in certain types of monocular instruments or as one of a pair to form a floating mark as in certain types of stereo scopes See also PARALLACTIC GRIDS.

Focal Length—See EQUIVALENT FOCAL LENGTH.

Focal Plane—*Aerial Photography*· The plane (perpendicular to the axis of the lens) in which images of points in the object field of the lens are focused. ***Focal Plane Plate***—A glass plate set in the camera so that the surface away from the lens coincides with the focal plane, for the purpose of locating the emulsion of the film in the focal plane when the film is pressed into contact with the glass plate mechanically. Also known as ***Contact Glass*** or ***Contact Plate.***

Focal Plane Plate—See under FOCAL PLANE.

Focal Plane Shutter—See under SHUTTER

Focus—The point toward which rays of light converge to form an image after passing through lens.

Fore and Aft Overlap—Not approved. See FORWARD LAP under OVERLAP.

Fore and Aft Tilt—Not preferred. See Y-TILT under TILT.

Foreground—*Photography*· That part of the landscape imaged in a horizontal or oblique photograph which is nearest the camera station.

Form Lines—Lines having the same appearance as contour lines but which have been sketched from visual observation to show the shape of the terrain rather than the elevation. These lines are based upon as many determined elevations as may be secured but too few to prescribe limits of accuracy for the resulting work.

Forward Lap—See under OVERLAP.

Forward Overlap—Not approved. See FORWARD LAP under OVERLAP.

Four Lens Camera—See under CAMERA.

Front Nodal Point—See under NODAL POINT—*Optics.*

Fusion—See STEREOSCOPIC FUSION under STEREOSCOPY.

Gamma—See under CHARACTERISTIC CURVE.

Gap—*Aerial Photography:* Any space where aerial photographs fail to meet minimum coverage requirements This might be a space not covered by any photograph or a space where the minimum specified overlap was not obtained

Geodesy—The science which treats of the determination of the size and figure of the earth (geoid) by direct measurements as triangulation, leveling, gravimetric observations. The applied science of geodesy is called ***Geodetic Surveying,*** i.e., surveying which takes account of the figure and size of the earth.

Geodetic Control—See under CONTROL

Geodetic Surveying—See under GEODESY.

Geographic Coordinates—See under COORDINATES.

Geoid—A theoretical surface of the earth as defined by the surface of the sea at mean elevation over the oceans and conceived as extending continuously through all the continents. ***Reference Spheroid***—A spheroid determined by revolving an ellipse about its shorter (polar) axis and used as a base for geodetic surveys of a large section of the earth, as the Clarke's Spheroid of 1866 used for geodetic surveys in the United States. The spheroid of reference is a theoretical figure, the dimensions of which approach closely the dimensions of the geoid, the exact dimensions being determined by various considerations of the section of the earth's surface concerned. ***Level Surface***—A surface which is at every point perpendicular to the direction of gravity; the geoid, or, in general, any surface parallel thereto. The surface of the sea, if changes in elevation due to tides, winds, etc., are neglected, is a level surface. A level surface is not a plane surface but is sometimes so regarded in surveys of limited areas. See also VERTICAL CONTROL DATUM. ***Horizontal Control Datum***—The position on the spheroid of reference assigned to the horizontal control (triangulation and traverse) of an area and defined by (1) the position (latitude and longitude) of one selected station in the area, and (2) the azimuth from the selected station to an adjoining station. The horizontal control datum may be for a continent or a small area A datum for a small area is usually called a ***Local Datum*** and is also given a proper name. The horizontal control datum for the North American continent is known as the ***North American Datum of 1927,*** the selected station for which is "Meades Ranch," Kansas with the azimuth to the adjoining station "Waldo." All geodetic positions on the North American Datum of 1927 depend on the position of "Meades Ranch" and the azimuth "Meades Ranch" to "Waldo." ***Vertical Control Datum***—Any level surface, as for example mean sea level, taken as a surface of reference from which to reckon elevations Also called the ***Datum Level.*** Although a level surface is not a plane, the vertical control datum is frequently referred to as the ***Datum Plane.*** See LEVEL SURFACE. ***Horizontal Plane***—A plane perpendicular to the direction of gravity; any plane tangent to the geoid or parallel to such a plane. ***Ground Plane***—*Photogrammetry:* The horizontal plane passing through the ground nadir of a camera station. ***Map Plane***—Any horizontal plane to which the planim-

etry and relief of an area are plotted or referenced. ***Hill Plane***—*Photogrammetry*: The plane containing the positions of three ground marks constituting control points. This may be, but rarely is, a horizontal plane.

Geometric Axis—Not approved when used in place of FIDUCIAL AXIS, which see

Goniometer—An instrument for measuring angles. ***Photogoniometer***—An instrument for measuring angles to any point on a photograph from the true perspective center.

Gradient—See under CHARACTERISTIC CURVE.

Gradient Speed—See under CHARACTERISTIC CURVE.

Grain†—*Photography:* One of the discrete light-sensitive particles of a photographic material, e g., a crystal of silver halide, or a particle, usually silver, resulting from the development of such a material. ***Granularity***—The inhomogeneity of a developed photographic image evident particularly on enlargement, and due to agglomerations of developed grains, or to an overlapping pattern of grains ***Graininess***—The subjective impression of granularity.

Graininess—See under GRAIN.

Granularity—See under GRAIN.

Graphic Ray Plot—Not approved. See RADIAL PLOT under RADIAL TRIANGULATION

Grid—See MAP GRID.

Grid Coordinates—See under COORDINATES.

Grid Method.—*Photogrammetry:* A method of plotting detail from oblique photographs by superimposing a perspective of a map grid on a photograph and transferring the detail by eye, the latter being guided by the corresponding lines of the map grid and its perspective See also PERSPECTIVE GRID.

Ground Camera—See under CAMERA.

Ground Control—See under CONTROL.

Ground Control Point—See under CONTROL POINT.

Ground Line—Not preferred. See GROUND PARALLEL under PRINCIPAL PLANE—*Photogrammetry.*

Ground Nadir—See under NADIR.

Ground Parallel—See under PRINCIPAL PLANE.—*Photogrammetry.*

Ground Photogrammetry—See under PHOTOGRAMMETRY.

Ground Photograph—Any photograph taken on the ground.

Ground Plane—See under GEOID.

Ground Plumb Point—GROUND NADIR is preferred. See under NADIR.

Ground Speed—*Air Navigation:* The velocity of an aircraft along its track with relation to the ground; the resultant of the heading and air speed of an aircraft and the direction and velocity of the wind. See also AIR SPEED.

Ground Survey—See under SURVEY.

Ground Trace—Not preferred. See GROUND PARALLEL under PRINCIPAL PLANE—*Photogrammetry.*

Gyroscopic Stabilization—Equilibrium obtained in a ship or airplane by the use of gyroscopes. The maintenance, by the use of gyroscopes, of an airplane on constant course in a horizontal position, the maintenance, by the use of gyroscopes. of a camera in a desired position within an aircraft.

Halation†—*Photography:* A spreading of a photographic image beyond its proper boundaries, particularly due to reflection from the side of the film or plate support opposite to that on which the emulsion it coated. It is particularly noticed in photographs of bright objects against a darker background.

Hand Templet—See under TEMPLET.

Hand Templet Method—See TEMPLET and RADIAL TRIANGULATION

Hand Templet Plot—See under RADIAL TRIANGULATION.

Hand Templet Triangulation—See under TEMPLET and also under RADIAL TRIANGULATION.

Haze—The presence of foreign matter in the atmosphere to an extent sufficient to reduce even slightly its transparency. Not aerial haze.

Heading—*Air Navigation:* The angular direction of the longitudinal axis of the aircraft with respect to the true meridian. In other words it is the course with drift correction applied. It is the true heading unless otherwise designated.

High Oblique Photograph—See under OBLIQUE PHOTOGRAPH.

Hill Plane—See under GEOID.

Homologue—A point or line in one system of points or lines corresponding to a point or line in another similar system; for example, homologous image points are the images on two or more photographs of the same object point.

Horizon—In general, the apparent or visible junction of earth and sky, as seen from any specific position. Also called the ***Apparent, Visible,*** or ***Local Horizon. Terrestrial Horizon***—A mathematical concept which is defined as the locus on the surface of the reference spheroid of the earth of the points of tangency of tangent rays from any given point of vision or perspective center. The APPARENT or VISIBLE HORIZON often approximates the TERRESTRIAL HORIZON. ***True Horizon***—A horizontal plane passing through a point of vision or perspective center. The APPARENT or VISIBLE HORIZON approximates the TRUE HORIZON only when the point of vision is very close to sea level.

Horizon Camera—See under CAMERA.

Horizon Photograph—*Aerial Photography:* A photograph of the horizon taken simultaneously with a vertical photograph for the sole purpose of obtaining an indication of the tilt of the vertical camera at the instant of exposure.

Horizontal Control—See under CONTROL

Horizontal Control Datum—See under GEOID.

Horizontal Coplane—See under COPLANAR.

Horizontal Parallax—See under PARALLAX.

Horizontal Photograph—A photograph taken with the camera axis horizontal.

Horizontal Plane—See under GEOID.

Horizon Trace—See under PRINCIPAL PLANE—*Photogrammetry.*

Humidity—Degree of wetness, especially of the atmosphere. ***Relative Humidity***—Ratio of aqueous vapor present in a space at a given temperature, as compared with the greatest amount it could possibly contain, at that temperature. ***Absolute Humidity***—The weight of water vapor contained in a given volume of air, as grains per cubic foot. ***Specific Humidity***—The weight of water vapor per unit weight of the moist air.

Hydrographic Map—See under MAP.

Hypsograph—An instrument of the slide rule type used to compute elevations from vertical angles and horizontal distances.

Hypsometry—In general, the determination of elevations above sea level. In particular, the determination of such elevations by observing the boiling point of water.

Incident Nodal Point—See under PRINCIPAL POINT—*Optics.*

Index Mark—See under FLOATING MARK.

Index of Refraction—See under SNELL'S LAW OF REFRACTION.

Infrared†—*Photography*· (adj., *Physics*) Pertaining to or designating those rays of light just beyond the red end of the visible spectrum, such as are emitted by a hot body. They are invisible and are detected by their thermal and photographic effects. Their wave lengths are longer than those of visible light and shorter than those of radio waves.

Interference Fringes—See under TEST GLASS.

Interior Orientation—The establishment of the principal distance, and the position of the principal point of a photograph with respect to the fiducial marks of the camera.

Interior Perspective Center—See under PERSPECTIVE CENTER.

Interocular Distance—Synonymous with EYE BASE, which see.

Interpolation—Determination of an intermediate value between fixed values from some known or assumed rate or system of change.

Interpupillary Distance—Synonymous with EYE BASE, which see.

Intervalometer—A timing device for automatically operating the shutter of a camera at any predetermined interval.

Iris Diaphragm—A continuously variable circular aperture in a lens which makes it possible to control the amount of light passing through the lens. Also called ***Stop***.

Isocenter—(1) The unique point common to the plane of a photograph, its principal plane, and the plane of an assumed truly vertical photograph taken from the same camera station and having an equal principal distance (2) The point of intersection on a photograph of the principal line and the isometric parallel. (3) The point on a photograph intersected by the bisector of the angle between the plumb line and the photograph perpendicular. The isocenter is significant because it is the center of radiation for displacements of images due to tilt.

Isocenter Plot—See under RADIAL TRIANGULATION.

Isocenter Triangulation—See under RADIAL TRIANGULATION.

Isometric Parallel—See under PRINCIPAL PLANE.

Isoradial—See under RADIAL.

Kodachrome†—*Photography:* A film for direct color photographic transparencies carrying three layers responding respectively to blue, green, and red light. In development, positive images are produced in each layer by reversal development, which forms dyes which are complementary in color to the spectral regions to which the layers respond. The dye-forming agents, called couplers, are in the developing solutions.

Kodacolor†—*Photography:* Originally, a motion picture color process using lenticular film. Now, a negative-positive color process in which the film and paper are each coated with three layers which respond respectively to blue, green and red light. The dye-forming couplers are incorporated in the layers, which are developed directly to give dye images of colors complementary to the spectral regions to which the layers respond. The negative in complementary colors is printed onto the paper to give a photograph in normal colors.

Kodacolor Aero Reversal Film†—*Photography:* A film of the Kodacolor type which is developed by the reversal method to give a positive transparency in natural colors

Kotavachrome†—*Photography:* A color print made on a white support by the Kodachrome process.

Latent Image†—*Photography:* An invisible image produced by a physical or chemical effect of light upon matter (usually silver halide or halides) which can be rendered visible by the subsequent chemical process of photographic development.

Lateral Chromatic Aberration—See under ABERRATION.

Lateral Level Element—*Photogrammetry*· See under CANT AND TORQUE.

Lateral Magnification—See under MAGNIFICATION.

Lateral Oblique Photograph—An oblique aerial photograph taken with the camera axis as nearly as possible normal to the flight line. Also called ***Lateral Oblique.***

Lateral Overlap—Not approved. See SIDE LAP under OVERLAP.

Lateral Tilt—Not preferred. See X-TILT under TILT.

Legend—A description, explanation, table of symbols and other information, which is printed on a map or chart, for a better understanding and interpretation of it. The title of a map or chart was formerly considered as part of the legend but this usage is obsolescent. The title should be considered as separate and not a part of the legend. Not legend reference.

Lens‡—*Optics*· A piece, or combination of pieces, of glass or other transparent material shaped to form an image by means of refraction.

Lens Component—See LENS ELEMENT.

Lens Distortion—See under ABERRATION.

Lens Element—One lens of a complex lens system. In a photographic lens, the terms front and rear elements are often used.

Lens Speed—See RELATIVE APERTURE.

Level Surface—See under GEOID.

Light Ray—See RAY OF LIGHT.

Light Slides—A thin plate, usually metal or fiber, rigid or flexible, which after insertion in a camera magazine renders it light tight. The employment of light slides makes it possible to interchange camera magazines in daylight.

Linear Magnification—See under MAGNIFICATION.

Linear Parallax—See under PARALLAX.

Line of Equal Scale—See ISOMETRIC PARALLEL under PRINCIPAL PLANE—*Photogrammetry.*

List—Not preferred See X-TILT under TILT.

Local Horizon—See HORIZON.

Locating Back—*Aerial Photography*· A plane surface in an aerial camera parallel to the plane of the lens) which is out of the focal plane by an amount equal to the thickness of the film used and against which the film is sucked by vacuum, or pressed by air pressure, in order to maintain the emulsion surface of the film in the focal plane at the instant of exposure. Locating backs are usually of metal and in both above cases are perforated or slotted to allow for the building up of a differential pressure or for the removal of air in the formation of a vacuum. A locating back which utilizes vacuum is known as a ***Vacuum Back,*** and one which utilizes pressure is known as a ***Pressure Back.***

Longitudinal Chromatic Aberration—See under ABERRATION.

Longitudinal Magnification—See under MAGNIFICATION.

Longitudinal Overlap—Not approved. See FORWARD LAP under OVERLAP.

Longitudinal Tilt—Not preferred. See Y-TILT under TILT.

Louver Shutter—See under SHUTTER.

Loxodromic Curve—Synonymous with RHUMB LINE, which see.

Low Oblique—See under OBLIQUE PHOTOGRAPH.

Magnetic Declination—The angle between the true (geographical) north and magnetic

north (direction of the compass needle). The magnetic declination is different for different places and changes continuously with respect to time.

Magnification—Optics: A quantity which states the ratio of the size of an image to the size of the object ***Linear Magnification***—The ratio of a linear quantity in the image to a corresponding linear quantity in the object. It may be Lateral Magnification or Longitudinal Magnification ***Lateral Magnification***—The ratio of a length in the image, perpendicular to the lens axis, to a corresponding length in the object. ***Longitudinal Magnification***—The ratio of a length in the image, along the axis, to a corresponding length in the object. ***Angular Magnification***—The ratio of the angle subtended at the eye by the image formed by an optical device, to the angle subtended at the eye by the object itself without the optical device This is convenient to use in cases where a distance in the object cannot be measured for expressing a Linear Magnification, as in the use of a telescope.

Manuscript Map—See under MAP.

Map*—A representation on a plane surface, at an established scale, of the physical features (natural, artificial, or both) of a part or the whole of the earth's surface, by means of signs and symbols, and with the means of orientation indicated. Also a similar representation of the heavenly bodies. A map may emphasize, generalize, or omit the representation of certain features to satisfy specific requirements The type of information which a map is designed primarily to convey is frequently used, in adjective form, to distinguish it from maps of other types. ***Topographic Map****—A map which presents the horizontal and vertical positions of the features represented; distinguished from a planimetric map by the addition of relief in measurable form. A topographic map usually shows the same features as a planimetric map, but uses contours or comparable symbols to show mountains, valleys, and plains; and in the case of hydrographic charts, symbols and numbers to show depths in bodies of water. ***Contour Map*****—A topographic map which portrays relief by means of contour lines. The term "contoured" is disapproved. ***Planimetric Map****—A map which presents the horizontal positions only for the features represented; distinguished from a topographic map by the omission of relief in measurable form. The natural features usually shown on a planimetric map include rivers, lakes, and seas; mountains, valleys, and plains; forests, prairies, marshes, and deserts. The cultural features include cities, farms, transportation routes, and public utility facilities; political and private boundary lines A planimetric map intended for special use may present only those features which are essential to the purpose to be served. ***Base Map***—1. A map showing certain fundamental information, copies of which are used to compile additional data of specialized nature. Often used to define a large scale planimetric map compiled from aerial photographs, copies of which are used for the addition of contours and other data by means of the planetable and/or photogrammetric methods. 2. A map showing all of the information from which maps showing specialized information can be prepared; a master map. ***Cadastral Map****—A map showing the boundaries of subdivisions of land, usually with the bearings and lengths thereof and the areas of individual tracts, for purposes of describing and recording ownership. A cadastral map may also show culture, drainage, and other features relating to the value and use of land. ***Hydrographic Map***—A map showing a portion of the waters of the earth, including shore lines, the topography along the shores and of the submerged portions, and as much of the topography of the surrounding country as is necessary for the purpose intended. See also Nautical Chart. ***Manuscript Map****—The original drawing of a map as compiled or constructed from various data as ground surveys, photographs, etc. ***Special Purpose Map*****—Any map designed primarily to meet specific requirements. Usually the map information portrayed on a special-purpose map is em-

phasized by omitting or subordinating other information of a general character which is not essential or is of less importance to the purpose to be served The special purposes for which maps are designed and used are numerous and are increasing with the trend toward the graphic portrayal of factual information in relation to the areas of origin or application. The map, in most cases, serves as a base on which special information is correlated. A word or phrase is usually employed to describe the type of information which the map is designed to present, as Route Maps, Tax Maps, Index Maps, etc.

Map Data—Collectively, the basic information which the surveyor obtains and the cartographer uses in the construction and compilation of a map.

Map Drawing—Not to be used for MANUSCRIPT MAP, which see under MAP.

Map Grid—Two sets of parallel lines at right angles drawn on a plane surface and used as a rectangular coordinate system (a reference system) for plotting positions and scaling distances and directions in surveying and mapping A map grid may or may not be based on a map projection For various classes of coordinate systems see under COORDINATES. See also MAP PROJECTION.

Map Parallel—See under PRINCIPAL PLANE—*Photogrammetry*.

Map Plane—See under GEOID.

Map Projector—A specially designed optical instrument by means of which the image of a photograph or a drawing is projected onto a table where it can be traced or compared with another drawing. The instrument is usually equipped with a mirror to erect the projected image and always is arranged so that the scale of the projected image can be varied The instrument does not provide for rectification of the photograph

Map Projection—A systematic drawing of lines on a plane surface to represent the parallels of latitude and the meridians of longitude of the earth or a section of the earth A map projection may be established by analytical computation or may be constructed geometrically by perspective projection. A map projection is frequently referred to as ***Projection*** but the complete term should be used unless the context clearly indicates the meaning.

Mapping Camera—Not preferred. See SURVEYING CAMERA under CAMERA.

Master Glass Negative—*Photogrammetry*: A glass negative exposed with the emulsion side in the same position as will be the emulsion on the film at the moment of exposure Such a negative is a record of the distance between the fiducial marks and is used for measuring film shrinkage.

Mechanical Templet—See under TEMPLET.

Mechanical Templet Plot—See under RADIAL TRIANGULATION.

Mechanical Templet Triangulation—See under RADIAL TRIANGULATION.

Meniscus Lens‡—*Optics:* A single-element lens, or cemented lens combination, concave on one side and convex on the other. For photographic purposes, a meniscus lens always has a small stop on the concave side and is frequently referred to as a landscape lens.

Mercury Barometer—See under BAROMETER.

Meridional Plane—See under POLAR BEARING.

Metapole—Not approved for isocenter, which see

Micron—One-thousandth part of a millimeter.

Millimicron—One-millionth part of a millimeter.

Minor Control—See under CONTROL.

Minor Control Plot—See RADIAL TRIANGULATION.

Mosaic—*Photogrammetry:* An assemblage of aerial photographs whose edges have been torn, or cut, and matched to form a continuous photographic representation of a portion of the earth's surface. Also called Aerial Mosaic though the adjective is unnecessary where the context clearly indicates the meaning ***Controlled Mosaic***—A mosaic laid on ground control to improve the accuracy of representation as regards distances and directions

Multiple Lens Camera—See under CAMERA.

Multiple Lens Photograph—A photograph made with a multiple lens camera. See also MULTIPLE LENS CAMERA under CAMERA.

Nadir—That point on the celestial sphere directly beneath the observer, and directly opposite to the zenith. ***Photograph Nadir***—*Photogrammetry* That point at which a vertical line through the perspective center of the camera lens pierces the plane of the photograph Also referred to as the ***Nadir Point. Ground Nadir***—The point on the ground vertically beneath the perspective center of the camera lens.

Nadir Point—See under NADIR.

Nadir Point Plot—See under RADIAL TRIANGULATION.

Nadir Point Slotted Templet Plot—See under RADIAL TRIANGULATION.

Nadir Point Triangulation—See under RADIAL TRIANGULATION.

Nadir Radial—See under RADIAL

Nautical Chart—A hydrographic or marine map. A map of a portion of the earth's surface which includes navigable waters and the adjacent or included land, if any, and on which are indicated depths of the water, marine obstructions, aids to navigation, and other information to aid the mariner in navigating.

Negative—*Photography:* A sensitized plate or film which has been exposed in a camera and which has the lights and shades in inverse order to those of the original subject. The plate or film does not become a negative until it is exposed, after which it may be an undeveloped or a developed negative.

Negative Lens—A lens which will diverge a beam of parallel light rays, no real focus being obtained. Also called ***Diverging Lens.***

Nine Lens Camera—See under CAMERA.

Nodal Plane—See under NODAL POINT—*Optics.*

Nodal Point—*Optics*· One of two points on the optical axis of a lens (or a system of lenses) such that when all object distances are measured from one point and all image distances are measured from the other they satisfy the simple lens relation $1/I = 1/O + 1/F$ (conjugate foci formula). Also a ray emergent from the second point is parallel to the ray incident at the first. The first nodal point is also referred to as the FRONT NODAL POINT, or INCIDENT NODAL POINT, and the second point as the REAR NODAL POINT, or EMERGENT NODAL POINT. Also called simply NODE, as front node. ***Nodal Plane***—A plane perpendicular to the optical axis at a nodal point. ***Principal Points***—When the initial and final media have different indices of refraction, another set of points are introduced known as PRINCIPAL POINTS, but in photogrammetry this condition seldom exists and the two sets of points are coincident.

Node—See under NODAL POINT—*Optics.*

North American Datum of 1927—See under GEOID.

Objective—The lens in a microscope or telescope which is nearest the object. Also the lens used in a camera.

Oblique Photograph—A photograph taken with the camera axis directed intentionally between the horizontal and the vertical. ***High Oblique***—An oblique photograph in which the apparent horizon is shown. ***Low Oblique***—An oblique photograph in which the apparent horizon is *not* shown.

Oblique Plotting Instrument—An instrument (usually monocular) for plotting from oblique photographs.

Occupy—*Surveying:* To set a surveying instrument over for the purpose of making observations, said of a station e.g. "occupy Greylock station" or "occupy Station B 16."

Opacity—See under CHARACTERISTIC CURVE.

Optical Axis—The optical axis of a lens element is a straight line which passes through the centers of curvature of the lens surfaces. In a compound lens if the centers of curvature of all the components were to lie in one straight line, this line would be the optical axis of such a lens. This exact condition is rarely obtained in practice.

Optical Flat—A piece of optical glass (usually a disk with parallel surfaces) the surfaces of which have been ground and polished plane to within a fraction of a wave-length of light Such a flat is used for testing the planeness of prism faces, mirrors, etc. Also called ***Optical Plane.***

Optical Plane—See OPTICAL FLAT.

Optically Flat—A surface is said to be optically flat when it has been ground and polished plane to within a fraction of a wave-length of light.

Origin—*Surveying:* The reference position from which angles or distances are reckoned See also under COORDINATES.

Orthochromatic†—*Photography:* (a) Of, or pertaining to, or producing tone values (of light or shade) in a photograph, corresponding to the tones of nature. (b) Incorrectly, designating a film made sensitive to blue and green, but not red, light.

Orthogonal—At right angles; rectangularly; meeting, crossing, or lying at right angles.

Overlap—*Photography:* Amount by which one photograph overlaps the area covered by another, customarily expressed as a percentage. The overlap between aerial photographs in the same flight is distinguished as the ***Forward Lap,*** and the overlap between photographs in adjacent parallel flights is called the ***Side Lap.***

Overlapping Pair—*Photogrammetry:* Two photographs taken at different exposure stations in such a manner that a portion of one photograph shows the same terrain as shown on a portion of the other photograph This term covers the general case and does not imply that the photographs were taken for stereoscopic examination. See also STEREOSCOPIC PAIR under STEREOSCOPY.

Overlay—*Mapping:* A record on a transparent medium to be superimposed on another record, Example—maps showing original land grants (or patents) prepared as tracing cloth overlays in order that they may be correlated with the maps showing present ownership, also the name overlay for a manuscript map.

Panchromatic†—*Photography:* Sensitive, as a film or plate emulsion, to light of all colors.

Parallactic Angle—See under PARALLAX.

Parallactic Grids—*Photogrammetry:* A uniform pattern of rectangular lines drawn or engraved on some transparent material, usually glass, and placed either over the photographs of a stereoscopic pair, or in the optical system of a stereoscope, to provide a continuous floating mark system.

Parallax—The apparent displacement of the position of a body with respect to a reference point or system caused by a shift in the point of observation. ***Absolute Stereoscopic Parallax***—*Photogrammetry:* Considering a pair of truly vertical photographs, of equal principal distances, taken from equal flight heights, or a pair of rectified photographs—the absolute stereoscopic parallax of a point is the algebraic difference, parallel to the air base, of the distances of the two images from their respective principal points. In photogrammetry the term ***Parallax*** is generally used to denote

absolute stereoscopic parallax and also to denote similar measurements when the above theoretical conditions are not strictly attained, as for example when measuring parallaxes on unrectified aerial photographs. ***Linear Parallax, X-Parallax*** and ***Horizontal Parallax*** are synonymous with absolute stereoscopic parallax but are not preferred ***Parallax Difference***—The difference in the absolute stereoscopic parallaxes of two points imaged on a pair of photographs Customarily used in the determination of the difference in elevations of the objects ***Y-Parallax***—*Photogrammetry:* The y-parallax of a point is the difference of the perpendicular distances of its two images from the vertical plane containing the air base The existence of y-parallax is an indication of tilt in either or both photographs and/or a difference in flying height and is confusing to stereoscopic examination of the pair Also called ***Vertical Parallax*** though the latter is not preferred ***Angular Parallax***—The angle subtended by the eye base of the observer at the object viewed. Also called ***Parallactic Angle*** or ***Angle of Convergence.***

Parallax Difference—See under PARALLAX.

Paraxial Ray—A paraxial ray is one whose path lies very near the axis of a lens and which intersects the lens surface at a point very close to its vertex and at nearly normal incidence.

Pass Point—A point whose horizontal and/or vertical position is determined from photographs by photogrammetrical methods, and which is intended for use after the manner of a ground control point in the orientation of other photographs.

Pencil of Light—See under RAY OF LIGHT.

Perspective Axis—Not approved for MAP PARALLEL, which see under PRINCIPAL PLANE—*Photogrammetry.*

Perspective Center—The point of origin or termination of bundles of perspective rays. The two such points usually associated with a survey photograph are the ***Interior Perspective Center*** and the ***Exterior Perspective Center.*** In a perfect lens-camera system, perspective rays from the interior perspective center to the photographic images enclose the same angle as do the corresponding rays from the exterior perspective center to the objects photographed. In a lens having distortion the above is true only for a particular zone of the photograph. In a perfectly adjusted lens-camera system the exterior and interior perspective centers correspond respectively to the front and rear nodal points of the camera lens.

Perspective Grid—*Photogrammetry:* A network of lines drawn or superimposed on a photograph which represents the perspective of a systematic network of lines on the ground or datum plane. See also GRID METHOD.

Perspective Plane—Any plane containing the perspective center Since the intersection of two planes is a straight line, it follows that the intersection of a perspective plane and the ground will always appear as a straight line on the photograph It also follows that any straight line in the object space will appear as a straight line on the photograph.

Perspective Ray—A line joining a perspective center and a point object

Photogoniometer—See under GONIOMETER.

Photogrammetric Camera—See under CAMERA.

Photogrammetric Control—See under CONTROL.

Photogrammetric Survey—See under SURVEY.

Photogrammetry—The science or art of obtaining reliable measurements by means of photography. ***Aerial Photogrammetry***—Photogrammetry with the aid of aerial photographs. ***Ground Photogrammetry***—Photogrammetry with the aid of ground

photographs. Also called ***Terrestrial Photogrammetry*** though this term is not preferred. ***Stereophotogrammetry***—Photogrammetry with the aid of stereoscopic equipment and methods.

Photograph—A general term for a positive or negative picture made by a camera on plate, film, or other medium. For specific types of photographs, see under the proper name as: AERIAL PHOTOGRAPH, MULTIPLE LENS PHOTOGRAPH, etc

Photograph Axis—See FIDUCIAL AXIS which is preferred

Photograph Center—The center of a photograph as indicated by the images of the fiducial mark or marks of the camera. In a perfectly adjusted camera the photograph center and the principal point are identical.

Photograph Coordinates—See under COORDINATES.

Photograph Meridian—See under PRINCIPAL PLANE—*Photogrammetry.*

Photograph Nadir—See under NADIR.

Photograph Parallel—See under PRINCIPAL PLANE—*Photogrammetry.*

Photograph Perpendicular—The perpendicular from the INTERIOR PERSPECTIVE CENTER to the plane of the photograph.

Photograph Plumb Point—PHOTOGRAPH NADIR is preferred. See under NADIR

Photograph Pyramid—A pyramid whose base is a triangle formed by three point images. on a photograph and whose apex is the interior perspective center of the photograph.

Phototheodolite—A ground surveying instrument combining a theodolite and a surveying camera in which the relationship between the camera axis and the line of collimation of the theodolite can be measured.

Picture Plane—A plane upon which is conceived to be projected a system of lines or rays from an object to form an image or picture. In perspective drawing, the system of rays is understood to converge to a single point. In photogrammetry the photograph is the picture plane.

Plane Control—See CONTROL.

Plane Coordinates—See under COORDINATES.

Plane Polar Coordinates—See under COORDINATES.

Plane Rectangular Coordinates—See under COORDINATES.

Planetable—An instrument by means of which points are located in the field directly on a map by graphical methods, the map being fastened to a table top supported by a tripod, from which the instrument derives its name.

Planimetric Map—See under MAP.

Plat—A diagram drawn to scale showing land boundaries and subdivisions, together with all data essential to the description of the several units. A plat differs from a map in that it does not show additional cultural, drainage and relief features. See also MAP.

Plate Axis—Not approved. See FIDUCIAL AXIS.

Plate Center—See PHOTOGRAPH CENTER which is preferred.

Plate Meridian—Not preferred. See PHOTOGRAPH MERIDIAN under PRINCIPAL PLANE—*Photogrammetry.*

Plate Parallel—Not preferred See PHOTOGRAPH PARALLEL under PRINCIPAL PLANE—*Photogrammetry.*

Plate Perpendicular—See PHOTOGRAPH PERPENDICULAR which is preferred.

Plate Plumb Point—Not preferred. See PHOTOGRAPH NADIR under NADIR.

Plumb Point—Synonymous with NADIR POINT which is preferred. See under NADIR

Plumb Point Triangulation—Not preferred See NADIR POINT TRIANGULATION under RADIAL TRIANGULATION.

Polar Axis—In a system of polar or spherical coordinates, the primary axis of direction

Polar Bearing—In a system of polar or spherical coordinates, the angle formed by the intersection of the reference meridional plane and the meridional plane containing the point. A ***Meridional Plane*** is defined as any plane containing the polar axis. See Fig. 1.

Polar Coordinates—See under COORDINATES.

Polar Distance—In a system of polar or spherical coordinates the angle at the origin between a line to a point and the pole. Also the arc of the great circle between the point and the pole

Polarized Light—According to the wave theory, ordinary light (unpolarized) is said to vibrate perpendicular to the direction of propagation in all planes. On passing through certain polarizing mediums (see POLAROID) ordinary light becomes *plane polarized,* that is, the vibrations are limited to single plane.

Polaroid—A manufactured plastic polarizing screen Ordinary light on passing through such a screen becomes plane polarized.

Porro Prism—This prism deviates the axis 180 degrees and inverts the image in the plane in which the reflection takes place. It may be considered to be two right angle prisms cemented together.

Positive Lens—A lens which will converge a beam of parallel light rays to a point focus. Also called ***Converging Lens.***

Positive—*Photography:* A photograph having approximately the same rendition of light and shade as the original subject.

Power of a Lens—See DIOPTER and also MAGNIFICATION

Precision**—Degree of refinement in the performance of an operation or in the statement of a result. Precision relates to the quality of execution, and is distinguished from accuracy which relates to the quality of the result. The term precision not only applies to the fidelity with which required operations are performed, but by custom has been applied to methods and instruments employed in obtaining results of a high order of accuracy. Precision is exemplified by the number of decimal places to which a computation is carried and a result stated. In a general way, the accuracy of a result should determine the precision of its expression. Precision is of no significance unless accuracy is also obtained.

Precision Camera—See under CAMERA.

Pressure Back—See LOCATING BACK.

Pressure Plate—*Photography:* A flat plate, usually of metal but frequently of glass or other substance, which by means of mechanical force, presses the film into contact with the focal plane plate of the camera.

Principal Distance—See under EQUIVALENT FOCAL LENGTH.

Principal Line—See under PRINCIPAL PLANE—*Photogrammetry.*

Principal Meridian—See under PRINCIPAL PLANE—*Photogrammetry.*

Principal Parallel—See under PRINCIPAL PLANE—*Photogrammetry,*

Principal Plane—*Optics:* See under PRINCIPAL POINT—*Optics.*

Principal Plane—*Photogrammetry:* The vertical plane through the internal perspective center containing the plate perpendicular of an oblique photograph, i.e., any photograph which is not a truly vertical photograph In Figure 3 the plane of the paper. In the case of a truly vertical photograph the principal plane and the other planes

and lines discussed below have no meaning. ***Principal Line***—The trace of the principal plane upon the photograph ***Horizon Trace***—An imaginary line on the plane of the photograph which represents the image of the true horizon. It corresponds to the intersection of the plane of the photograph and the horizontal plane containing the internal perspective center or rear nodal point of the lens In Figure 3, the line through the point "t" perpendicular to the plane of the paper. ***Vanishing Point***—The image on the plane of the photograph of the point towards which a system of parallel lines in the object space converge. Since any system of parallel lines in the object space will meet at infinity, the image of the meeting point will be formed by the ray through the perspective center, *O* in Figure 3, parallel to the system. The vanishing points of all systems of parallel lines parallel to one plane will lie on a straight line on the photograph called a ***Vanishing Line.*** The vanishing line for all systems of horizontal parallel lines in the object space is the horizon trace. ***Photograph Meridian***—The image on the photograph of any horizontal line in the object space which is parallel to the principal plane. Since all such lines meet at infinity, the image of the meeting point is at the intersection of the principal line and the horizon trace, the point "t" in Figure 3, and all photograph meridians pass through that point. The principal line sometimes called the ***Principal Meridian*** is the only photograph meridian which is perpendicular to the photograph parallels. ***Photograph Parallel***—Any horizontal line on the photograph. All photograph parallels are perpendicular to the principal line. The photograph parallel passing through the principal point is the ***Principal Parallel,*** and that passing through the isocenter is the ***Isometric Parallel.*** Thus the isometric parallel is the intersecting line between the plane of the photograph and a horizontal plane having an equal perpendicular distance from the same perspective center. Also called ***Line of Equal Scale. Axis of Tilt***—A line through the perspective center perpendicular to the principal plane The term is arbitrarily restricted to this definition. The axis of tilt could be any one of several lines in space, as for example the Isometric Parallel or the Ground Line but the present definition is the only one which permits the concept of tilting the photograph without upsetting the positional elements of exterior orientation. ***Map Parallel***—The intersection of the plane of the photograph with the plane of the map. ***Ground Parallel***—The intersection of the plane of the photograph with the plane of reference of the ground.

Principal Point Assumption—See under RADIAL TRIANGULATION.

Principal Point Method—Not recommended. See RADIAL TRIANGULATION.

Principal Point—*Optics:* See under NODAL POINT—*Optics.*

Principal Point—*Photogrammetry:* The foot of the perpendicular from the interior perspective center to the plane of the photograph, i e., the foot of the photograph perpendicular.

Principal Point Radial—See RADIAL.

Principal Point Triangulation—See RADIAL TRIANGULATION.

Principal Torque—*Photogrammetry:* See under CANT AND TORQUE.

Print—*Photography:* A photographic copy made by projection or contact printing from a photographic negative or from a transparent drawing as in blue printing. ***Contact Print***—A print made with the negative or transparent drawing in contact with the sensitized surface. ***Ratio Print***—A print, the scale of which has been changed from that of the negative by photographic enlargement or reduction.

Process Lens‡—A lens for photo-mechanical or enlarging or projection purposes; usually of low aperture (f/10 approximately) and of symmetrical construction.

Projection—In geometry, the extension of lines or planes to intersect a given surface;

the transfer of a point from one surface to a corresponding position on another surface by graphical or analytical methods *Perspective Projection*—The projection of points by straight lines drawn through them from some given point to an intersection with the plane of projection. Unless otherwise indicated the point of projection is understood to be within a finite distance of the plane of projection For example, a photograph is formed by a perspective projection of light rays from the rear node of the lens (the point of projection) to the negative (the plane of projection). The term PERSPECTIVE PROJECTION is preferable to the term CONIC PROJECTION as applied to the geometry of a photograph. *Orthographic Projection*—A perspective projection of points by straight lines from a point of projection at an infinite distance from the plane of the drawing. It is regularly used in mechanical drawing and when so used, the two vertical planes are revolved about their respective lines of intersection with the horizontal plane so as to show all three views on the plane of the paper. See also MAP PROJECTION.

Radial—Photogrammetry A line or direction from the radial center to any point on a photograph. The radial center is assumed to be the principal point unless otherwise designated as for example *Nadir Radial*—a radial from the nadir point or *Isoradial*—a radial from the isocenter.

Radial Center—The selected point on a photograph from which radials (directions) to various image points are drawn or measured, i.e., the origin of radials The radial center is either the principal point, the nadir point, the isocenter, or a substitute center.

Radial Direction—Not preferred. See RADIAL.

Radial Line—Not preferred. See RADIAL.

Radial Line Method—See RADIAL TRIANGULATION.

Radial Line Plot—Not preferred. See RADIAL PLOT under RADIAL TRIANGULATION.

Radial Plot—See RADIAL TRIANGULATION.

Radial Triangulation—Photogrammetry: A method of triangulation either analytic or graphic, utilizing overlapping vertical, nearly vertical, or oblique aerial photographs for the location of points, imaged on the photographs, in their correct relative position to one another. The center of each vertical photograph (radial center), or the approximate nadir point of each oblique, serves as a station from which directions to points imaged on the photograph are traced, or measured, and used to extend the triangulation by intersection and by resection. A radial triangulation is also correctly called a ***Radial Plot*** or a ***Minor Control Plot.*** If made by analytic methods it is an ***Analytic Radial Triangulation.*** A radial triangulation is assumed to be graphic unless prefixed by the word analytic. A graphic radial triangulation is usually laid out directly onto ground control plotted on a map, map projection, or map grid, but may be first laid out independently of such control and later adjusted to it as a unit. In the latter case the scale and azimuth of the radial triangulation unit are not known until it is adjusted to the ground control. The radial center for near vertical photographs may be the principal point, the nadir point, or the isocenter. A radial triangulation is assumed to be made with the principal points as radial centers unless the definitive term designates otherwise, as for example, ***Nadir Point Triangulation*** or ***Nadir Point Plot*** and ***Isocenter Triangulation*** or ***Isocenter Plot.*** The adjective "radial" is not necessary in the preceding four terms. The adjective "analytic" is required to designate that the triangulation is by analytic and not graphic methods as ***Analytic Nadir Point Triangulation.*** A graphic radial triangulation may be made by several methods as follows: ***Slotted Templet Triangulation*** or ***Slotted Templet Plot***—A graphic radial triangulation using slotted templets. ***Spider Templet Triangulation*** or ***Spider Templet Plot***—

A graphic radial triangulation using spider templets. ***Mechanical Templet Triangulation*** or ***Mechanical Templet Plot***—A graphic radial triangulation using either slotted or spider templets or any form of mechanical templet. ***Hand Templet Triangulation*** or ***Hand Templet Plot***—A graphic radial triangulation using any form of hand templets. In the preceding eight terms it is assumed that the radial center is the principal point unless the term includes the words "Nadir Point" or "Isocenter" as ***Nadir Point Slotted Templet Plot*** or unless the context states that a radial center other than the principal point was used For definitions of various templets see under TEMPLET. ***Direct Radial Triangulation*** or ***Direct Radial Plot***—A graphic radial triangulation made by tracing the directions from successive radial centers directly onto a transparent plotting sheet rather than laying the triangulation by the templet method ***Strip Radial Triangulation*** or ***Strip Radial Plot***—A Direct Radial Triangulation in which the photographs are plotted in flight strips without reference to ground control and the strips later adjusted together and adjusted to the ground control ***Principal Point Assumption***—The assumption with respect to near vertical photographs that radial directions are correct if measured from the Principal Point.

Random Acceleration—*Air Navigation:* Unintentional acceleration, or deceleration, usually of short duration, of an aircraft in flight caused by uncontrollable changes in speed or direction while attempting to maintain constant horizontal flight.

Rate of Climb—The rate of ascent from the earth's surface, the vertical component of the velocity of the center of gravity of an aircraft, usually expressed in feet per minute.

Ratio Print—See under PRINT.

Ray—Not approved for RADIAL, which see

Ray Center—Not approved. See RADIAL CENTER.

Ray Direction—Not approved for RADIAL, which see.

Ray of Light—The geometrical conception of a single element of light propagated in a straight line and of infinitesimal cross section used in tracing analytically the path of light through an optical system. ***Pencil of Light***—A bundle of rays originating at, or directed to, a single point. ***Beam of Light***—A group of pencils of light, as those originating at the many points of an illuminated surface. A beam of parallel light rays is a special case in which each pencil is of such a small cross section that it may be regarded as a ray.

Ray Tracing—*Optics:* A trigonometric calculation of the path of a light ray through an optical system.

Rear Nodal Point—See under PRINCIPAL POINT—*Optics.*

Reconnaissance—A general examination or survey of a region with reference to its main features, usually as a preliminary to a more detailed survey.

Recording Statoscope—See under STATOSCOPE.

Rectification—*Photogrammetry:* The process of projecting a tilted or oblique photograph onto a horizontal reference plane, the angular relation between the photograph and the plane being determined by ground measurements. The special process of rectifying the oblique images from a multiple lens camera to equivalent vertical images by projection into a plane perpendicular to the camera axis is called ***Transformation.*** In this case the projection is into a plane determined by the angular relations of the camera axes and not necessarily into a horizontal plane. ***Transforming Printer***—A specially designed projection printer for use with a particular multiple lens camera for transformation of the oblique (wing) negatives taken by that camera.

Reference Spheroid—See under GEOID

Reflecting Prism—A prism in which deviation of a light beam is produced by reflection within the prism. Practically all prisms used in photogrammetric instruments are of this type.

Refracting Prism—A prism which deviates a beam of light by refraction The angular deviation caused by this type of prism is a function of the wave-length of light, therefore, if the beam being deviated is composed of white light, the prism will spread the beam into a spectrum. Refracting prisms are satisfactory only for small deviations. See WEDGE.

Refraction—The bending of light rays when light passes from one transparent medium into another having a different index of refraction. The ***Angle of Refraction*** is the angle the refracted ray makes with the normal to the surface separating the two media See also SNELL'S LAW.

Relative Aperture—The relative aperture of a photographic or telescopic lens is defined as the ratio of the equivalent focal length to the diameter of the entrance pupil. Expressed as—f·4.5 etc Also called ***f-Number*** or ***Speed.***

Relative Humidity—See under HUMIDITY.

Relative Orientation—See under EXTERIOR ORIENTATION.

Relative Tilt—In near vertical photography, the tilt of a photograph with reference to an arbitrary plane, not necessarily a horizontal plane, such as the preceding or subsequent photograph in a strip. Also defined as the tilt of a photograph with respect to a polar axis parallel to the plate perpendicular of another photograph such as the preceding or subsequent photograph in a strip.

Reproduction—*Mapping:* 1. The summation of all of the processes involved in printing copies from an original drawing. The principal processes are photography, lithography or engraving, and printing. 2 A printed copy of an original drawing made by the processes of reproduction.

Resolving Power—A mathematical expression of lens definition, usually stated as the maximum number of lines per millimeter that can just be resolved (that is, seen as separate lines) in the image.

Reticle—See under FLOATING MARK.

Revert—*Optics:* To interchange the right and left sides of an image without altering the relative positions of the top and bottom as accomplished by certain prisms and mirrors. ***Reversion***—*Optics·* The act of reverting.

Rhomboidal Prism—A prism which only displaces the axis of the beam of light laterally.

Rhumb Line—A line (curved) on the surface of the earth, crossing all meridians at a constant angle. Also called a ***Loxodromic Curve.*** On a Mercator projection the rhumb line is represented by a straight line.

Right Angle Prism—A prism which deviates the axis of the beam 90 degrees and reverts the image.

Roof Prism—Any type of prism in which reflection is produced at two internal surfaces inclined at 90 degrees to each other so as to form a little "roof." The term is often used to refer particularly to AMICI PRISM, which see.

Rotating Prism—See DOVE PRISM.

Service Ceiling—*Air Navigation:* The height in the standard atmosphere above which the maximum rate of climb of an aircraft is less than a given value. The highest altitude at which an airplane will perform, with working load, without excessive loss in efficiency. Not FLIGHT CEILING

Seven Lens Camera—See under CAMERA.

Shutter—*Photography:* The mechanism of a camera which, when set in motion, permits

light to reach the sensitized surface of the film or plate for a predetermined length of time. ***Focal Plane Shutter***—A shutter located near the focal plane and consisting of a curtain with a slot which is pulled across the focal plane to make the exposure. ***Between the Lens Shutter***—A shutter located between the lens elements of a camera and usually consisting of thin metal leaves which open and close or revolve to make the exposure. ***Louver Shutter***—A shutter consisting of a number of thin metal strips of louvers which operate like a Venetian blind to make the exposure. Usually located just in front of or just behind the lens.

Side Lap—See under OVERLAP.

Side Overlap—Not approved. See SIDE LAP under OVERLAP.

Single Lens Camera—See under CAMERA.

Skewed Photograph—*Aerial Photography*. Not recommended. See CRAB.

Slotted Templet Plot—See under RADIAL TRIANGULATION.

Slotted Templet—See under TEMPLET.

Spider Templet—See under TEMPLET.

Slotted Templet Method—See TEMPLET and RADIAL TRIANGULATION.

Slotted Templet Triangulation—See under RADIAL TRIANGULATION.

Snell's Law of Refraction—This law states that the sine of the angle of incidence divided by the sine of the angle of refraction equals a constant called the INDEX OF REFRACTION when one of the media is air. The index of refraction can also be explained as the ratio of the velocity of light in one medium to that in another. For air-glass this ratio is approximately 1.5 to 1.7. Also see REFRACTION.

Solarization—*Photography*: A reversal of gradation sequence in the (usually very dense) image obtained on the normal development of films, plates, and papers after giving a very intense or long continued exposure. A greater exposure than this appears to restore the original sequence of gradation.

Space Coordinates—See under COORDINATES.

Special Purpose Map—See under MAP.

Specific Humidity—See under HUMIDITY.

Specular Reflection—*Optics:* The type of reflection such as obtained from a highly polished plane surface in which all rays are reflected in approximately the same direction.

Speed—*Photography:* See under CHARACTERISTIC CURVE.

Speed of a Lens—See RELATIVE APERTURE.

Spherical Aberration—See under ABERRATION.

Spherical Coordinates—See under COORDINATES.

Spherical Lens—A lens in which the surfaces are segments of spheres. Practically all photographic lenses belong in this class. Also see ASPHERICAL LENS.

Spheroid of Reference—See under GEOID.

Spider Templet Plot—See under RADIAL TRIANGULATION.

Spider Templet Triangulation—See under RADIAL TRIANGULATION.

Standard**—An exact value, or concept thereof established by authority, custom, or common consent, to serve as a model or rule in the measurement of quantity, or in the establishment of a practice or procedure.

State Plane Coordinate Systems—See under COORDINATES.

Station—*Surveying*: 1. A point whose position has been or is to be determined. A station may be a marked station, i.e., a point more or less permanently marked for recovery,

or an unmarked station, one which is not recoverable. 2. A length of 100 feet, measured along a given line, which may be straight, broken, or curved 3. Any point on a straight, broken, or curved line, whose position is indicated by its total distance from a starting point, or zero point, as station 4+47.2 meaning 447 2 feet from zero. See also CAMERA STATION.

Statoscope—A sensitive form of barometer used in aerial photography for measuring small differences in altitude between successive air stations. ***Recording Statoscope***—A statoscope equipped with a recording camera whose shutter is synchronized with that of the aerial camera.

Stereocomparator—*Photogrammetry* A stereoscopic instrument for measuring parallax and sometimes including a means of measuring photograph coordinates of image points

Stereogram—See under STEREOSCOPY.

Stereophotogrammetry—See under PHOTOGRAMMETRY.

Stereoscope—See under STEREOSCOPY.

Stereoscopic Correspondence—See CORRESPONDENCE.

Stereoscopic Fusion—See under STEREOSCOPY.

Stereoscopic Image—See under STEREOSCOPY.

Stereoscopic Pair—See under STEREOSCOPY

Stereoscopic Vision—See under STEREOSCOPY.

Stereoscopy—The science and art which deals with stereoscopic effects and the methods by which they are produced. ***Stereoscope***—An optical instrument for assisting the observer to view two properly prepared photographs, or diagrams, to obtain the mental impression of a three dimensional model. ***Binocular Vision***—Simultaneous vision with both eyes. ***Stereoscopic Vision***—That particular application of binocular vision which enables the observer to view an object or two different perspectives of an object (as two photographs taken from different camera stations) to obtain therefrom the mental impression of a three dimensional model. ***Stereoscopic Fusion***—That mental process which combines the two perspective images on the retinas of eyes in such a manner as to give a mental impression of a three dimensional model. ***Stereoscopic Image***—That mental impression of a three dimensional model which results from stereoscopic fusion. Also called STEREOSCOPIC MODEL. ***Stereoscopic Pair***—*Photogrammetry:* Two photographs of the same area taken from different camera stations in such a manner as to afford stereoscopic vision. ***Stereogram***—A stereoscopic pair of photographs or drawings correctly oriented and mounted for stereoscopic viewing.

Stop—See IRIS DIAPHRAGM and also APERTURE STOP.

Striae—*Optics:* Threadlike filaments within a piece of glass caused by improper mixing of the molten glass during manufacture. Actually these filaments are composed of glass of slightly different index of refraction than the surrounding glass. The extreme fineness of striae often makes their detection difficult.

Strip Air Coordinates—See under COORDINATES.

Strip Radial Triangulation—See under RADIAL TRIANGULATION.

Substitute Center—A point which because of its ease of identification on overlapping photographs is used as a radial center in lieu of the principal point.

Survey—The act or operation of making measurements for determining the relative positions of points on or beneath the earth's surface; also the results of such operations; also an organization for making surveys. ***Photogrammetric Survey***—A survey utilizing either ground photographs or aerial photographs. ***Aerial Survey***—A survey

utilizing aerial photographs as part of the surveying operations, also the taking of aerial photographs for surveying purposes, also the photographs taken of an area for surveying purposes. ***Ground Survey***—A survey made by ground methods as distinguished from an Aerial Survey A ground survey may, or may not, include the use of ground photographs but does not include the use of aerial photographs.

Surveying Camera—See under CAMERA.

Surveyor—One who surveys See SURVEY.

Swing—*Photogrammetry:* The rotation of a photograph in its own plane around the photograph perpendicular. Also, the angle at the principal point of a photograph measured clockwise from the positive y-axis to the principal line.

Symbol—A diagram, design, letter or abbreviation, placed on maps and charts, which by convention, usage, or reference to a legend, is understood to stand for or represent a specific characteristic or object (Not conventional sign) Standard symbols (for the United States) have been adopted by the Federal Board of Surveys and Maps, 1938.

Symmetrical Lens—A lens whose front group of elements and rear group of elements correspond in every detail, i e , radii of curvature, thickness, spacings, diameters, indices of refraction, dispersive powers, focal lengths, and respective positions on opposite sides of the lens diaphragm.

Target—*Optics:* See TEST CHART

Telephoto Lens‡—*Optics:* A lens comprised of a positive front element and a negative rear element, the focal length of the combination being greater than the distance from the front lens surface to the focal plane. This construction is used to make relatively compact long-focus cameras.

Templet—*Photogrammetry·* A templet used in radial triangulation to represent the aerial photograph; the templet is a record of the directions or radials taken from the photograph ***Hand Templet***—A templet made by tracing the radials from the photograph onto a transparent medium, as on celluloid; hand templets are laid out and adjusted by hand to form the radial triangulation ***Celluloid Templet***—A hand templet made on celluloid ***Mechanical Templet***—Any templet which is manipulated and adjusted mechanically in laying out the radial triangulation ***Slotted Templet***—A mechanical templet on which the radials are represented by slots cut in a sheet of cardboard, metal or other material. ***Spider Templet***—A mechanical templet which is formed by attaching slotted steel arms representing radials to a center core The spider templet is characterized by the fact that it can be disassembled and the parts used again.

Templet Method—See general description under RADIAL TRIANGULATION, and also TEMPLET.

Terrain—An area of ground considered as to its extent and topography.

Terrestrial Camera—See GROUND CAMERA under CAMERA, which is preferred.

Terrestrial Horizon—See under HORIZON.

Terrestrial Photogrammetry—Not preferred. See GROUND PHOTOGRAMMETRY under PHOTOGRAMMETRY.

Terrestrial Photograph—Not preferred. See GROUND PHOTOGRAPH.

Test Chart—A chart for testing the performance of photographic lenses. The design usually consists of ruled lines or squares of various sizes so arranged that by examining the image of such a chart the quality of the lens for various parts of the field may be determined. Also called a ***Target.***

Test Glass—An optical element used for checking the curvature of lens surfaces during

the final polishing operation. The test glass has a curvature equal and opposite to that desired on the lens. When the two surfaces are placed in contact, interference fringes are formed. This fringe pattern (also called Newton's rings) is really a contour map of the air film between the two glasses, the contour interval being one-half a wave length of light (about 0.00001 inch)

Thick Lens—A term used in geometrical optics to indicate that the thickness of a lens is considered, all distances being measured from the nodal points instead of the lens center.

Thin Lens—A term used in geometrical optics to indicate that the thickness of a lens is ignored, all distances being measured from the lens center. Used for approximate computations.

Three Lens Camera—See under CAMERA.

Tilt—*Photogrammetry:* 1. The angle between the plate perpendicular and a vertical through the air station 2. Also expressed as the dihedral angle between the plane of the photograph and a horizontal plane normal to the plumb line 3. Also expressed in terms of spherical coordinates as the polar distance of the plate perpendicular where the polar axis is a vertical through the air station. See Fig 1. ***X-Tilt and Y-Tilt***—Tilt expressed as resultant rotations about each of two stationary rectangular axes lying in a horizontal plane—the X-Tilt being the resultant rotation about the X-axis and Y-Tilt the resultant rotation about the Y-axis

Tilt Line—Not approved. See PRINCIPAL LINE under PRINCIPAL PLANE—*Photogrammetry.*

Tip—Not preferred. See Y-TILT under TILT.

Tolerance—The allowable variation from a standard or from specified conditions.

Topographic Feature—See under TOPOGRAPHY.

Topographic Map—See under MAP.

Topography—The features of the actual surface of the earth considered collectively as to form. A single feature as a mountain or valley is called a ***Topographic Feature.***

Torque—*Photogrammetry:* See under CANT and TORQUE.

Track—*Air Navigation:* The actual path of an aircraft over the surface of the earth. The angle contained between a meridian and a line representing the actual path of an aircraft relative to the earth. It is referrred to the true meridian unless otherwise stated.

Transformation—See under RECTIFICATION.

Transformed Print—A photographic print made by projection in a transforming printer. See also MULTIPLE LENS CAMERA under CAMERA and TRANSFORMING PRINTER under RECTIFICATION

Transformer—Not approved for TRANSFORMING PRINTER, which see under RECTIFICATION.

Transforming Printer—*Photogrammetry:* See under RECTIFICATION

Traverse—A method of surveying whereby the lengths and directions of lines connecting a series of stations are measured. A traverse may be closed or open according to whether it does or does not return to the starting point or end on a known position. Traverses may be of many kinds of stadia, compass, transit, etc.

Trimming and Mounting Diagram—*Photography:* A sketch which indicates how the prints of a transformed multiple lens photograph should be connected in order to obtain, in effect, a photograph made by a single lens. The information is given in the form of distances referred to the fiducial marks on the photograph, and is the result of the calibration test for the particular camera used.

True Horizon—See under HORIZON.

Vacuum Back—See under LOCATING BACK.

Vanishing Line—See under PRINCIPAL PLANE—*Photogrammetry.*

Vanishing Point—See under PRINCIPAL PLANE—*Photogrammetry.*

Vectograph—A print or transparency in which the two views of a stereoscopic pair are rendered not in terms of a silver or pigment image but in terms of degree of polarization. When such a print is examined through *polaroid* spectacles a three-dimensional model is seen.

Venturi Tube—A short tube with flaring ends connected by a constricted section, into which is a side tube connection. It is installed in the air stream of an airplane and when air flows through it, there is a reduction of pressure in the constricted neck which partial vacuum may be utilized to operate small instruments.

Vertical Control—See under CONTROL.

Vertical Control Datum—See under GEOID.

Vertical Parallax—Not preferred. See Y-PARALLAX under PARALLAX.

Vertical Photograph—*Aerial Photography:* An aerial photograph made with the camera axis vertical or as nearly vertical as practicable in an aircraft.

Vignetting—The interference of the lens mounting with oblique rays, thus causing a reduction in the effective diaphragm area.

Visibility—*Air Navigation:* The greatest distance toward the horizon that prominent objects, such as mountains, buildings, towers, etc. can be seen and identified by the unaided eye.

Visible Horizon—See under HORIZON.

Wandering Mark—Not preferred. See FLOATING MARK.

Wedge—*Optics:* 1. A prism of very small deviation, such as those used in the eyepieces of some stereoscopes. 2. A plate of glass whose transparency diminishes from one edge to the other. Such a wedge is often used as a "comparison wedge" in determining the density of negatives.

Wide Angle Lens—A photographic lens is said to be a wide angle lens if its angular field is unusually large. There is no definite division point between an ordinary and a wide angle lens, but in general it can be said that a wide angle lens has an angular field of greater than 80 degrees.

Wing Photograph—A photograph taken by one of the side or wing lenses of a multiple lens camera. See MULTIPLE LENS CAMERA under CAMERA.

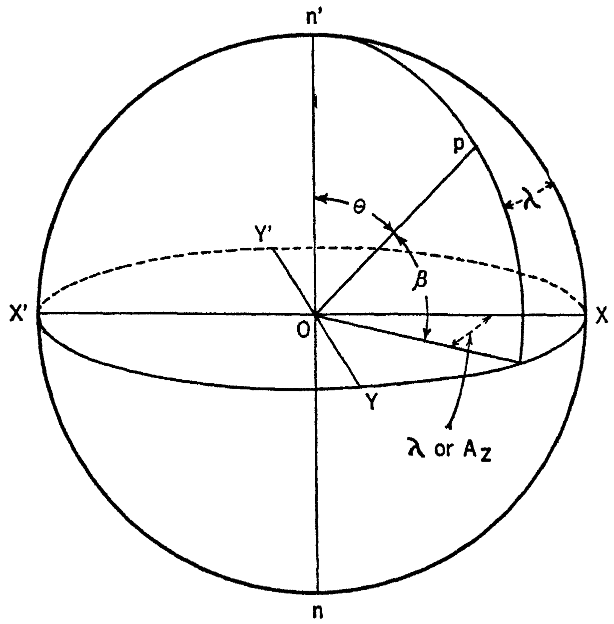
X-Parallax—See under PARALLAX.

X-Tilt—See under TILT.

Y-Parallax—See under PARALLAX.

Y-Tilt—See under TILT.

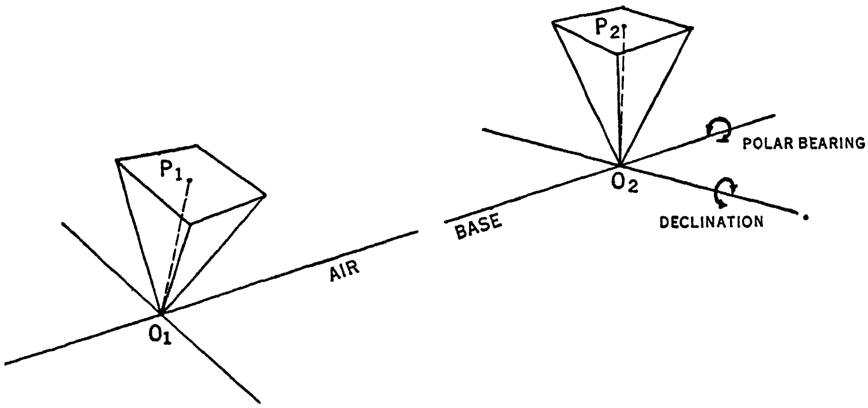
Figure 1



ORIENTATION OF A SINGLE PHOTOGRAPH
EXPRESSED IN TERMS OF SPHERICAL COORDINATES

O = Perspective center of the photograph
p = Principal point.
Op = Photograph perpendicular
XYX'Y' = Equator (horizontal).
n n' = Polar axis (vertical).
θ = Polar distance = tilt.
β = Declination.
λ = Polar bearing, azimuth of principal plane with reference to meridional plane (A_Z).
XnX'n' = Reference meridional plane.
Opn' = Principal plane.

Figure 2a



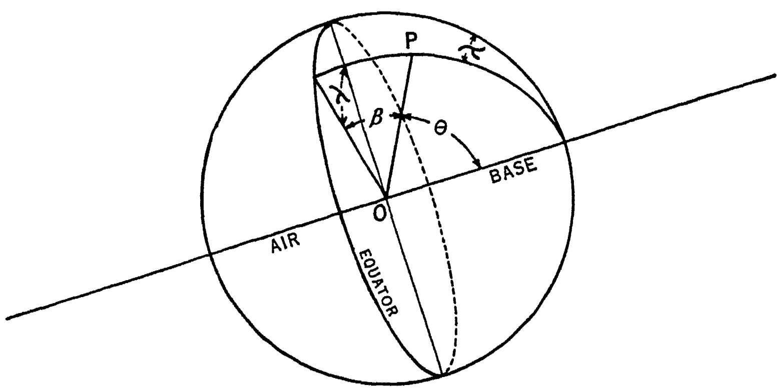


Figure 2b

ORIENTATION OF A PAIR OF PHOTOGRAPHS
EXPRESSED IN TERMS OF SPHERICAL COORDINATES

O = Perspective center of photograph.
P = Principal point.
θ = Polar distance.
β = Declination.
λ = Polar bearing.

The plane of the paper in Fig. 2b corresponds to the reference meridional or epipolar plane. The subscripts 1 and 2 refer to the first and second photographs, respectively, of a pair.

GEOMETRY OF THE TILTED PHOTOGRAPH IN THE PRINCIPAL PLANE

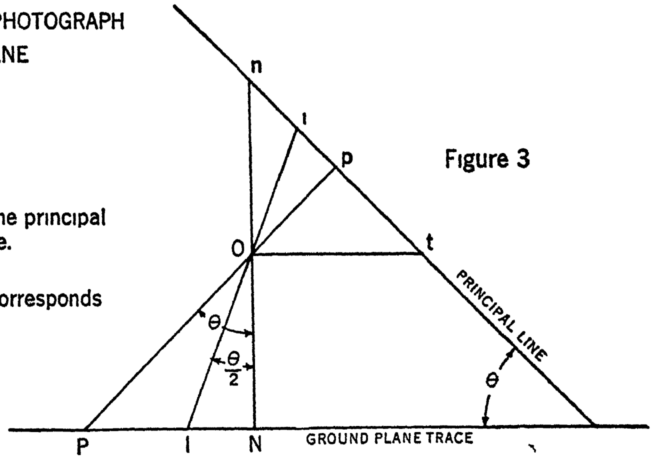
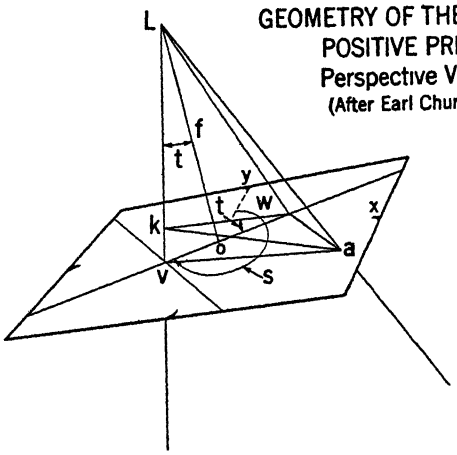


Figure 3

n = Nadir point.
i = Isocenter.
p = Principal point
t = Point of intersection of the principal line and the horizon trace.
O = Perspective center.
The plane of the paper corresponds to the principal plane.

GEOMETRY OF THE TILTED POSITIVE PRINT
Perspective View
(After Earl Church)



L = Perspective center
o = Principal point
v = Nadir point
a = An image
t = Tilt
s = Swing
vow = Principal line
x,y = Fiducial marks
Lo = Photograph perpendicular = f
Lv is vertical
kwa is a horizontal right triangle
wkv is a right triangle
wka is a true horizontal angle

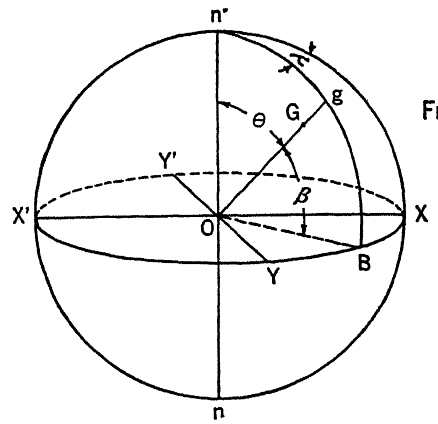


Figure 4

POLAR AND SPHERICAL COORDINATES

XYX'Y' = Equator.
nn' = Polar axis.
n'X nX' = Reference meridianal plane (plane of the paper).
The polar coordinates of point G are the distance OG, the polar bearing λ and the polar distance θ
The spherical coordinates of point g on the surface of a sphere of radius Og are the polar bearing λ and declination β

READY REFERENCE LIST

Compiled by Ensign Dorothy M. Hagen, U S N.R.

BIBLIOGRAPHY

Cobb, G. C. "Bibliography on the Interpretation of Aerial Photographs and Recent Bibliographies on Aerial Photography and Related Subjects" Bulletin of the Geological Society of America. V. 54, Aug. 1, 1943. N. Y. The Geological Society of America.

"Photogrammetric Engineering," Vol. II, No 4, October-November-December, 1936. "Bibliography of Photogrammetry." Washington, D. C. American Society of Photogrammetry.

Steinmann, J. V. "Aerial Photography, a Book List." The Booklist. V. 37, June 1941. Chicago, American Library Association.

PERIODICALS

"Monthly Abstract Bulletin." Eastman Kodak Co., Rochester, N Y.

"Photogrammetric Engineering," published quarterly by the American Society of Photogrammetry, P O. Box 18, Benjamin Franklin Station, Washington, D. C.

"Royal Society of Aeronautics, Journal," published monthly by The Society, 4 Hamilton Pl, Piccadilly, London, W. I

"The Geographical Review," published by the American Geographical Society. Broadway at 156th Street, N. Y., N. Y.

"The Military Engineer," published by The Military Engineer, 808 Mills Building, Washington, D. C.

GENERAL WORKS ON PHOTOGRAMMETRY

Anderson, R. O. "Applied Photogrammetry." Ann Arbor, Mich., Edwards Brothers, Inc., 1941

Anderson, R. O. "Rigorous Analysis of the Scale Point and Tilt Formulas." Ann Arbor, Mich., Edwards Brothers, Inc, 1941.

Anderson, R O. "Tilt of the Aerial Photograph by Graphical Resection." Ann Arbor, Mich., Edwards Brothers, Inc, 1941.

Bagley, J. W. "Aerophotography and Aerosurveying" N. Y, McGraw-Hill Book Co., Inc, 1941.

Breed, C. B. and Hosmer, G. L. "Higher Surveying" N. Y., John Wiley and Sons, Inc., 1941.

British Columbia. Lands Dept, Forest Branch. "Procedure and Notes. Plotting Vertical Air Photographs by Radial Intersection (planimetry only)." By W. Hall Air Survey Section, British Columbia Forest Service., Victoria, B. C., 1939.

Church, E. "Elements of Aerial Photogrammetry." Syracuse, N. Y., Syracuse University Press, 1944.

Clark, W "Photography by Infrared: Its Principles and Applications." N. Y., John Wiley and Sons, 1939.

Curwen, E. C. "Air-Photography and the Evolution of the Corn Field." London Published for the Economic History Society by A. & C. Black, Ltd., 1938.

Davis, R. "Dimensional Changes in Aerial Photographic Films and Papers." U. S. National Bureau of Standards. Journal of Research of National Bureau of Standards. Washington, U. S. Govt. Print. Off., 1937.

Davis, R. E. and Foote, F. S. "Surveying." N. Y., McGraw-Hill Book Co., Inc., 1940.

Desjardins, L. and Hower, S. G. "A Technique for Geologic, Topographic and Structural Mapping from Aerial Photographs." Tulsa, Okla., Aero Exploration Co.

Dry, C J. "Aerial Photos. How to Make and Sell Them." Canton, O., The Fomo Publishing Co., 1940.

Eardley, A. J "Aerial Photographs Their Use and Interpretation." N. Y , Harper & Brothers, 1942.

Eardley, A. J. "Interpretation of Geologic Maps and Aerial Photographs " Ann Arbor, Mich., Edwards Brothers, Inc , 1941

Field, R. M. and Stetson, H. T "Map Reading and Avigation." N. Y , Van Nostrand, 1942.

Flexner, W. W and Walker, G. L. "Military and Naval Maps and Grids " N Y , The Dryden Press, 1943

Gardner, I. C. "Relation of Camera Error to Photogrammetric Mapping " U S. National Bureau of Standards Journal of Research of the National Bureau of Standards. Washington, U. S Govt Print Off., 1939

Gruber, O. Von, "ed." "Photogrammetry. Collected Lectures and Essays." Boston American Photographic Publishing Co , 1942.

Hart, C. A. "Air Photography Applied to Surveying." N Y , Longmans, Green and Co , 1940.

Heavey, W. F "Map and Aerial Photo Reading Simplified." Harrisburg, Pa., The Military Service Publishing Co 1943

Henney, K and Dudley, B , "eds " "Handbook of Photography." N. Y. McGraw-Hill Book Co., Inc 1939.

Lahee, F. H. "Field Geology " 4th ed. N. Y., McGraw-Hill Book Co , Inc., 1941.

Light, R. U "Forces on Africa " N. Y , American Geographical Society, Special Publication No 25, 1941.

Lobeck, A. K. "Geomorphology, an Introduction to the Study of Landscapes " N Y , McGraw-Hill Book Co , Inc., 1939.

Lobeck, A K and Tellington, W. J. "Military Maps and Air Photographs, their use and interpretation." N. Y , McGraw-Hill Book Co., Inc , 1944

MacClintock, P. Outlines for Courses in Map and Aerial Photography Interpretation. Geology Dept., Princeton University. "How to Examine Aerial Photographs" and "Geology of the Terrain as Shown on Aerial Photographs." Princeton, N J , Princeton University Press, 1942

Mack, J. E. and Martin, M. J. "The Photographic Press." N Y., McGraw-Hill Book Co., Inc., 1939.

Maclean, N. F. "Manual for Instructions in Military Maps and Aerial Photographs " N. Y., Harper Brothers, 1943.

Military Service Publishing Co. "Map reading, Interpretation of Aerial Photographs, Military Sketching." 2d ed , Harrisburg, Pa., The Author, 1938

Military Service Publishing Co. "Map and Aerial Photo Reading Simplified " 4th ed. Harrisburg, Pa , The Author, 1942.

Military Service Publishing Co. "Map and Aerial Photograph Reading—Complete." Harrisburg, Pa., The Author, 1943.

Neblette, C. B. "Photography, Its Principles and Practice." N. Y., D. Van Nostrand Co., 1938.

New York (State) Executive Dept. Division of State Planning "Aerial Photographic Mapping in New York State." Albany, N Y., 1937.

Nuguid, A. S. "Analytical Investigation of the Use of Unrectified Photographs in a Plotting Stereo-Comparator " Syracuse, N. Y., Syracuse University, 1941.

Oregon. State Planning Board "Mapping Program for Oregon. Status of Topographic and Aerial Mapping with Recommendations for Completion. A Report from the Advisory Committee on Aerial Surveys and Maps to the State Planning Board." Portland, 1937.

Oregon. State Planning Board. "Reference Data Showing Aerial Surveys in Oregon" Portland, 1937.
Putnam, W. C. "Map Interpretation with Military Applications." N. Y., McGraw-Hill Book Co., Inc, 1943.
Raisz, E. "General Cartography." N. Y., McGraw-Hill Book Co., Inc., 1938
Raynor, W. H. "Advanced Surveying." N. Y., D. Van Nostrand Co, Inc, 1941.
Rich, J. L. "The Face of South America, An Aerial Traverse" N. Y, American Geographical Society, 1942. Special Publication No. 26.
Root, R. R. "Camouflage With Planting." Chicago. R. F. Seymour, 1942.
Salt, J S. A "A Simple Method of Surveying from Air Photographs" London H. M. Stationery Off., 1939.
Schmidt, E. F. "Flights over Ancient Cities of Iran." Chicago, University of Chicago Press, 1940.
Sharp, H. O. "Photogrammetry." 3rd ed., N. Y., Wiley and Sons, Inc., 1943.
Sharp, H. O. "Photogrammetry." 3rd textbook ed., N. Y., Rensselaer Polytechnic Institute, 1940.
Smith, H. T. U. "Aerial Photographs and Their Applications." N. Y, Appleton-Century, 1943.
Syracuse University. "Bulletins on Aerial Photogrammetry." Syracuse, N. Y., Syracuse University, 1937–1943. Bulletins by Earl Church as follows:
  No. 8—Orientation of Aerial Photographs in the Multiplex Projector from Computed Data 1937.
  No. 9—Property Surveys by Means of Aerial Photographs. 1938.
  No. 10—Photogrammetric Methods for Planimetric and Property Surveys 1939.
  No. 11—Analytical Method for the Extension of a Photogrammetric Survey with No Ground Control Except for the Initial Photograph. 1940
  No 12—Two New Analytical Methods of Space Resection in Aerial Photogrammetry. 1941.
  No 13—Determination of the Scale Data for Two Aerial Photographs from One Control Line and Three Additional Elevations 1942.
  Bulletins by Graduate Students as Follows
  No 1—Investigation of Methods of Determining the Tilts of Aerial Photographs. R. J Heredia 1939.
  No 2—Investigation of the Magnitude of Errors in Ground Positions Caused by Errors in the Focal Length or in the Position of The Principle Point of the Aerial Camera. E. D. Sewell. 1940.
  No. 3—Analytical Bridging and Adjustment of Secondary Control Through a Strip of Aerial Photographs. H. E. Sewell. 1940.
  No. 4—Data for a Fictitious Set of Overlapping Aerial Photographs for Use in Problems of Investigation. H. E. and E. D. Sewell. 1940.
  No. 5—Vertical Scale Correction for the Multiplex Projector. G. C. Tewinkel. 1940.
  No. 6—Analytical Investigation of the Use of Unrectified Photographs in a Plotting Stereo-Comparator. A. S. Nuguid. 1941.
  No 7—Student Reports on Three Research Problems in Aerial Photogrammetry. T. Ates, M. Tugal and A. Keretli. 1943.
Talley, B. B. "Engineering Applications of Aerial and Terrestrial Photogrammetry." N. Y., Pitman Publishing Corp., 1938.
Tobin, E. "Aerial Maps, Their Manufacture and Application." San Antonio, Edgar Tobin Aerial Surveys, 1939.
U. S. Army Air Forces Advanced Flying School. "Aerial Photography and Photograph Reading." Brooks Field, Texas, 1942.
U. S. Cavalry School. "Map and Aerial Photograph Reading." Ft. Riley, Kansas, 1939.

U. S Dept of Agriculture. Flood Control Advisory Committee. "Proceedings of Seminar on Aerial Photography in Flood Control Surveys, Dec 6–8, 1939. 1940 Appendix A. Preliminary Draft. The Use of Aerial Photographs and Mosaics With Particular Reference to Flood Control Surveys; Appendix B, Wilson R. C. Preliminary Instructions for Vegetation Type Map and Cultivated Land Classification in California Forest Survey." Washington, U. S Govt. Print. Off , 1939–40.

U. S. Quartermaster School. "Instructor's Guide, Map Reading and Interpretation of Foreign Maps." Camp Lee, Va., 1942.

U. S. Quartermaster School. "Lesson Plans and Instructional Material; Map Reading and Interpretation of Foreign Maps." Camp Lee, Va., 1942.

U. S. Signal Corps Replacement Training Center. "Map Reading." Instructor's Lesson Plans. Clerks, Specialists Division, Map Reading Department Fort Monmouth, Red Bank, N. J , 1942.

U. S Signal Corps Replacement Training Center. "Map Reading " Instructor's Lesson Plans CSD-14 Fort Monmouth, N J., 1943.

Washer, F. E. "Locating the Principal Point of Precision Airplane Mapping Cameras " U. S. National Bureau of Standards Journal of Research of the National Bureau of Standards Washington, U. S. Govt Print Off , 1941.

Washer, F. E. "Resolving Power and Distortion of Typical Airplane-camera Lens." U. S. National Bureau of Standards. Journal of Research of the National Bureau of Standards. Washington, U. S. Govt. Print. Off., 1939.

White, Z. W. "Aerial Photography and Its Use in Forest Management." Master of Forestry Thesis. New Haven, Conn., Yale University School of Forestry, Feb 1938.

Whitmore, G. D. "Elements of Photogrammetry." Scranton, Pa., International Textbook Co., 1941.

Whitmore, G. D "Elements of Photogrammetry, Prepared Especially for Home Study." Scranton, Pa., International Correspondence Schools, 1941

UNITED STATES WAR DEPARTMENT MANUALS

Basic Field Manual, FM 21–25. "Elementary Map and Aerial Photograph Reading." 1941.

Basic Field Manual, FM 21–26. "Advanced Map and Aerial Photograph Reading." 1941.

Field Manual, FM 1–35. "Aerial Photography." 1942.

Field Manual, FM 30–21. "Military Intelligence—Role of Aerial Photography." 1940.

Technical Manual, TM 1–219. "Basic Photography." 1941.

Technical Manual, TM 1–220. "Aerial Photography." 1942.

Technical Manual, TM 5–230. "Topographic Drafting." 1940.

Technical Manual, TM 5–235. "Surveying." 1940.

Technical Manual, TM 5–236. "Surveying Tables." 1940.

Technical Manual, TM 5–240. "Aerial Photography." 1941.

Technical Manual, TM 5–244. "Multiplex Mapping Equipment." 1943.

Technical Manual, TM 5–245. "Topography and Surveying Map Reproduction in the Field." 1942.

Technical Manual, TM 5–246. "Interpretation of Aerial Photographs." 1942.

Technical Manual, TM 9–2653. "Instruction Guide, Stereoscopic Tester MIAI." 1942.

Training Manual, TM 2170–6. "Air Corps: Aerial Photography." 1938.

Training Manual, TM 2180–37. "Topography and Surveying, Map Reproduction in the Field." 1940.

MISCELLANEOUS PUBLICATIONS

Engineer Detachment, Wright Field, Dayton, O. "The Multiplex Manual." 1941.

Engineer School, Fort Belvoir, Va. "Instruction Manual for Stereocomparagraph" 1940.

Fairchild Aviation Corp., N. Y. "Practical Stereoscopy for the Beginner." 1943

Great Britain—General Staff. "Manual of Map Reading, Photo Reading and Field Sketching." London. H M. Stationery Off., 1939

Kelsh, T. T. "The Slotted Template Method for Controlling Maps Made from Aerial Photographs" Washington, U. S. Department of Agriculture, 1940 Miscel. Pub. No 404.

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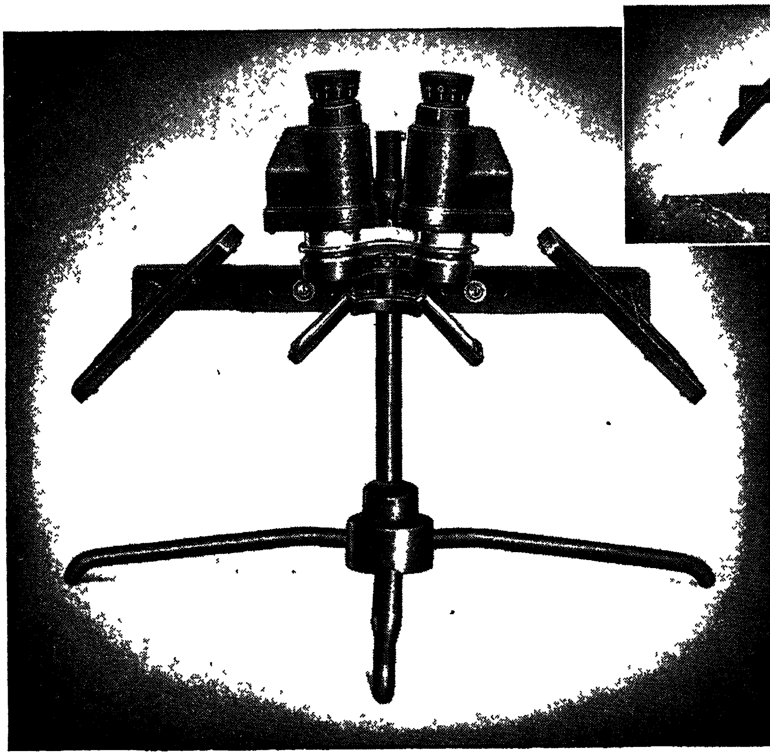
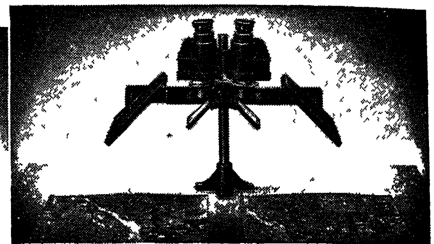
REFLECTING STEREOSCOPE





Above: M-11 STEREOSCOPE on Bracket Support can be screwed to any table-top. It will accommodate photos up to 10x12 inches Shown is a pair of 10x10-inch pictures.

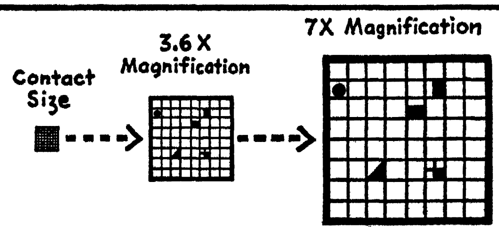
Left: M-11 STEREOSCOPE on Folding Tripod Support is ideal for tough field use . accommodates photos up to 10 x 10 inches . . . "takes down" easily for storage in rugged, trunk-type case.

The New RYKER STEREOSCOPE...MODEL M-11
with MAGNIFICATION RANGE *from* 3.6X *to* 7X

The M-11 Stereoscope permits *faster, far more accurate* identification of detail in aerial photographs.

Vertical adjustment of the instrument on its support rod, coupled with the wide focusing range of our Binoculars and use of simple, supplementary lens-inserts gives an uninterrupted magnification range from 3.6X to 7X.

Since steroscopic perception varies as the square of the magnification, *ability to surely identify fine detail is over three times greater at 7X than at 4X* (the standard magnification of older military type instruments).



Viewed at a distance of about 6 ft. this drawing roughly illustrates the increase in perceptible detail due to magnification.

A simple adjustment permits the distances from eyes to photos to be made unequal, thus varying the magnification through each eyepiece, and permitting fusion of picture-pairs differing in scale up to about 15%.

The field of view without Binoculars is extremely wide . . . accommodating the entire 70% overlap area of a pair of 9x9-inch photos.

Substitution of a sturdy Tripod Support for the awkward folding legs of former models, makes the M-11 Stereoscope handier to use . . . assures a steady stance under all conditions . . . leaves space beneath the instrument unobstructed for plotting.

Available either with or without Binocular Magnifier, which may be purchased as optional equipment. Details and prices will be mailed on request.

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★

1000 Ashby Ave. Berkeley 2, Calif.

Eyes in the Air

FEET ON THE GROUND

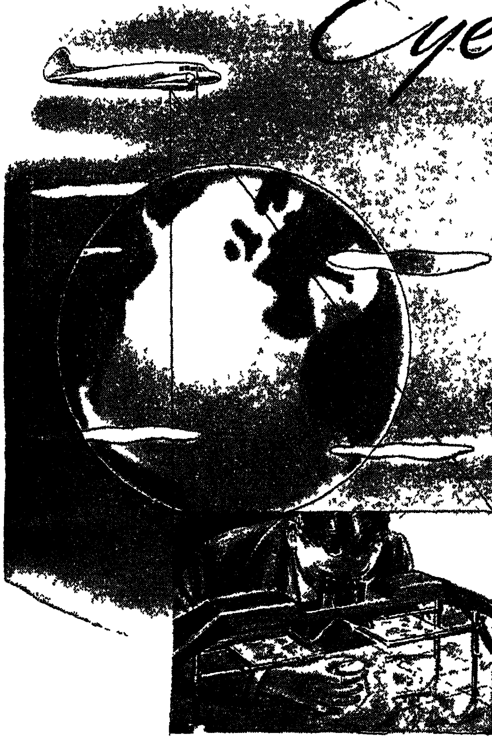

PHOTO-PLANE AND PLOTTING-ROOM, JOINED IN EFFECTIVE PARTNERSHIP, OFFER PROMISE OF VAST ACCOMPLISHMENT IN YEARS TO COME.

As a pioneer in the development and manufacture of every type of aerial photographic equipment, and many types of stereoscopic plotting devices, Fairchild has been privileged, for over two decades, to work in close association with every factor in the field of Photogrammetry.

All of us . . . in government, military, naval or commercial groups . . . can well be proud of past achievements.

That, however, is not enough. We are faced with a new challenge· As expressed by a speaker before our national society, "In years to come, Photogrammetry will make possible utilization of all the waste spaces of the world"

This is a vast undertaking . and great opportunity. Merely to complete the topographic mapping of our own Country would take twenty years . . using *all of the industry's wartime personnel and equipment.*

In the future advance of the science of Photogrammetry, Fairchild Camera and Instrument Corporation will continue to pioneer new and improved equipment. The units at right, typical of present production, are but a promise of greater developments to come.

"THE HISTORY OF PHOTOGRAMMETRY IS ... THE STORY OF FAIRCHILD CAMERAS"

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